Breakdown of the Landau-Ginzburg-Wilson paradigm at quantum phase transitions

Science **303**, 1490 (2004); cond-mat/0312617
cond-mat/0401041

Leon Balents (UCSB)
Matthew Fisher (UCSB)
S. Sachdev (Yale)
T. Senthil (MIT)
Ashvin Vishwanath (MIT)

Outline

A. Magnetic quantum phase transitions in “dimerized” Mott insulators

Landau-Ginzburg-Wilson (LGW) theory

B. Mott insulators with spin S=1/2 per unit cell

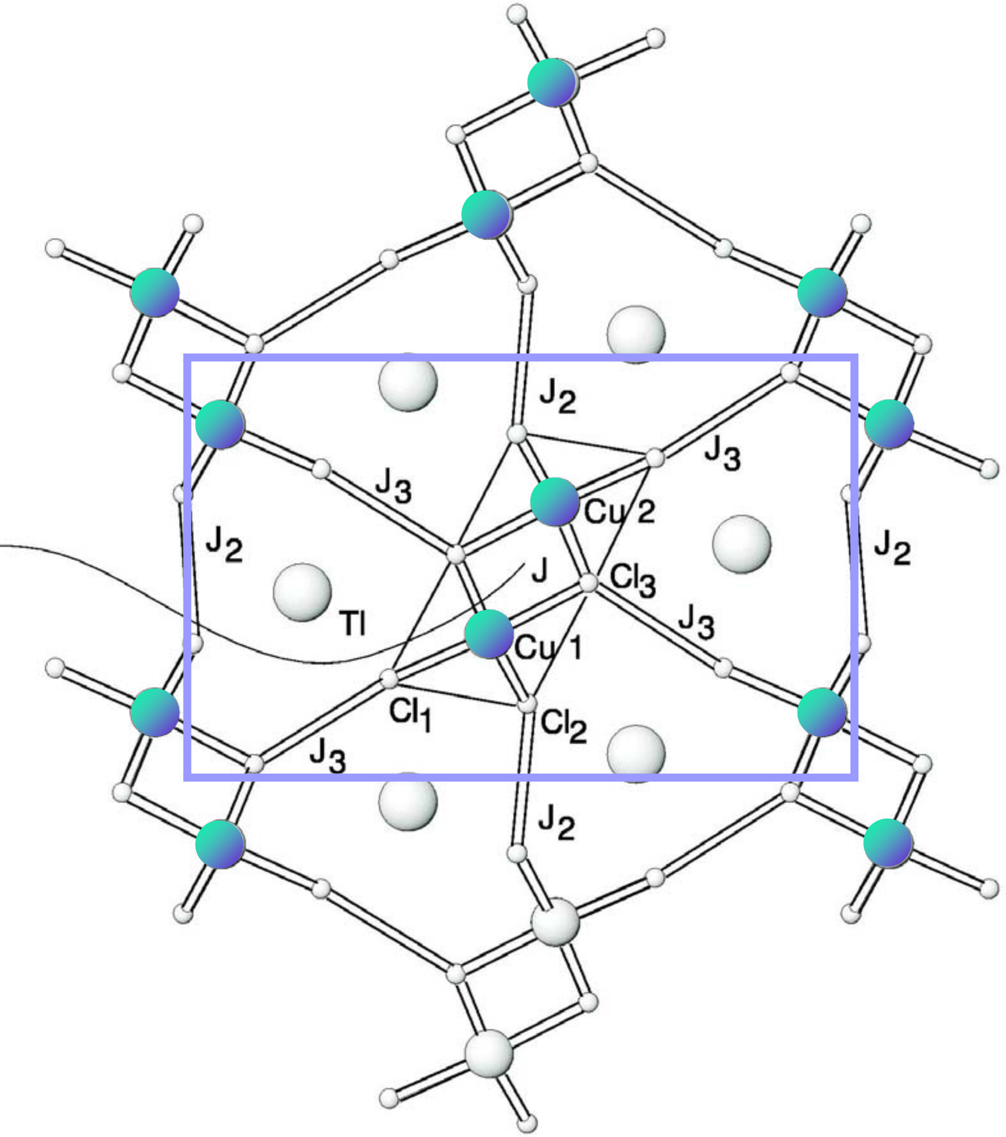
Berry phases, bond order, and the breakdown of the LGW paradigm

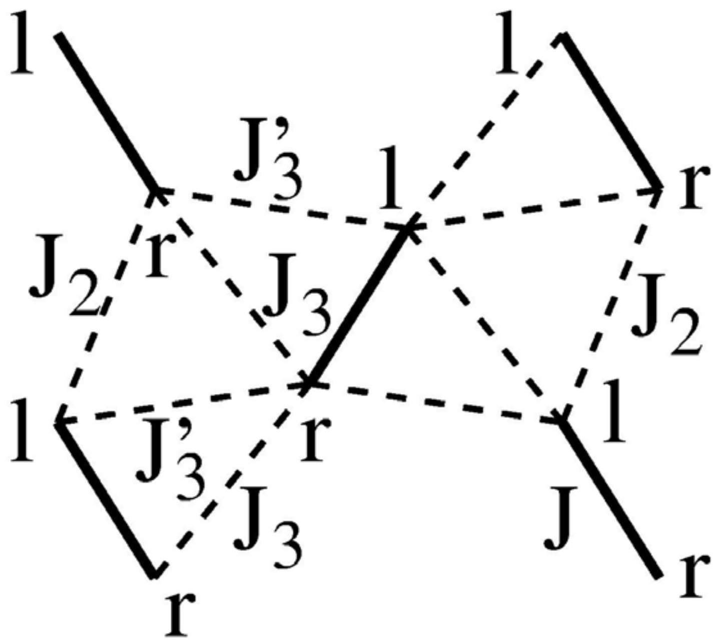
A. Magnetic quantum phase transitions in "dimerized" Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***

$TlCuCl_3$



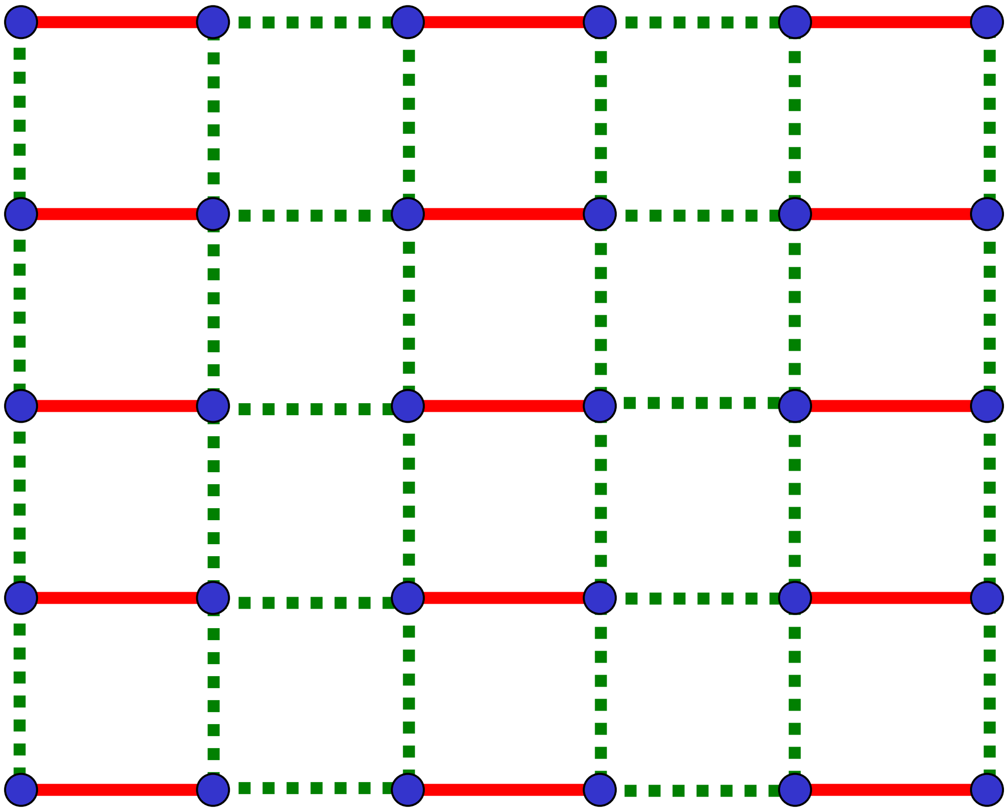


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

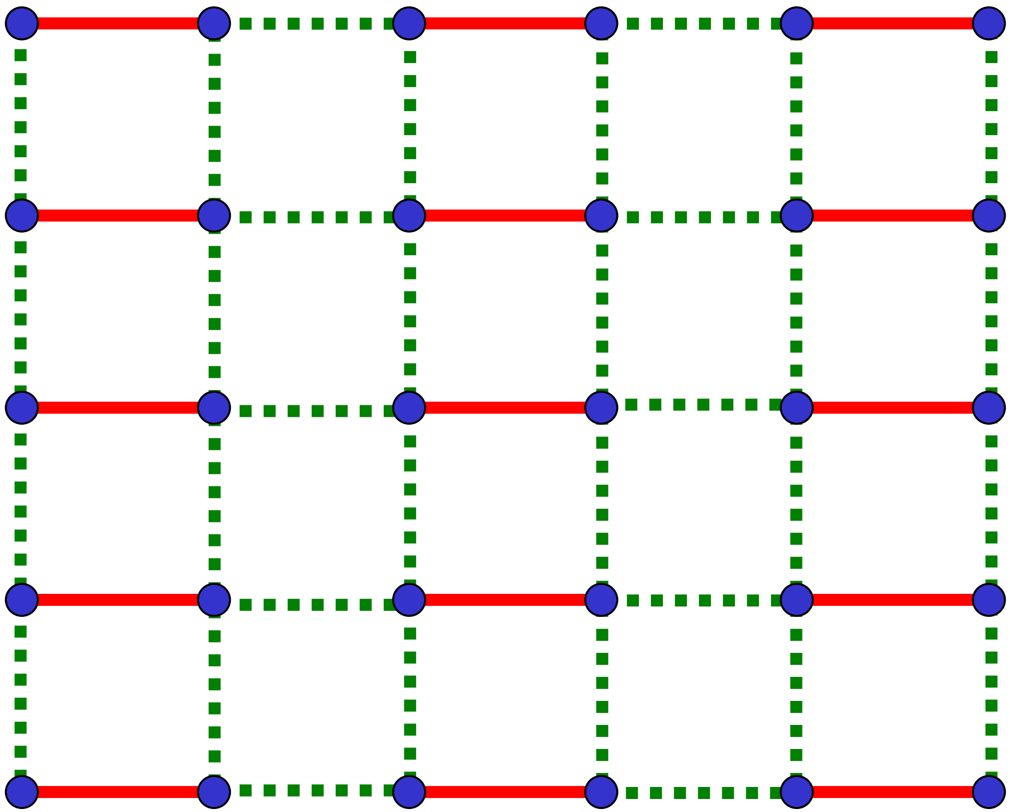
S=1/2 spins on coupled dimers



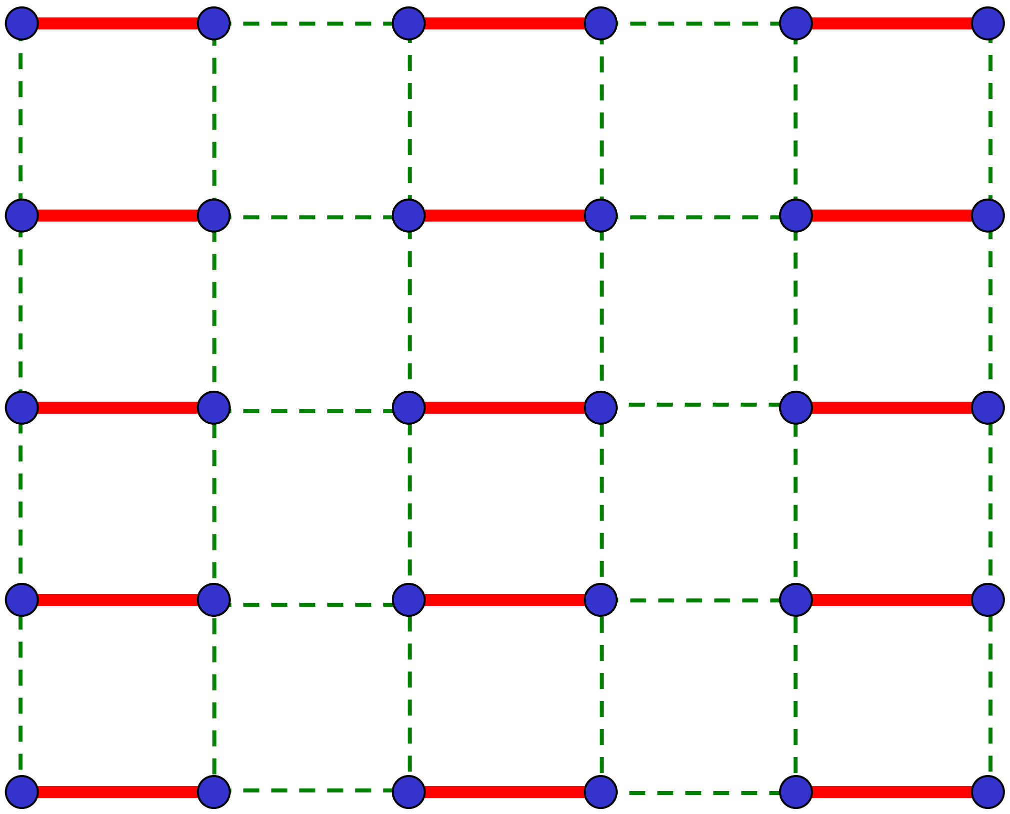
J λJ

$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

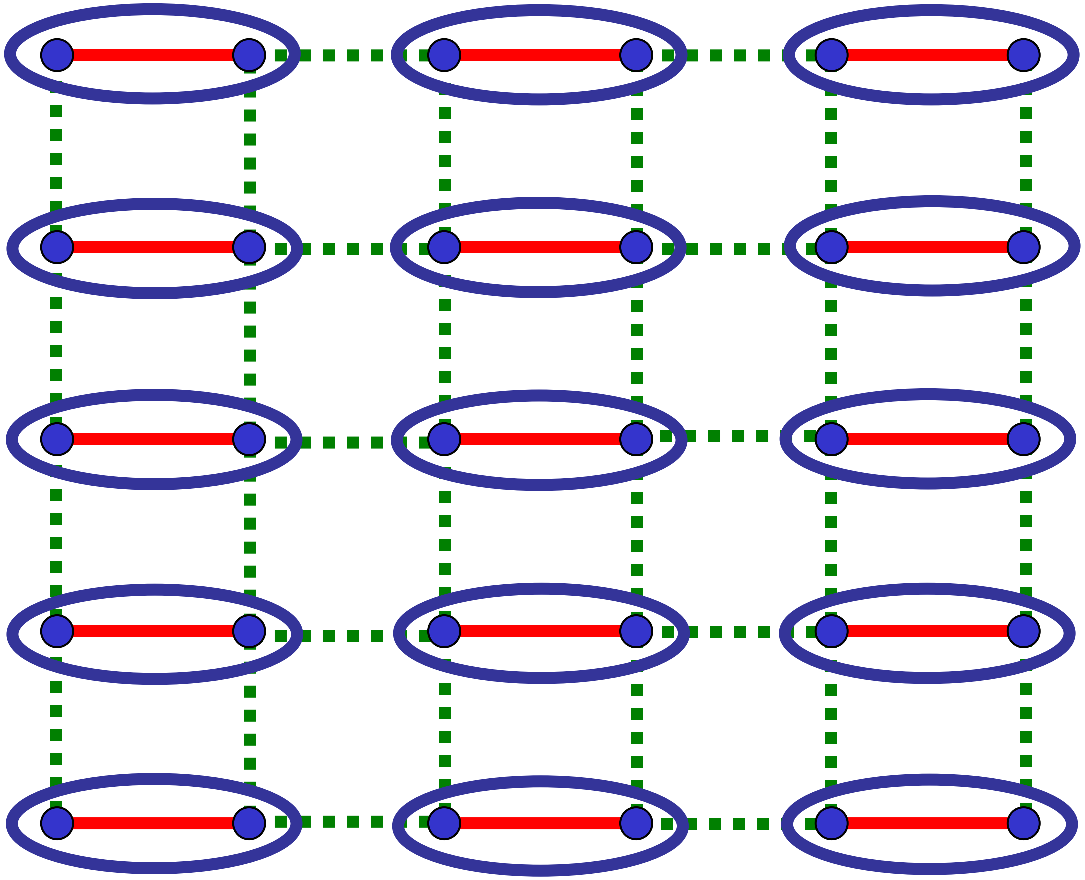


$\boxed{\lambda \text{ close to } 0}$ Weakly coupled dimers

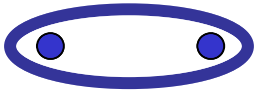


λ close to 0

Weakly coupled dimers



Paramagnetic ground state

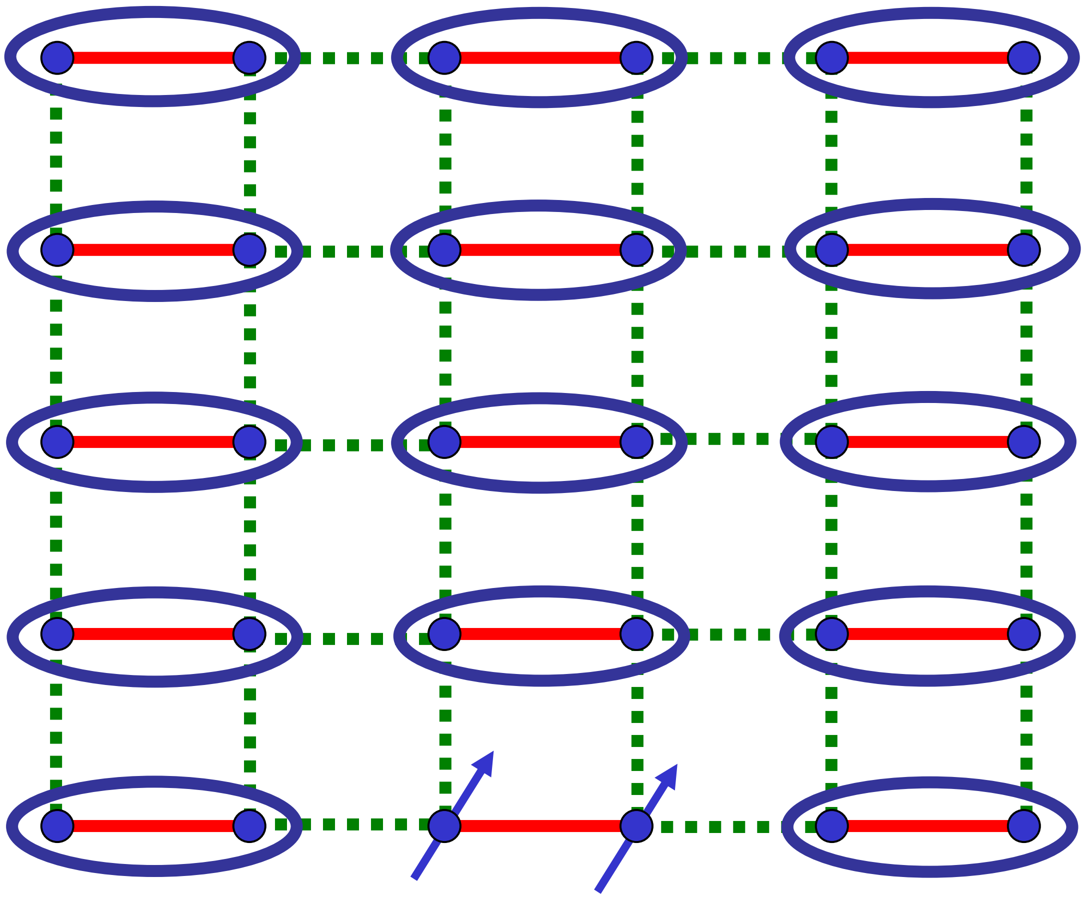


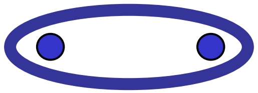
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

$$\left\langle \vec{S}_i \right\rangle = 0$$

$\boxed{\lambda \text{ close to } 0}$

Weakly coupled dimers

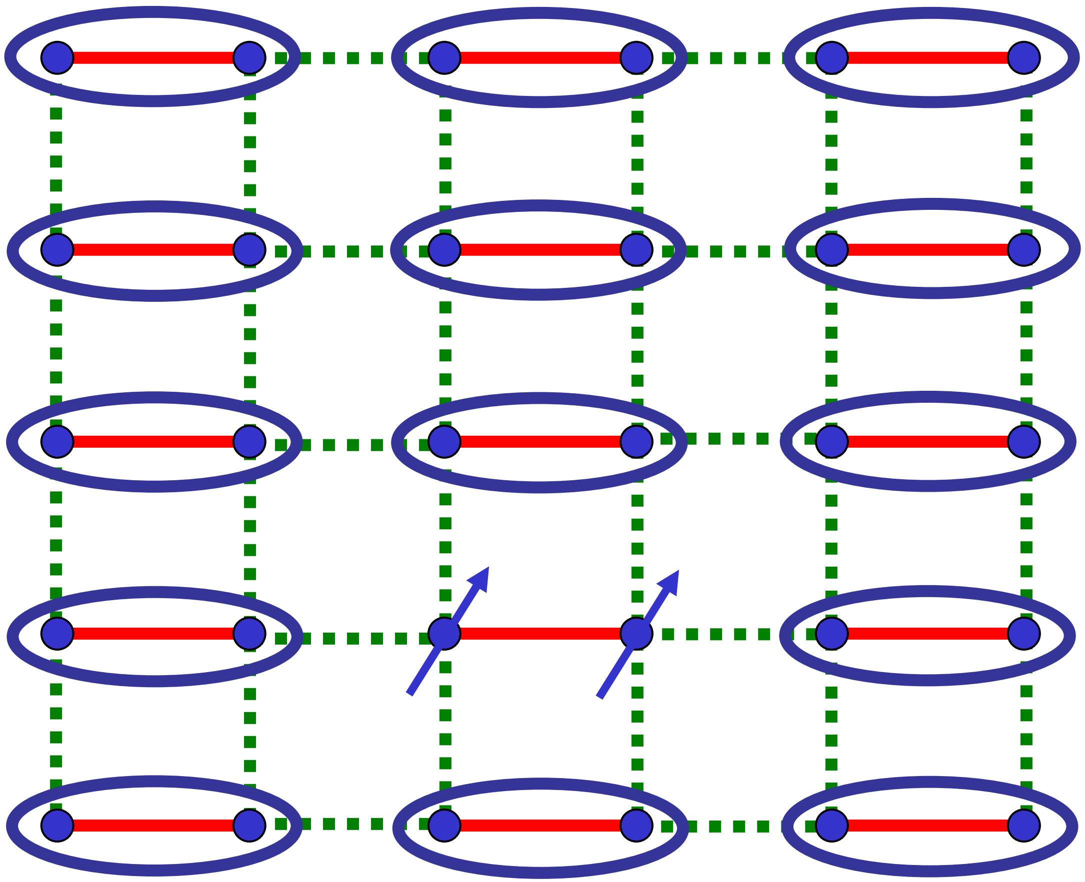


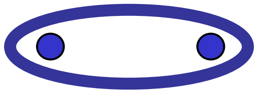
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers



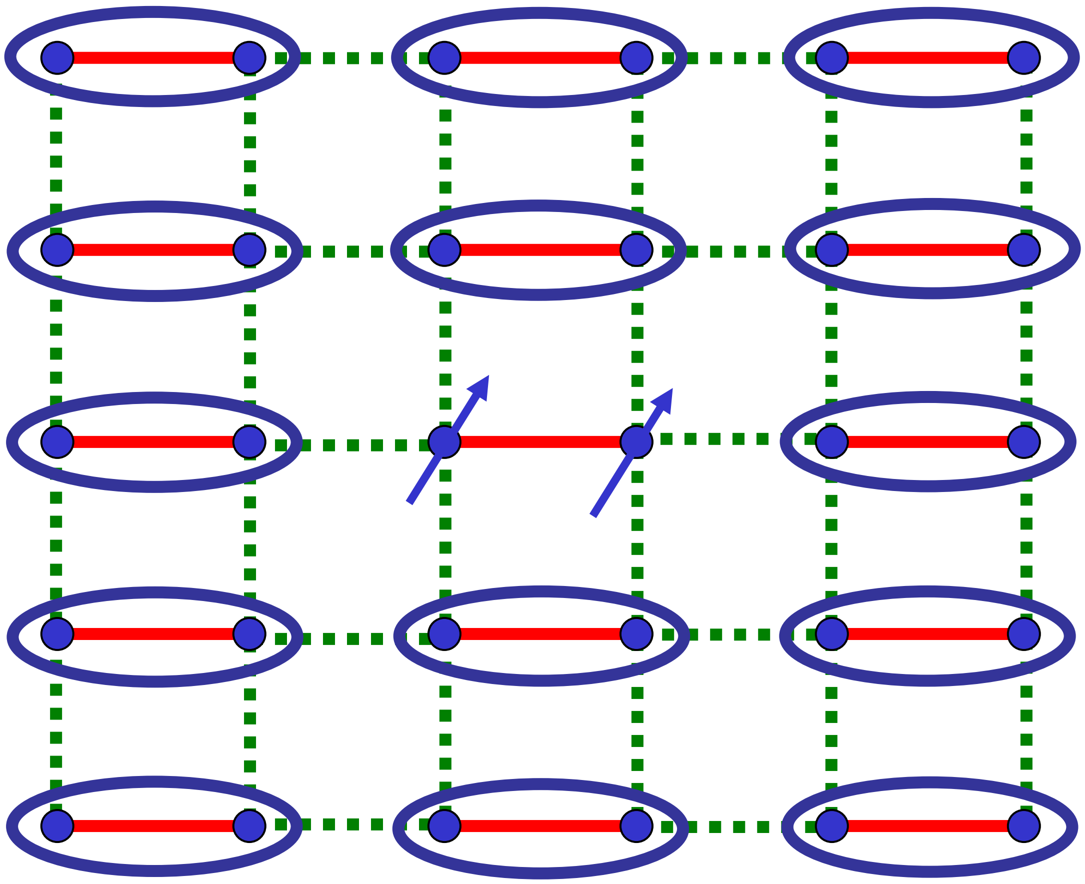


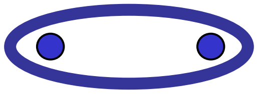
$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

$\boxed{\lambda \text{ close to } 0}$

Weakly coupled dimers

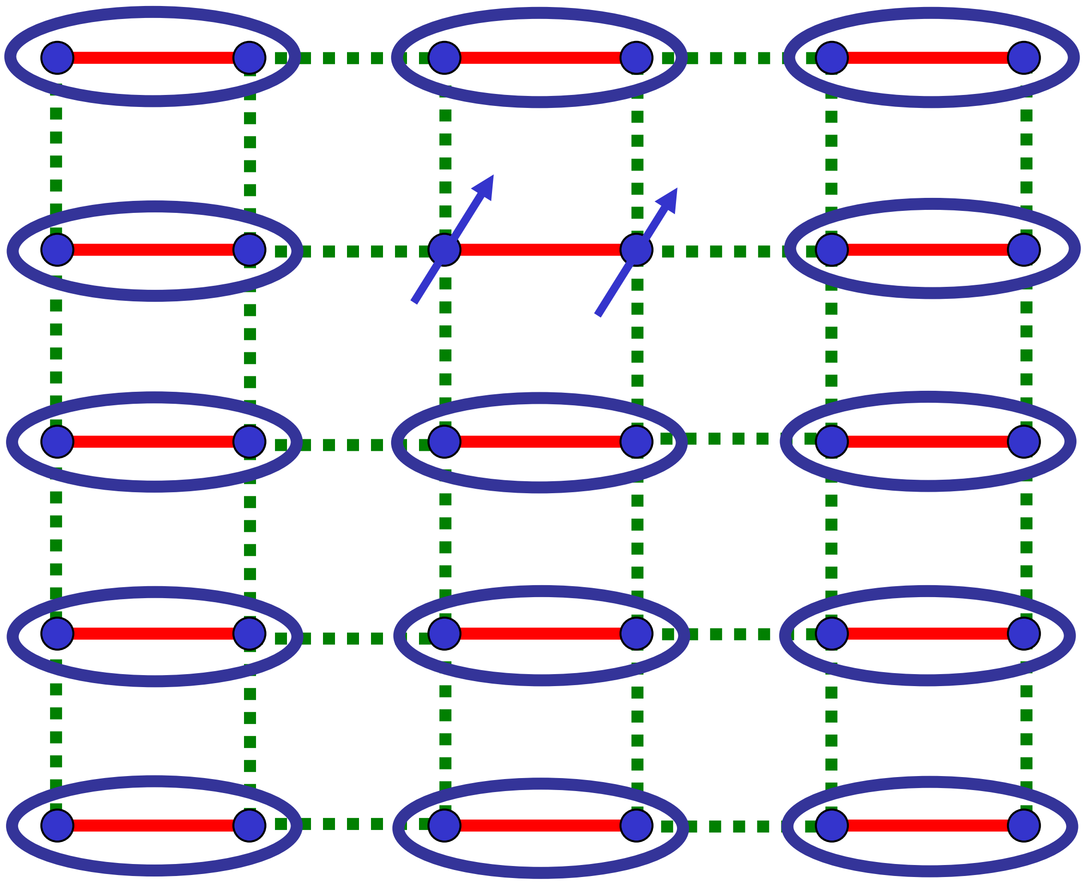


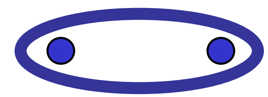

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers

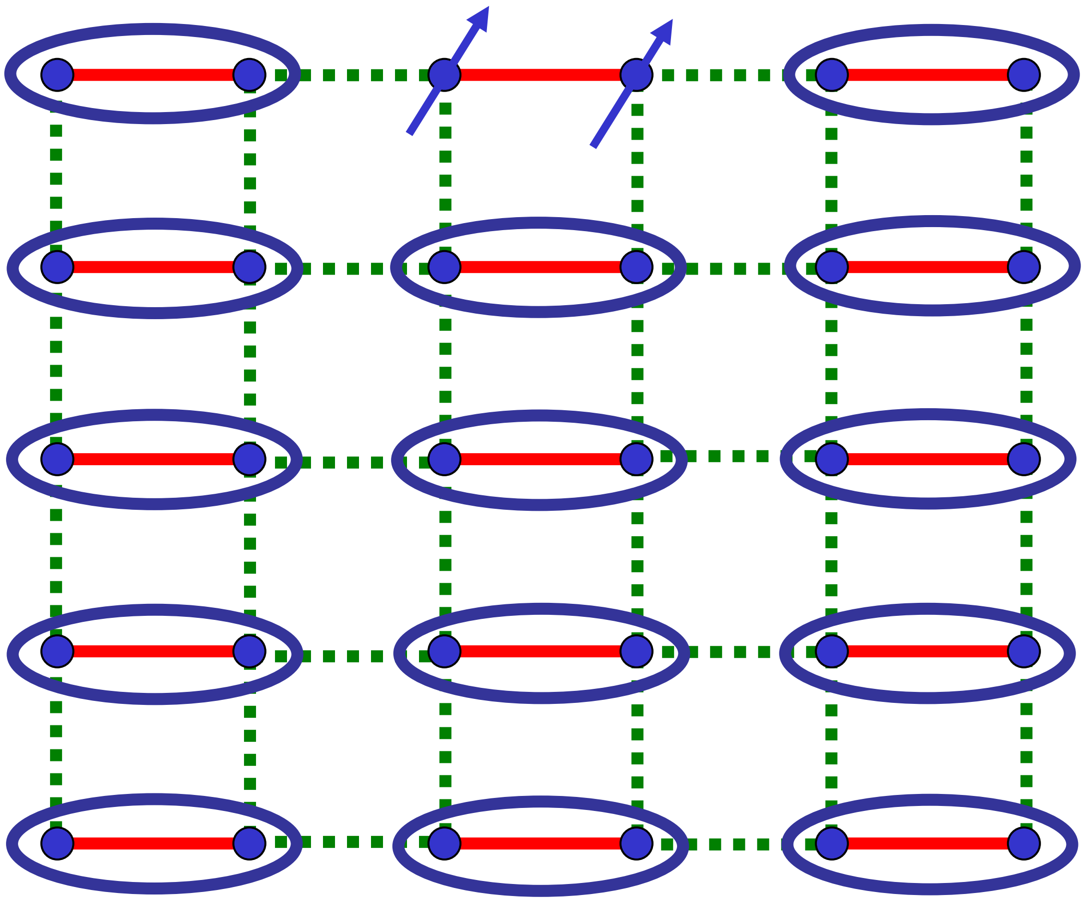


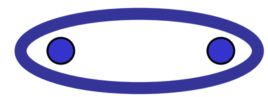
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers

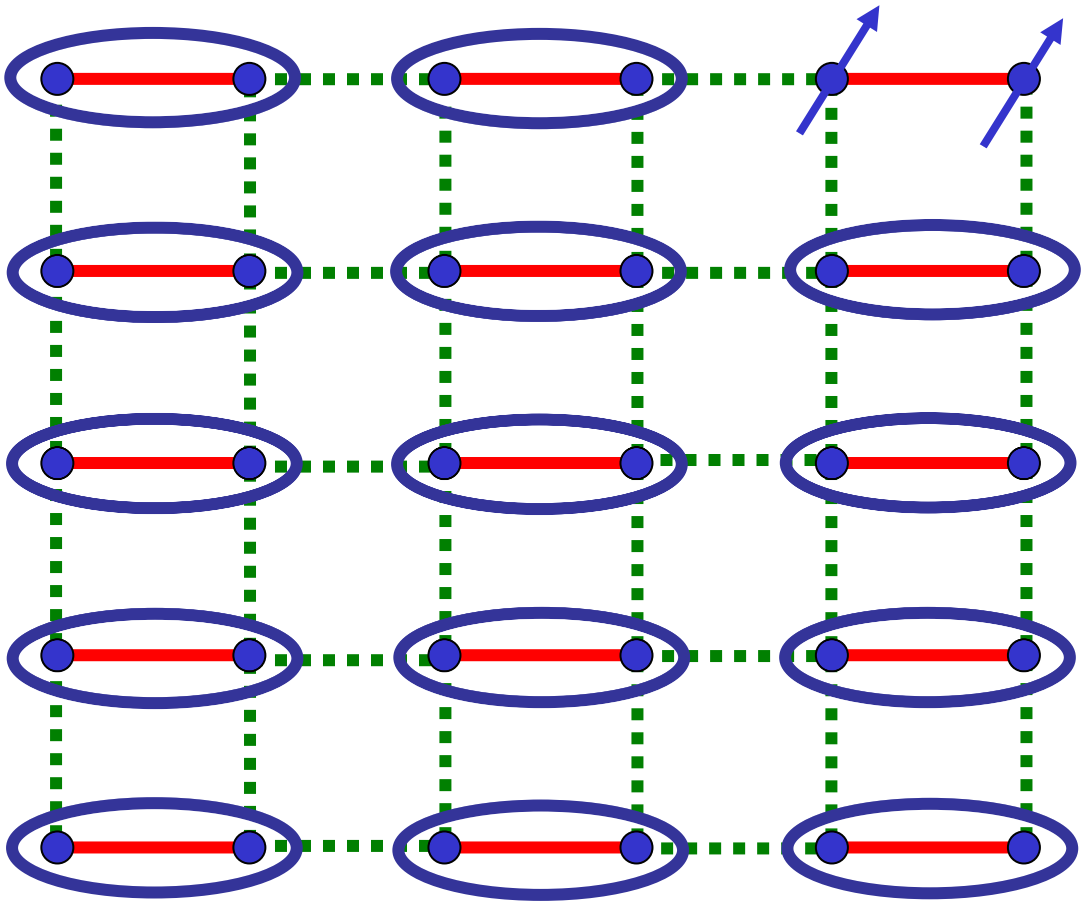


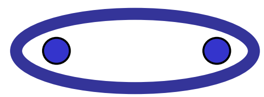

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

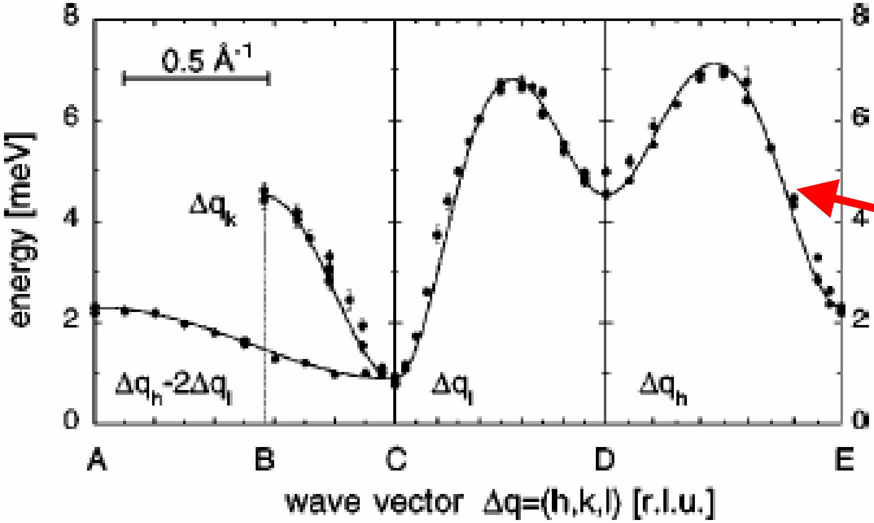
(***exciton***, spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

$TlCuCl_3$



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

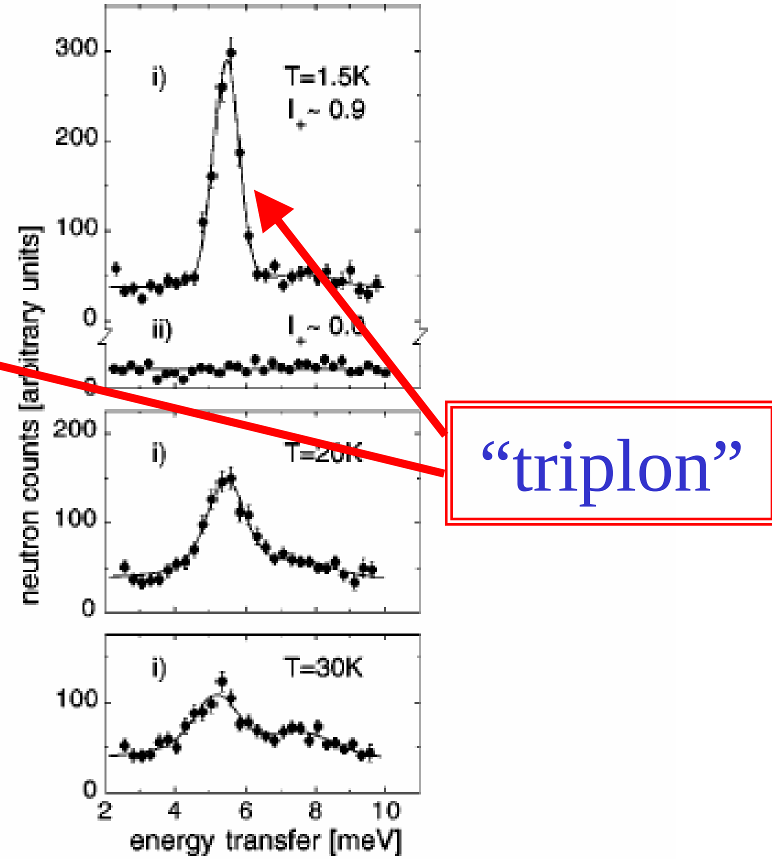
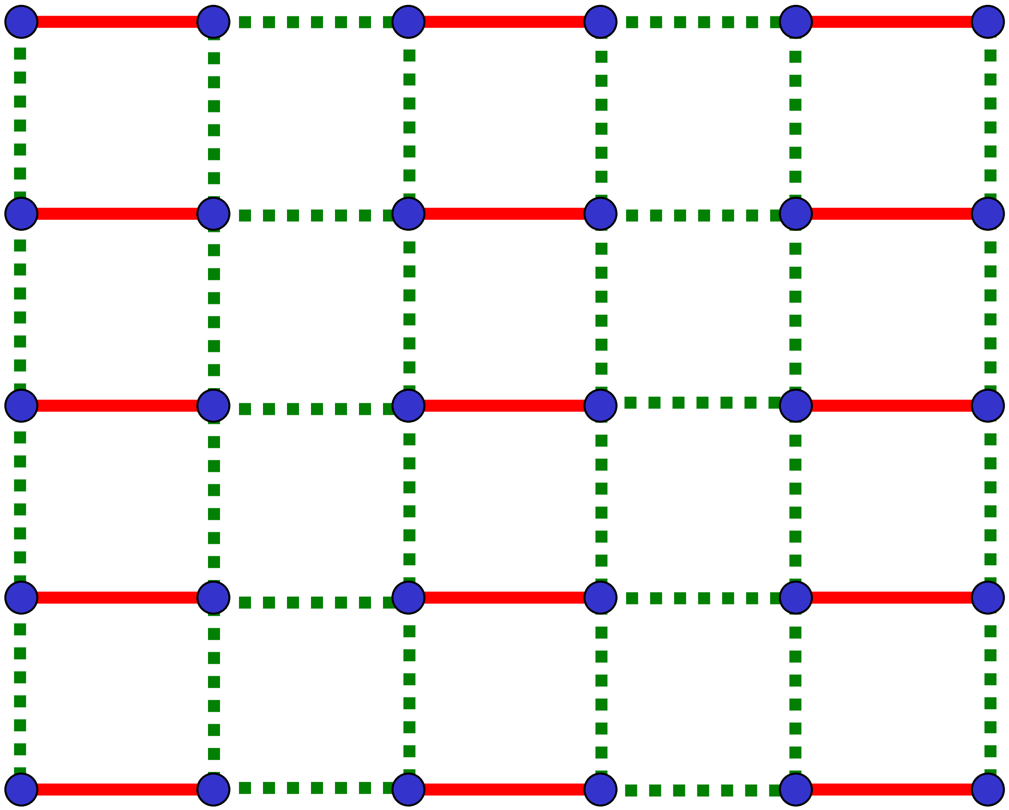


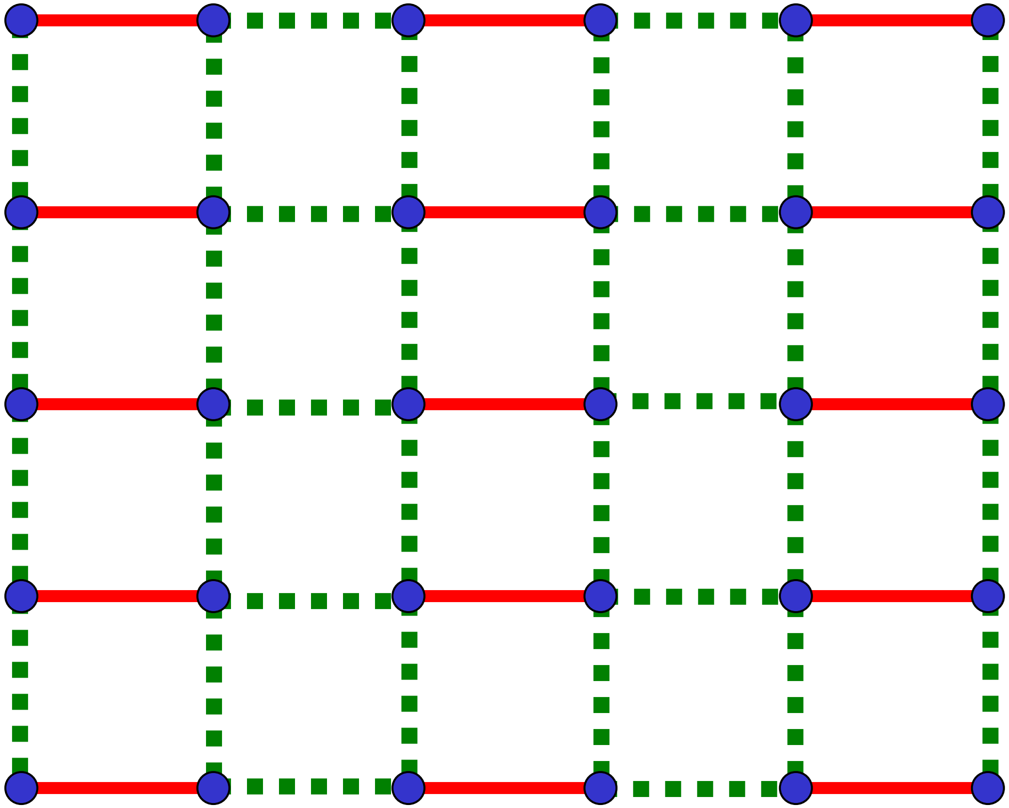
FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

Coupled Dimer Antiferromagnet



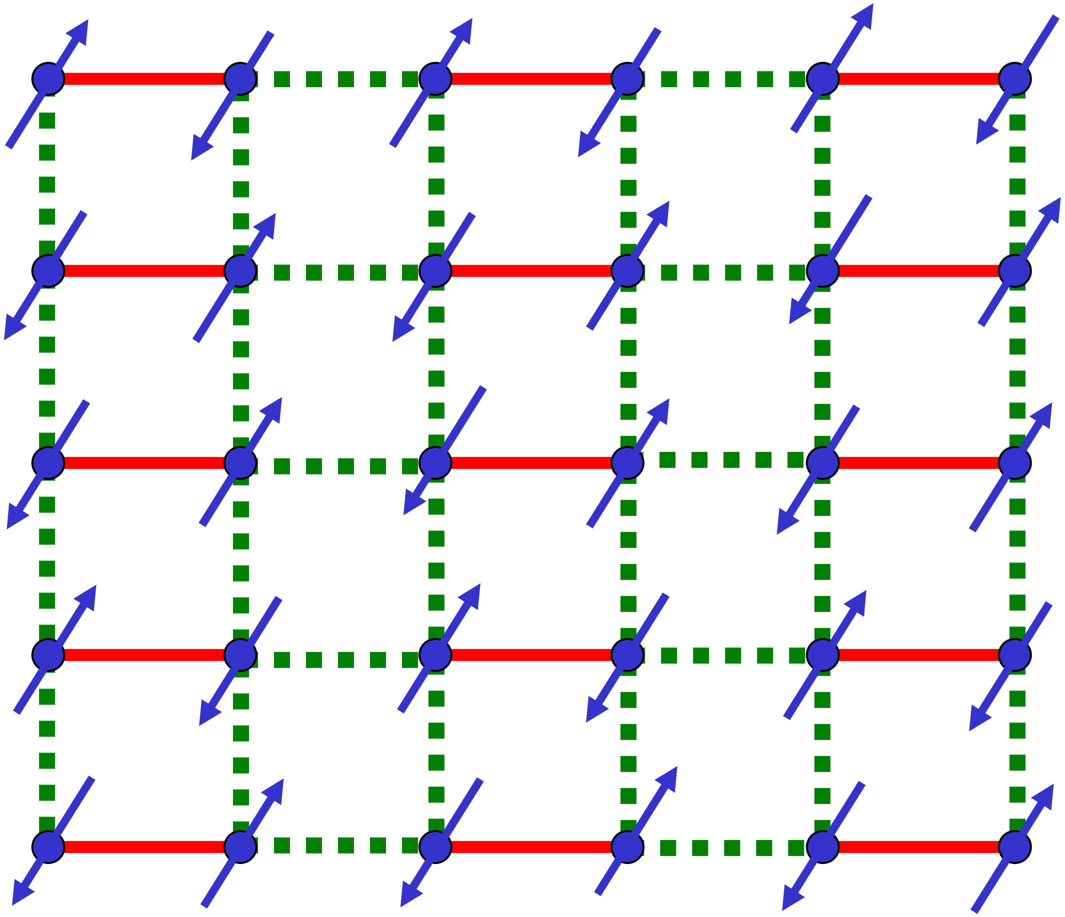
$\boxed{\lambda \text{ close to } 1}$

Weakly dimerized square lattice



$\boxed{\lambda \text{ close to } 1}$

Weakly dimerized square lattice



Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^{\,2} p_x^{\,2} + c_y^{\,2} p_y^{\,2}}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

$$\boxed{\left\langle \vec{\varphi} \right\rangle \neq 0}$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \dfrac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

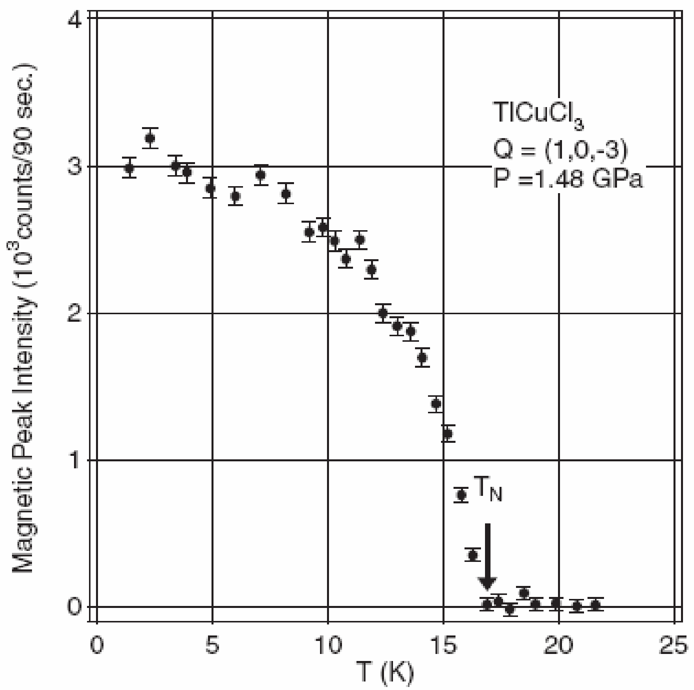


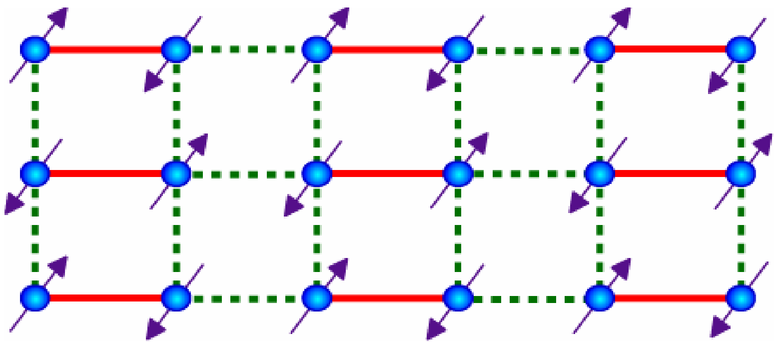
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

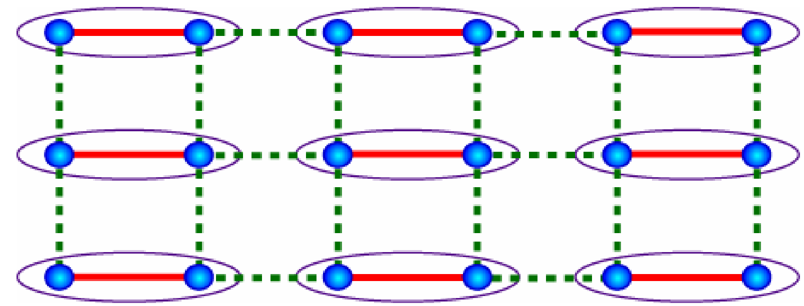
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



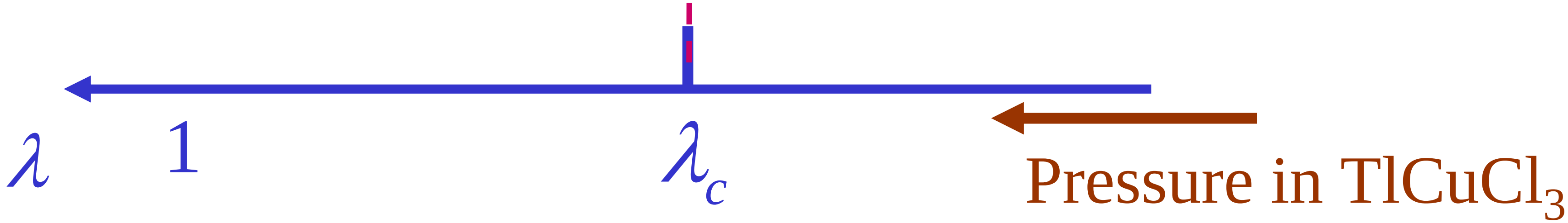
Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$



The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev*. B **39**, 2344 (1989)

LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$, oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ constitute the *triplon* excitation

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

Key reason for validity of LGW theory

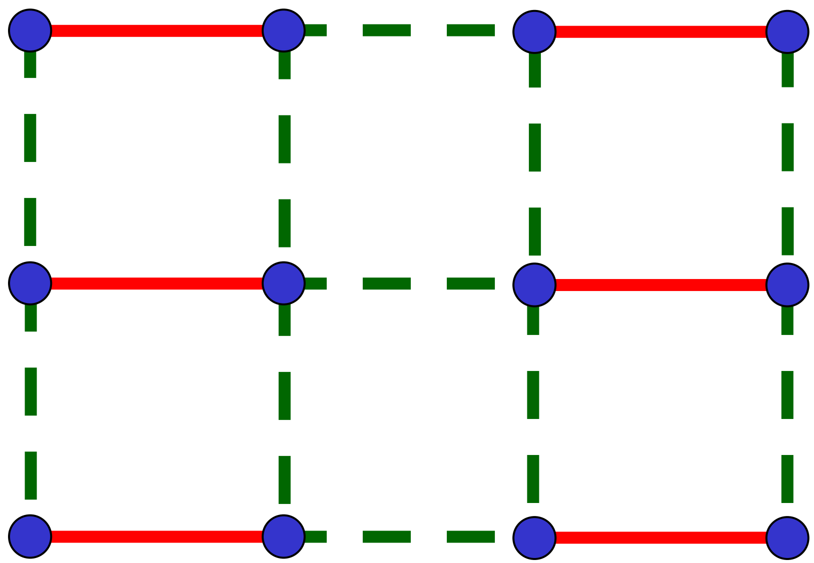
Classical statistical mechanics: There is a simple high temperature disordered state with $\langle \vec{\varphi} \rangle = 0$ and exponentially decaying correlations

Quantum mechanics: There is a "*quantum disordered*" non-degenerate ground state with $\langle \vec{\varphi} \rangle = 0$ and an energy gap to all excitations

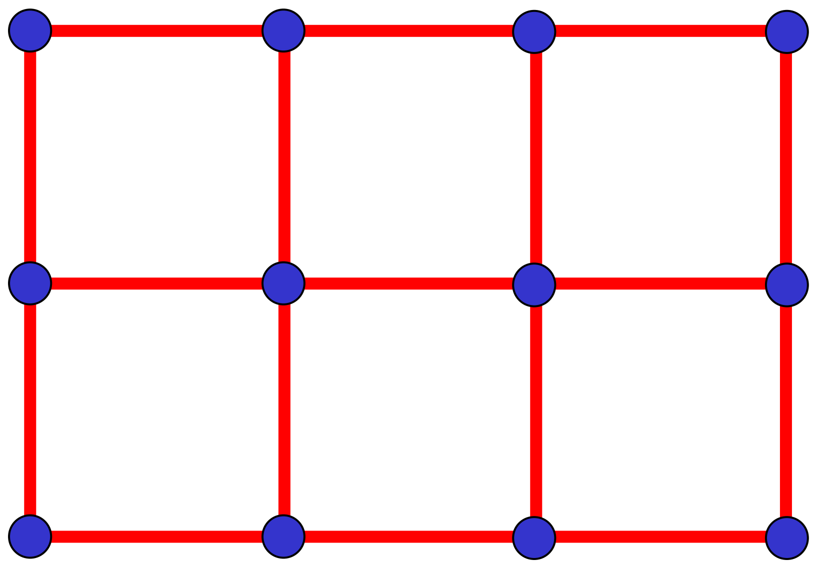
B. Mott insulators with
spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

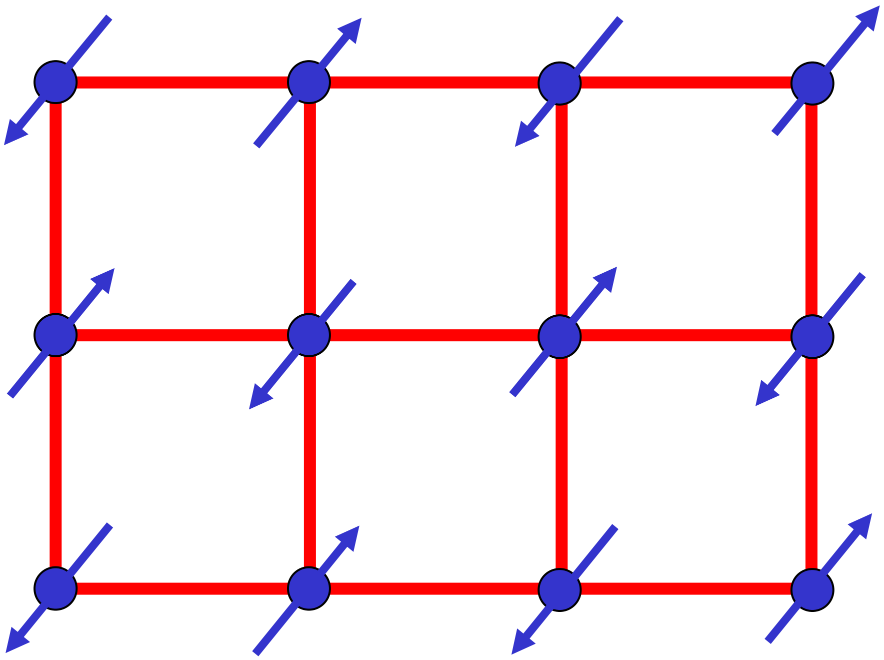
Mott insulator with two S=1/2 spins per unit cell



Mott insulator with one S=1/2 spin per unit cell

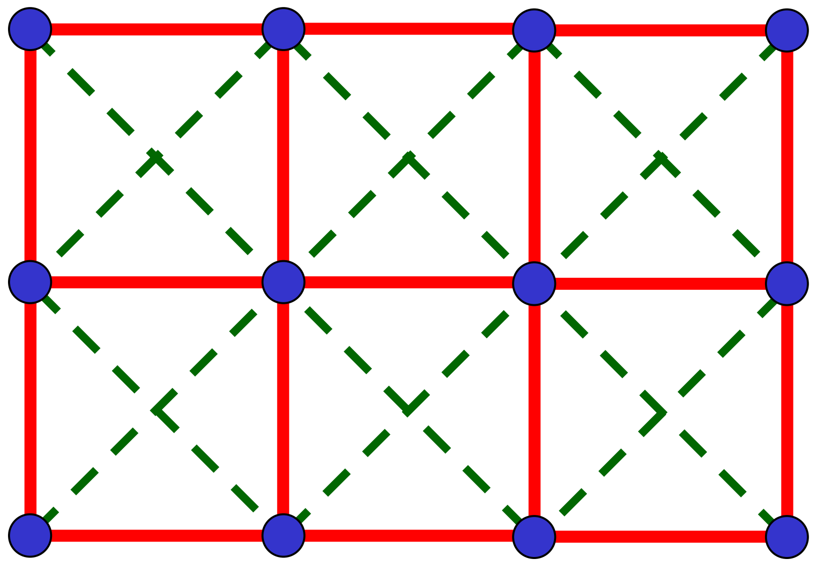


Mott insulator with one S=1/2 spin per unit cell



Ground state has Neel order with $\vec{\varphi} \neq 0$

Mott insulator with one S=1/2 spin per unit cell

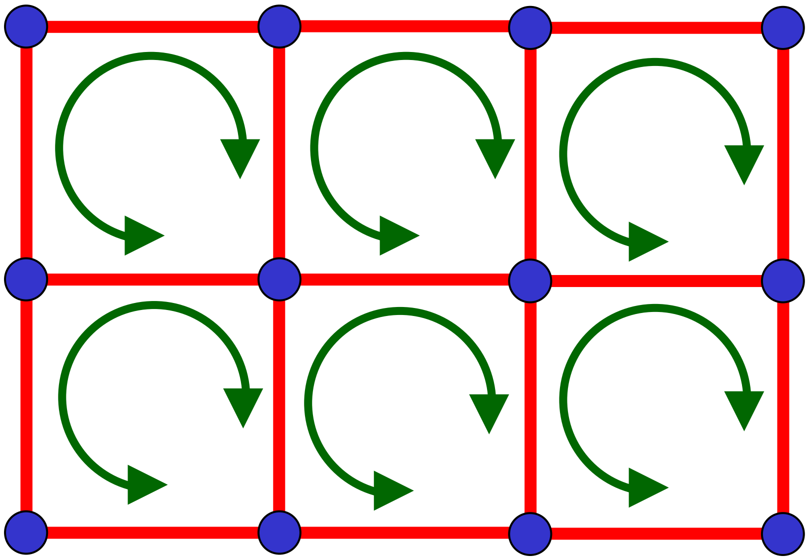


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange. The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell

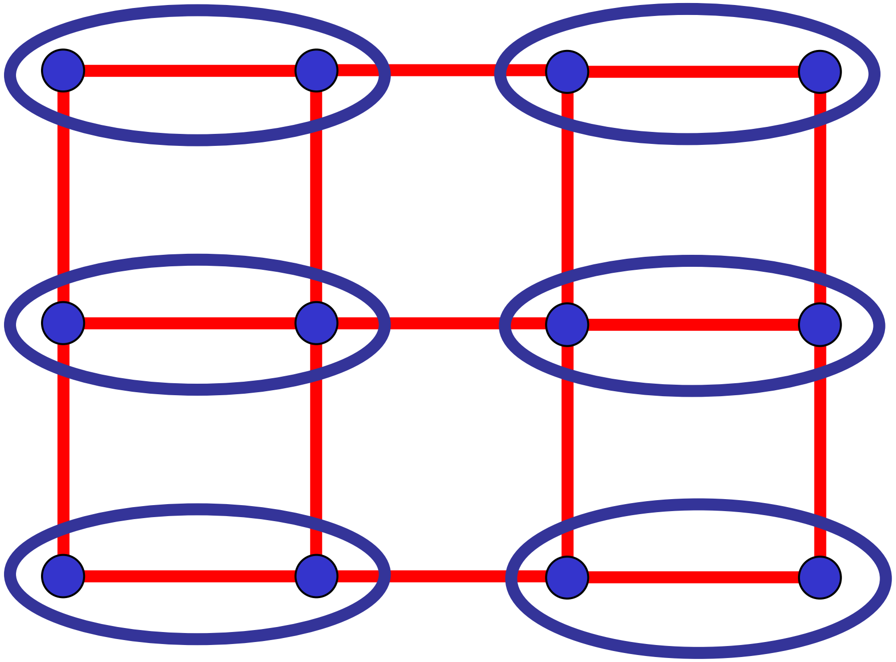


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.
The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

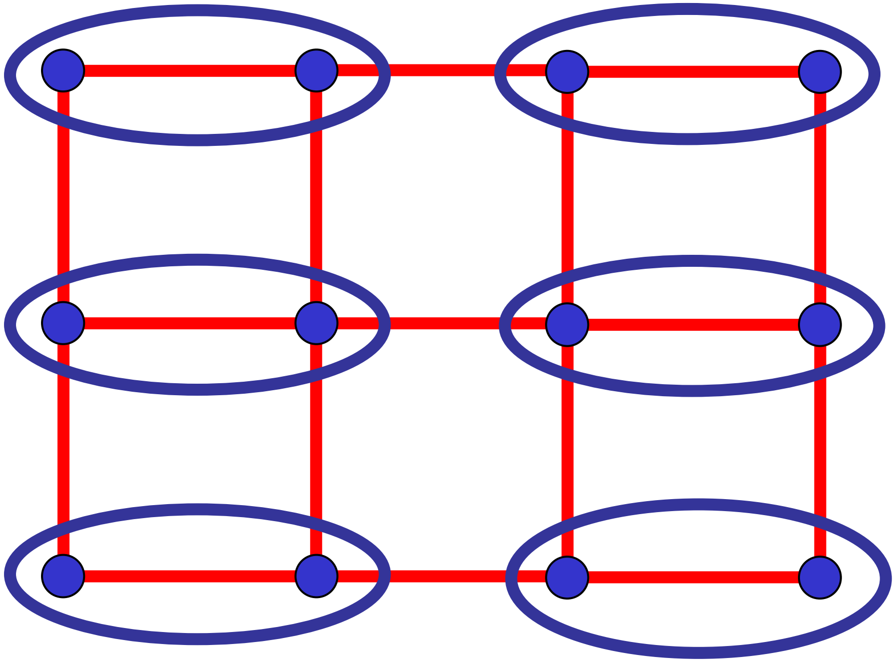
Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

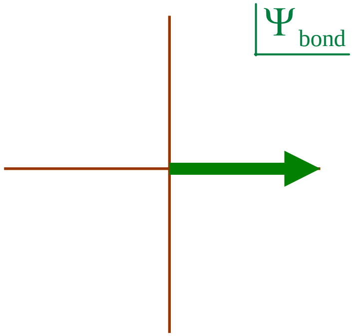
Mott insulator with one S=1/2 spin per unit cell



Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell



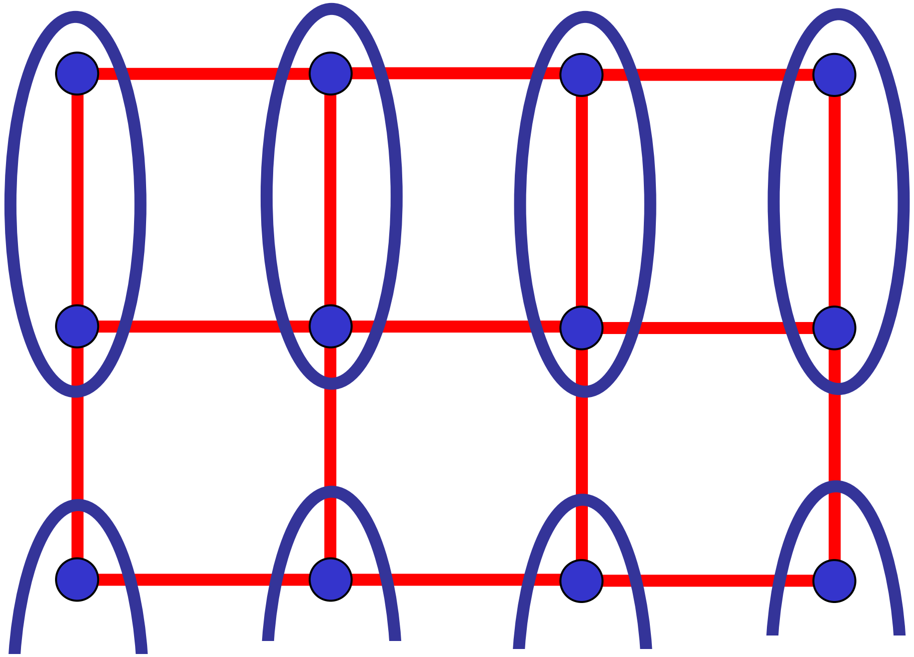
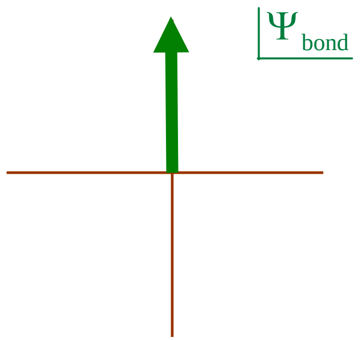


Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell

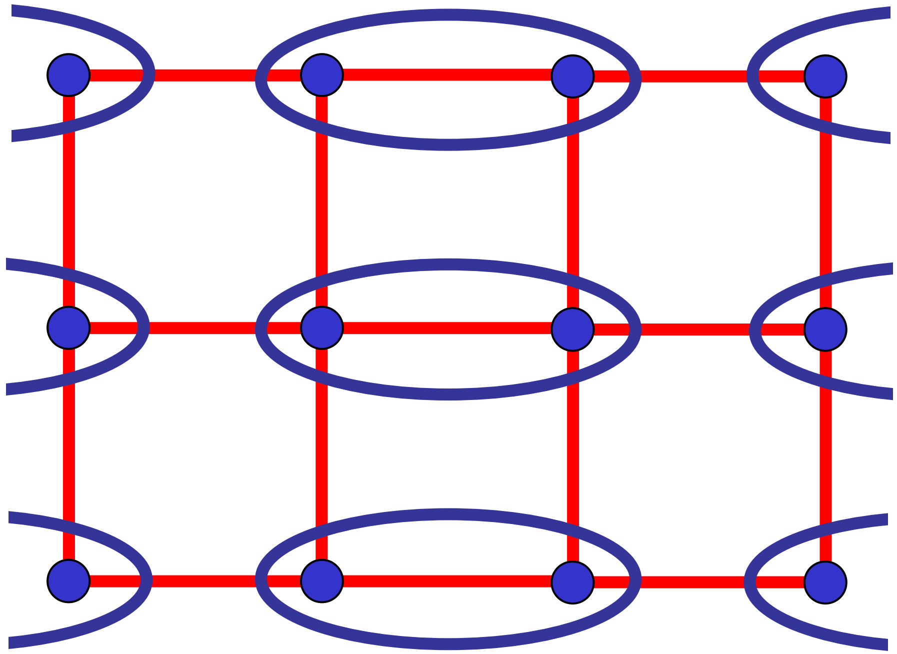



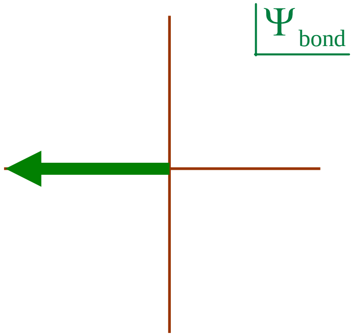
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan\left(\boldsymbol{r}_j - \boldsymbol{r}_i\right)}$$

Mott insulator with one S=1/2 spin per unit cell



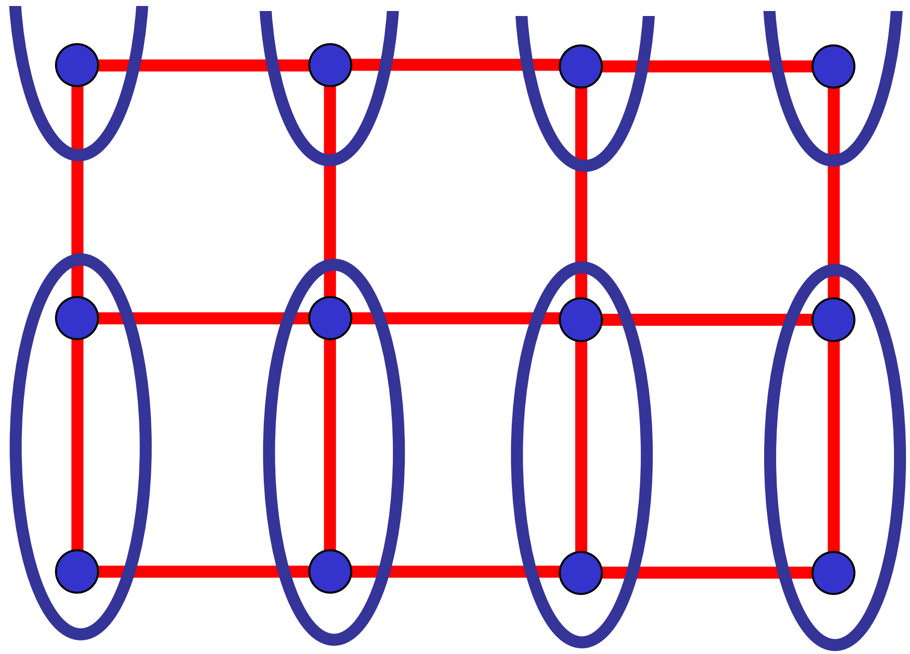


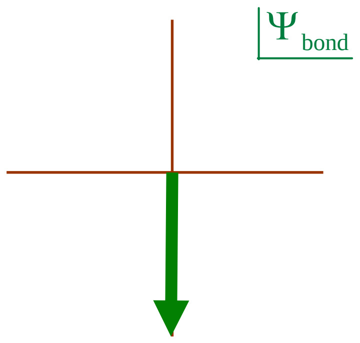
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



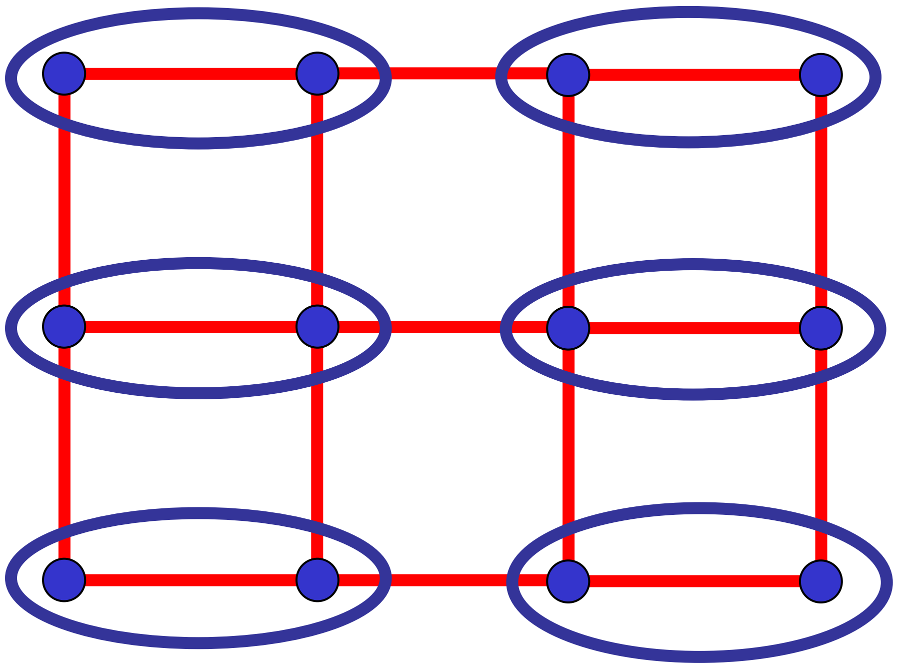


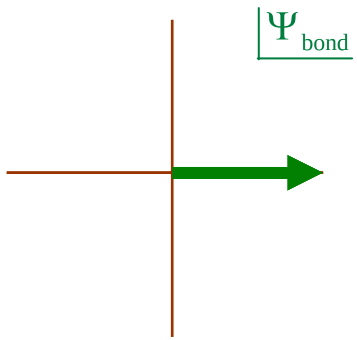
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



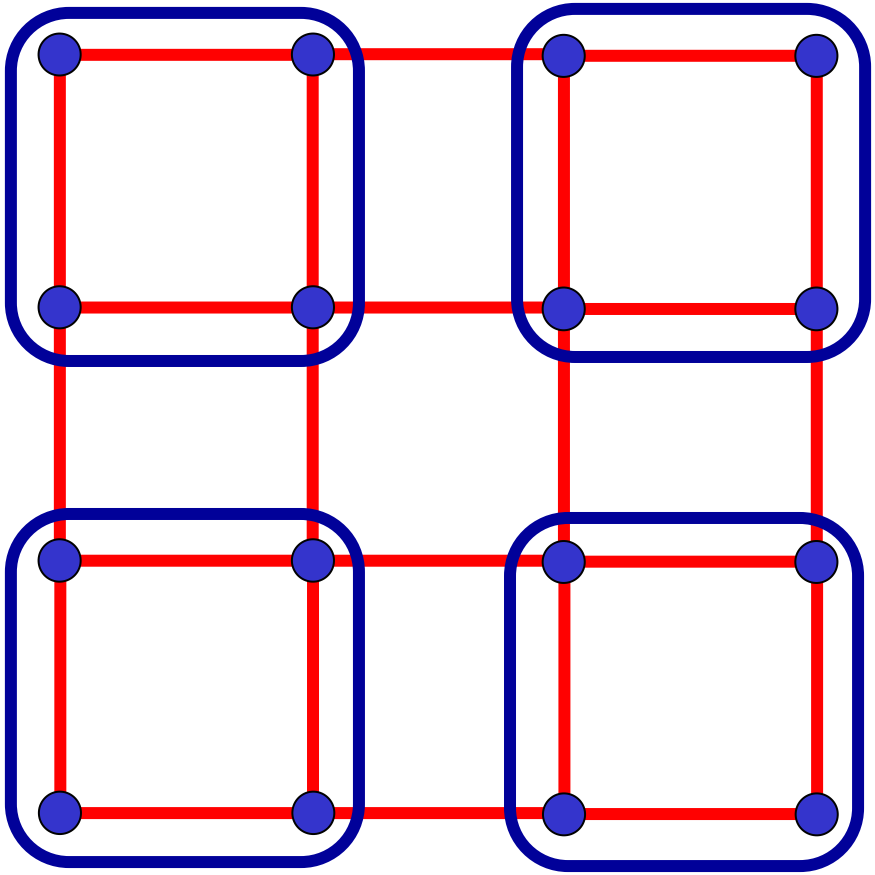


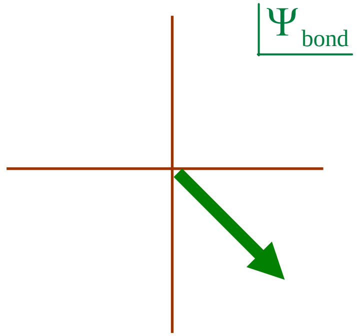
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan\left(\boldsymbol{r}_j - \boldsymbol{r}_i\right)}$$

Mott insulator with one S=1/2 spin per unit cell



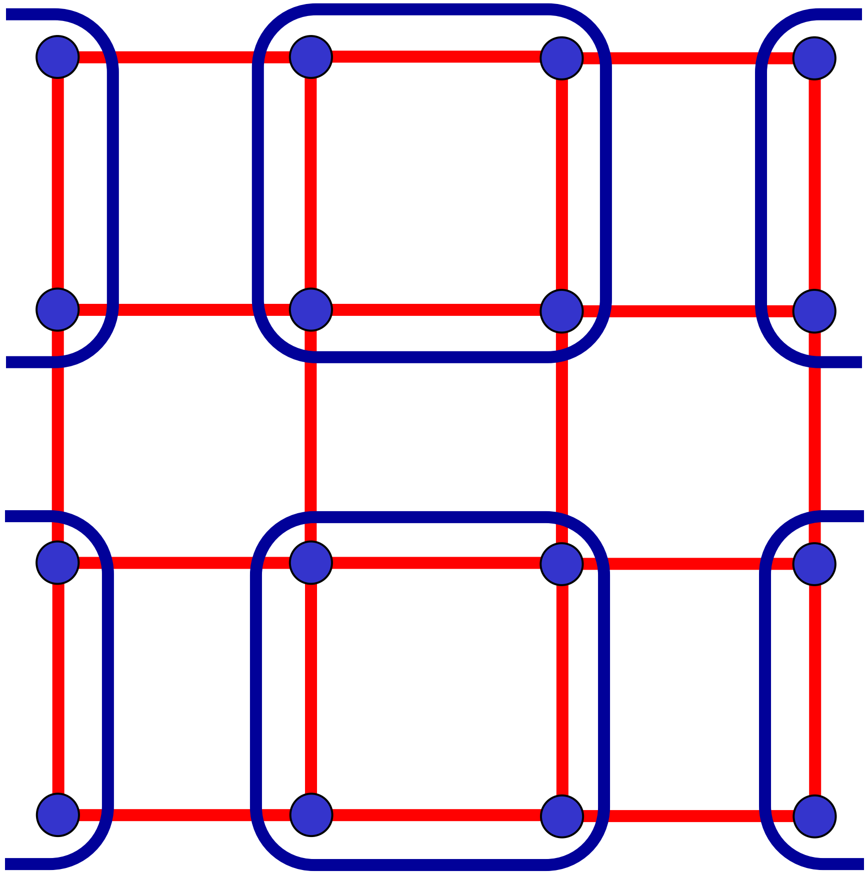


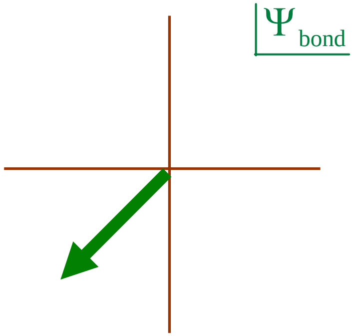
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Another state breaking the symmetry of rotations by $n\pi/2$ about lattice sites, which also has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



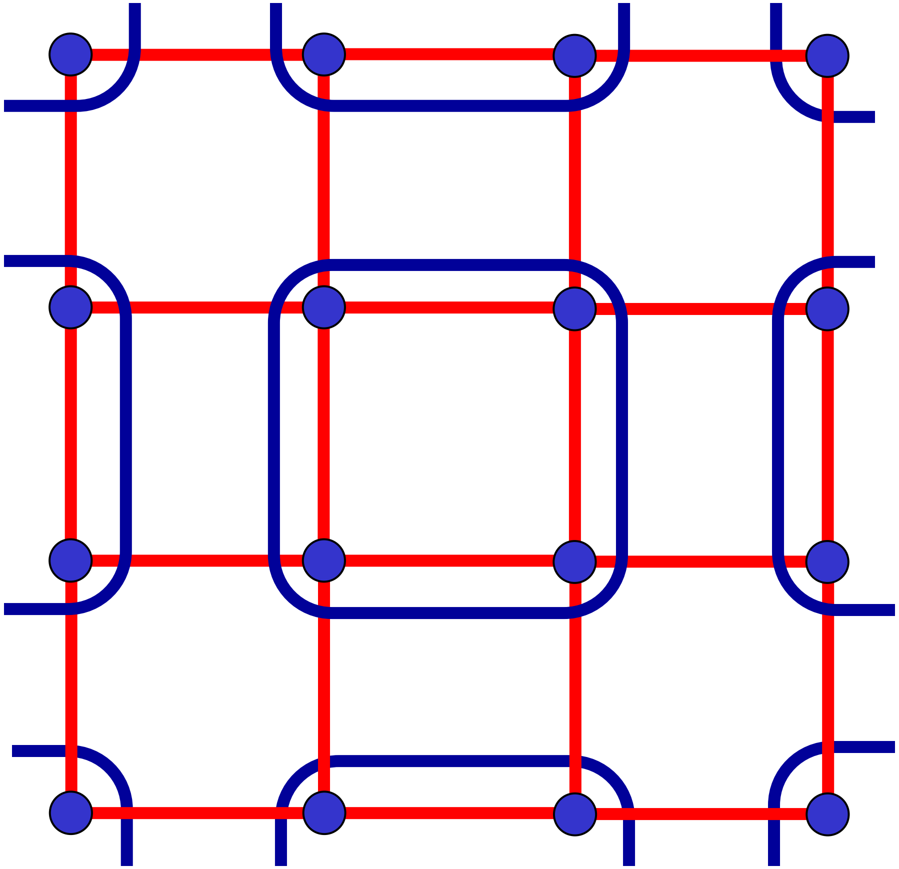


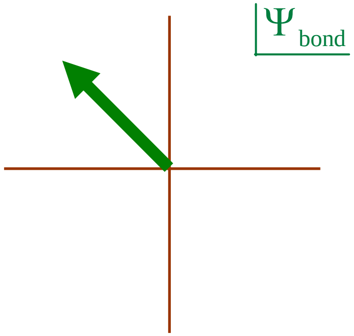
Possible large g paramagnetic ground state with $\left\langle \vec{\varphi} \right\rangle = 0$

Another state breaking the symmetry of rotations by $n\pi / 2$ about lattice sites, which also has $\left\langle \Psi_{\text{bond}} \right\rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}\left(i\right) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan\left(\boldsymbol{r}_j - \boldsymbol{r}_i\right)}$$

Mott insulator with one S=1/2 spin per unit cell



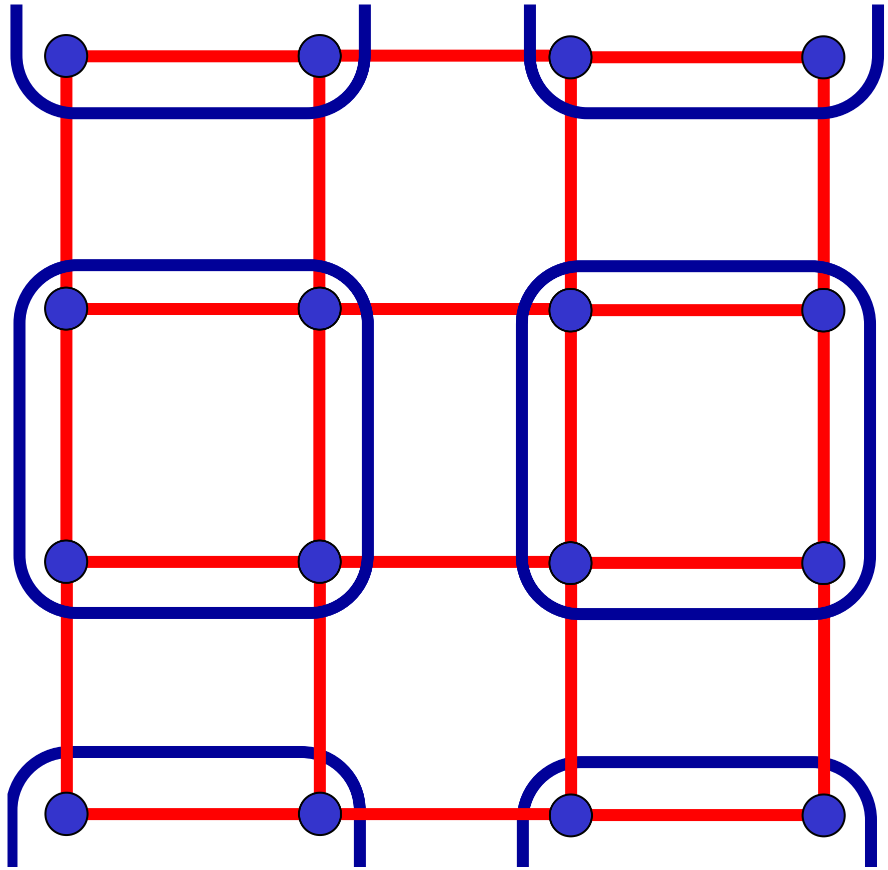


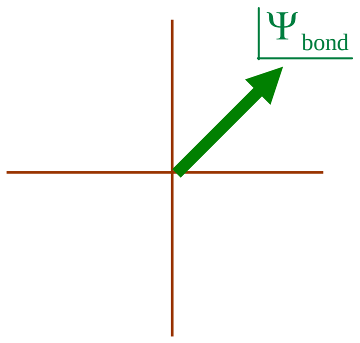
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Another state breaking the symmetry of rotations by $n\pi / 2$ about lattice sites, which also has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



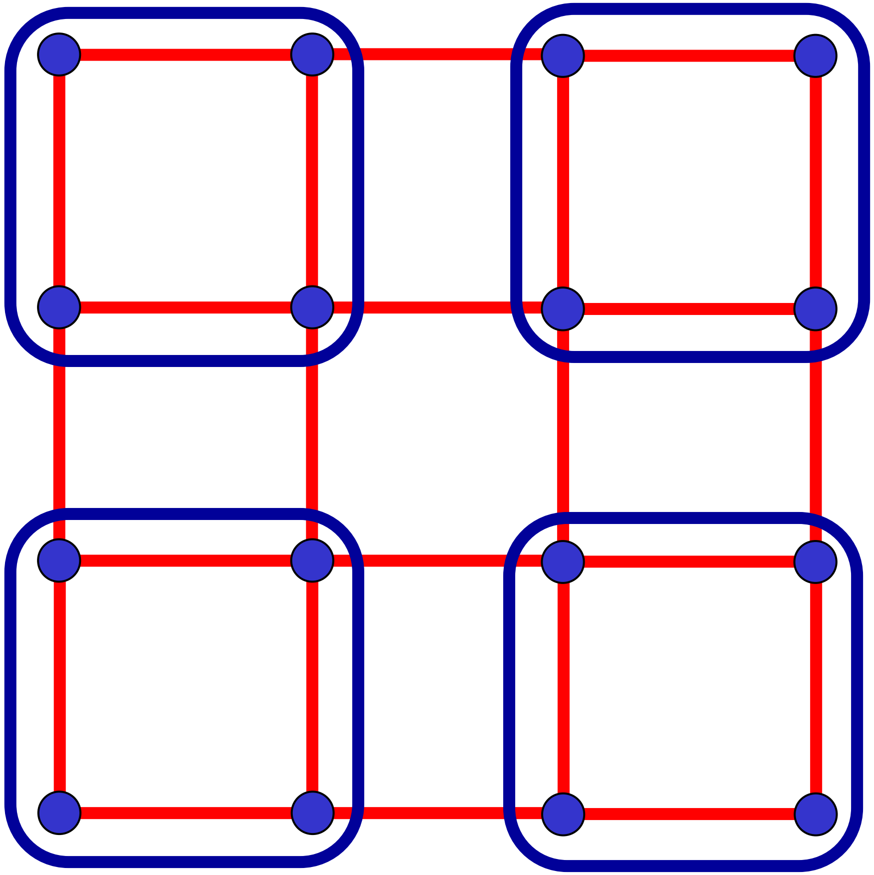


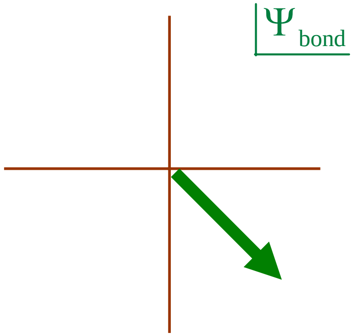
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Another state breaking the symmetry of rotations by $n\pi / 2$ about lattice sites, which also has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell





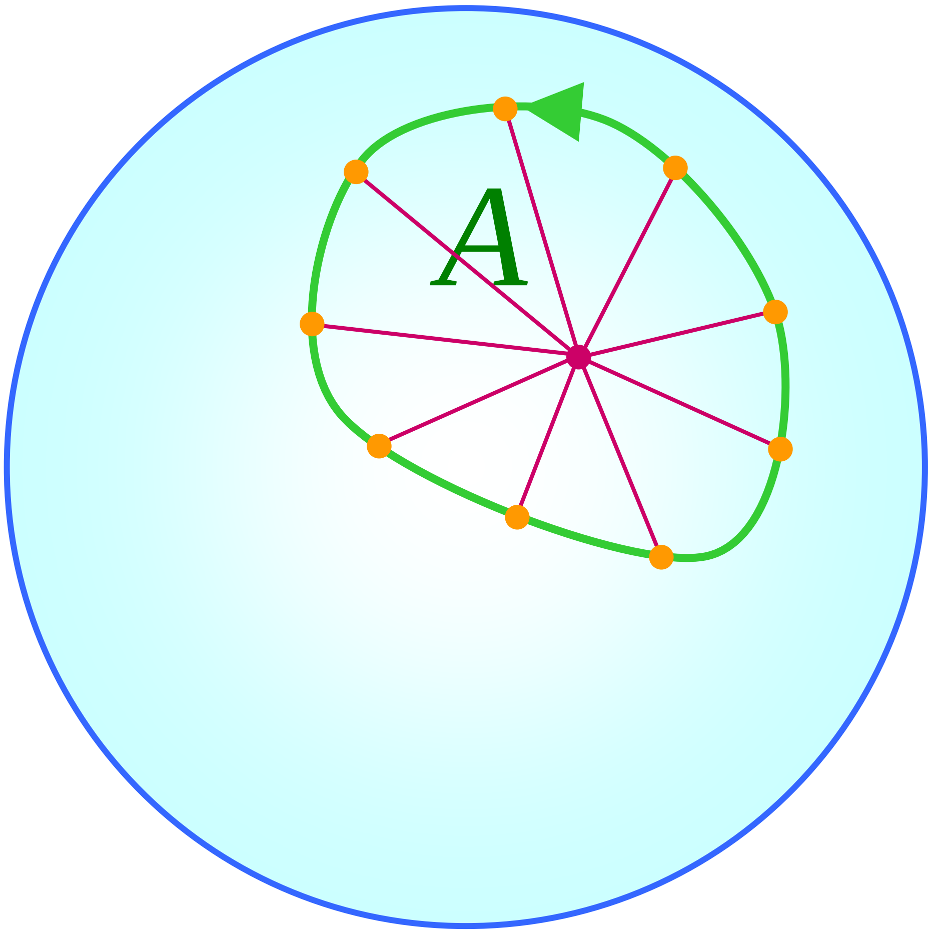
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Another state breaking the symmetry of rotations by $n\pi/2$ about lattice sites, which also has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Quantum theory for destruction of Neel order

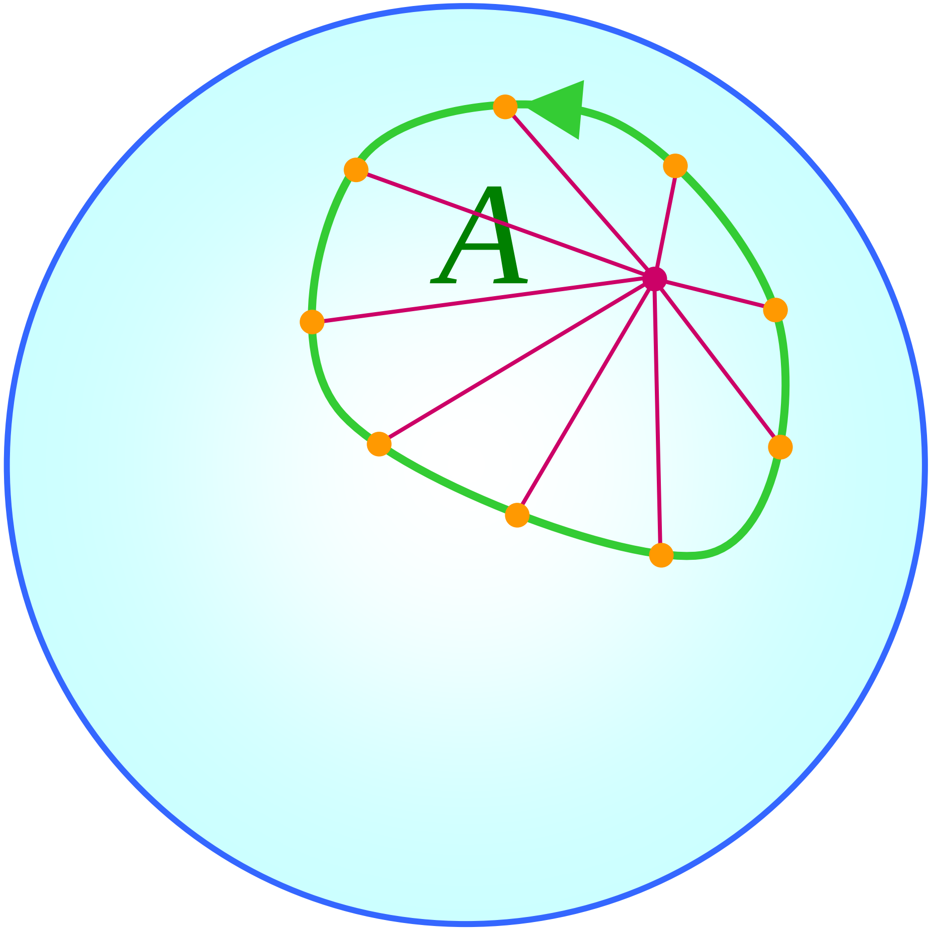
Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

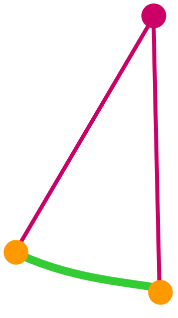
Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



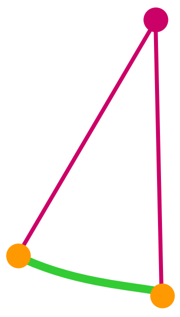
$$e^{iSA}$$

Quantum theory for destruction of Neel order



Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

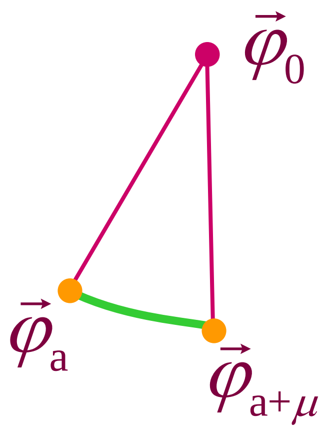


Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\mathrm{a}} = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_{\mathrm{a}}$ = (0,0,1) in classical Neel state;

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices ;



$\left(\mu = x, y, \tau\right)$

Quantum theory for destruction of Neel order

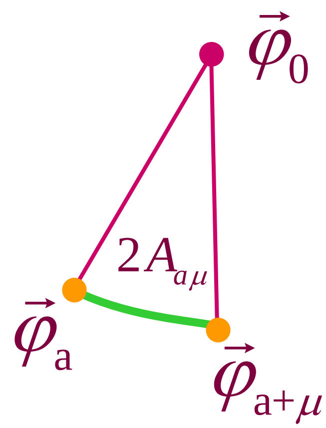
Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\rm a} = 2\eta_a \vec{S}_a \to \vec{\varphi}_{\rm a} = (0,0,1)$ in classical Neel state;

$\eta_{\rm a} \to \pm 1$ on two square sublattices ;

$A_{a\mu} \to$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_{\rm a}$, $\vec{\varphi}_{{\rm a}+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$



Quantum theory for destruction of Neel order

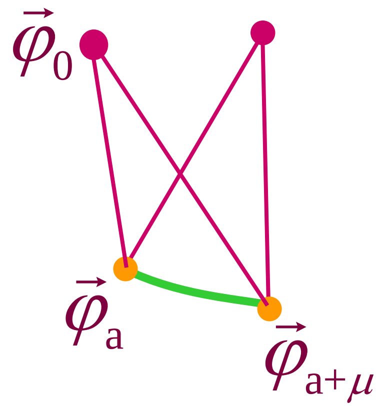
Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\rm a} = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_{\rm a} = (0,0,1)$ in classical Neel state;

$\eta_{\rm a} \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_{\rm a}$, $\vec{\varphi}_{{\rm a}+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$



Quantum theory for destruction of Neel order

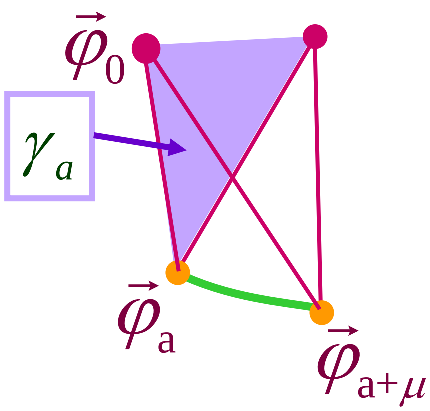
Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\mathrm{a}} = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_{\mathrm{a}} = (0,0,1)$ in classical Neel state;

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_{\mathrm{a}}$, $\vec{\varphi}_{\mathrm{a}+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$



<u>**Quantum theory for destruction of Neel order**</u>

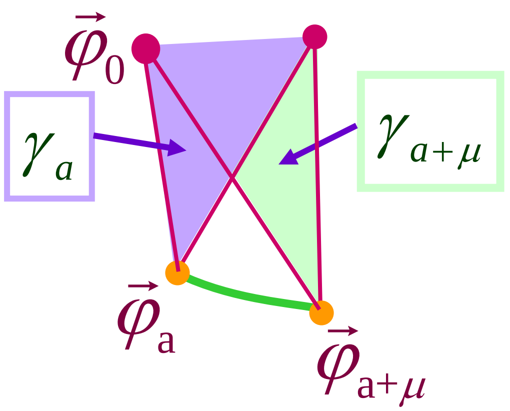
Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points a

Recall $\quad \vec{\varphi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_a = (0,0,1)$ in classical Neel state;

$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$



Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points a

Recall $\vec{\varphi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_a = (0,0,1)$ in classical Neel state;

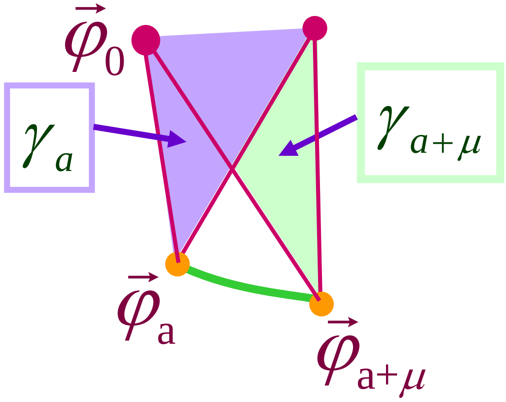
$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a “gauge transformation”



Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points a

Recall $\vec{\varphi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_a = (0,0,1)$ in classical Neel state;

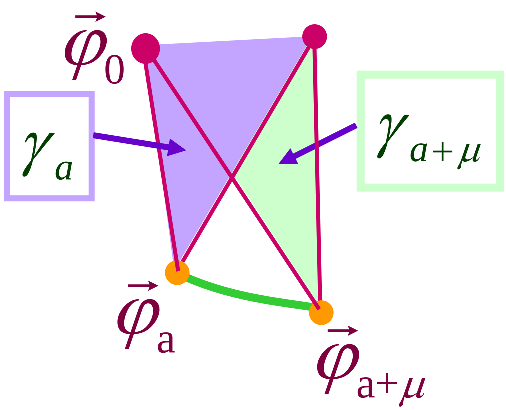
$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

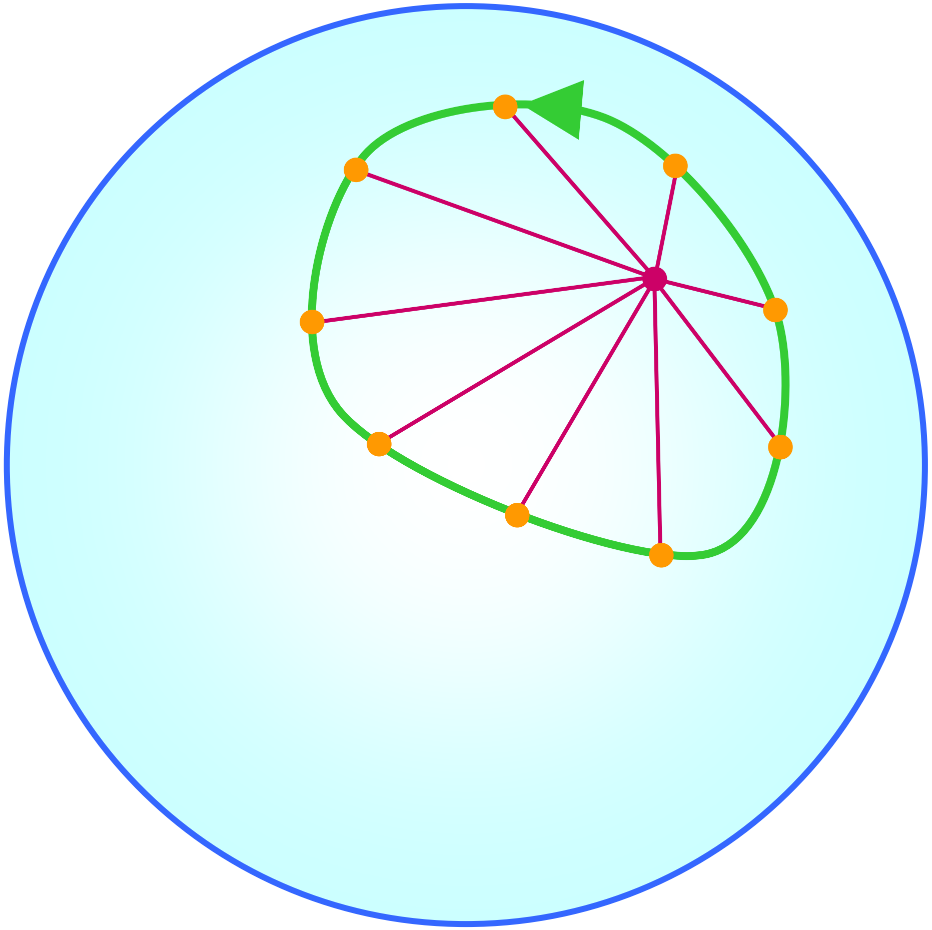
Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"



The area of the triangle is uncertain modulo 4π, and the action has to be invariant under $A_{a\mu} \rightarrow A_{a\mu} + 2\pi$

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$\exp\left(i \sum_a \eta_a A_{a\tau} \right)$$

Sum of Berry phases of all spins on the square lattice.

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} \right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\left\langle \vec{\varphi} \right\rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\left\langle \vec{\varphi} \right\rangle = 0$

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories $\rightarrow$ need an effective action for $A_{a\mu}$ at large g

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

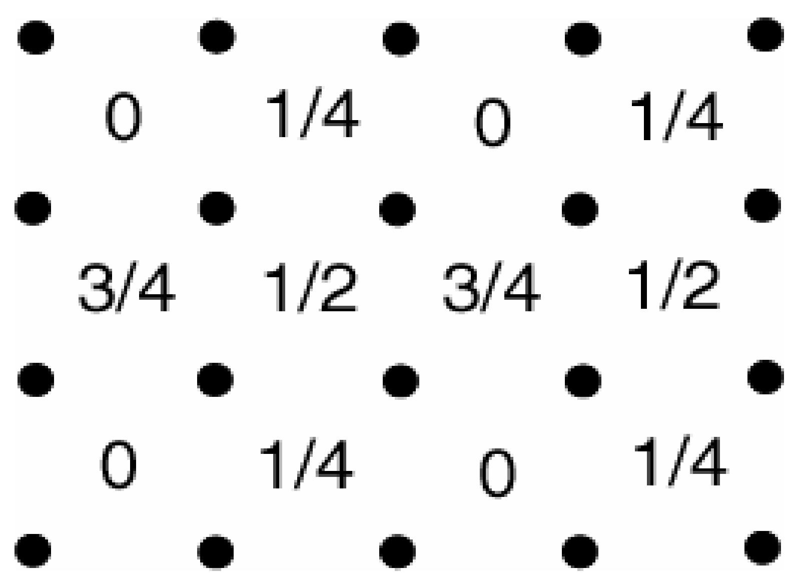
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields a representation in terms of a Coulomb gas of monopoles

$$Z_{\rm dual} = \sum_{\{m_{\bar{\jmath}}\}} \exp\left(-\frac{\pi}{2e^2} \sum_{\bar{\jmath},\bar{\jmath}'} \frac{m_{\bar{\jmath}} m_{\bar{\jmath}'}}{|r_{\bar{\jmath}} - r_{\bar{\jmath}'}|} + 2\pi i \sum_{\bar{\jmath}} m_{\bar{\jmath}} \mathcal{X}_{\bar{\jmath}} \right)$$

with the $m_{\bar{\jmath}}$ integer monopole charges. Each monopole carries a Berry phase (F.D.M. Haldane, *Phys. Rev. Lett.* **61**, 1029 (1988)) determined by the fixed $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



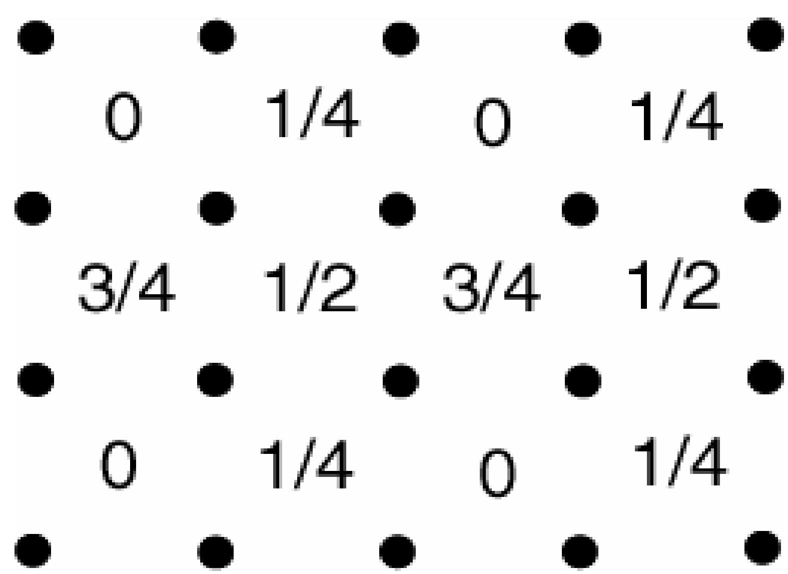
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Alternative representation is in terms of a "height" model

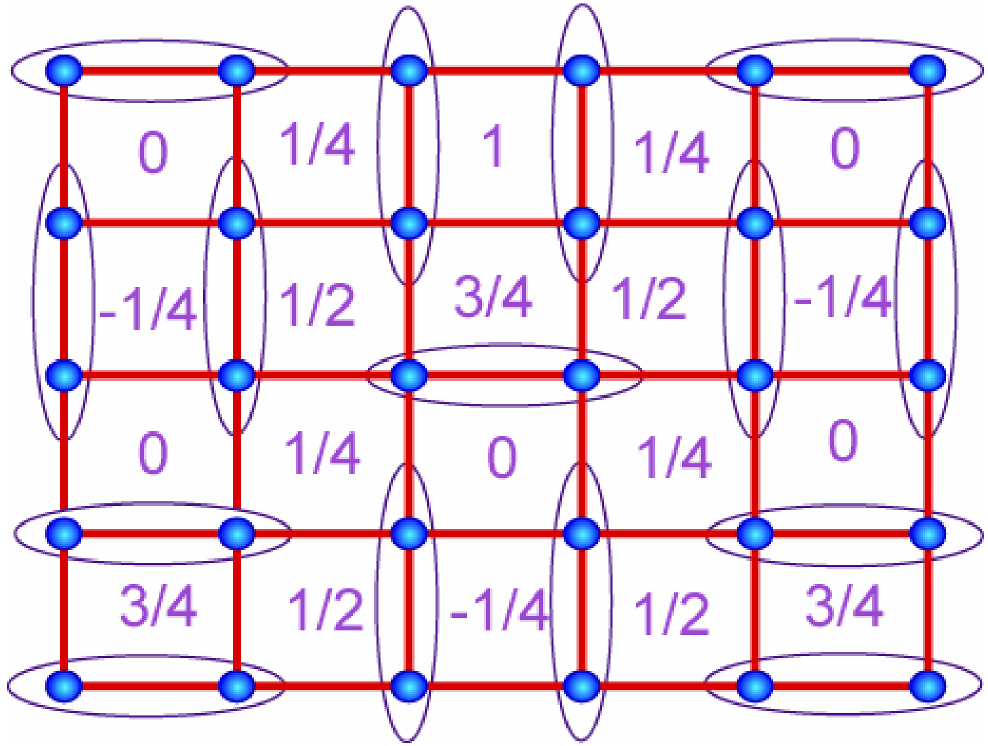
$$Z_{\text{dual}} = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2} \sum_{\bar{\jmath}} \left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}} \right)^2 \right)$$

with the $h_{\bar{\jmath}}$ integer heights.

The Berry phases now lead to height 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with nearest-neighbor valence bond pairings of the sites square lattice



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

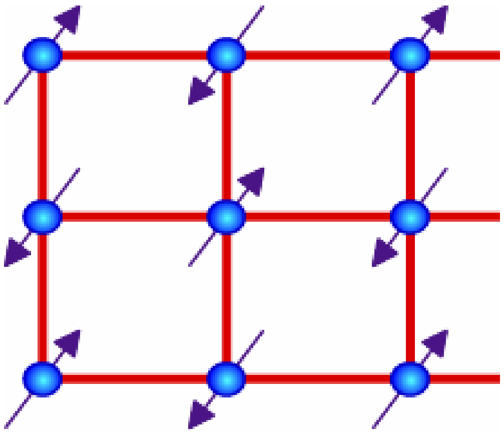
D.S. Fisher and J.D. Weeks, *Phys. Rev. Lett.* **50**, 1077 (1983).

$\Rightarrow$ There is a definite average height of the interface

$\Rightarrow$ **Ground state has bond order.**

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

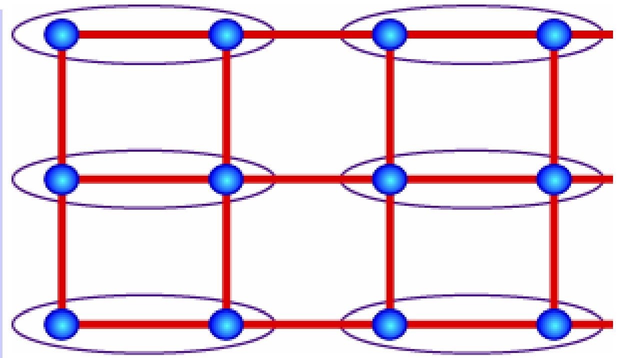
$$Z = \prod_a \int d\vec{\varphi}_a \,\delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$



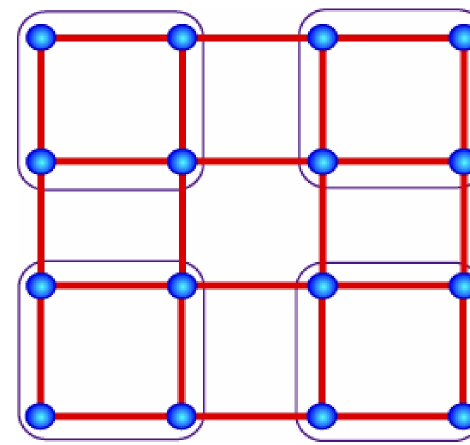
Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



or



Bond order

$\langle \Psi_{\mathrm{bond}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0

g