Competing orders and quantum criticality in the cuprate superconductors

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Outline

I. Coupled Ladder Antiferromagnet
   A. Ground states in limiting regimes
   B. Coherent state path integral
   C. Quantum field theory for critical point

II. Berry phases and duality in one dimension
    S=1/2 quantum XY model.

III. Berry phases and duality in two dimensions
     Bond-centered charge ("spin-Peierls") order.

IV. Magnetic transitions in $d$-wave superconductors
    A. Theory of SC+SDW to SC quantum transition
    B. Phase diagrams of Mott insulators and superconductors in an applied magnetic field
    C. Comparison of predictions with experiments

V. Conclusions
**I.A Coupled Ladder Antiferromagnet**

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, cond-mat/0107115.

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

0 \leq \lambda \leq 1
Square lattice antiferromagnet

Experimental realization: $La_2CuO_4$

Ground state has long-range magnetic (Neel) order

$$\langle \vec{S}_i \rangle = (-1)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves

$$\varepsilon_k = \sqrt{c_x^2 k_x^2 + c_y^2 k_y^2}$$
\( \lambda \) close to 0

Weakly coupled ladders

\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

Paramagnetic ground state

\( \langle \vec{S}_i \rangle = 0 \)

Excitation: \( S=1 \) exciton (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

\[ \epsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2 \Delta} \]
Quantum paramagnet $\langle \vec{S} \rangle = 0$

Neel state $\langle \vec{S} \rangle = \pm N_0 \neq 0$

Spin gap $\Delta$

Neel order $N_0$
### I.B Coherent state path integral


Path integral for a single spin

\[
Z = \text{Tr}\left( e^{-H[S]/T} \right)
= \int \mathcal{D} N(\tau) \delta(N^2 - 1) \exp \left( -iS \int A_\tau(\tau) d\tau - \int d\tau H\left[ SN(\tau)\right] \right)
\]

\[A_\tau(\tau)d\tau = \text{Oriented area of triangle on surface of unit sphere bounded by } N(\tau), N(\tau + d\tau), \text{ and a fixed reference } N_0\]

Action for lattice antiferromagnet

\[N_j(\tau) = \eta_j n(x_j, \tau) + L(x_j, \tau)\]

\[\eta_j = \pm 1 \text{ identifies sublattices}\]

\[n \text{ and } L \text{ vary slowly in space and time}\]
Integrate out $L$ and take the continuum limit

$$Z = \int \mathcal{D} \mathbf{n}(x, \tau) \delta(n^2 - 1) \exp \left( -iS \sum_j \int \eta_j A_\tau(x_j, \tau) d\tau \right)$$

$$- \frac{1}{2g} \int d^2 x d\tau \left( (\partial_\tau \mathbf{n})^2 + c^2 (\nabla_x \mathbf{n})^2 \right)$$

$\eta_j = \pm 1$ identifies sublattices

Small $g$: long-range Neel order and spin-wave theory

Large $g$: duality mapping to the Quantum Dimer Model
I.C Quantum field theory for critical point

λ close to λ_c: neglect Berry phases and use “soft spin” field

\[ S_b = \int d^2x d\tau \left[ \frac{1}{2} \left( (\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right] \]

\[ \phi_\alpha \rightarrow \text{3-component antiferromagnetic order parameter} \]

Oscillations of \(\phi_\alpha\) about zero (for \(r > 0\))

\( \rightarrow \text{spin-1 collective mode} \)

\[ \text{Im} \chi (k, \omega) \]

\[ \epsilon = \Delta + \frac{c^2 k^2}{2 \Delta} \]

\(T=0\) spectrum
Critical coupling ($\lambda = \lambda_c$)

Dynamic spectrum at the critical point

$\text{Im } \chi(k, \omega) \sim (\omega - c|k|)^{-(2-\eta)}$

No quasiparticles --- dissipative critical continuum
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II. Quantum XY model in one dimension  S. Sachdev and K. Park, cond-mat/0108214

\[ H_{XY} = J_1 \sum_j (S_{x,j} S_{x,j+1} + S_{y,j} S_{y,j+1} + \lambda S_{z,j} S_{z,j+1}) \]

\[ + J_2 \sum_j (S_{x,j} S_{x,j+2} + S_{y,j} S_{y,j+2} + \lambda S_{z,j} S_{z,j+2}) \]

Write

\[ \mathbf{n}_j = (\cos(\theta_j), \sin(\theta_j), 0) \]

where \( j \) is now a discrete spacetime index upon a square lattice.

Structure of Berry phase terms.

\[ S \sum_j \eta_{\hat{\jmath}} A_{j\tau} = S \sum_j \ell_{\hat{\jmath}} \epsilon_{\mu\nu} \Delta_\mu A_{j\nu} \]

where \( \ell_{\hat{\jmath}} = 0 \) (\( \ell_{\hat{\jmath}} = 1 \)) on even (odd) columns.

For XY model, \( S\epsilon_{\mu\nu} \Delta_\mu A_{j\nu} = (2\pi S) \times \) vortex number

Vortices in odd columns carry a factor \((-1)^{2S}\).
$S$ integer.

$$Z'_{XY} = \prod_j \int d\theta_j \exp \left( \frac{1}{g} \sum_j \cos(\Delta_\mu \theta_j) \right)$$

where $\mu = x, \tau$.

This is the action for a “classical” $XY$ model in $D = 2$.
Displays Kosterlitz-Thouless transition.
Dual height model for KT transition:

$$Z'_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp \left( -\frac{1}{2g} \sum_j (\Delta_\mu \theta_j - 2\pi m_{j\mu})^2 \right)$$

This is the Villain periodic Gaussian form. Poisson summation leads to

$$Z'_{XY} = \sum_{\{p_j\}} \exp \left( -\frac{g}{2} \sum_j (\Delta_\mu p_j)^2 \right)$$

Height (or roughening) model on the square lattice
All heights are integers.
For XY model, the A 'flux' measures vortex number. In the periodic Gaussian formulation

$$S\epsilon_{\mu\nu}\Delta_\mu A_{j\nu} = \pi\epsilon_{\mu\nu}\Delta_\mu m_{j\nu}$$

So with Berry phases partition function of $S = 1/2$ quantum XY model is

$$Z_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp \left( -\frac{1}{2g} \sum_j (\Delta_\mu \theta_j - 2\pi m_{j\mu})^2 + i\pi \sum_j \ell_j \epsilon_{\mu\nu}\Delta_\mu m_{j\nu} \right)$$

Vortices in odd columns contribute a factor $(-1)$ to the partition function. Weights are not positive.

Dual height model for $Z_{XY}$

$$Z_{XY} = \sum_{\{p_j\}} \exp \left( -\frac{g}{2} \sum_j \left( \Delta_\mu p_j - \frac{1}{2} \Delta_\mu \ell_j \right)^2 \right)$$

Height model in which the heights are integers (half-integers) on even (odd) columns.
Phases of height model

Rough interface
Tomonaga-Luttinger liquid with power-law spin correlations:

\[ \langle n_{ix}n_{jx} \rangle = \langle n_{iy}n_{jy} \rangle \sim \frac{1}{|i-j|^{g/(2\pi)}} \]
\[ \langle n_{iz}n_{jz} \rangle \sim \frac{1}{|i-j|^{2\pi/g}} \]

Smooth interface
All correlations decay exponentially → there is a gap to all excitations
\[ \langle p_j - \ell_j/2 \rangle \] has a definite value: any such definite value breaks a discrete symmetry of the Hamiltonian.
\[ \langle p_j - \ell_j/2 \rangle = 0, 1/2 \text{ (plus integer)} \] are states with bond-centered charge order i.e. neighboring links have valence bonds with distinct probabilities

\[ \langle p_j - \ell_j/2 \rangle = 1/4, 3/4 \text{ (plus integer)} \] are the two states with Ising antiferromagnetic order.
Excitations of paramagnet with bond-charge-order

Deconfined $S=1/2$ spinons

Phase diagram

Luttinger liquid
(quasi-long-range XY order)
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III. Berry phases and the square lattice antiferromagnet

\[ H = \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \]

Action:

\[ S_b = \int d^2 x d\tau \left[ \frac{1}{2} \left( (\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 \right) + V(\phi_\alpha^2) \right] \]


**Missing: Spin Berry Phases**

Berry phases induce bond charge order in quantum “disordered” phase with \( \langle \phi_\alpha \rangle = 0 \);

“Dual order parameter”

\[ H = \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \]

Square lattice with first (\(J_1\)) and second (\(J_2\)) neighbor exchange interactions


S. Sachdev and K. Park, cond-mat/0108214.
Quantum dimer model —

Quantum “entropic” effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left. These effects lead to a broken square lattice symmetry near the transition to the Neel state.

Properties of paramagnet with bond-charge-order

Stable $S=1$ spin exciton – quanta of 3-component $\phi_\alpha$

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2 \Delta}$$

$\Delta \rightarrow$ Spin gap

$S=1/2$ spinons are *confined* by a linear potential.
Effect of static non-magnetic impurities (Zn or Li)

Spinon confinement implies that free $S=1/2$ moments form near each impurity

$$\chi_{\text{impurity}} (T \to 0) = \frac{S(S+1)}{3k_B T}$$

Field theory of bond order

Discretize coherent state path integral on a cubic lattice in spacetime:

\[
Z = \prod_j \int d\mathbf{n}_j \delta(\mathbf{n}_j^2 - 1) \exp \left( -\frac{1}{2g} \sum_{j,\mu} \mathbf{n}_j \cdot \mathbf{n}_{j+\hat{\mu}} - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)
\]

where \( \mu = x, y, \tau \), and we assume henceforth that \( S = 1/2 \).

For large \( g \), perform a "high temperature" expansion to obtain an effective action for the \( A_{j\mu} \). This action must be invariant under the 'gauge' transformation

\[
A_{j\mu} \to A_{j\mu} - \Delta_{\mu} \gamma_j
\]

associated with the change in choice of \( n_0 \) (\( \gamma_j \) is the oriented area of the spherical triangle formed by \( n_j \) and the two choices for \( n_0 \)). Also, it should be invariant under

\[
A_{j\mu} \to A_{j\mu} + 4\pi
\]

because area of triangle is uncertain modulo \( 4\pi \).

Simplest large \( g \) effective model

\[
Z = \prod_j \int dA_{j\mu} \exp \left( \frac{1}{e^2} \sum_{\square} \cos \left( \frac{1}{2} \epsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda} \right) - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)
\]

with \( e^2 \sim g^2 \).

This is compact QED in 2+1 dimensions with Berry phases.
Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields

\[ Z = \sum_{\{h_j\}} \exp \left( -\frac{e^2}{2} \sum \left( \Delta_\mu h_j - \Delta_\mu x_j \right)^2 \right) \]

with \( h_j \) integer.

Height model in 2+1 dimensions with ‘offsets’ \( x_j = 0, 1/4, 1/2, 3/4 \) on the four dual sublattices.
For large $e^2$, low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice.

⇒ 2+1 dimensional height model is the path integral of the quantum dimer model.

There is no roughening transition for three dimensional interfaces, which are smooth for all couplings.

⇒ There is a definite average height of the interface.

⇒ Ground state has bond-charge order.
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Parent compound of the high temperature superconductors: \( \text{La}_2\text{CuO}_4 \)

Mott insulator: square lattice antiferromagnet

Ground state has long-range magnetic (Néel) order

Néel order parameter: \( \phi_\alpha = (-1)^{i_x+i_y} S_{i\alpha} ; \quad \alpha = x, y, z \)

\[ \langle \phi_\alpha \rangle \neq 0 \]
Introduce mobile carriers of density $\delta$ by substitutional doping of out-of-plane ions e.g. $\text{La}_{2-\delta}\text{Sr}_{\delta}\text{CuO}_4$

Exhibits superconductivity below a high critical temperature $T_c$

Superconductivity in a doped Mott insulator

$\uparrow\downarrow$ ?

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid

Quantum numbers of ground state and low energy quasiparticles are the same, but characteristics of the Mott insulator are revealed in the vortices.


Zero temperature phases of the cuprate superconductors as a function of hole density

SDW along (1,1) + localized holes

Neel LRO

~0.05 ~0.12

H

SC+SDW -> SC

Theory for a system with strong interactions: describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

J. E. Sonier et al., cond-mat/0108479.
Concentration of mobile carriers $\delta$ in e.g. $24\text{La CuO}_2$.

Further neighbor magnetic couplings.

Universal properties of magnetic quantum phase transition change little along this line.

$\langle \bar{S} \rangle = 0$

Experiments

Superconductor (SC)

Magnetic ordering transitions in the insulator

Square lattice with first ($J_1$) and second ($J_2$) neighbor exchange interactions (say)

$$H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$


See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204.

Spin-Peierls (or plaquette) state

"Bond-centered charge order"

$$\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Neel state

$J_2 / J_1$
Properties of paramagnet with bond-charge-order

Stable \( S=1 \) spin exciton – quanta of 3-component \( \phi_\alpha \)

\[ \varepsilon_k = \Delta + \frac{c_x^2k_x^2 + c_y^2k_y^2}{2\Delta} \]

\( \Delta \rightarrow \) Spin gap

\( S=1/2 \) spinons are confined by a linear potential.

Transition to Neel state \( \Rightarrow \) Bose condensation of \( \phi_\alpha \)

Develop quantum theory of SC+SDW to SC transition driven by condensation of a \( S=1 \) boson (spin exciton)
Doping the paramagnetic Mott insulator

“Large $N$” theory in region with preserved spin rotation symmetry

See also J. Zaanen, *Physica C* 217, 317 (1999),
IV.A Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

\[ S_\alpha (r) = \Phi_\alpha (r) e^{iK \cdot r} + \text{c.c.} \]

\[ \Phi_\alpha (r) \] is a complex field (except for \( K=(\pi,\pi) \) when \( e^{iK \cdot r} = (-1)^{r_x+r_y} \))

Associated “charge” density wave order

\[ \delta \rho (r) \propto S^2_\alpha (r) = \sum_\alpha \Phi^2_\alpha (r) e^{i2K \cdot r} + \text{c.c.} \]

Action for SDW ordering transition in the superconductor

\[ S = \int d^2r d\tau \left[ |\nabla_r \Phi_\alpha|^2 + c^2 |\partial_\tau \Phi_\alpha|^2 + V(\Phi_\alpha) \right] \]

Similar terms present in action for SDW ordering in the insulator

Coupling to the \( S=1/2 \) Bogoliubov quasiparticles of the \( d \)-wave superconductor

Trilinear “Yukawa” coupling

\[ \int d^2r d\tau \Phi_\alpha \Psi \Psi \]

is prohibited unless ordering wavevector is fine-tuned.

\[ \kappa \sum_\alpha \int d^2r d\tau |\Phi_\alpha|^2 \Psi^\dagger \Psi \text{ is allowed} \]

Scaling dimension of \( \kappa = (1/\nu - 2) < 0 \Rightarrow \text{irrelevant.} \)
Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field


Peaks at \((0.5,0.5)\pm(0.125,0)\)
and \((0.5,0.5)\pm(0,0.125)\)
⇒ dynamic SDW of period 8

Neutron scattering off \(\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4\) \((\delta = 0.163, \text{SC phase})\)
at low temperatures in \(H=0\) (red dots) and \(H=7.5\text{T}\) (blue dots)


Using a picture of “dynamically fluctuating spins in the vortices”, the amplitude of the field-induced signal, and the volume-fraction of vortex cores (~10%), Lake *et al.* estimated that in such a model each spin in the vortex core would have a low-frequency moment equal to that in the insulating state at $\delta=0$ (0.6 $\mu_B$).

*Observed field-induced signal is much larger than anticipated.*
Concentration of mobile carriers $\delta$ in e.g. $\text{La}_2\text{CuO}_4$

Further neighbor magnetic couplings

Insulator with localized holes

$\langle \bar{S} \rangle = 0$

Magnetic order

$\langle \bar{S} \rangle \neq 0$

Superconductor (SC)

Concentration of mobile carriers $\delta$

in e.g. $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$

**IV.B Phase diagrams in a magnetic field.**

A. Effect of magnetic field on onset of SDW on insulator

B. Effect of magnetic field on SC+SDW to SC transition
IV.B Phase diagrams in a magnetic field.

A. Effect of magnetic field on onset of SDW in the insulator

$H$ couples via the Zeeman term

$$\left| \partial_\tau \Phi_\alpha \right|^2 \Rightarrow \left( \partial_\tau \Phi_\alpha^* - i\epsilon_{\alpha\sigma\rho} H_\sigma \Phi_\rho \right) \left( \partial_\tau \Phi_\alpha - i\epsilon_{\alpha\beta\gamma} H_\beta \Phi_\gamma \right)$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Related theory applies to spin gap systems in a field and to double layer quantum Hall systems at $v=2$
### IV.B Phase diagrams in a magnetic field.

#### Infinite diamagnetic susceptibility of non-critical superconductivity leads to a strong effect.

- Theory should account for *dynamic* quantum spin fluctuations.
- All effects are $\sim H^2$ except those associated with $H$ induced superflow.
- Can treat SC order in a *static* Ginzburg-Landau theory.

#### Formulas

- **$S_b$**:
  \[
  S_b = \int d^2r \int_0^{1/T} d\tau \left[ |\nabla_r \Phi_\alpha|^2 + c^2 |\partial_\tau \Phi_\alpha|^2 + s |\Phi_\alpha|^2 + \frac{g_1}{2} \left( |\Phi_\alpha|^2 \right)^2 + \frac{g_2}{2} |\Phi_\alpha|^2 \right]
  \]

- **$S_c$**:
  \[
  S_c = \int d^2r d\tau \left[ \frac{\nu}{2} |\Phi_\alpha|^2 |\psi|^2 \right]
  \]

- **$F_{GL}$**:
  \[
  F_{GL} = \int d^2r \left[ -|\psi|^2 + \frac{|\psi|^4}{2} + |(\nabla_r - iA)\psi|^2 \right]
  \]

- **Z**:
  \[
  Z[\psi(r)] = \int D\Phi(r, \tau) e^{-F_{GL} - S_b - S_c}
  \]

- **$\delta \ln Z[\psi(r)]/\delta \psi(r) = 0$**
Envelope of lowest energy spin-exciton eigenmode $\Phi_\alpha$

after including exciton interactions:

$V(r) = V_0(r) + g \langle |\Phi_\alpha(r)|^2 \rangle$

Potential $V_0(r) = s + \nabla|\psi(r)|^2$

acting on excitons

---

Strongly relevant repulsive interactions between excitons imply that low energy excitons must be extended.

Dominant effect: uniform softening of spin excitations by superflow kinetic energy

\[ \nu_s \sim \frac{1}{r} \]

Spatially averaged superflow kinetic energy

\[ \sim \langle \nu_s^2 \rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H} \]

Tuning parameter \( s \) replaced by \( s_{\text{eff}} (H) = s - C \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right) \)
Main results

Elastic scattering intensity

\[ I(H) = I(0) + a \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right) \]

• All functional forms are exact.

Structure of long-range SDW order in SC+SDW phase

Computation in a self-consistent “large $N$” theory

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343

$$\left\langle \Phi_\alpha (r) \right\rangle$$

Dynamic structure factor

$$S(k, \omega) = (2\pi)^3 \delta(\omega) \sum_G |f_G|^2 \delta(k-G) + \cdots$$

$G \rightarrow$ reciprocal lattice vectors of vortex lattice.

$k$ measures deviation from SDW ordering wavevector $K$
Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La$_2$CuO$_{4+y}$


Solid line --- fit to:
\[
\frac{I(H)}{I(0)} = 1 + a \frac{H}{H_{c2}} \ln \left( \frac{3.0H_{c2}}{H} \right)
\]

$a$ is the only fitting parameter
Best fit value - $a = 2.4$ with $H_{c2} = 60$ T
Neutron scattering of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ at $x=0.1$


Solid line - fit to: $I(H) = a \frac{H}{H_{c2}} \ln\left( \frac{H_{c2}}{H} \right)$
Dynamic SDW fluctuations in the SC phase

Computation of spin susceptibility $\chi''(k, \omega)$ in self-consistent large $N$ theory of $\Phi_\alpha$ fluctuations

Field $H$ chosen to place the system close to boundary to SC+SDW phase

$2\pi / (\text{vortex lattice spacing})$
Prediction of static CDW order by vortex cores in SC phase, with dynamic SDW correlations

“Spin gap” state in vortex core appears by a “local quantum disordering transition” of magnetic order: by our generalized phase diagram, charge order should appear in this region.

K. Park and S. Sachdev
Pinning of static CDW order by vortex cores in SC phase, with dynamic SDW correlations

A.Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329
Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343

Superflow reduces energy of dynamic spin exciton, but action so far does not lead to static CDW order because all terms are invariant under the “sliding” symmetry:

$$\Phi_\alpha (r) \rightarrow \Phi_\alpha (r) e^{i\theta}$$

Small vortex cores break this sliding symmetry on the lattice scale, and lead to a pinning term, which picks particular phase of the local CDW order

$$S_{\text{pin}} = \zeta \sum_{\text{All } r_v \text{ where } \psi(r_v)=0} \int_0^{1/T} d\tau [\Phi_\alpha^2 (r_v) e^{i\theta} + \text{c.c.}]$$

With this term, SC phase has static CDW but dynamic SDW

$$\langle \Phi_\alpha^2 (r) \rangle \neq 0 ; \quad \langle \Phi_\alpha (r) \rangle = 0$$

$$\delta \rho (r) \propto \sum_\alpha \Phi_\alpha^2 (r) e^{i2K \cdot r} + \text{c.c.} ; \quad S_\alpha (r) = \Phi_\alpha (r) e^{iK \cdot r} + \text{c.c.}$$

“Friedel oscillations of a doped spin-gap antiferromagnet”
Pinning of CDW order by vortex cores in SC phase

Computation in self-consistent large $N$ theory

$$\left\langle \Phi_{\alpha}^2 (r, \tau) \right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_{\alpha} (r, \tau) \Phi_{\alpha}^* (r_v, \tau_1) \right\rangle^2$$

- $\square \rightarrow$ low magnetic field
- $\triangle \rightarrow$ high magnetic field near the boundary to the SC+SDW phase

Diagram:
- Envelope of static charge order
- Wavefunction of dynamically fluctuating exciton above the spin gap

Graphical data points:
- $\langle \Phi_{\alpha}^2 (r, \tau) \rangle$
Simplified theoretical computation of modulation in local density of states at low energy due to CDW order induced by superflow and pinned by vortex core

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329

\[
H = \sum_{ij} \left( -t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \Delta_{ij} c_{i\sigma}^\dagger c_{j,-\sigma} + \text{h.c.} \right) + \sum_i \left[ v(r_i) - \mu \right] c_{i\sigma}^\dagger c_{i\sigma},
\]

\[
v(r) = v_1 \left\{ \cos \left[ K_{ex} \cdot (r - r_0) \right] + \cos \left[ K_{ey} \cdot (r - r_0) \right] \right\} e^{-|r-r_0|/\xi_c} \left( |r - r_0|^2 + 1 \right)^{-3/4}
\]
STM around vortices induced by a magnetic field in the superconducting state


**Local density of states**

![Graph showing differential conductance vs. sample bias](image)

1Å spatial resolution image of integrated LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

Vortex-induced LDOS integrated from 1meV to 12meV

Fourier Transform of Vortex-Induced LDOS map

Distances in k-space have units of $2\pi/a_0$

$a_0=3.83$ Å is Cu-Cu distance

Doping the paramagnetic Mott insulator

“Large $N$” theory in region with preserved spin rotation symmetry

See also J. Zaanen, *Physica C* 217, 317 (1999),
Main results

Neutron scattering observation of SDW order enhanced by superflow.

STM observation of CDW fluctuations enhanced by superflow and pinned by vortex cores.

Prospects for studying quantum critical point between SC and SC+SDW phases by tuning H?
Conclusion: Framework for spin/charge order in cuprate superconductors

Confined, paramagnetic Mott insulator has

1. Stable $S=1$ spin exciton $\phi_\alpha$.
3. $S=1/2$ moments near non-magnetic impurities

Further neighbor magnetic couplings

La$_2$CuO$_4$

Magnetic order $\langle \bar{S} \rangle \neq 0$

Experiments

Concentration of mobile carriers $\delta$

$\langle \bar{S} \rangle = 0$

$T=0$

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations