Quantum phases and critical points of correlated metals

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Outline

I. **Kondo lattice models**
   Doniach's phase diagram and its quantum critical point

II. A new phase: FL*
   Paramagnetic states of quantum antiferromagnets:
   (A) Bond order, (B) Topological order.

III. Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments

IV. Extended phase diagram and its critical points

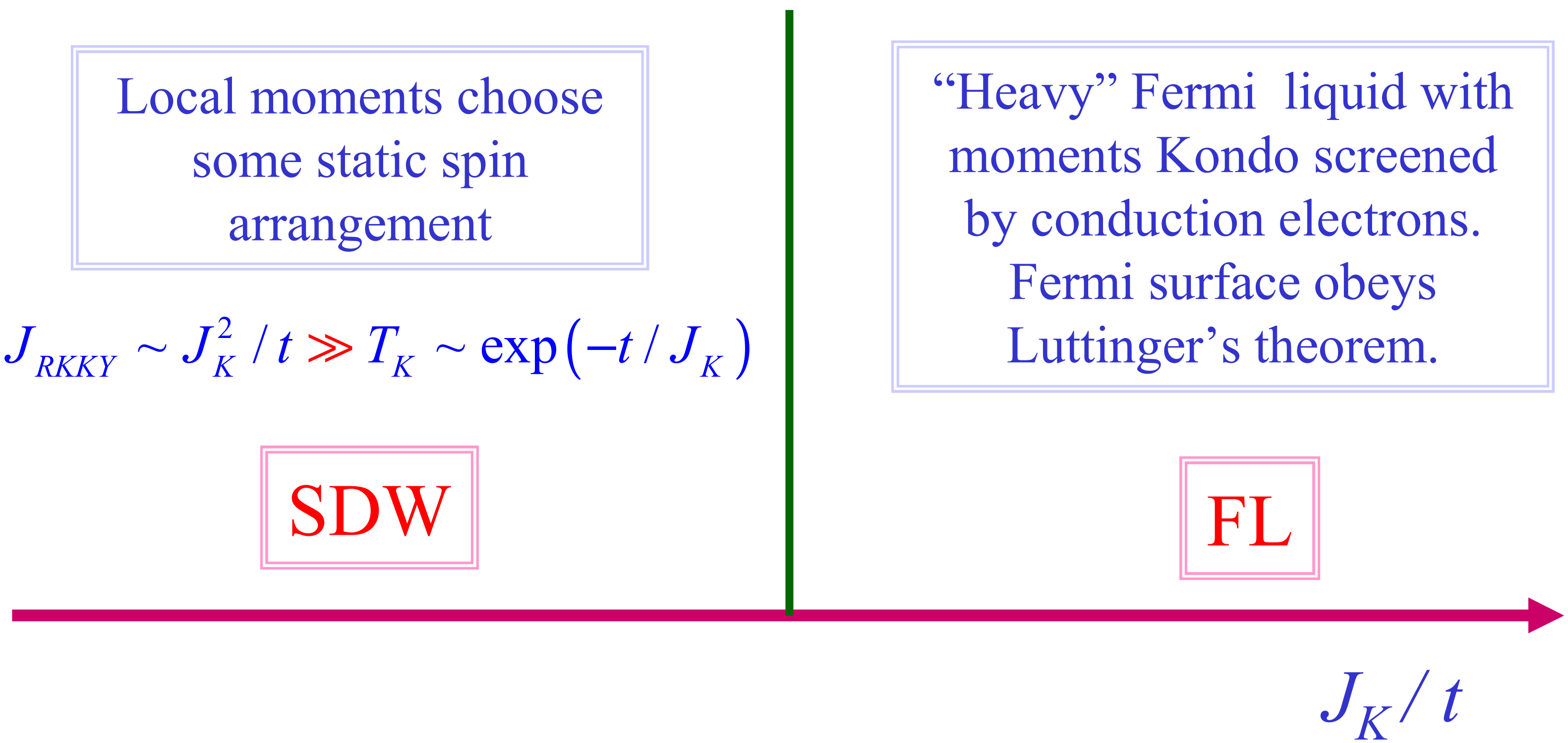
V. Conclusions

I. Doniach's $T=0$ phase diagram for the Kondo lattice

$$H = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_i \left(J_K c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right)$$

$c_{i\sigma} \rightarrow$ Conduction electrons;

$\vec{S}_{fi} \rightarrow$ localized $f_{i\sigma}$ moments (assumed S=1/2, for specificity)



Luttinger's theorem on a d-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

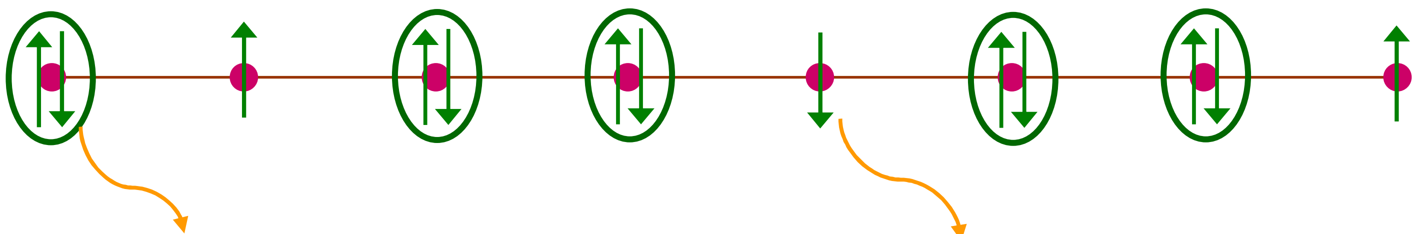
$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \left(\text{mod } 2 \right)$$

A "large" Fermi surface

Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)\left|0\right\rangle$$

$$f_{i\downarrow}^{\dagger}\left|0\right\rangle, \ S=1/2 \text{ hole}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1-n_c\right) = \left(1+n_c\right) \bmod 2\end{aligned}$$

Arguments for the Fermi surface volume of the FL phase

Alternatively:

Formulate Kondo lattice as the large U limit of the Anderson model

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(V c_{i\sigma}^{\dagger} f_{i\sigma} + V f_{i\sigma}^{\dagger} c_{i\sigma} + \varepsilon_f \left(n_{fi\uparrow} + n_{fi\downarrow} \right) + U n_{fi\uparrow} n_{fi\downarrow} \right) + \cdots$$

$$n_T = n_f + n_c$$

For small U, Fermi surface volume = $\left(n_f + n_c \right) \bmod 2$.

This is adiabatically connected to the large U limit where $n_f = 1$

Quantum critical point between SDW and FL phases

Spin fluctuations of renormalized S=1/2 fermionic quasiparticles, h_σ
(*loosely speaking*, T_K remains finite at the quantum critical point)

Gaussian theory of paramagnon fluctuations: $\vec{\phi} \sim h^\dagger_\sigma \vec{\tau}_{\sigma\sigma'} h_{\sigma'}$

Action: $$S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\phi}(q,\omega)\right|^2 \left(q^2 + |\omega| + \Gamma(\delta,T)\right)$$

J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).

Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal *corrections* to scaling, generated by $\vec{\phi}^4$ term.

J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968);
T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985)
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985);
A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

Quantum critical point between SDW and FL phases

Additional singular corrections to quasiparticle self energy in d=2

Ar. Abanov and A. V. Chubukov *Phys. Rev. Lett*. **84**, 5608 (2000);
A. Rosch *Phys. Rev*. B **64**, 174407 (2001).

....

Critical point *not* described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega / k_B T$
In such a theory, paramagnon scattering amplitude would be determined by $k_B T$ alone, and not by value of microscopic paramagnon interaction term.

S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).

(Contrary opinions: P. Coleman, Q. Si……….)

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(A) Bond order, (B) Topological order.

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Reconsider Doniach phase diagram

II. A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has "*topological order*" and associated neutral excitations. The topological order can be easily detected by the violation of Luttinger's theorem. It can only appear in dimensions $d > 1$

$$2\times\frac{v_0}{(2\pi)^d}\left(\text{Volume enclosed by Fermi surface}\right)$$

$$=\left(n_T-1\right)\left(\text{mod } 2\right)$$

Precursors: L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev.* B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, Phys. Rev. B **62**, 7850 (2000);
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev.* B **66**, 045111 (2002).

It is more convenient to consider the Kondo-Heiseberg model:

$$H = \sum_{i<j} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i} \left(J_K c^{\dagger}_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Work in the regime $J_H > J_K$

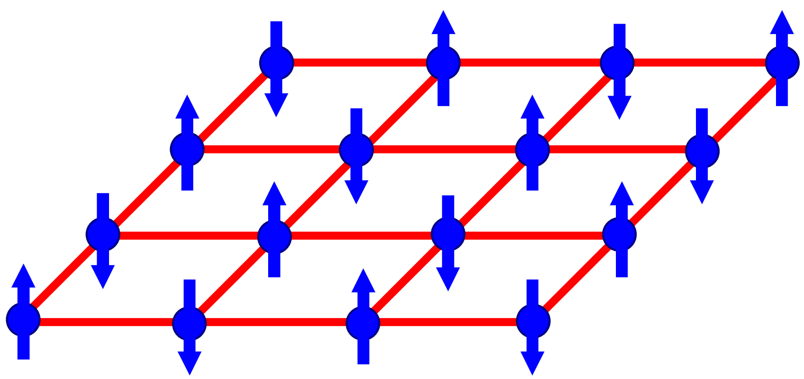
Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Ground states of quantum antiferromagnets

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance

Two classes of ordered states:

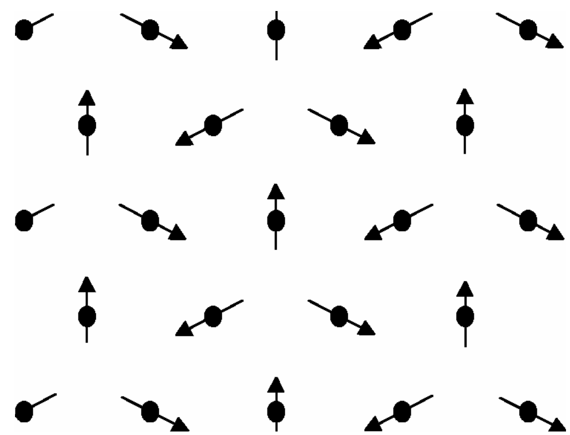
(A) Collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = (\pi, \pi); \ \vec{N}^2 = 1$$

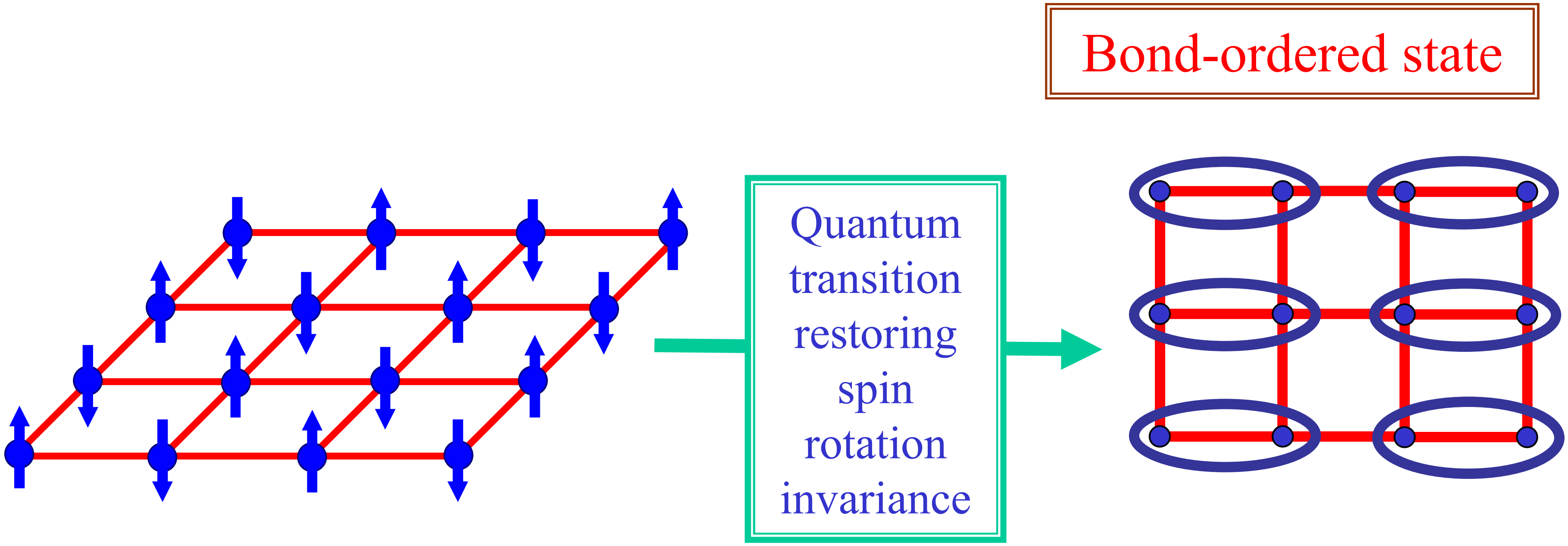
(B) Non-collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q} \bullet \boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q} \bullet \boldsymbol{r})$$

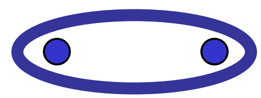
$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \ \vec{N}_1^2 = \vec{N}_2^2 = 1 ; \ \vec{N}_1 \bullet \vec{N}_2 = 0$$

(A) Collinear spins, bond order, and confinement



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

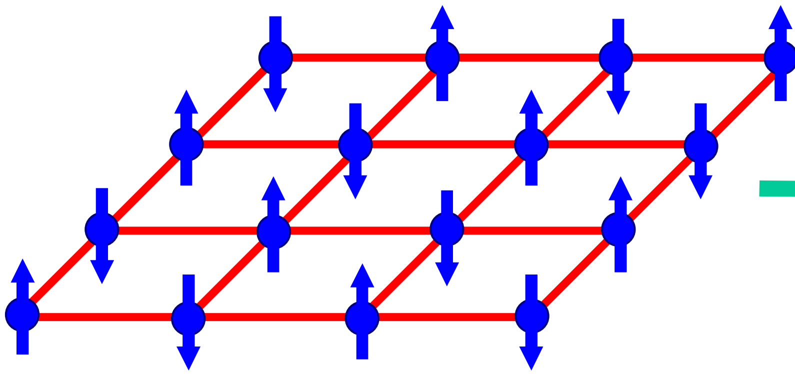
$$\boldsymbol{Q} = (\pi, \pi); \ \vec{N}^2 = 1$$

 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

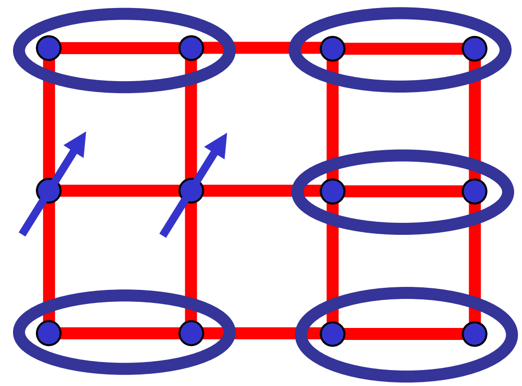
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

(A) Collinear spins, bond order, and confinement

Bond-ordered state

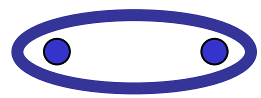


Quantum transition restoring spin rotation invariance



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = (\pi, \pi); \; \vec{N}^2 = 1$$



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

$S = 1$ excitation is gapped $\vec{N}$ particle

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

State of conduction electrons

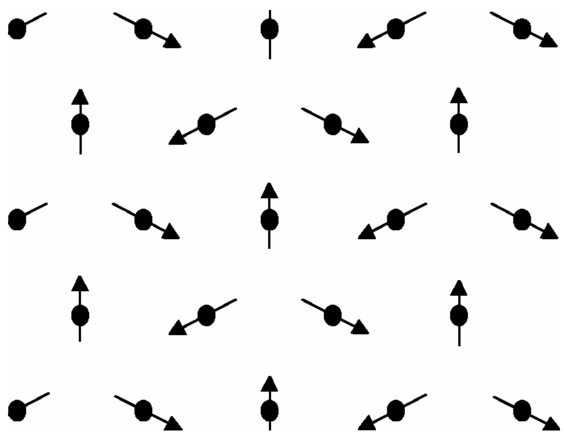
At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c

Perturbation theory in J_K is regular and so this state will be stable for finite J_K

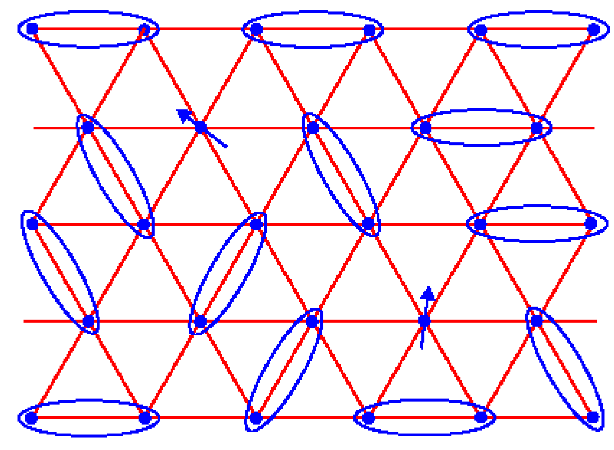
However, because n_f=2 (per unit cell of ground state)
$\boldsymbol{n_T = n_f + n_c = n_c}$ **(mod 2),** and Luttinger's theorem is obeyed.

FL state with bond order

(B) Non-collinear spins, deconfined spinons, Z_2 gauge theory, and topological order



Quantum transition restoring spin rotation invariance



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q}\bullet\boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q}\bullet\boldsymbol{r})$$

$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \ \vec{N}_1^{\,2} = \vec{N}_2^{\,2} = 1 \, ; \ \vec{N}_1 \bullet \vec{N}_2 = 0$$

RVB state with free spinons

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991) – Z_2 gauge theory
A.V. Chubukov, T. Senthil and S. Sachdev, *Phys. Rev. Lett.* **72**, 2089 (1994).

$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q}\bullet\boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q}\bullet\boldsymbol{r})$$

$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \; \vec{N}_1^2 = \vec{N}_2^2 = 1 \, ; \; \vec{N}_1 \bullet \vec{N}_2 = 0$$

Solve constraints by writing:

$$\vec{N}_1 + i\vec{N}_2 = \varepsilon_{ac} z_c \vec{\sigma}_{ab} z_b$$

where $z_{1,2}$ are two complex numbers with

$$|z_1|^2 + |z_2|^2 = 1$$

Order parameter space: S_3/Z_2

Physical observables are invariant under the Z_2 gauge transformation $z_a \to \pm z_a$

Other approaches to a Z_2 gauge theory:

R. Jalabert and S. Sachdev, *Phys. Rev.* B **44**, 686 (1991); S. Sachdev and M. Vojta, *J. Phys. Soc. Jpn* **69,** Suppl. B, 1 (2000).

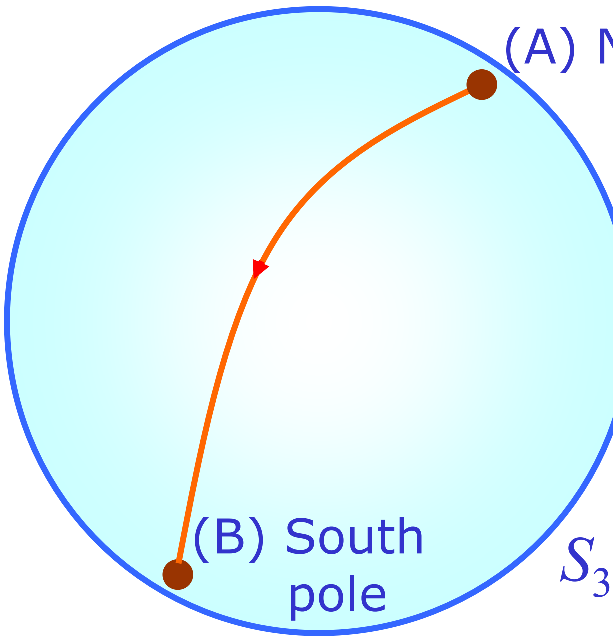
X. G. Wen, *Phys. Rev.* B **44**, 2664 (1991).

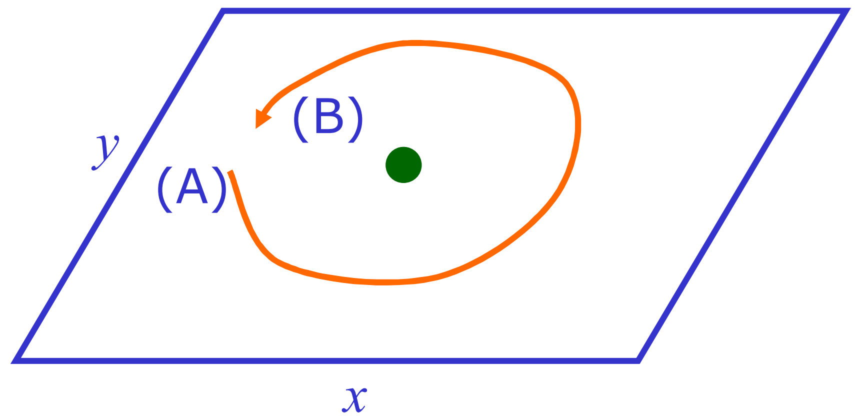
T. Senthil and M.P.A. Fisher, *Phys. Rev.* B **62**, 7850 (2000).

R. Moessner, S. L. Sondhi, and E. Fradkin, *Phys. Rev.* B **65**, 024504 (2002).

L. B. Ioffe, M.V. Feigel'man, A. Ioselevich, D. Ivanov, M. Troyer and G. Blatter, *Nature* **415**, 503 (2002).

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$





Can also consider vortex excitation in phase without magnetic order, $\left\langle \vec{S}(\boldsymbol{r}) \right\rangle = 0$: **vison**

A paramagnetic phase with vison excitations suppressed has topological order.
Suppression of visons also allows z_a quanta to propagate – these are the spinons.

State with spinons must have topological order

State of conduction electrons

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c

Perturbation theory in J_K is regular, and topological order is robust, and so this state will be stable for finite J_K

So volume of Fermi surface is determined by **$(n_T - 1) = n_c (\mathrm{mod}\ 2)$,** and Luttinger's theorem is violated.

The FL* state

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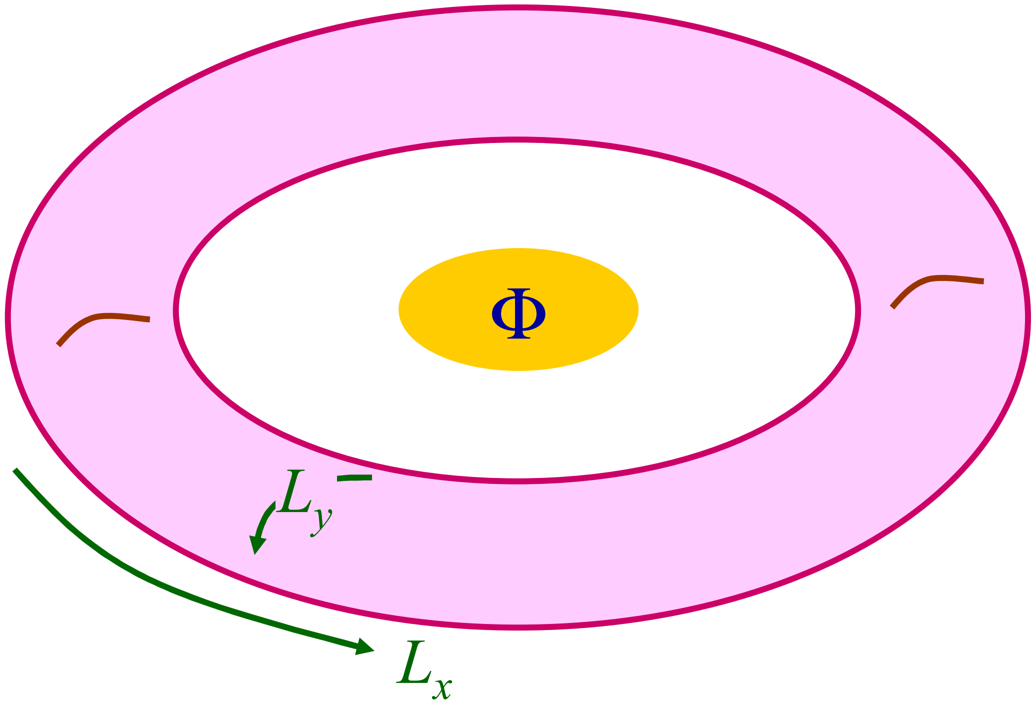


Unit cell a_x, a_y.
L_x/a_x, L_y/a_y
coprime integers

Adiabatically insert flux $\Phi = 2\pi$ (units $\hbar$=c=e=1) acting on $\uparrow$ electrons.

State changes from $|\Psi\rangle$ to $|\Psi'\rangle$, and $UH(0)U^{-1} = H(\Phi)$, where

$$U = \exp\left[\frac{2\pi i}{L_x}\sum_{r} x\,\hat{n}_{Tr\uparrow}\right].$$

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000).

Adiabatic process commutes with the translation operator T_x, so momentum P_x is conserved.

$$\text{However } U^{-1} T_x U = T_x \exp\left[\frac{2\pi i}{L_x} \sum_{\boldsymbol{r}} \hat{n}_{T\boldsymbol{r}\uparrow} \right];$$

so shift in momentum ΔP_x between states $U|\Psi'\rangle$ and $|\Psi\rangle$ is

$$\Delta P_x = \frac{\pi L_y}{v_0} n_T \left(\operatorname{mod} \frac{2\pi}{a_x} \right) \qquad (1).$$

Alternatively, we can compute ΔP_x by assuming it is absorbed by quasiparticles of a Fermi liquid. Each quasiparticle has its momentum shifted by $2\pi/L_x$, and so

$$\Delta P_x = \frac{2\pi}{L_x} \frac{(\text{Volume enclosed by Fermi surface})}{(2\pi)^2 / (L_x L_y)} \left(\operatorname{mod} \frac{2\pi}{a_x} \right) \qquad (2).$$

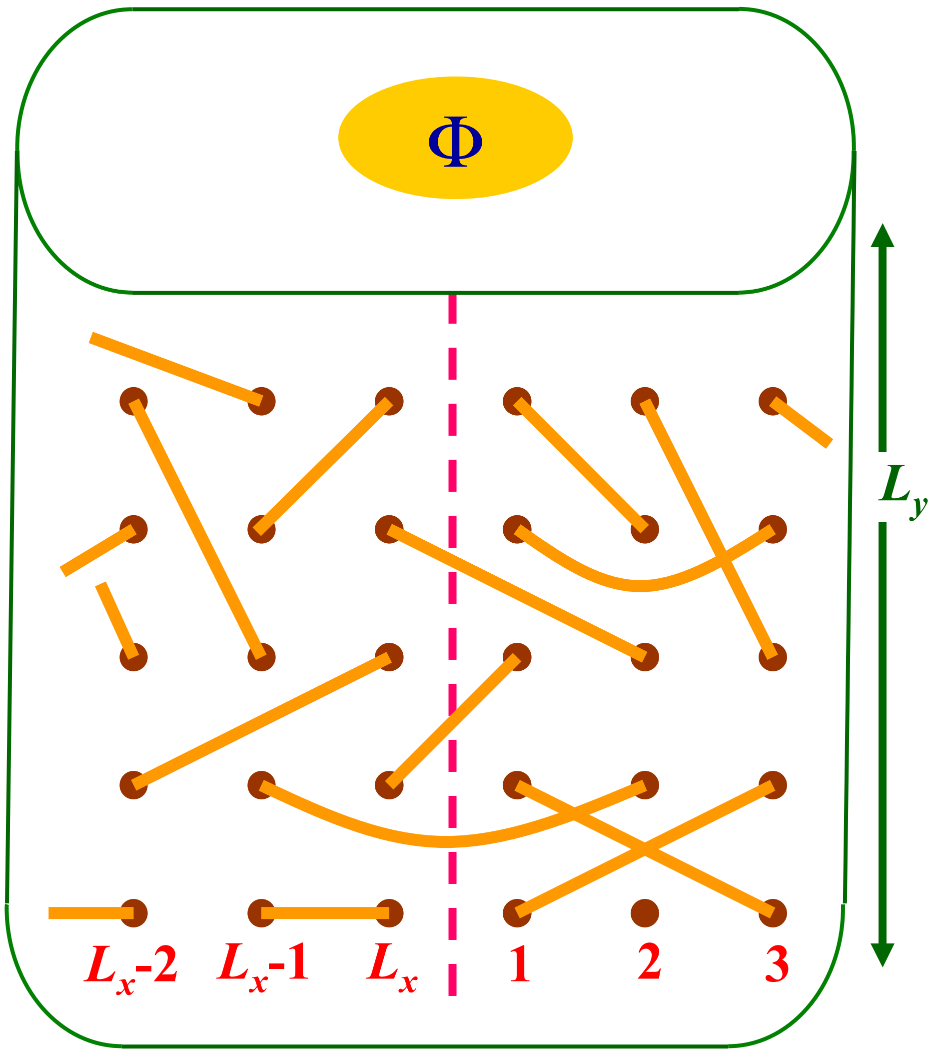
From (1) and (2), same argument in y direction, using coprime $L_x/a_x, L_y/a_y$:

$$2 \times \frac{v_0}{(2\pi)^2} (\text{Volume enclosed by Fermi surface}) = n_T \; (\operatorname{mod} 2)$$

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000).

Effect of flux-piercing on a topologically ordered quantum paramagnet

$$|D\rangle =$$



N. E. Bonesteel, *Phys*. *Rev*. B **40**, 8954 (1989).
G. Misguich, C. Lhuillier, M. Mambrini, and P. Sindzingre, *Eur*. *Phys*. *J*. B **26**, 167 (2002).

$$|\Psi\rangle = \sum_D a_D |D\rangle$$

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N. E. Bonesteel,
Phys. Rev. B **40**, 8954 (1989).
G. Misguich, C. Lhuillier,
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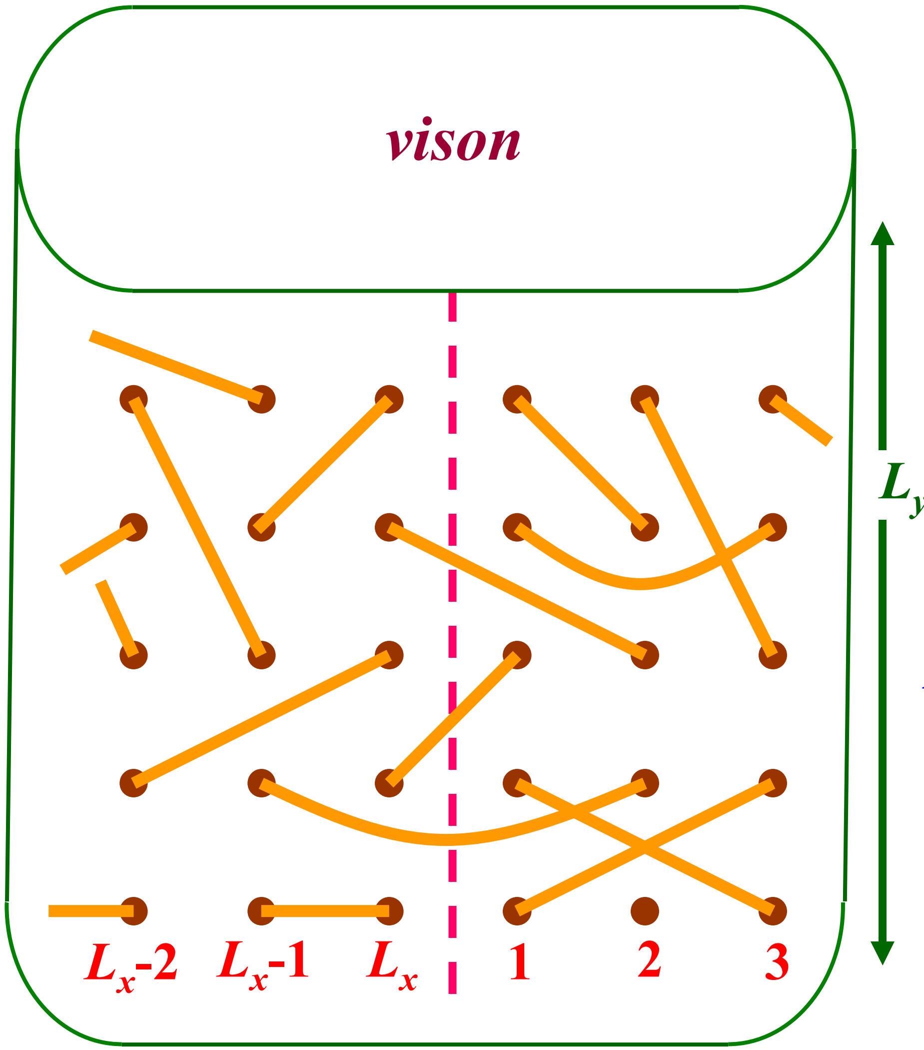
Eur. Phys. J. B **26**, 167 (2002).

$|D\rangle =$



$$|\Psi\rangle = \sum_D a_D |D\rangle$$

After flux insertion $|D\rangle \Rightarrow$

$$(-1)^{\substack{\text{Number of bonds} \\ \text{cutting dashed line}}} |D\rangle;$$

Equivalent to inserting a ***vison*** inside hole of the torus.

Vison carries momentum $\pi L_y / v_0$

Flux piercing argument in Kondo lattice

Shift in momentum is carried by n_T electrons, where

$$n_T = n_f + n_c$$

In topologically ordered, state, momentum associated with n_f=1 electron is absorbed by creation of vison. The remaining momentum is absorbed by Fermi surface quasiparticles, which enclose a volume associated with n_c electrons.

The FL* state.

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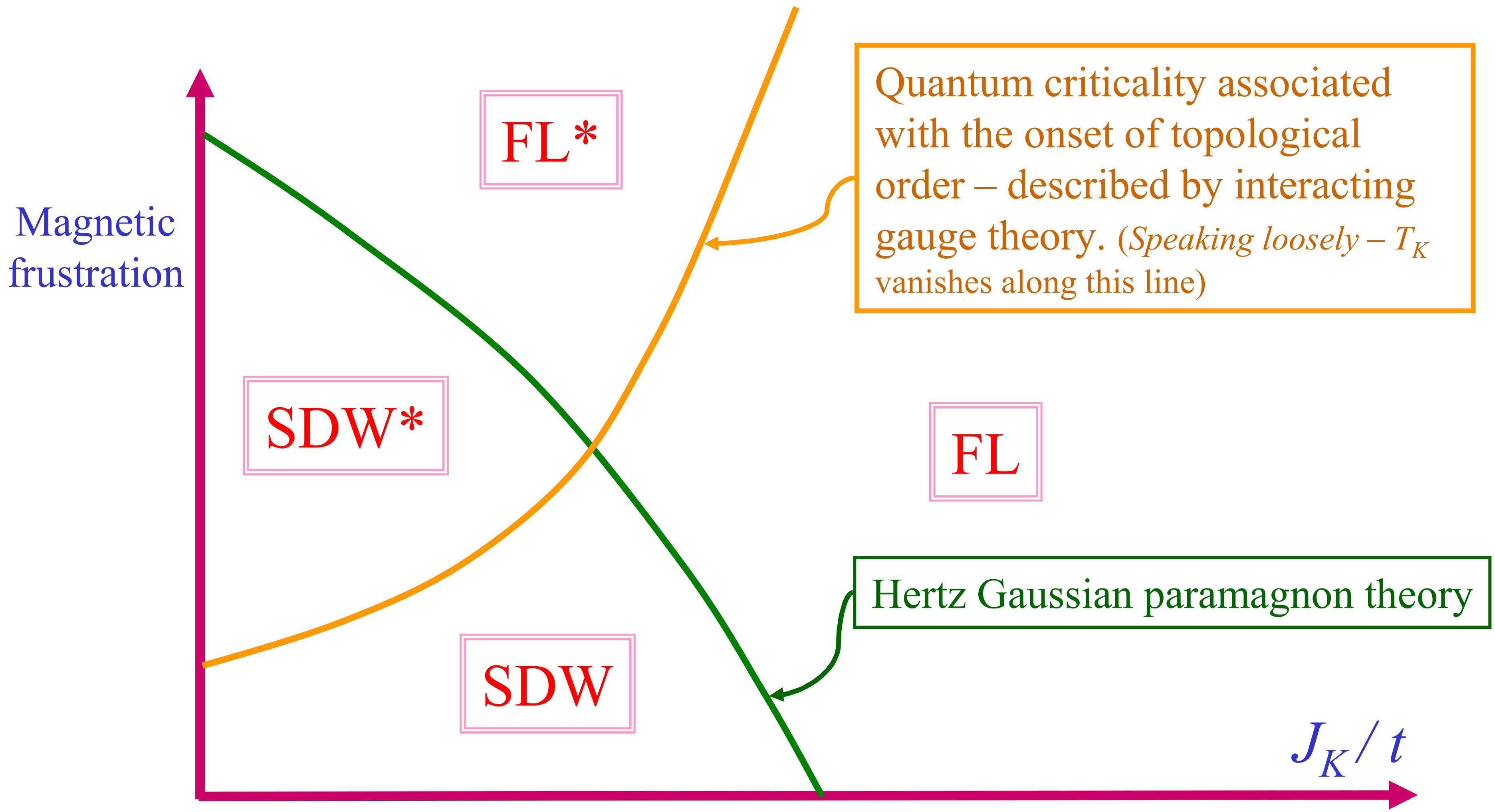
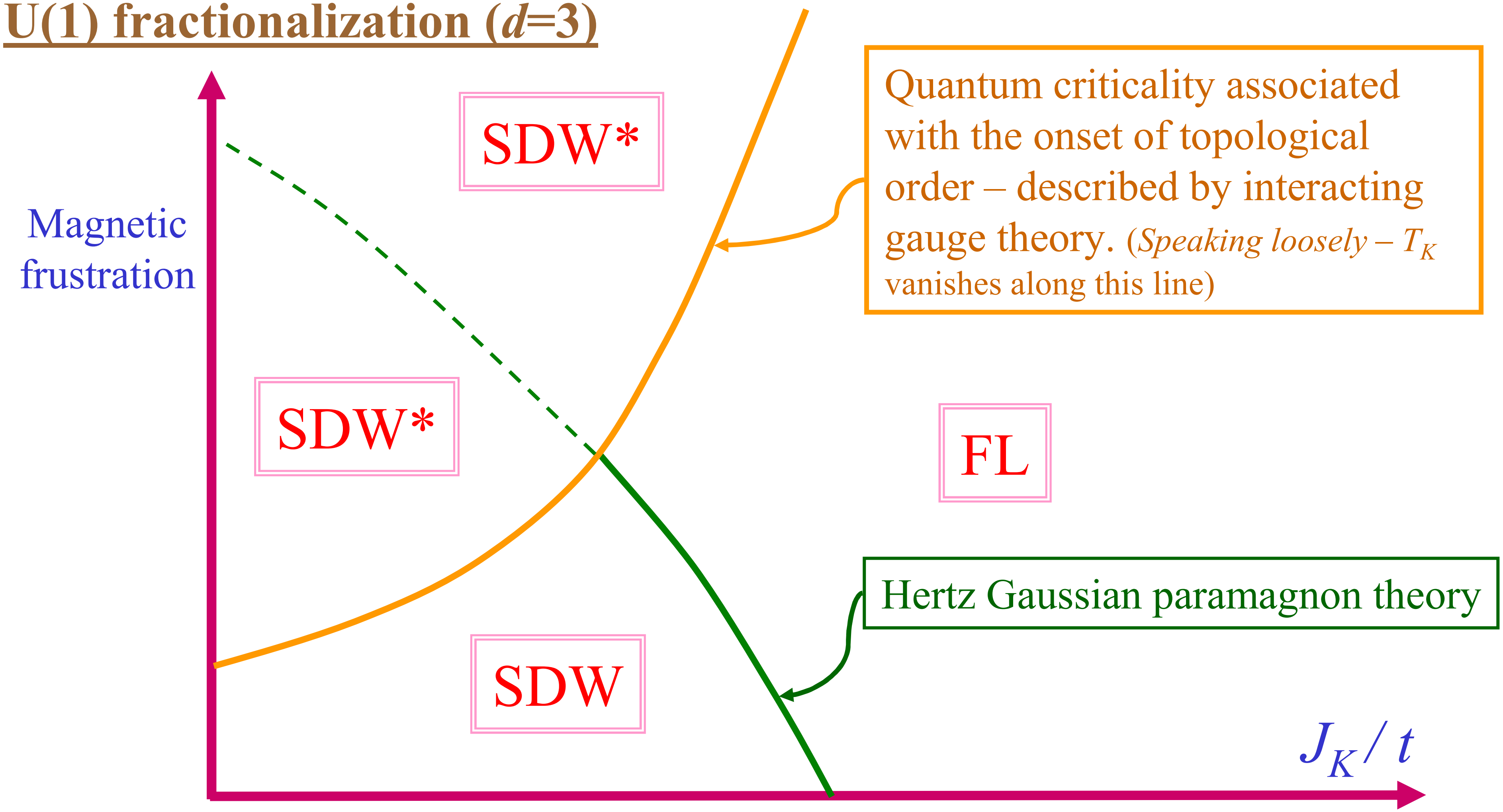


- $*$ phases have spinons with Z_2 (d=2,3) or U(1) (d=3) gauge charges, and associated gauge fields.
- Fermi surface volume does *not* distinguish SDW and SDW* phases.

IV. Extended *T=0* phase diagram for the Kondo lattice

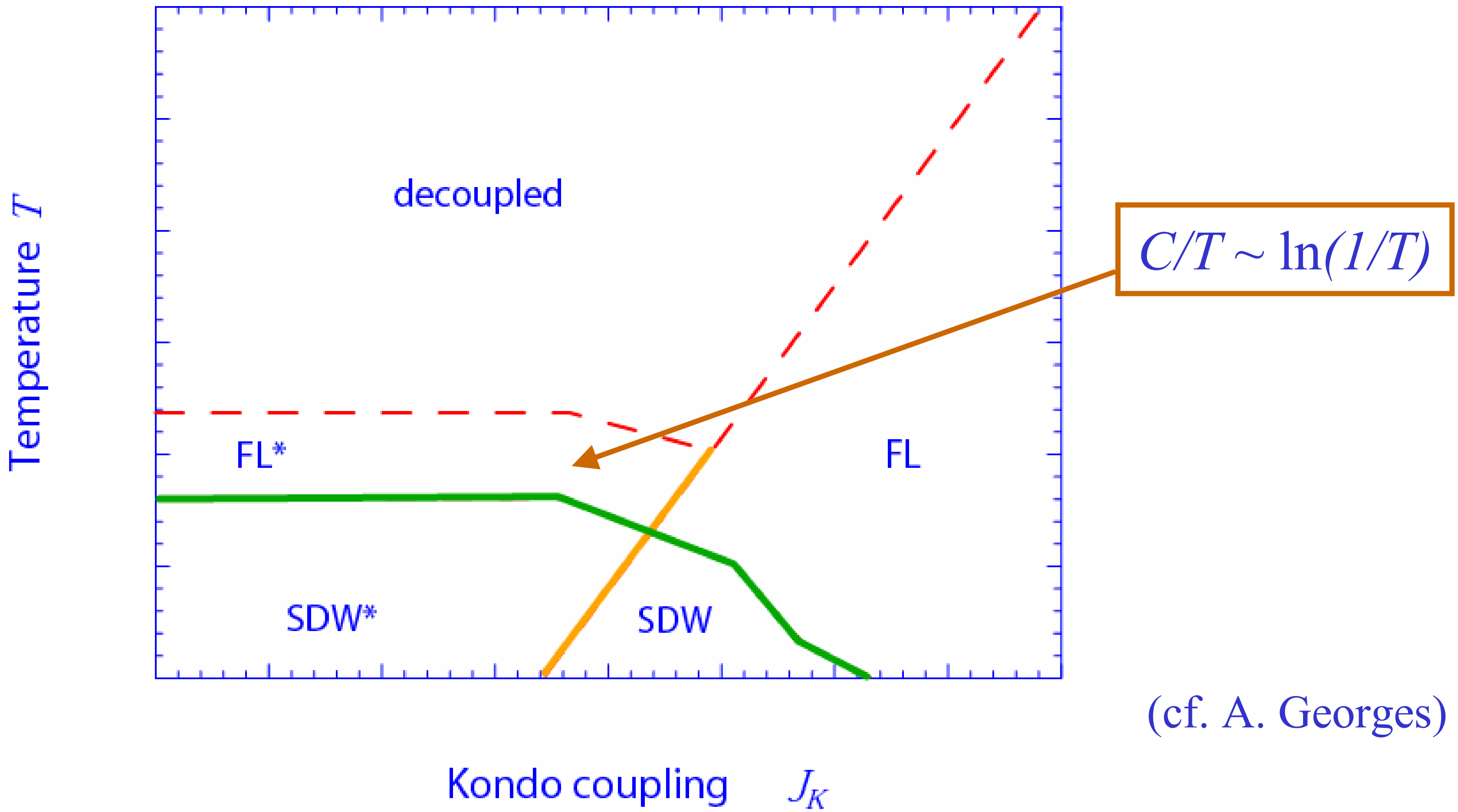
U(1) fractionalization (*d*=3)



- Because of strong gauge fluctuations, U(1)-FL* may be unstable to U(1)-SDW* at low temperatures.
- Only phases at *T*=0: FL, SDW, U(1)-SDW*.

U(1) fractionalization (d=3)

Mean-field phase diagram



(cf. A. Georges)

- Because of strong gauge fluctuations, U(1)-FL* may be unstable to U(1)-SDW* at low temperatures.
- Only phases at T=0: FL, SDW, U(1)-SDW*.
- Quantum criticality dominated by a T=0 FL-FL* transition.

Strongly coupled quantum criticality with a topological or spin-glass order parameter

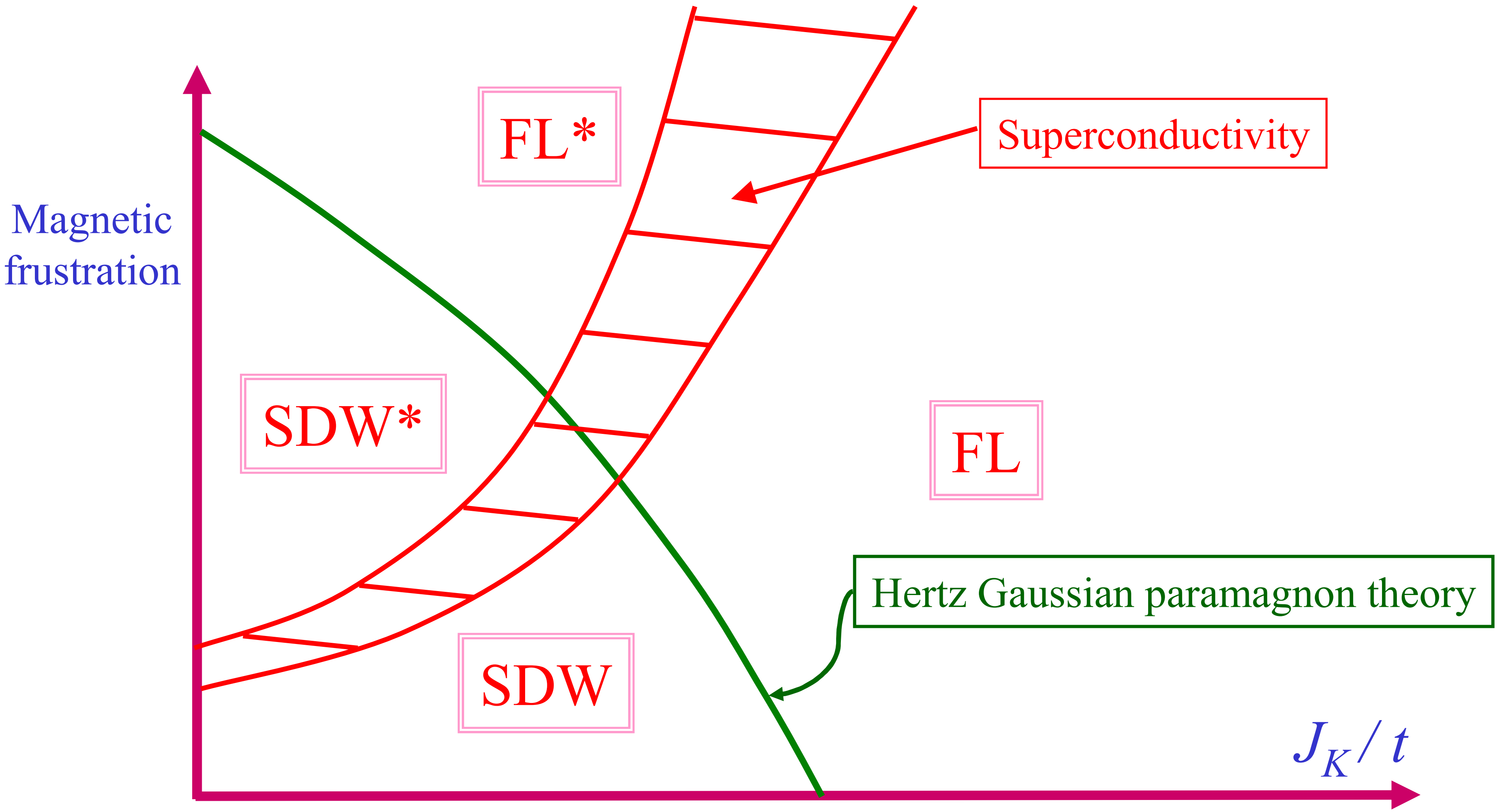
Order parameter does not couple directly to simple observables

Dynamic spin susceptiblity

$$\chi(q,\omega) = \frac{1}{-i\gamma\omega + A(q-Q)^2 + B + T^{\alpha}\Phi\left(\frac{\hbar\omega}{k_B T}\right)}$$

Non-trivial universal scaling function which is a property of a bulk *d*-dimensional quantum field theory describing "hidden" order parameter.

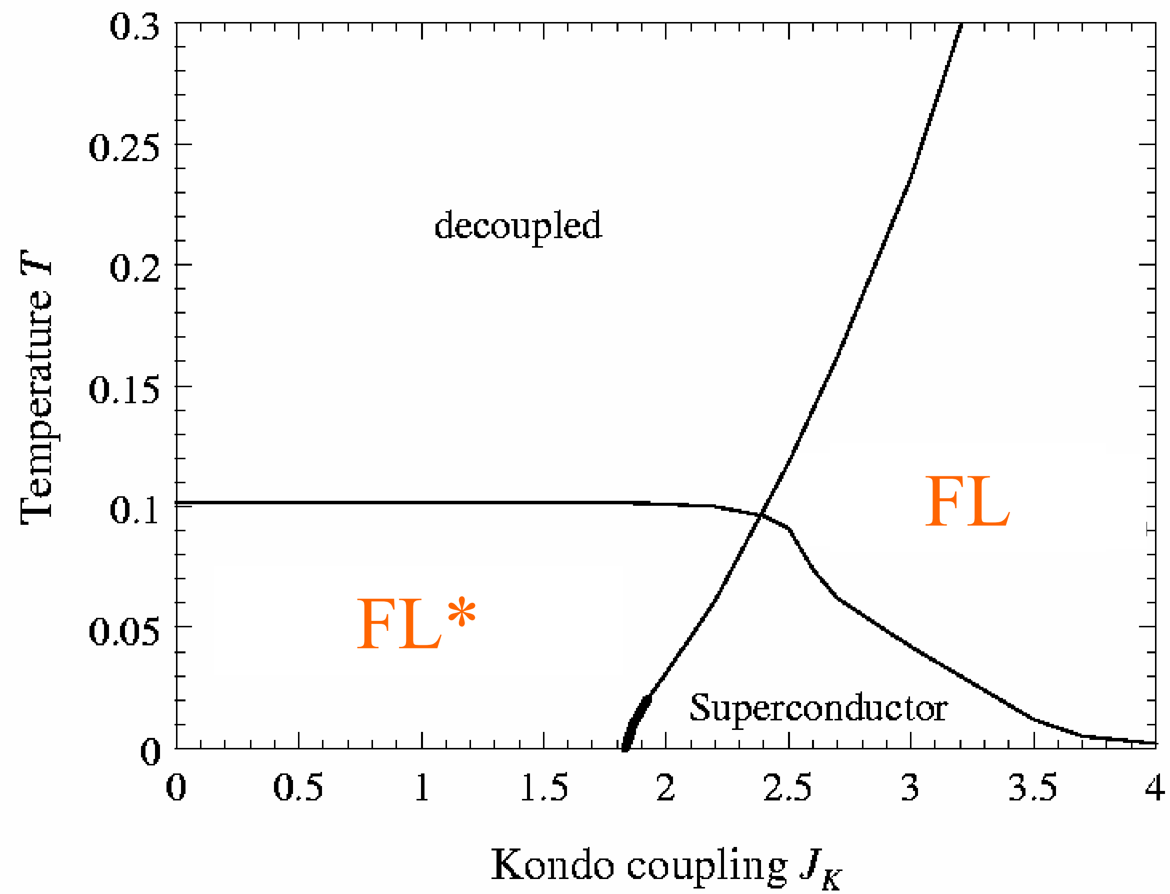
Z_2 fractionalization



• Superconductivity is generic between FL and Z_2 FL* phases.

Z_2 fractionalization

Mean-field phase diagram



Pairing of spinons in small Fermi surface state induces superconductivity at the confinement transition

Small Fermi surface state can also exhibit a second-order metamagnetic transition in an applied magnetic field, associated with vanishing of a spinon gap.

Conclusions

- New phase diagram as a paradigm for clean metals with local moments.
- Topologically ordered (*) phases lead to novel quantum criticality.
- New FL* allows easy detection of topological order by Fermi surface volume