Quantum phase transitions: from antiferromagnets and superconductors to black holes

Reviews:
arXiv:0907.0008
arXiv:0810.3005 (with Markus Mueller)

Talk online: sachaev.physics.harvard.edu
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Outline

1. Coupled dimer antiferromagnets
   Order parameters and Landau-Ginzburg criticality

2. Graphene
   ’Topological’ Fermi surface transitions

3. Quantum criticality and black holes
   $AdS_4$ theory of compressible quantum liquids

4. Quantum criticality in the cuprates
   Global phase diagram and the spin density wave transition in metals
1. Coupled dimer antiferromagnets
   *Order parameters and Landau-Ginzburg criticality*

2. Graphene
   *Topological* Fermi surface transitions

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   *AdS*$_4$ *theory of compressible quantum liquids*

4. Quantum criticality in the cuprates
   *Global phase diagram and the spin density wave transition in metals*
An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in Néel state.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ \frac{1}{\sqrt{2}} (|\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle) \]
Pressure in TICuCl$_3$

\[
\rho = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow)
\]
Quantum critical point with non-local entanglement in spin wavefunction

\[ \frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right) \]
Excitation spectrum in the paramagnetic phase
Excitation spectrum in the paramagnetic phase
Excitation spectrum in the paramagnetic phase
Excitation spectrum in the paramagnetic phase
TlCuCl$_3$ at ambient pressure

![Graph of energy vs. wave vector](image)

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$
for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer
$\text{TlCuCl}_3$ at ambient pressure

Sharp spin 1 particle excitation above an energy gap (spin gap)

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer
Excitation spectrum in the Néel phase

Tuesday, November 3, 2009
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Description using Landau-Ginzburg field theory

\[ S = \int d^2 r d\tau \left[ (\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right] \]
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \]

\[ \lambda > \lambda_c \]

Spin \( S = 1 \) "triplon"
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c)\varphi^2 + u(\varphi^2)^2 \]

\[ \lambda > \lambda_c \]
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \]

\[ \lambda > \lambda_c \]
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\phi}) = (\lambda - \lambda_c) \vec{\phi}^2 + u (\vec{\phi}^2)^2 \]

\[ \lambda > \lambda_c \]
Excitation spectrum in the paramagnetic phase

$$V(\vec{\phi}) = (\lambda - \lambda_c) \vec{\phi}^2 + u (\vec{\phi}^2)^2$$

$$\lambda > \lambda_c$$

Spin $S = 1$ “triplon”
Excitation spectrum in the Néel phase

\[ \lambda \]
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$$\lambda < \lambda_c$$

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Excitation spectrum in the Néel phase

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle.

\[ V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2 \]

\[ \lambda < \lambda_c \]
Excitation spectrum in the Néel phase

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal Higgs-Englert-Brout particle

\[ V(\bar{\varphi}) = (\lambda - \lambda_c)\varphi^2 + u(\bar{\varphi}^2)^2 \]

\[ \lambda < \lambda_c \]
Observation of $3 \rightarrow 2$ low energy modes, emergence of new Higgs-Englert-Brout particle in the Néel phase, and vanishing of Néel temperature at the quantum critical point

Prediction of quantum field theory

Potential for $\vec{\phi}$ fluctuations:  $V(\vec{\phi}) = (\lambda - \lambda_c)\vec{\phi}^2 + u (\vec{\phi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\phi} = 0$:

$$V(\vec{\phi}) \approx (\lambda - \lambda_c)\vec{\phi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$
Prediction of quantum field theory

Potential for $\vec{\phi}$ fluctuations: $V(\vec{\phi}) = (\lambda - \lambda_c) \bar{\phi}^2 + u (\bar{\phi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\bar{\phi} = 0$:

$$V(\vec{\phi}) \approx (\lambda - \lambda_c) \bar{\phi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{\lambda - \lambda_c}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\phi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \vec{\phi}_1$:

$$V(\vec{\phi}) \approx 2(\lambda_c - \lambda) \phi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs-Englert-Brout particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$
**Prediction of quantum field theory**

**Energy of Higgs-Englert-Brout particle**

Energy of triplon

\[
V(\bar{\varphi}) = (\lambda - \lambda_c)\varphi^2 + u(\varphi^2)^2
\]

\[
\sqrt{2} = \frac{\text{Energy of triplon}}{\text{Energy of Higgs-Englert-Brout particle}}
\]

---

**TLCuCl₃**

\( p_c = 1.07 \text{ kbar} \)

\( T = 1.85 \text{ K} \)

\( \lambda = 0.04 \)

\( \lambda_c = 0.03 \)

\( u = 0.01 \)

\( \varphi = 0.1 \)

**Graph**

Energy \( \sqrt{2}E(p < p_c), E(p > p_c) \) [meV]

Pressure \( |(p - p_c)| \) [kbar]

\( Q = (0, 4, 0) \)

\( L(p < p_c) \)

\( L(p > p_c) \)

\( Q = (0, 0, 1) \)

\( L, T_1 (p < p_c) \)

\( L(p > p_c) \)

\( \times \) unscaled

---

$O(3)$ order parameter $\vec{\varphi}$

\[
S = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]
\]
Classical spin waves

Quantum critical

Dilute triplon gas

Neel order
Classical dynamics of spin waves

Neel order
Classical spin waves

Quantum critical

Dilute triplon gas

Classical Boltzmann equation for $S=1$ particles

Neel order
Classical spin waves

Quantum critical

Dilute triplon gas

Neel order

CFT3 at $T>0$
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Graphene
Graphene

Conical Dirac dispersion

\[ \varepsilon_{\vec{k}} = \hbar v_F |\vec{k}| \]
Quantum phase transition in graphene tuned by a gate voltage
Quantum phase transition in graphene tuned by a gate voltage

Hole Fermi surface

Electron Fermi surface

\[ \mu < 0 \]

\[ \mu > 0 \]
Quantum phase transition in graphene tuned by a gate voltage

There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point.

$\mu < 0$

Hole Fermi surface

Electron Fermi surface

$\mu > 0$
Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, \( \psi_\sigma \), \( \sigma = 1 \ldots 4 \), interacting with a \( 1/r \) Coulomb interaction

\[
S = \int d^2 r d\tau \psi_\sigma^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\
+ \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma (r) \frac{1}{|r - r'|} \psi_\sigma^\dagger (r') \psi_\sigma' (r')
\]
Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, $\psi_\sigma$, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$
S = \int d^2rd^2\tau \psi_\sigma^\dagger \left( \partial_\tau - iv_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\
+ \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r-r'|} \psi_\sigma^\dagger(\sigma', r')
$$

Dimensionless “fine-structure” constant $\alpha = e^2/(\hbar v_F)$.

RG flow of $\alpha$:

$$
\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots
$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$
Quantum phase transition in graphene

$T(K)$

Quantum critical

Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

$\mu < 0$

$\mu > 0$

$n \times 10^{12} / m^2$
Quantum critical transport

Quantum "perfect fluid" with shortest possible relaxation time, $\tau_R$

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

Quantum critical transport

Transport coefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

\[ \sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)] \]

Quantum critical transport

Transport coefficients not determined by collision rate, but by universal constants of nature

Momentum transport

\[ \eta \equiv \frac{\text{viscosity}}{s} = \frac{\hbar}{k_B} \times [\text{Universal constant } O(1)] \]

Quantum critical transport in graphene

\[ \sigma(\omega) = \begin{cases} 
\frac{e^2}{\hbar} \left[ \frac{\pi}{2} + \mathcal{O} \left( \frac{1}{\ln(\Lambda/\omega)} \right) \right] & , \quad \hbar \omega \gg k_B T \\
\frac{e^2}{\hbar \alpha^2(T)} \left[ 0.760 + \mathcal{O} \left( \frac{1}{|\ln(\alpha(T))|} \right) \right] & , \quad \hbar \omega \ll k_B T \alpha^2(T) 
\end{cases} \]

where the “fine structure constant” is

\[
\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130
\]


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   \textit{AdS}_4 \textit{theory of compressible quantum liquids}

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Global phase diagram and the spin density wave transition in metals
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

Maldacena, Gubser, Klebanov, Polyakov, Witten
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
**AdS/CFT correspondence**

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

- **3+1 dimensional AdS space**
- **Quantum criticality in 2+1 dimensions**

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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AdS/CFT correspondence

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AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
Conformal field theory in 2+1 dimensions at $T = 0$
Conformal field theory in 2+1 dimensions at $T > 0$

Einstein gravity on AdS$_4$ with a Schwarzschild black hole
Conformal field theory in 2+1 dimensions at $T > 0$, with a non-zero chemical potential, $\mu$ and applied magnetic field, $B$.

Einstein gravity on AdS$_4$ with a Reissner-Nordstrom black hole carrying electric and magnetic charges.
Examine free energy and Green’s function of a probe particle

F. Denef, S. Hartnoll, and S. Sachdev, to appear
Short time behavior depends upon conformal $\text{AdS}_4$ geometry near boundary.
Long time behavior depends upon near-horizon geometry of black hole

F. Denef, S. Hartnoll, and S. Sachdev, to appear
Radial direction of gravity theory is measure of energy scale in CFT

F. Denef, S. Hartnoll, and S. Sachdev, to appear
Infrared physics of Fermi surface is linked to the near horizon $\text{AdS}_2$ geometry of Reissner-Nordstrom black hole
Geometric interpretation of RG flow
Geometric interpretation of RG flow

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

$$\sigma = \sigma_Q + \epsilon^* \rho^2 v^2 \varepsilon + P \pi \delta z \omega$$

where $\sigma_Q$ is the universal conductivity of the CFT, $\rho$ is the charge density, $\epsilon$ is the energy density and $P$ is the pressure.

The same quantities also determine the thermal conductivity, $\kappa$:

$$\kappa = \sigma_Q \left( k_B T \epsilon^* \rho \right)^2$$

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport coefficients in the quantum critical regime.

The same results were later obtained from the equations of generalized relativistic magnetohydrodynamics.

So the results apply to experiments on graphene, and to the dynamics of black holes.

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

As a simple example, in zero magnetic field, we can write the electrical conductivity as

\[ \sigma = \sigma_Q + \frac{e^*2 \rho^2 v^2}{\varepsilon + P} \pi \delta(\omega) \]

where \( \sigma_Q \) is the universal conductivity of the CFT, \( \rho \) is the charge density, \( \varepsilon \) is the energy density and \( P \) is the pressure.

The same quantities also determine the thermal conductivity, \( \kappa \):

\[ \kappa = \sigma_Q \left( \frac{k_B T}{e^*2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 \]
Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport coefficients in the quantum critical regime.

A second example: In an applied magnetic field $B$, the dynamic transport coefficients exhibit a hydrodynamic cyclotron resonance at a frequency $\omega_c$

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and damping constant $\gamma$

$$\gamma = \sigma Q \frac{B^2 v^2}{c^2(\varepsilon + P)}.$$

The same constants determine the quasinormal frequency of the Reissner-Nordstrom black hole.
Green’s function of a fermion

\[ G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega \theta(k)} \]

Green’s function of a fermion

\[ G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega \theta(k)} \]

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the “quasinormal frequencies”, $\zeta_\ell$, of the black hole. Here $\ell$ represents any set of quantum numbers:

$$ F_{\text{boson}} = -T \sum_\ell \ln \left( \frac{|\zeta_\ell|}{2\pi T} \left| \Gamma \left( \frac{iz_\ell}{2\pi T} \right) \right|^2 \right) $$

$$ F_{\text{fermion}} = T \sum_\ell \ln \left( \left| \Gamma \left( \frac{iz_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right) $$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period ($2\pi/(\text{Fermi surface area})$) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

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The cuprate superconductors
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface
Antiferromagnetism

Fermi surface

d-wave superconductivity
Crossovers in transport properties of hole-doped cuprates

Crossovers in transport properties of hole-doped cuprates

Strange metal

Pseudo-gap

d-wave SC

Hole doping $x$
Antiferromagnetism

Fermi surface

d-wave superconductivity
Antiferromagnetism

Fermi surface

d-wave superconductivity
Canonical quantum critical phase diagram of coupled-dimer antiferromagnet

Classical spin waves

Quantum critical

Dilute triplon gas

Neel order

Pressure in TlCuCl$_3$


Crossovers in transport properties of hole-doped cuprates

Strange metal

Pseudo-gap

d-wave SC

Hole doping $x$

$T$ (K)
Crossovers in transport properties of hole-doped cuprates

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?


Only candidate quantum critical point observed at low $T$

Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates
Antiferromagnetism

Fermi surface

d-wave superconductivity
Antiferromagnetism

Fermi surface

d-wave superconductivity
The electron spin polarization obeys
\[ \langle \vec{S}(r, \tau) \rangle = \vec{\varphi}(r, \tau)e^{i \vec{K} \cdot r} \]
where \( \vec{K} \) is the ordering wavevector.
Hole-doped cuprates

Increasing SDW order

Hole-doped cuprates

Increasing SDW order

Hole-doped cuprates

Increasing SDW order

Hot spots

Hole-doped cuprates

Fermi surface breaks up at hot spots into electron and hole “pockets”

Hole-doped cuprates

Increasing SDW order

Hole pockets

Electron pockets

Hot spots

Fermi surface breaks up at hot spots into electron and hole “pockets”

Theory of quantum criticality in the cuprates

Underlying SDW ordering quantum critical point in metal at $x = x_m$
Antiferromagnetism

Fermi surface

d-wave superconductivity
Antiferromagnetism

Fermi surface

d-wave superconductivity
Theory of quantum criticality in the cuprates

Underlying SDW ordering quantum critical point in metal at $x = x_m$
Theory of quantum criticality in the cuprates

Onset of $d$-wave superconductivity hides the critical point $x = x_m$
Theory of quantum criticality in the cuprates

Fluctuating, paired Fermi pockets

Strange Metal

Large Fermi surface

d-wave superconductor

Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$. 


Theory of quantum criticality in the cuprates

Fluctuating, paired Fermi pockets

Strange Metal

Large Fermi surface

d-wave superconductor

Spin density wave (SDW)


Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$. 
Theory of quantum criticality in the cuprates

Fluctuating, paired Fermi pockets

Thermally fluctuating SDW

Magnetic quantum criticality

Spin gap

d-wave superconductor

Strange Metal

Large Fermi surface

Spin density wave (SDW)

$x_s$ $x_m$

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.


Theory of quantum criticality in the cuprates

Many neutron scattering and quantum oscillation experiments in high magnetic fields are explained by this phase diagram.


Theory of quantum criticality in the cuprates

Many neutron scattering and quantum oscillation experiments in high magnetic fields are explained by this phase diagram.
Theory of quantum criticality in the cuprates

- Fluctuating, paired Fermi pockets
- Large Fermi surface
- Strange Metal
- d-wave superconductor


Many neutron scattering and quantum oscillation experiments in high magnetic fields are explained by this phase diagram.
Theory of quantum criticality in the cuprates

Many neutron scattering and quantum oscillation experiments in high magnetic fields are explained by this phase diagram.


Theory of quantum criticality in the cuprates

Underlying SDW ordering quantum critical point in metal at $x = x_m$
Fluctuations about mean field theory

SDW fluctuation $\vec{\varphi}$

Fermions near connected hot spots
Fluctuations about mean field theory

SDW fluctuation $\vec{\phi}$

Fermions near connected hot spots

Turn $\vec{\phi}$ lines into doubled particle-holes lines, and add dotted lines for fermion loops

Sung-Sik Lee, Phys. Rev. B 80, 165102 (2009); M. Metlitski and S. Sachdev, to appear
Fluctuations about mean field theory

SDW fluctuation \( \vec{\varphi} \)

Fermions near connected hot spots

All planar graphs contain the dominant singularity, and have to be resummed for a consistent theory
Fluctuations about mean field theory

SDW fluctuation $\vec{\varphi}$

Fermions near connected hot spots

A string theory for the Fermi surface?

Sung-Sik Lee, Phys. Rev. B 80, 165102 (2009); M. Metlitski and S. Sachdev, to appear
Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors
Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density.
Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal.

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity.