

# Quantum phase transitions and the Luttinger theorem.

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and to appear



# Outline

## A. Bose-Fermi mixtures

*Depleting the Bose-Einstein condensate in trapped ultracold atoms*

## B. The Kondo Lattice

*The heavy Fermi liquid (FL) and the fractionalized Fermi liquid (FL\*)*

## C. *Detour*: Deconfined criticality in insulators

*Landau forbidden quantum transitions*

## D. Deconfined criticality in the Kondo lattice ?

## A. Bose-Fermi mixtures

*Depleting the Bose-Einstein condensate  
in trapped ultracold atoms*

# Mixture of bosons $b$ and fermions $f$

(e.g.  ${}^7\text{Li}+{}^6\text{Li}$ ,  ${}^{23}\text{Na}+{}^6\text{Li}$ ,  ${}^{87}\text{Rb}+{}^{40}\text{K}$ )

Tune to the vicinity of a Feshbach resonance  
associated with a molecular state  $\psi$

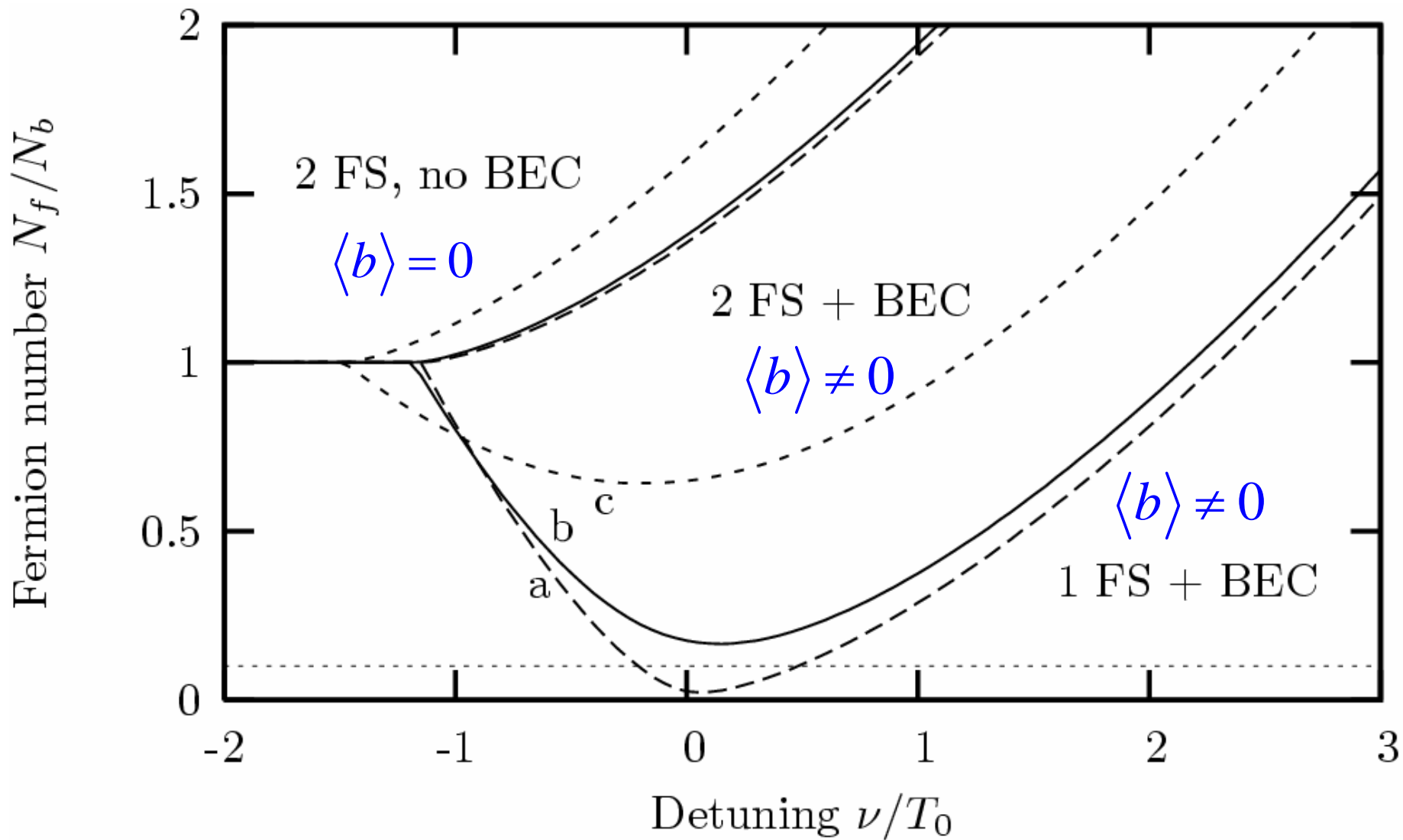
Conservation laws:

