Universal theory of complex SYK models and extremal charged black holes

Subir Sachdev

Chaos and Order: from Strongly Correlated Systems to Black Holes, Kavli Institute for Theoretical Physics, University of California, Santa Barbara, December 7, 2018
1. Quantum matter without quasiparticles: the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian

4. Supersymmetric models
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The complex SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

\[
H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i
\]

\[
c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j c_i^\dagger = \delta_{ij}
\]

\[
Q = \frac{1}{N} \sum_i c_i^\dagger c_i
\]

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|U_{ij;kl}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The complex SYK model

Feynman graph expansion in $U_{ijkl}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

\[
G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)}, \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)
\]

$G(\tau = 0^-) = Q$.

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$G(\tau = 0^-) = Q$.

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The complex SYK model

Solution of these equations and corresponding evaluation of the free energy yields the following universal results (i.e. all results are quantitatively unchanged by adding additional higher $q$ fermion terms):

• At long times, and at $T = 0$,
  \[ G(\tau) \sim |\tau|^2 \]
  with $\tau = 1/q$ (indicating there are no quasiparticles)

• At general charge $Q$, there is a spectral symmetry determined by a parameter $E$:
  \[ G(\tau) \sim \begin{cases} 
  \tau^2 & \tau > 0 \\
  e^{2\pi E}(\tau) & \tau < 0
  \end{cases} \]

• There is a universal 'Luttinger relation' between $1 < E < 1$ and the total charge $0 < Q < 1$
  \[ e^{2\pi E} = \sin(\pi + \pi) \sin(\pi) \]
  \[ \sin(2\pi) \sin(2\pi) \]
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\end{cases}, \quad T = 0$$

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\end{cases}, \quad T = 0
\]

- There is a universal ‘Luttinger relation’ between \(-\infty < \mathcal{E} < \infty\) and the total charge \(0 < Q < 1\)

\[
e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}
\]

\[
Q = \frac{1}{2} - \frac{\theta}{\pi} + \left(\Delta - \frac{1}{2}\right) \frac{\sin(2\theta)}{\sin(2\pi\Delta)}
\]

A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
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Solution of these equations and corresponding evaluation of the free energy yields the following universal results (i.e. all results are quantitatively unchanged by adding additional higher $q$ fermion terms):

- There is a non-vanishing entropy in the zero temperature limit

\[ S(T \rightarrow 0) = N s_0 + \ldots \]

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- There is a non-vanishing entropy in the zero temperature limit
  \[
  S(T \to 0) = N s_0 + \ldots
  \]

- The saddle point equations imply the relation
  \[
  \frac{d s_0}{d Q} = 2\pi \mathcal{E}
  \]

Integrating this relation from \( s_0 = 0, Q = 0 \), allows us to compute \( s_0 \) as a function of \( Q \).

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There are \(2^N\) many body levels with energy \(E\). Shown are all values of \(E\) for a single cluster of size \(N = 12\). The \(T \rightarrow 0\) state has an entropy \(S_{GPS} = Ns_0\), where \(s_0 < \ln 2\) is determined by integrating

\[
\frac{ds_0}{dQ} = 2\pi \mathcal{E}.
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At \(Q = 1/2\),

\[
s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots
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where \(G\) is Catalan’s constant.

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Solution of these equations and corresponding evaluation of the free energy yields the following universal results (i.e. all results are quantitatively unchanged by adding additional higher $q$ fermion terms):

- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi E T \tau}}{\sqrt{1 + e^{-4\pi E}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2}, \quad 0 < \tau < 1/T,$$

where the ‘particle-hole asymmetry’ is determined by $E$

A. Georges and O. Parcollet PRB 59, 5341 (1999)
S. Sachdev, PRX 5, 041025 (2015)
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Charged black holes

\[ S_{EM} = \frac{1}{2\kappa^2} \int d^{d+2}x \sqrt{-g} \left( R + \frac{6}{L^2} - \frac{L^2}{g_F^2} F^2 \right) \]

Black hole horizon of radius \( r_0 \)

Solutions of \( S_{EM} \) have metric and gauge field \((F = dA)\)

\[ ds^2 = -V(r)dt^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad A = \mu \left( 1 - \frac{r_0^{d-1}}{r^{d-1}} \right) dt \]

\[ V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}. \]

where \( d\Omega_d^2 \) is the metric of the \( d \)-sphere. All parameters of the solution are determined in terms of the chemical potential \( \mu \), and the Hawking temperature of horizon, \( T \).

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD 60, 064018 (1999)
Charged black holes

In the $T \to 0$ limit, at fixed $\mu$, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of $R_h$.

- The total charge in the black hole is

$$Q = \frac{R_h^{d-1} \sqrt{2d [(d+1)R_h^2 + (d-1)L^2]}}{\kappa^2 g_F}$$
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$$Q = \frac{R_h^{d-1} \sqrt{2d [(d+1)R_h^2 + (d-1)L^2]}}{\kappa^2 g_F}$$

- The Bekenstein-Hawking entropy remains finite as $T \to 0$ ($s_d$ is the area of the $d$-dimensional surface of a unit sphere)

$$S(T \to 0) = s_0 + \ldots , \quad s_0 = \frac{2\pi s_d}{\kappa^2} R_h^d$$

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- In the near-horizon region, we change co-ordinates from $r$ to $\zeta$ so that

$$r - R_h = \frac{R_2^2}{\zeta}, \quad R_2 = \frac{LR_h}{\sqrt{d(d + 1)R_h^2 + (d - 1)^2L^2}}.$$

Then the near-horizon metric becomes $\text{AdS}_2 \times \text{S}_d$, with

$$ds^2 = R_2^2 \left[ -dt^2 + \frac{d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_d^2, \quad A = \frac{\mathcal{E}}{\zeta} dt.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d \left[ (d + 1)R_h^2 + (d - 1)L^2 \right]}}{2 \left[ d(d + 1)R_h^2 + (d - 1)^2L^2 \right]}.$$
Charged black holes

Black hole horizon of radius $R_h$ and entropy $s_0$

Black hole horizon of radius $R_h$ and entropy $s_0$

\[ ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2 \]

Gauge field: \[ A = (\mathcal{E}/\zeta)dt \]

- The entropy $s_0$, the charge $Q$, and the dimensionless electric field $\mathcal{E}$ obey

\[ \frac{ds_0}{dQ} = 2\pi \mathcal{E} \]

A. Sen, JHEP 0509, 038 (2005)
Charged black holes

In the $T \to 0$ limit, at fixed $\mu$, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of $R_h$.

- A probe fermion has a near-horizon Green’s function with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2}, \quad 0 < \tau < 1/T,$$

where the ‘particle-hole asymmetry’ is determined by $\mathcal{E}$. This is identical to the complex SYK model.

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The fluctuations can be expressed as a path integral over a Green’s function $G(\tau_1, \tau_2)$ which is bilocal in time. At frequencies $\ll U$, the action is invariant under reparameterizations $f(\tau)$ and gauge transformations $\phi(\tau)$

$$\tau \rightarrow f(\sigma)$$

$$G(\tau_1, \tau_2) \rightarrow [f'(\sigma_1)f'(\sigma_2)]^{-\Delta} e^{-i(\phi(\sigma_1) - \phi(\sigma_2))} G(\sigma_1, \sigma_2)$$
Fluctuations

• The saddle-point

\[ G(\tau_1 - \tau_2) = -A \frac{e^{-2\pi \mathcal{E} T (\tau_1 - \tau_2)}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left( \frac{T}{\sin(\pi T (\tau_1 - \tau_2))} \right)^{2\Delta} \]

is invariant only under PSL(2, R) transformations which map the thermal circle onto itself, and an associated gauge transformation

\[ \tan(\pi T f(\tau)) = \frac{a \tan(\pi T \tau)}{\pi T} + b \quad \frac{\pi T}{c \tan(\pi T \tau)} + d \], \quad ad - bc = 1,

\[ -i \phi(\tau) = -i \phi_0 + 2\pi \mathcal{E} T (\tau - f(\tau)) \]

A. Kitaev, 2015

Fluctuations

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f, \phi] = \frac{NK}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau f)^2 - \frac{N\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings $K, \gamma, \text{and } \mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left( \frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}} \left[ f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.
Fluctuations

An *exact* path integral over the effective action leads to the following physical consequences

- The ground state energy with fermion number $NQ + p$ ($p$ integer) varies as

  $$E_p = E_0 + \frac{p^2}{2NK}$$

  This identifies $K$ with the compressibility $K = dQ/d\mu$ at $T = 0$. 
**Fluctuations**

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- The low temperature corrections to the entropy at fixed $Q$ are

$$S(T \rightarrow 0, Q) = N \left[ s_0 + \gamma T + \ldots \right] + 2 \ln(U/T) \ldots$$

This defines $\gamma$ as the co-efficient of the linear-in-$T$ specific heat (at fixed $Q$)
Fluctuations

An exact path integral over the effective action leads to the following physical consequences

• The many-body density of states, $D(E)$, is related to the grand potential, $\Omega(T)$ by

$$Z = e^{-\Omega(T)/T} = \int_{-\infty}^{\infty} dED(E)e^{-E/T}$$

We obtain

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi pE} d(E - E_p)$$

where $NQ + p$ is the integer fermion number,

$$d(E) \sim \exp(Ns_0) \sinh\left(\sqrt{2N\gamma E}\right) \quad , \quad E > 0 \quad , \quad e^{-cN} \ll \gamma E \ll N$$

There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in $N$. 

The complex SYK model

There are $2^N$ many body levels with energy $E$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = Ns_0$, where $s_0 < \ln 2$ is determined by integrating

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At $Q = 1/2$,

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where $G$ is Catalan’s constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
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  D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p E} d(E - E_p)
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  where $NQ + p$ is the integer fermion number,
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  d(E) \sim \exp(Ns_0) \sinh\left(\sqrt{2N\gamma E}\right) , \quad E > 0 , \quad e^{-cN} \ll \gamma E \ll N
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  There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in $N$. 

Fluctuations

Fluctuations

An exact path integral over the effective action leads to the following physical consequences

- At charge $NQ + p$, the prefactor of the $\sinh(\sqrt{2N\gamma(E - E_p)})$ term is

  $$\exp[Ns_0(Q) + 2\pi p\mathcal{E}] \approx \exp[Ns_0(Q + p/N)]$$

using

$$\frac{ds_0}{dQ} = 2\pi\mathcal{E}$$
The Schwarzian theory and black holes

- Reparameterization invariance is a defining property of Einstein’s theory of gravity

- In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

\[ ds^2 = \frac{(d\tau^2 + d\zeta^2)}{\zeta^2} \] is invariant under

\[ \tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \] with \( ad - bc = 1 \).
Reparameterization invariance is a defining property of Einstein’s theory of gravity.

In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R).

Semiclassical fluctuations about the saddle-point of Einstein-Maxwell theory of a charged black holes in $d \geq 2$ spatial dimensions lead to the same Schwarzian+phase theory of fluctuations.

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
The Schwarzian theory and black holes

- The Einstein-Maxwell theory leads to the following parameters for the Schwarzian+phase theory

\[ K = \left. \frac{dQ}{d\mu} \right|_{T=0} = \frac{2(d - 1)L^2 s_d R_h^{d-3} \left[ d(d + 1)R_h^2 + (d - 1)^2 L^2 \right]}{(d + 1)g_F^2 \kappa^2} \]

\[ S(T \to 0, Q) = s_0 + \gamma T + \ldots \]

\[ \gamma = \frac{4\pi^2 d s_d L^2 R_h^{d+1}}{\kappa^2 (d(d + 1)R_h^2 + (d - 1)^2 L^2)} . \]
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SYK with $\mathcal{N} = 2$ SUSY

$$Q = i \sum_{1 \leq i < j < k \leq N} C_{ijk} c_{i} c_{j} c_{k}$$

$$H = QQ^\dagger + Q^\dagger Q = \sum_{i, j, k, \ell} U_{ij; k\ell} c_{i}^\dagger c_{j}^\dagger c_{k} c_{\ell}$$

$C_{ijk}$ are independent random variables with $\overline{C_{ijk}} = 0$ and $|C_{ijk}|^2 = 2U/N^2$.
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$C_{ijk}$ are independent random variables with $C_{ijk} = 0$ and $|C_{ijk}|^2 = 2U/N^2$.

There is a $U(1)_R$ charge

$$Q_R = \frac{1}{3} \sum_i \left( c_i^\dagger c_i - \frac{1}{2} \right)$$

The smallest absolute values of are $Q_R = 0$ (for $N$ even) and $Q_R = \pm 1/6$ (for $N$ odd) and we focus on these cases: $\mathcal{N} = 2$ SUSY is preserved by the ground states. Cases with extensive $Q_R$ charge, $\lim_{N \to \infty} Q_R/N \neq 0$, break SUSY and require a more complex analysis.

SYK with $\mathcal{N} = 2$ SUSY

- There is an exact, exponentially large degeneracy of zero energy states for any $C_{ijk}$. Consequently, $e^{Ns_0} \equiv g(N, Q_R)$ is an integer for all $N$ (there was no such requirement for the non-SUSY case). The exact non-zero values for small $Q_R$ are

\[
g(N, 0) = 2 \times 3^{N/2 - 1}, \quad g(N, \pm 1/3) = 3^{N/2 - 1}, \quad \text{for } N \text{ even}
g(N, \pm 1/6) = 3^{(N-1)/2}, \quad \text{for } N = 3 \mod 4\g(N, \pm 1/6) = 3^{N/2 - 1}, \quad g(N, \pm 1/2) = 1 \text{ or } 3, \quad \text{for } N = 1 \mod 4
\]

There are no zero energy states for other values of $Q_R$. So in the large $N$ limit, the entropy $s_0 = (1/2) \ln 3$ for $\lim_{N \to \infty} Q_R/N = 0$. 

SYK with $\mathcal{N} = 2$ SUSY

- Fluctuations are described by a super-Schwarzian theory for $\lim_{N \to \infty} Q_R/N = 0$. The density of states has delta function at zero energy, and a continuum contribution at non-zero energies, both of order $e^{s_0N}$. There is a gap between the ground states and the excited states $\sim 1/N$ for even $N$.

D. Stanford and E. Witten, arXiv:1703.04612

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- Similar results are expected to $\mathcal{N} = 2$ black holes with AdS$_2$ horizons obtained from string theory (Sen, Dabholkar, Gomes, Murthy ...).
SYK with $\mathcal{N} = 2$ SUSY

- Fluctuations are described by a super-Schwarzian theory for $\lim_{N \to \infty} \mathcal{Q}_R/N = 0$. The density of states has delta function at zero energy, and a continuum contribution at non-zero energies, both of order $e^{s_0 N}$. There is a gap between the ground states and the excited states $\sim 1/N$ for even $N$.

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- For $\lim_{N \to \infty} \mathcal{Q}_R/N \neq 0$, we expect that the density of states of the generic complex SYK model will apply.
Quantum matter without quasiparticles

- Planckian dynamics is realized in the ‘solvable’ SYK models.

- Black holes thermalize in a time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS$_2$ horizons