Strange metals and black holes

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Quantum entanglement
The double slit experiment

Interference of water waves
The double slit experiment

Bullets
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Send electrons through the slits
The double slit experiment

Interference of electrons
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Is the electron a wave?

Interference of electrons
The double slit experiment

Unlike water waves, electrons arrive one-by-one (so is it like a particle?)

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons

No interference when you watch the electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Each electron passes through both slits!

Interference of electrons
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.

Actual state of each electron is $|L\rangle + |R\rangle$. 

Principles of Quantum Mechanics: 1. Quantum Superposition
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ \begin{array}{c}
\text{Hydrogen molecule:}
\end{array} \]

\[ = \frac{1}{\sqrt{2}} ( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Quantum Entanglement: quantum superposition with more than one particle
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Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away
Quantum entanglement
Quantum entanglement

Strange metals
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
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Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time.
High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
Quantum matter without quasiparticles

Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

Figure: K. Fujita and J. C. Seamus Davis
“Strange”,

“Bad”,

or “Incoherent”,

metal has a resistivity, \( \rho \), which obeys

\[ \rho \sim T, \]

and

in some cases \( \rho \gg h/e^2 \)

(in two dimensions),

where \( h/e^2 \) is the quantum unit of resistance.
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model
The Sachdev-Ye-Kitaev (SYK) model

Entangle electrons pairwise randomly
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Entangle electrons pairwise randomly
The SYK model has “nothing but entanglement”
The Sachdev-Ye-Kitaev (SYK) model

This describes both a strange metal and a black hole!
A strongly correlated metal built from Sachdev-Ye-Kitaev models
See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Low ‘coherence’ scale

\[ E_c \sim \frac{t^2}{U} \]

For \( E_c < T < U \), the resistivity

\[ \rho \sim \frac{h}{e^2} \left( \frac{T}{E_c} \right) . \]
Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.
Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

Thermal equilibration into a chaotic quantum state happens very rapidly in systems without quasiparticle excitations: it happens in a shortest possible time of order $\frac{\hbar}{k_B T}$.

(SS 1999, Maldacena, Shenker, Stanford 2015)
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius $R = \frac{2GM}{c^2}$
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away 0.1 seconds later!
The ring-down is predicted by General Relativity to happen in a time \( \frac{8\pi GM}{c^3} \) \( \sim \) 8 milliseconds. Curiously this happens to equal \( k_B T_H \): so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate.
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
• Black holes have an entropy and a temperature, $T_H = \frac{\hbar c^3}{8\pi GMk_B}$.

• The entropy is proportional to their surface area.
• The ring-down is predicted by General Relativity to happen in a time \( \frac{8\pi GM}{c^3} \approx 8 \text{ milliseconds}. \)
• The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$, so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!
Black holes have an entropy and a temperature, \( T_H = \frac{\hbar c^3}{8\pi G M k_B} \).

The entropy is proportional to their surface area.

They relax to thermal equilibrium in a time \( \sim \frac{\hbar}{k_B T_H} \).
Quantum entanglement

Black holes

Strange metals
Strange metals
Quantum entanglement

Black holes
Strange metals

The SYK model is both a strange metal and a black hole!
SYK and black holes
The SYK model has “dual” description in which an extra spatial dimension, $\zeta$, emerges. The curvature of this “emergent” spacetime is described by Einstein’s theory of general relativity.
There is a black hole in the emergent spacetime at the same temperature, $T_H$, as the SYK model.

The duality explains:

(i) why they have a common thermalization time $\hbar/(k_B T_H)$. 
There is a black hole in the emergent spacetime at the same temperature, $T_H$, as the SYK model. The duality explains:

(ii) why the black hole entropy is proportional to its surface area.
Tensor network of hierarchical entanglement

$\mathbf{x}$

$D$-dimensional space

depth of entanglement
Quantum entanglement leads to an emergent spatial dimension.
SYK and black holes

An extra spatial dimension emerges from quantum entanglement!

SS 2010; A. Kitaev, 2015
The SYK model is both a strange metal and a black hole!
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$. 
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order $\frac{\hbar}{(k_B T)}$.
- These are also characteristics of black holes in quantum gravity.