$$b^\dagger b + \psi^\dagger \psi = N_b$$

$$f^\dagger f + \psi^\dagger \psi = N_f$$

$$f^\dagger f - b^\dagger b = N_f - N_b$$

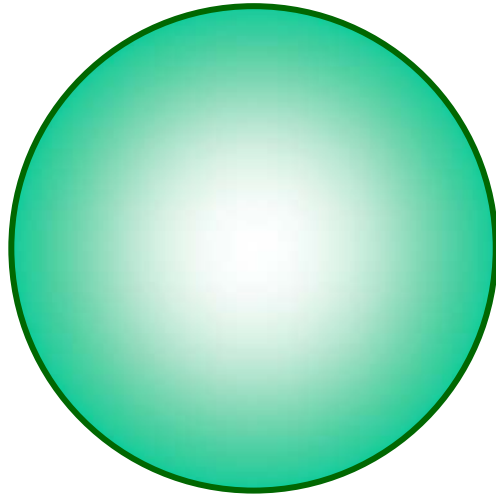
# Phase diagram



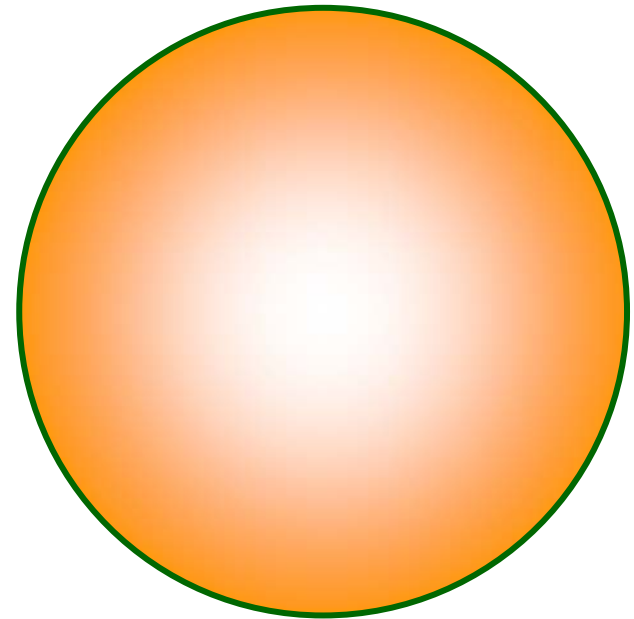
## 2 FS, no BEC phase

“atomic” Fermi surface

“molecular” Fermi surface



$$\langle b \rangle = 0$$

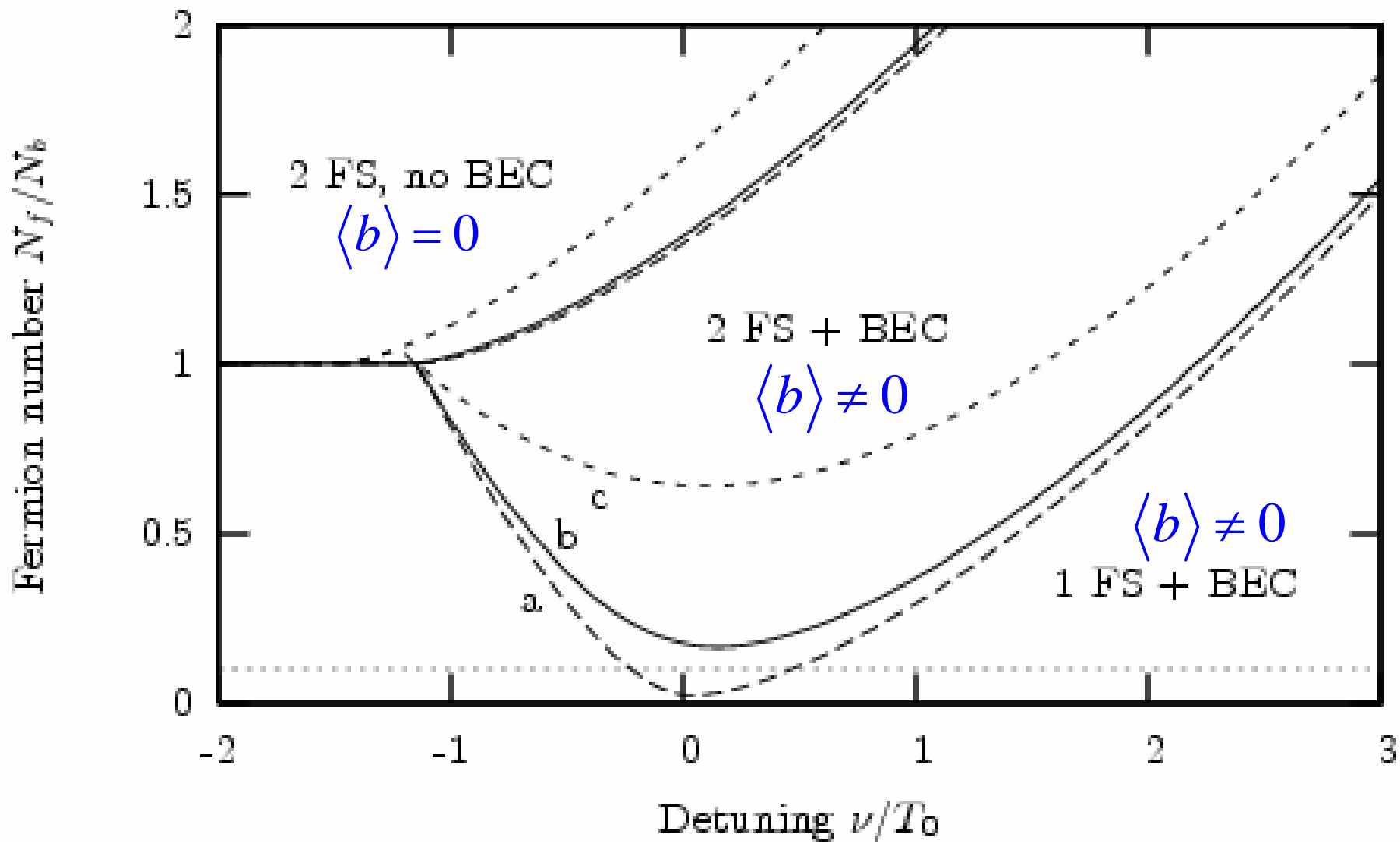


$$\text{Volume} = N_b$$

$$\text{Volume} = N_f - N_b$$

2 Luttinger theorems; volume within both  
Fermi surfaces is conserved

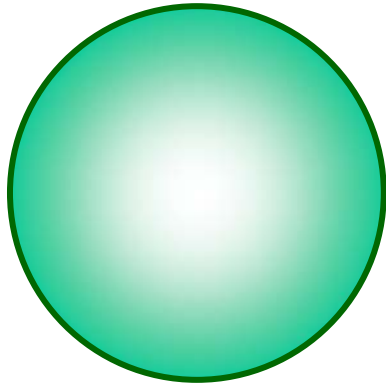
# Phase diagram



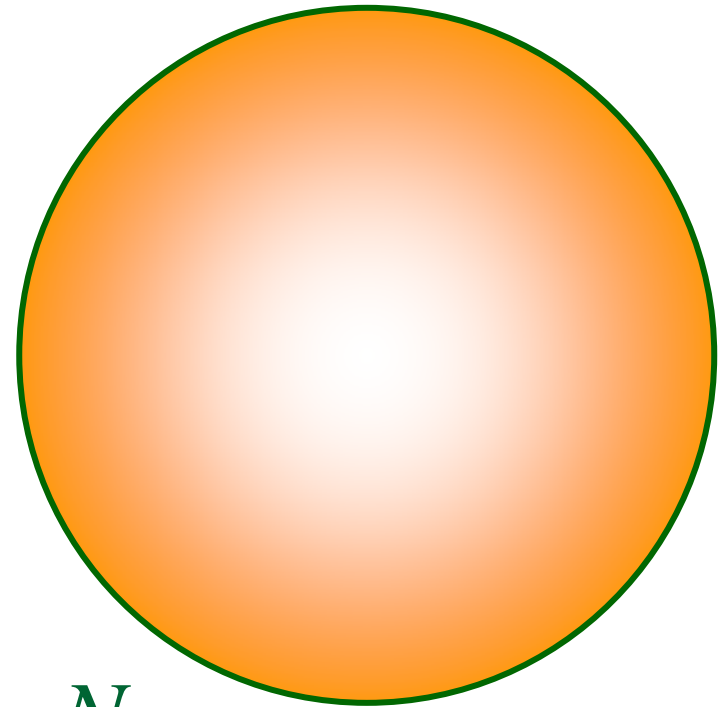
## 2 FS + BEC phase

“atomic” Fermi surface

“molecular” Fermi surface



$$\langle b \rangle \neq 0$$

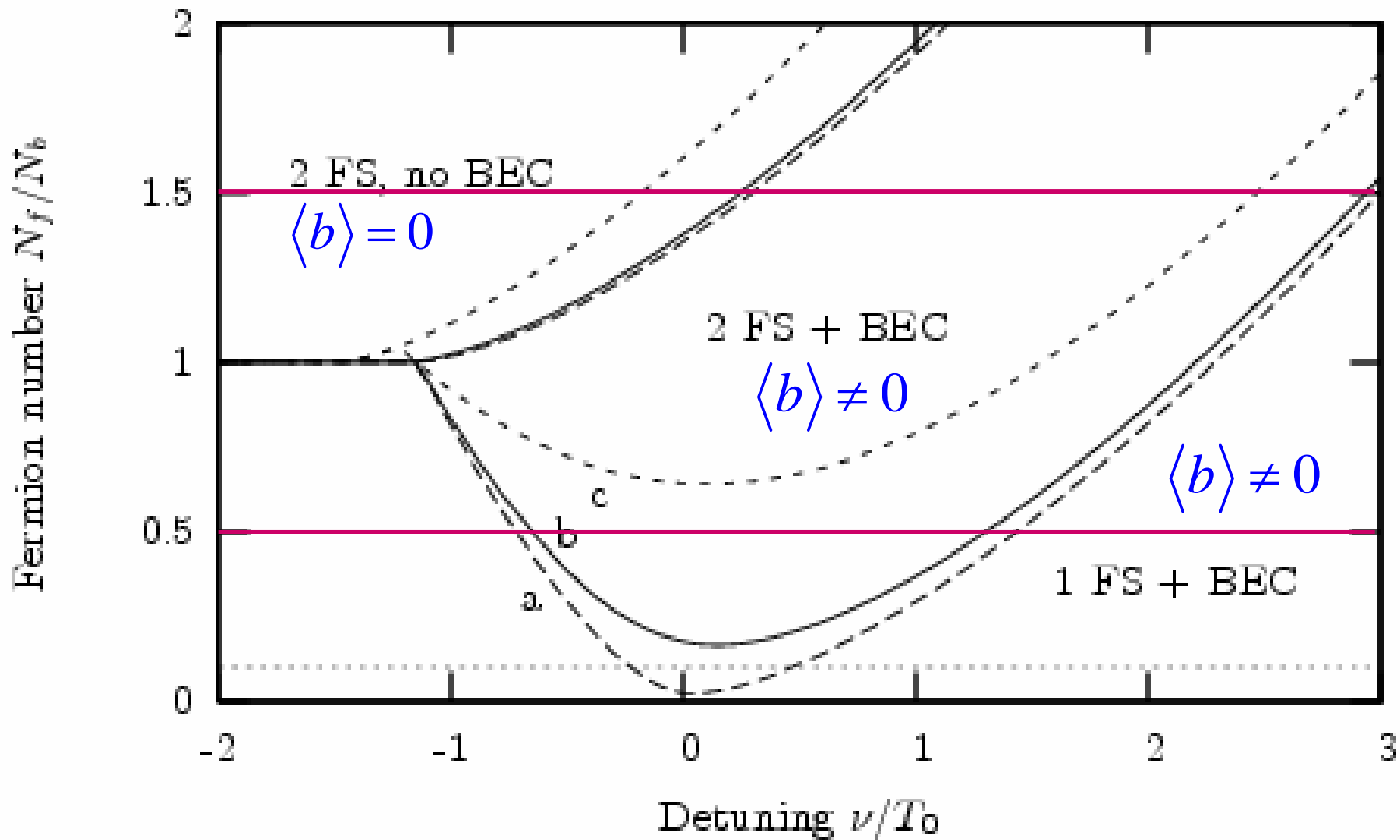


$$\text{Total volume} = N_f$$

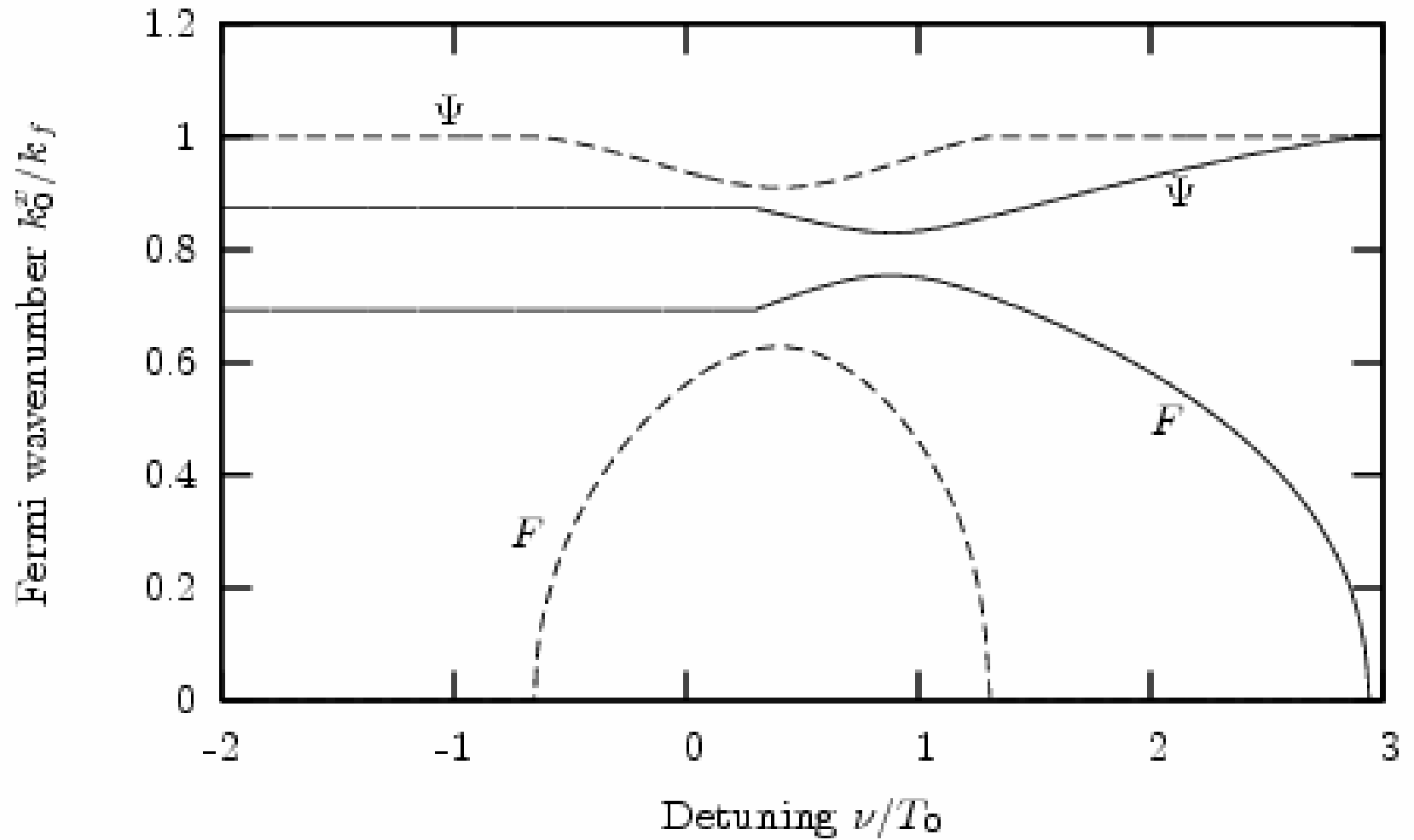
1 Luttinger theorem; only total volume within Fermi surfaces is conserved



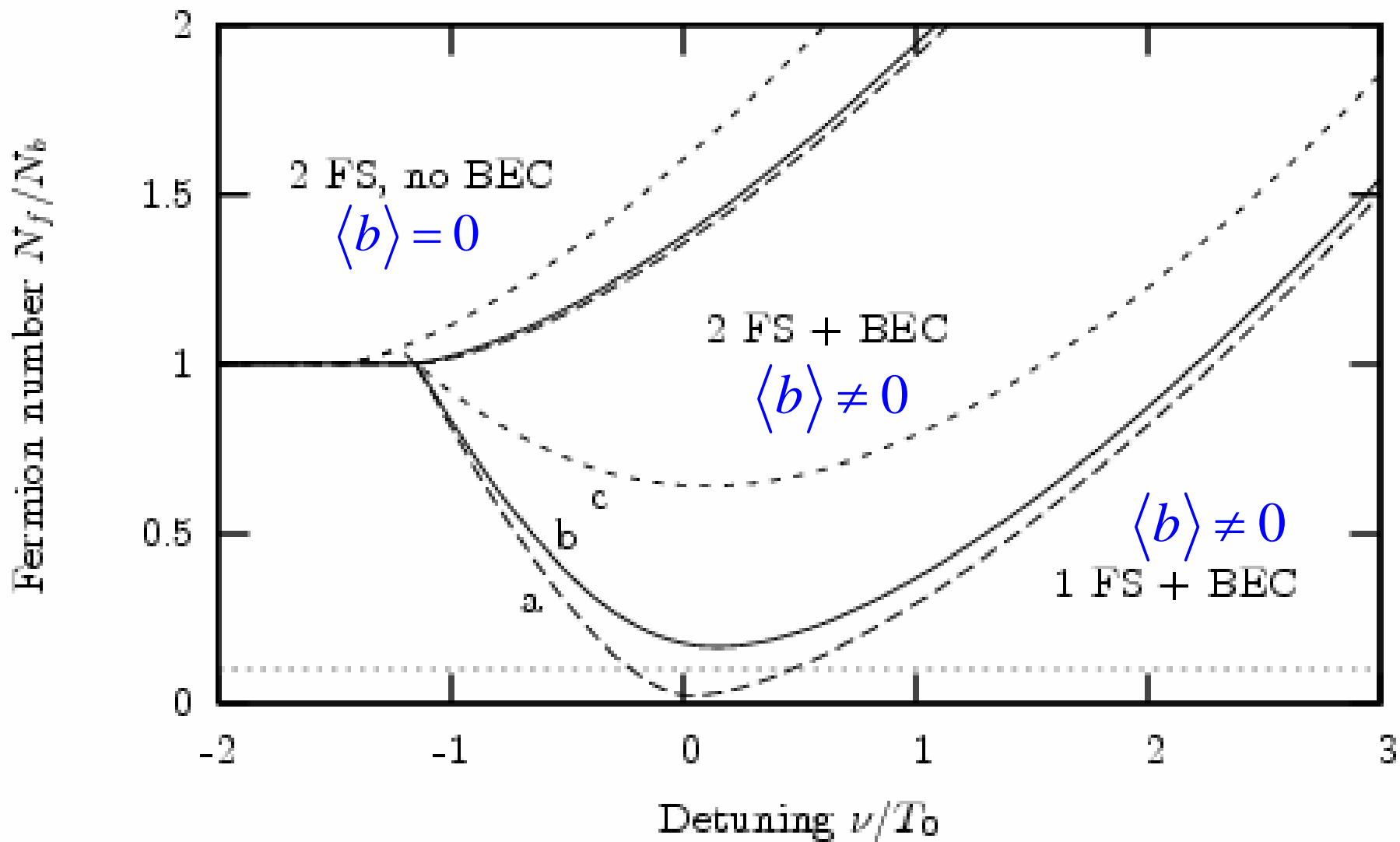
# Phase diagram



# Fermi wavevectors



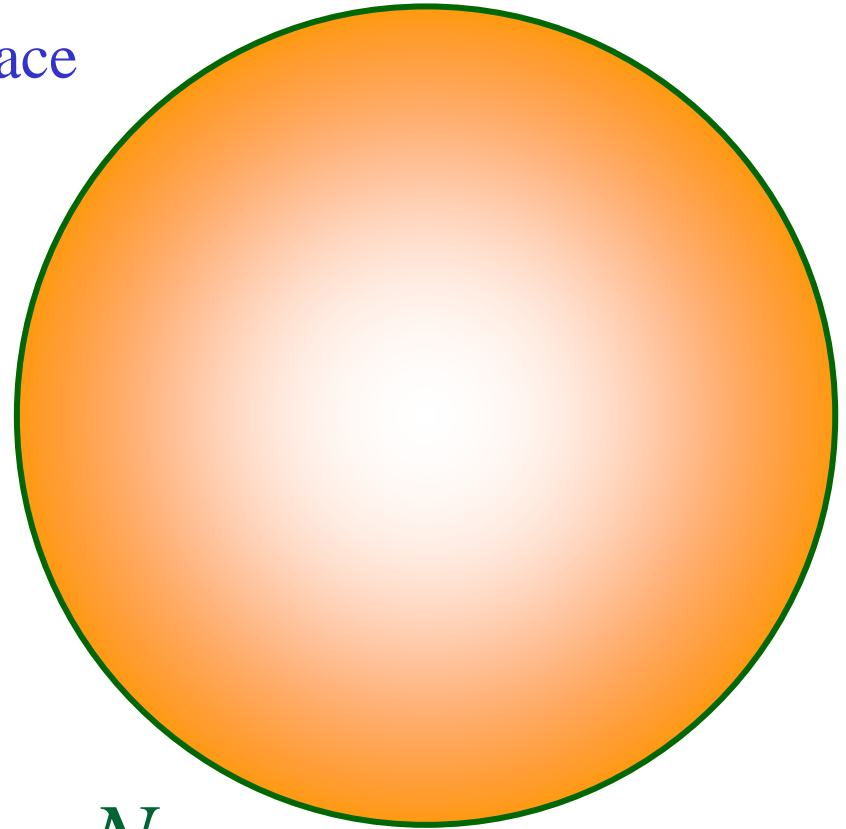
# Phase diagram



# 1 FS + BEC phase

“atomic” Fermi surface

$$\langle b \rangle \neq 0$$



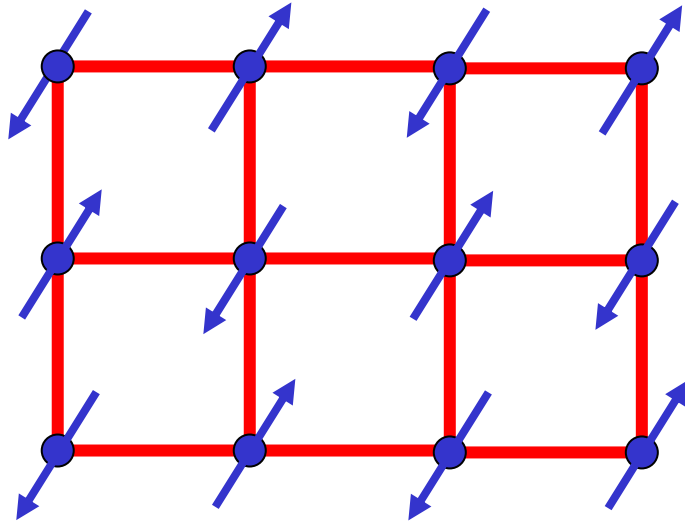
$$\text{Total volume} = N_f$$

1 Luttinger theorem; only total volume within Fermi surfaces is conserved

## B. The Kondo Lattice

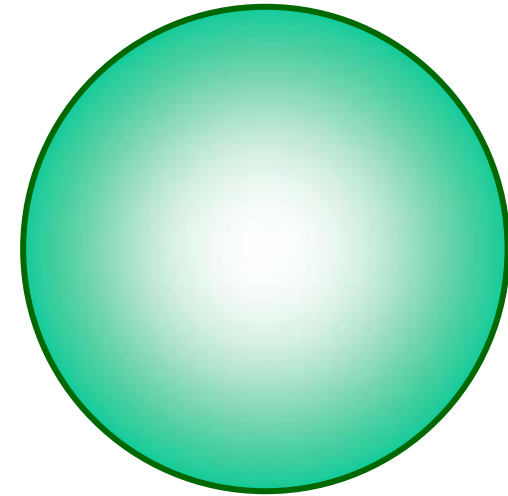
*The heavy Fermi liquid (FL) and the  
fractionalized Fermi liquid (FL\*)*

# The Kondo lattice



Local moments  $f_\sigma$

+



Conduction electrons  $c_\sigma$

$$H_K = \sum_{i < j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + J_K \sum_i c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of  $f$  electrons per unit cell =  $n_f = 1$

Number of  $c$  electrons per unit cell =  $n_c$

Define a bosonic field which measures the hybridization between the two bands:

$$b_i \sim \sum_{\sigma} c_{i\sigma}^{\dagger} f_{i\sigma}$$

Analogy with Bose-Fermi mixture problem:

$c_{i\sigma}$  is the analog of the "molecule"  $\psi$

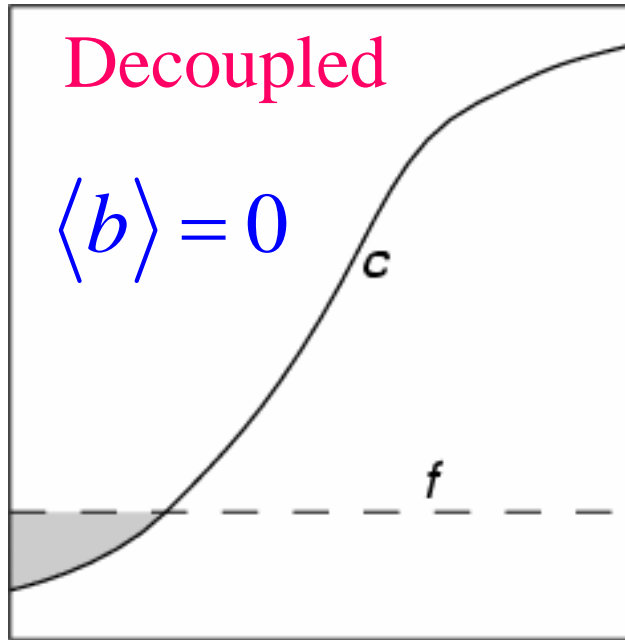
Conservation laws:

$$f_{\sigma}^{\dagger} f_{\sigma} + c_{\sigma}^{\dagger} c_{\sigma} = 1 + n_c \quad (\text{Global})$$

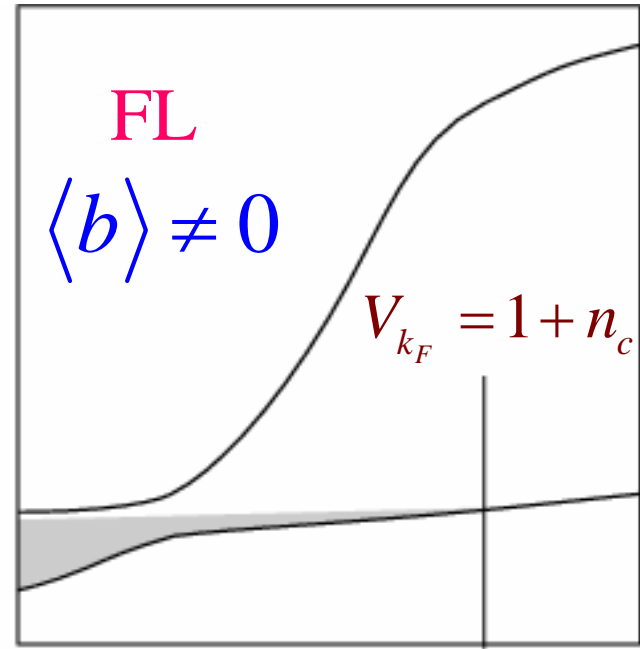
$$f_{\sigma}^{\dagger} f_{\sigma} + b^{\dagger} b = 1 \quad (\text{Local})$$

Main difference: second conservation law is *local* so there is a U(1) gauge field.

1 FS + BEC  $\Leftrightarrow$  Heavy Fermi liquid (FL)  $\Leftrightarrow$  Higgs phase



(a)  $k$

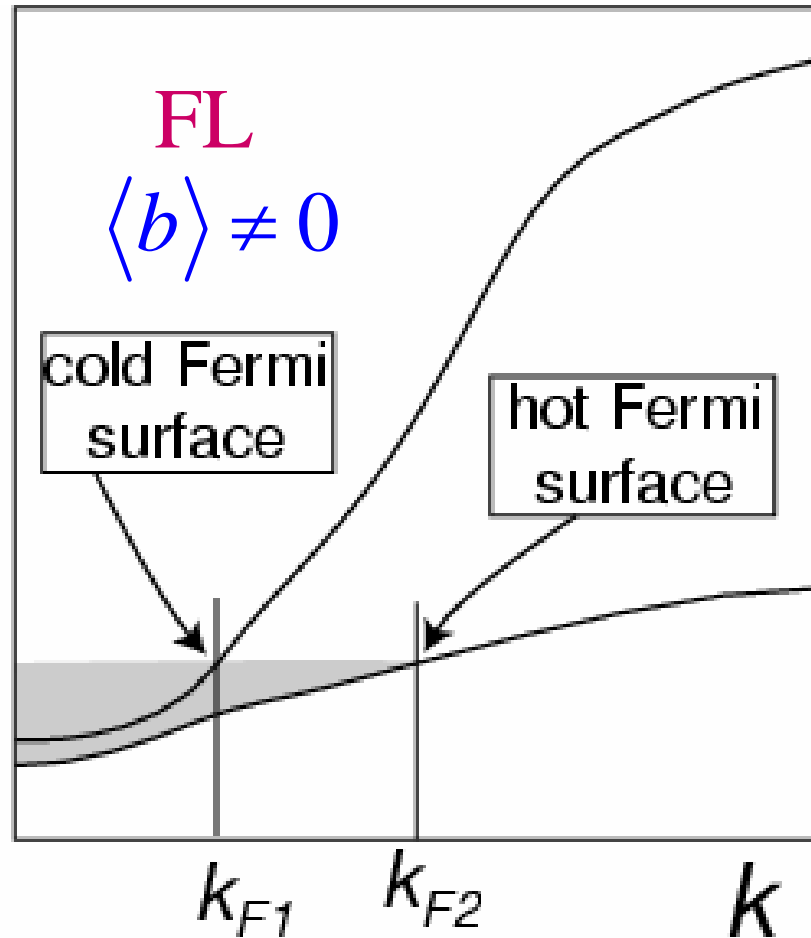


(b)  $k_F$   $k$

If the  $f$  band is dispersionless in the decoupled case, the ground state is always in the 1 FS FL phase.



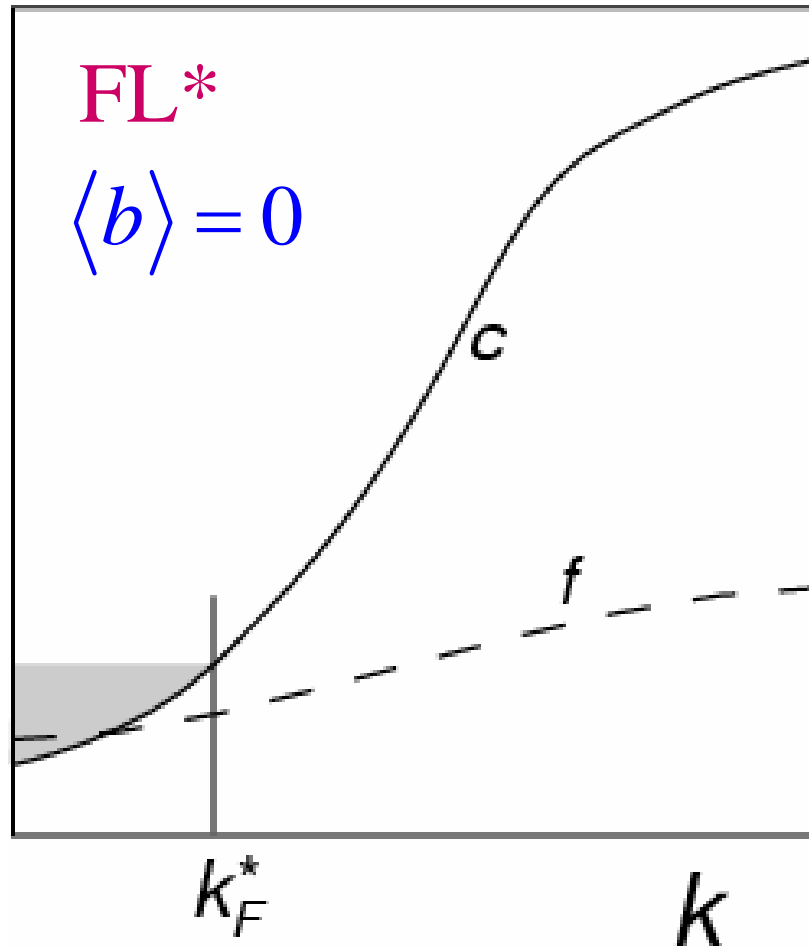
2 FS + BEC  $\Leftrightarrow$  Heavy Fermi liquid (FL)  $\Leftrightarrow$  Higgs phase



A bare  $f$  dispersion (from the RKKY couplings) allows a 2 FS FL phase.

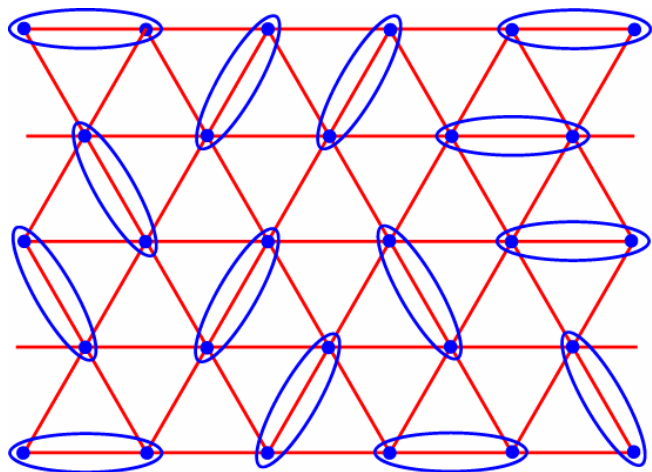
2 FS, no BEC  $\Leftrightarrow$  Fractionalized Fermi liquid (FL\*)

$\Leftrightarrow$  Deconfined phase

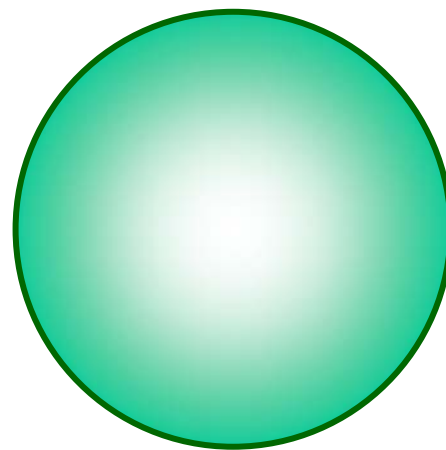


The  $f$  band “Fermi surface” realizes a spin liquid  
(because of the local constraint)

## Another perspective on the FL\* phase



+



Conduction electrons  $c_\sigma$

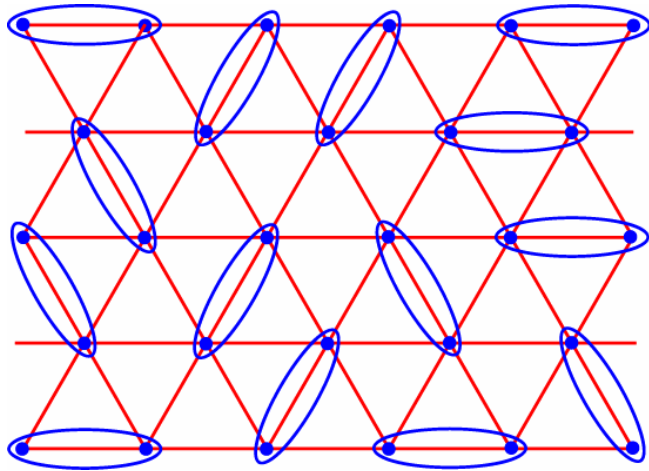
Local moments  $f_\sigma$

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma'} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by  $J_H$ , and then couple to conduction electrons by  $J_K$

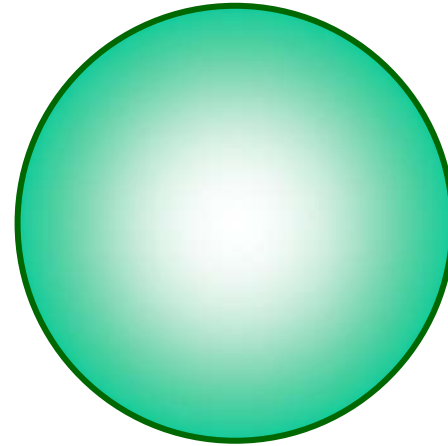
Choose  $J_H$  so that ground state of antiferromagnet is a  $Z_2$  or  $U(1)$  spin liquid

## Influence of conduction electrons



Local moments  $f_\sigma$

+



Conduction electrons  $c_\sigma$

At  $J_K=0$  the conduction electrons form a Fermi surface on their own with volume determined by  $n_c$ .

Perturbation theory in  $J_K$  is regular, and so this state will be stable for finite  $J_K$ .

So volume of Fermi surface is determined by  $(n_c+n_f-1)=n_c \pmod{2}$ , and does not equal the Luttinger value.

The (U(1) or  $Z_2$ ) FL\* state

## A new phase: FL\*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has “*topological order*” and associated neutral excitations. The topological order can be detected by the violation of Luttinger’s Fermi surface volume. It can only appear in dimensions  $d > 1$

$$2 \times \frac{V_0}{(2\pi)^d} (\text{Volume enclosed by Fermi surface}) \\ = n_c \pmod{2}$$

Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett.* **62**, 595 (1989).

Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica B* **182**, 201 (1992).

Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).

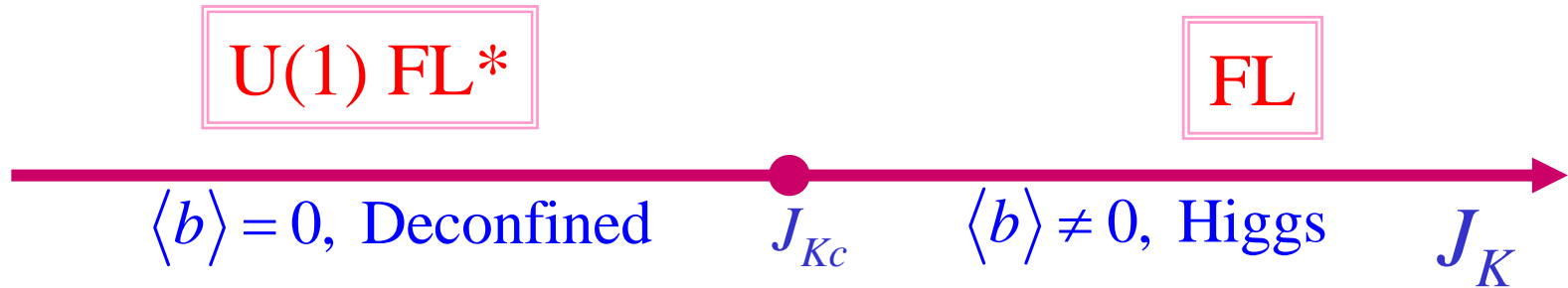
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev. B* **66**, 045111 (2002).

L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev. B* **60**, 1654, (1999);

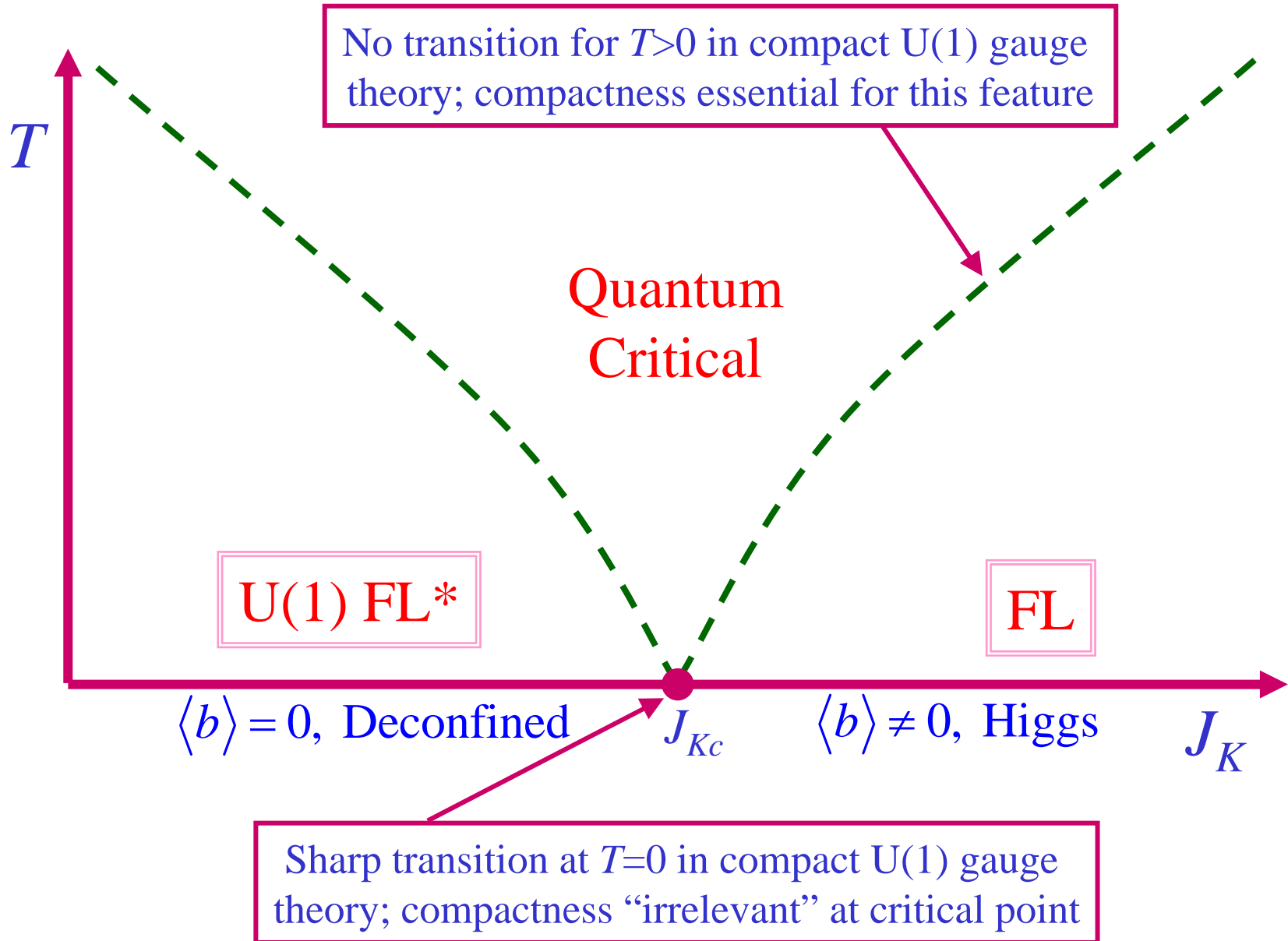
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **62**, 7850 (2000).

F. H. L. Essler and A. M. Tsvelik, *Phys. Rev. B* **65**, 115117 (2002).

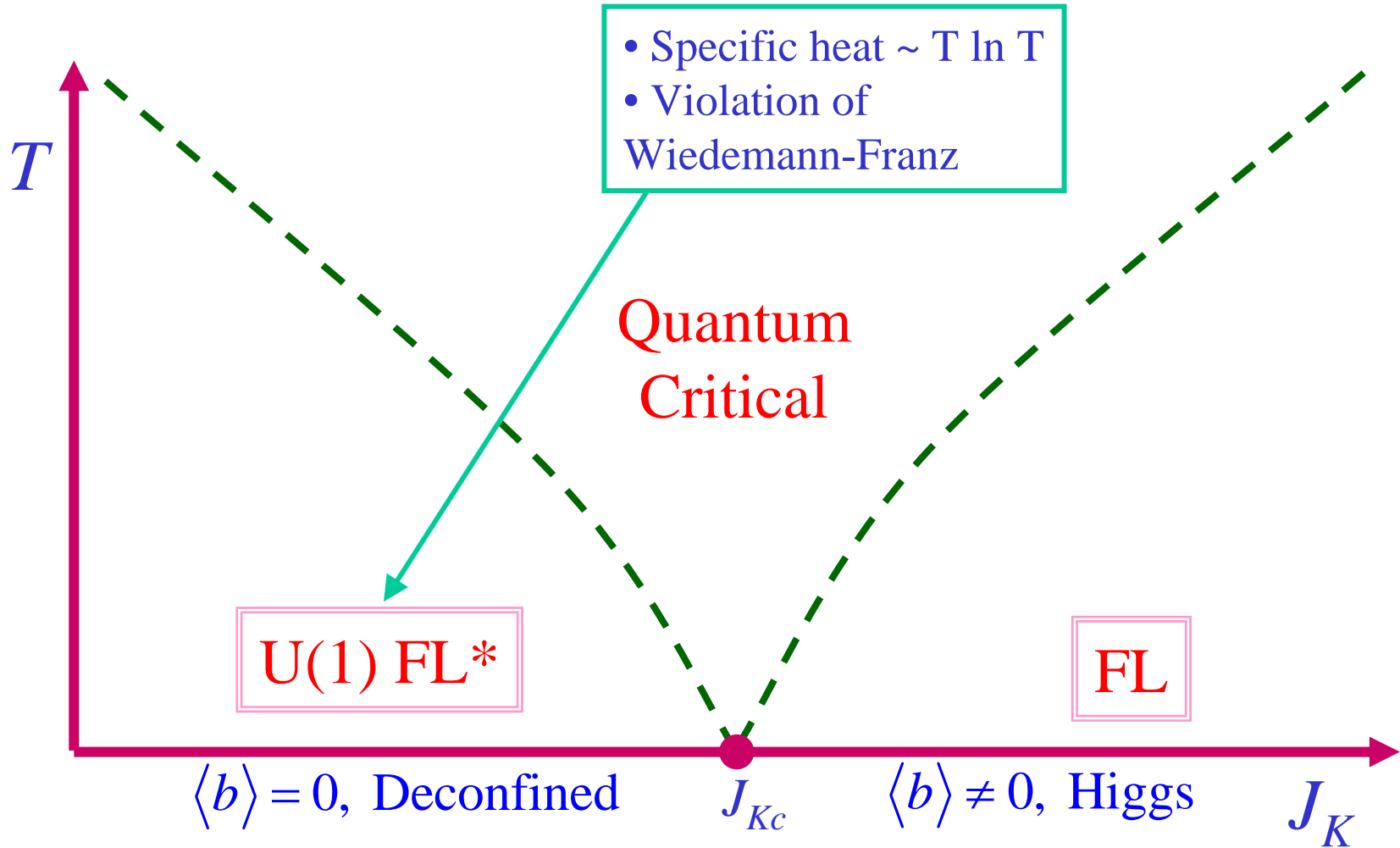
# Phase diagram



# Phase diagram

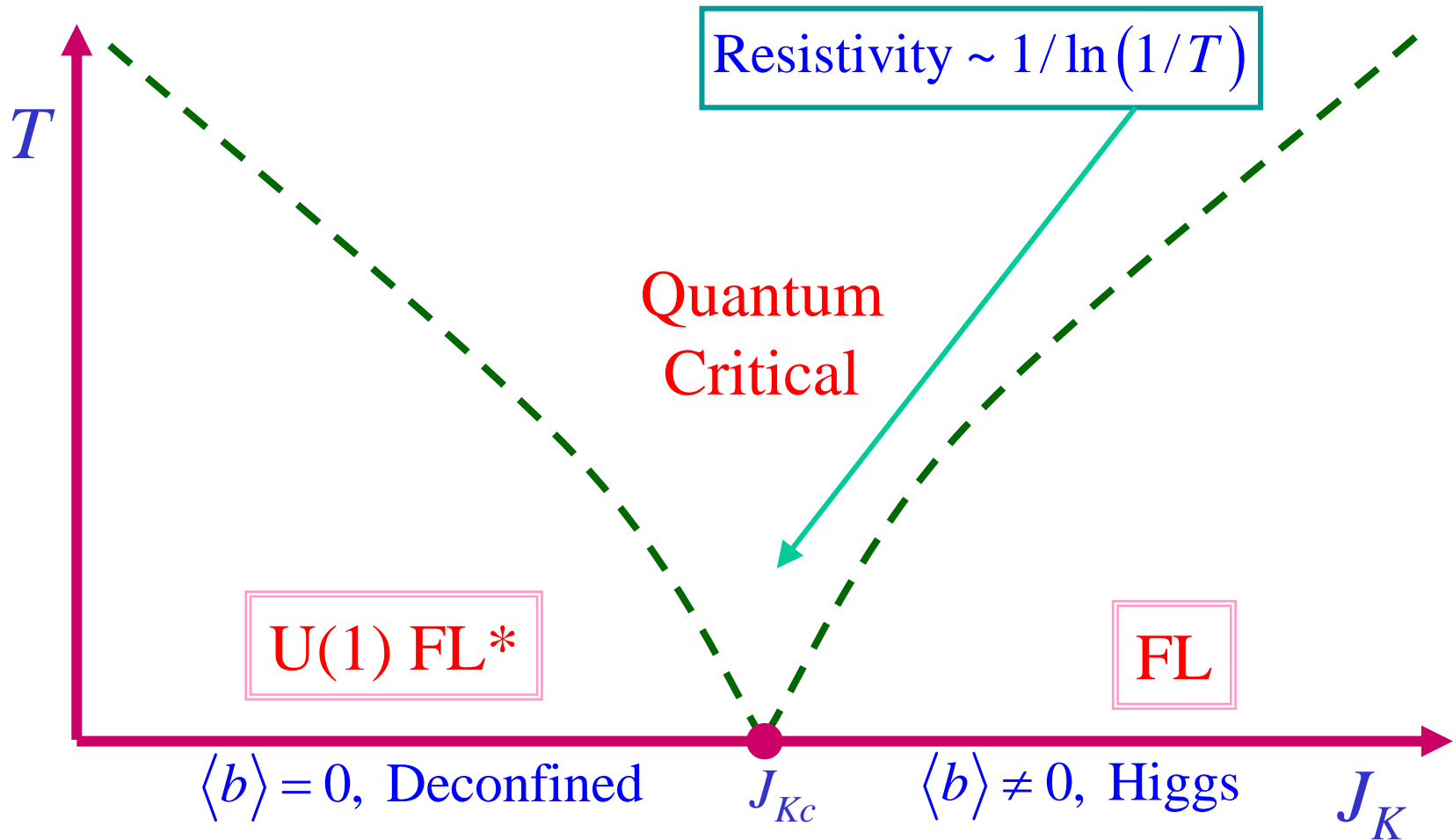


# Phase diagram





# Phase diagram

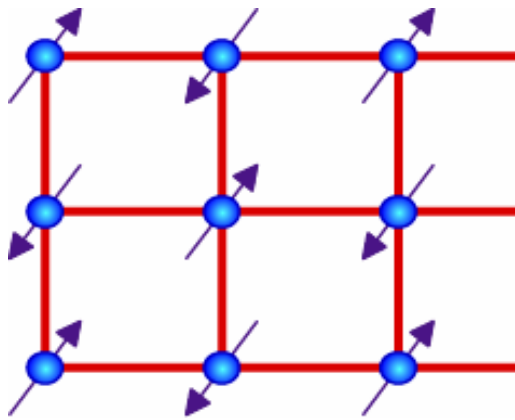


Is the  $U(1) FL^*$  phase unstable to the LMM metal at the lowest energy scales ?

*C. Detour:* Deconfined criticality in  
insulating antiferromagnets

*Landau forbidden quantum transitions*

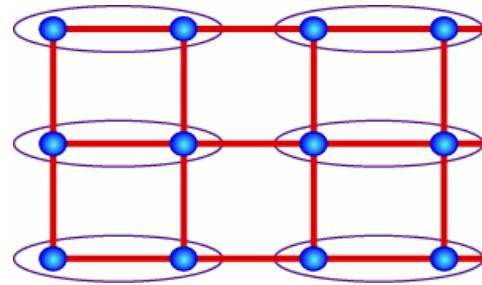
# Phase diagram of S=1/2 square lattice antiferromagnet



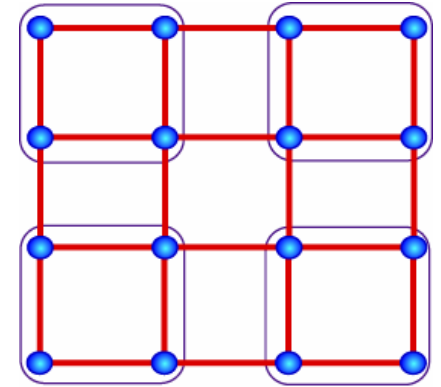
Neel order

$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \neq 0$$

(Higgs)



or



VBS order  $\Psi_{\text{VBS}} \neq 0$ ,

$S = 1/2$  spinons  $z_\alpha$  confined,

$S = 1$  triplon excitations



Deconfined critical point described by a theory of spinons

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

Landau-forbidden transition between phases which break  
“unrelated” symmetries

## Attempted theory for the destruction of Néel order

Express Néel order  $\vec{\varphi}$  in terms of  $S = 1/2$  bosonic spinons  $z_\alpha$  by

$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta.$$

This introduces a U(1) gauge invariance under  $z_\alpha \rightarrow z_\alpha e^{i\phi(x,\tau)}$ .

Field theory for the  $z_\alpha$  spinons:

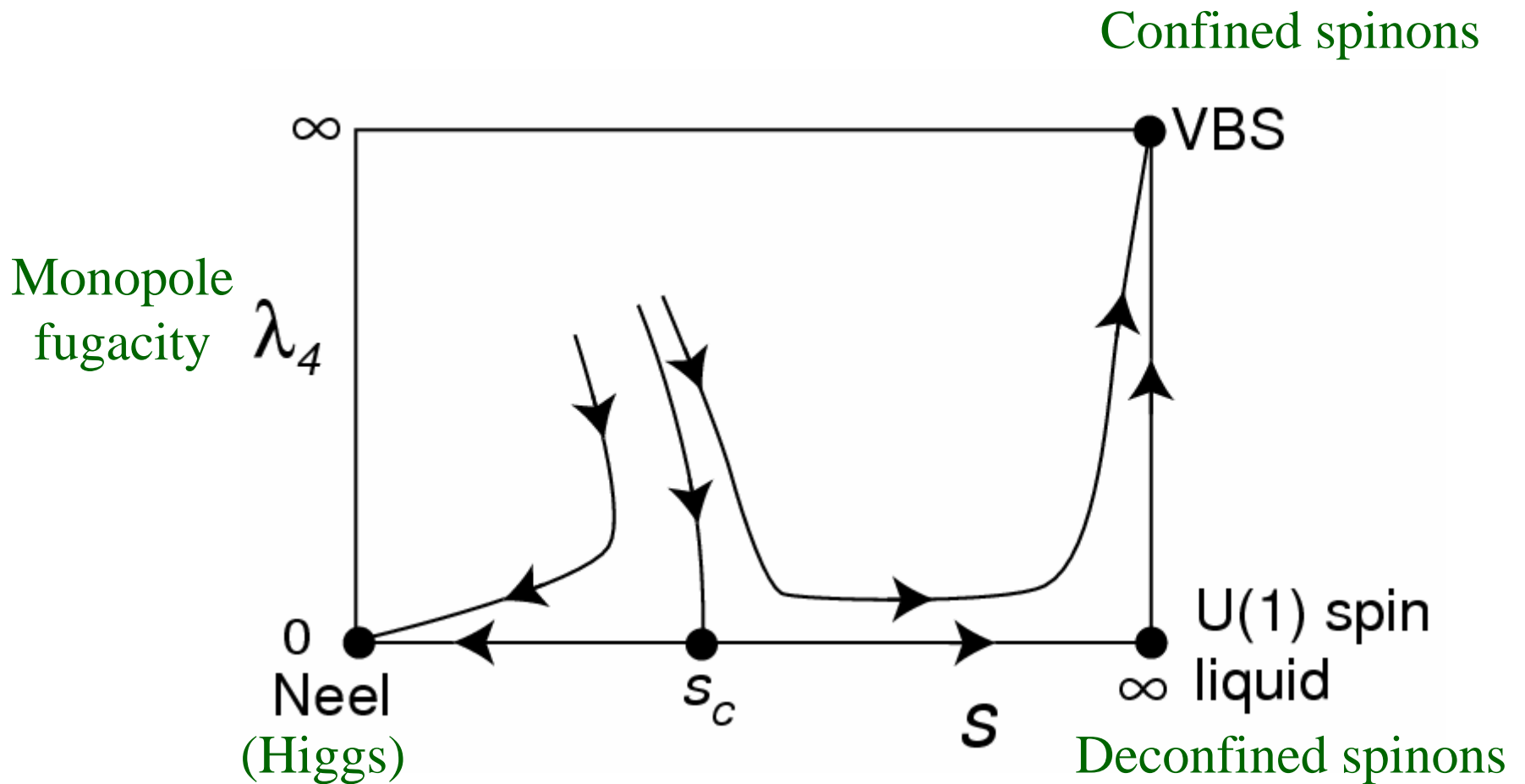
$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

where  $A_\mu$  is a U(1) gauge field.

### Phases of theory

$s < s_c \Rightarrow$  Néel (Higgs) phase with  $\langle z_\alpha \rangle \neq 0$

$s > s_c \Rightarrow$  Deconfined U(1) spin liquid with  $\langle z_\alpha \rangle = 0$



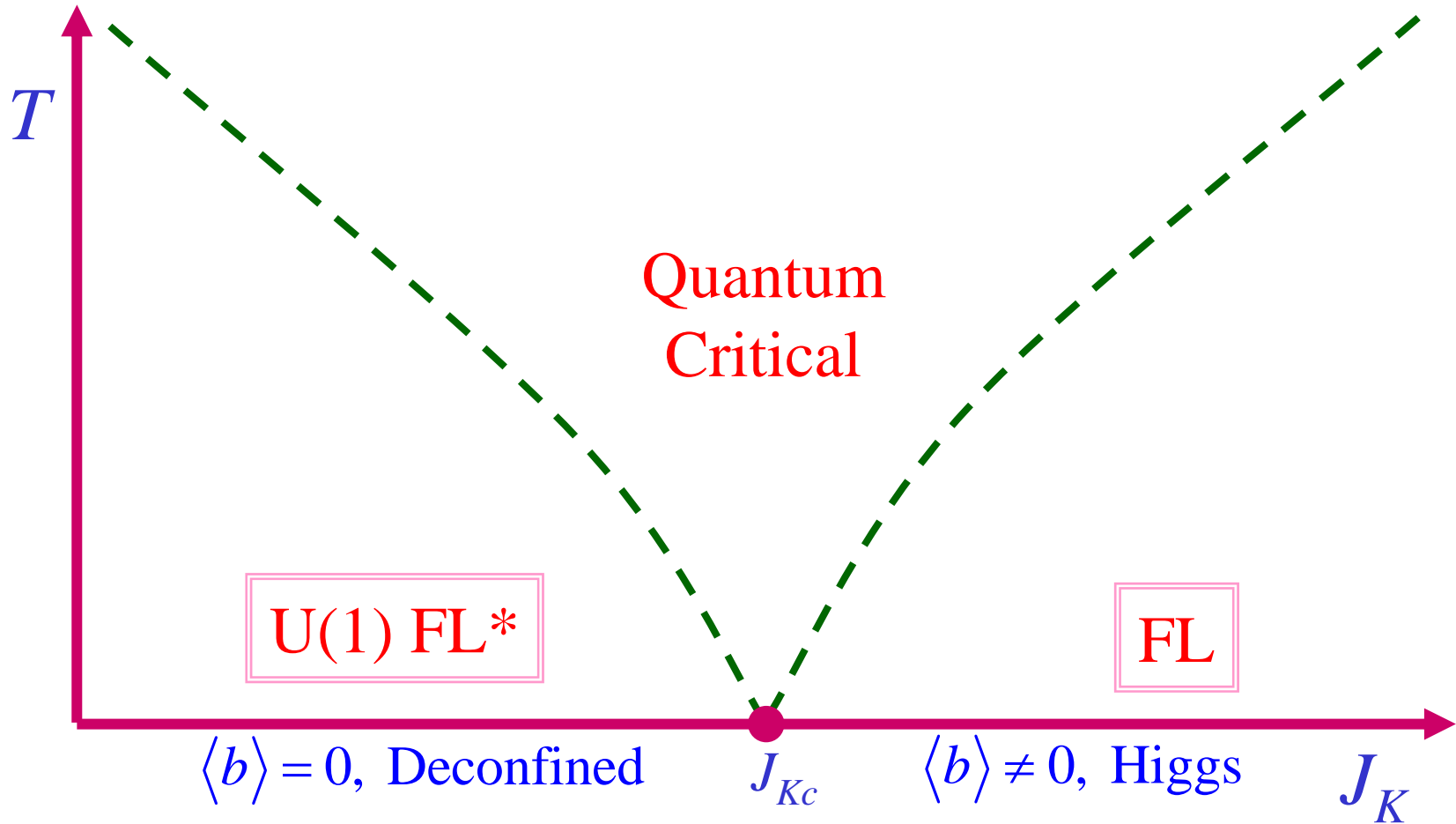
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

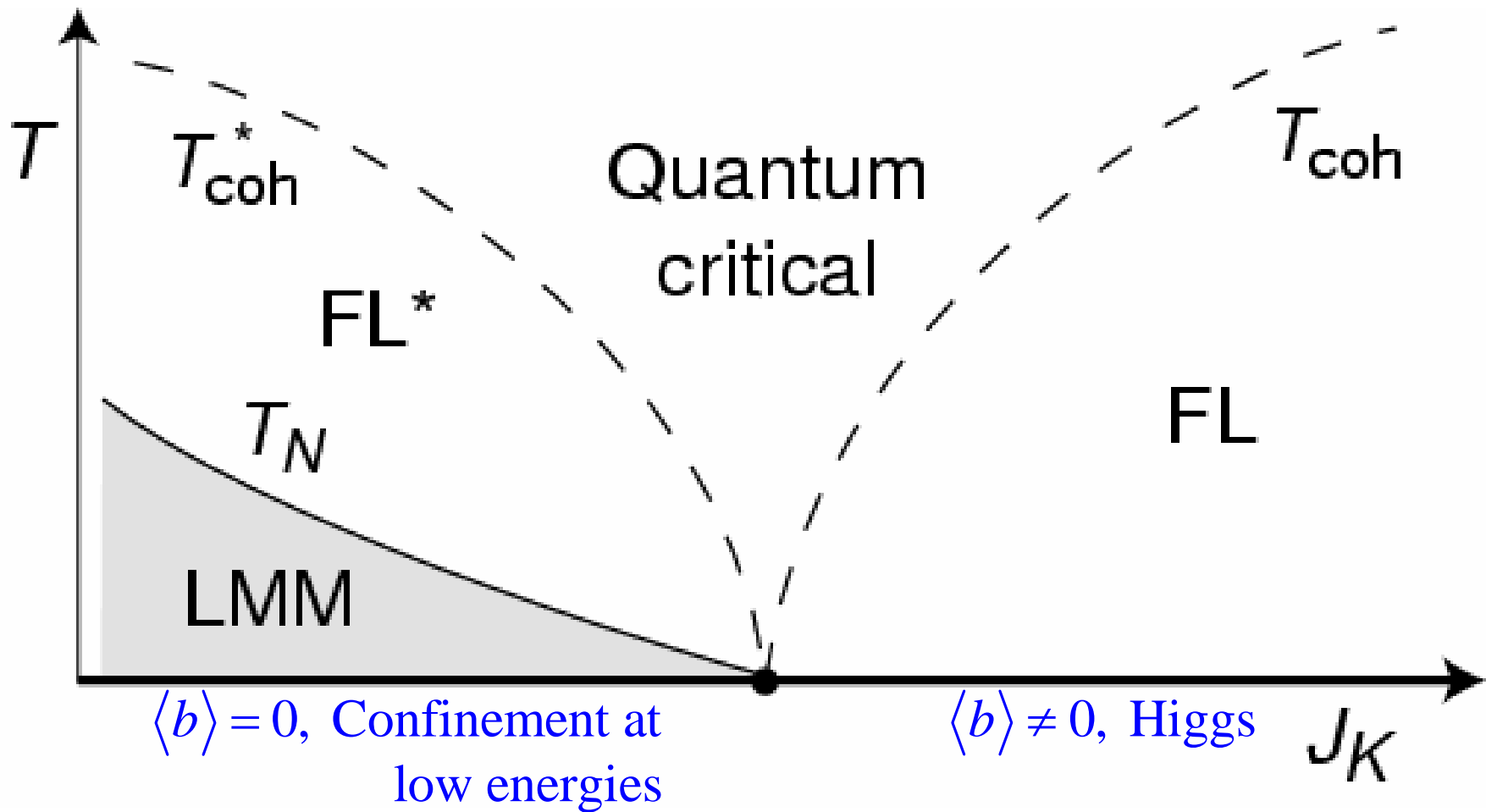
F. Deconfined criticality in the Kondo lattice ?

# Phase diagram



Is the U(1) FL\* phase unstable to the LMM metal at the lowest energy scales ?

## Phase diagram ?



U(1) FL\* phase generates magnetism at energies much lower than the critical energy of the FL to FL\* transition