Quantum phase transitions in d-wave superconductors

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cond-mat/0007170
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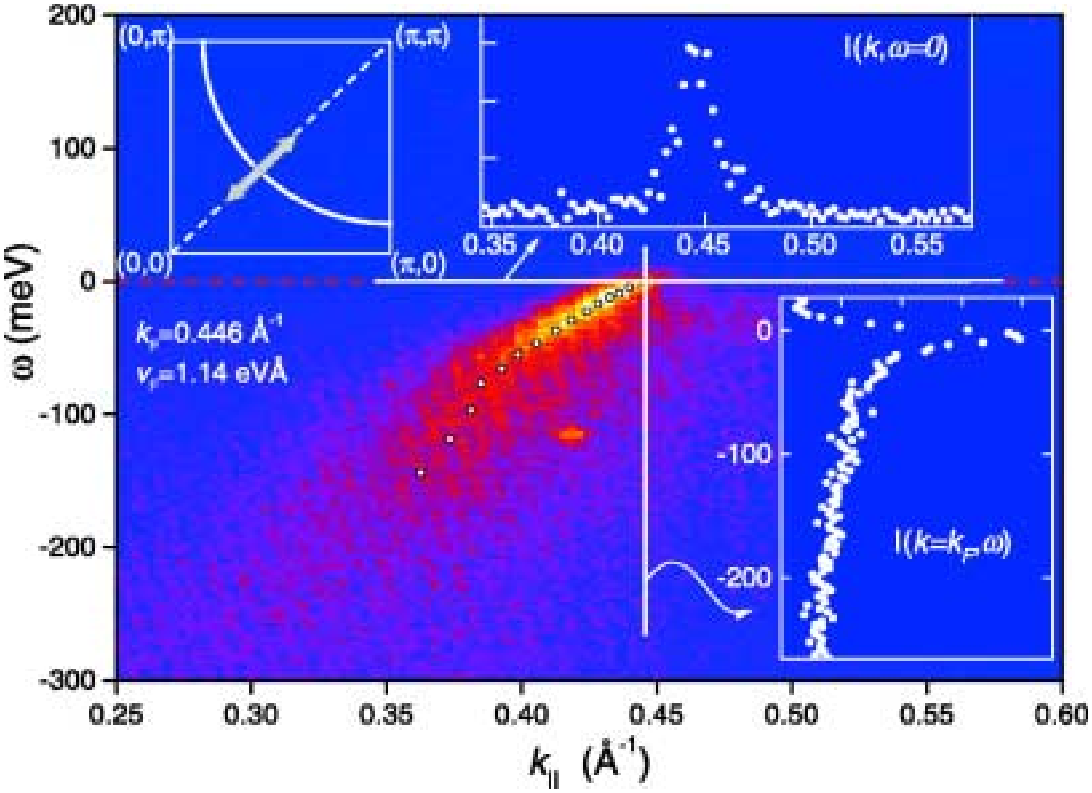
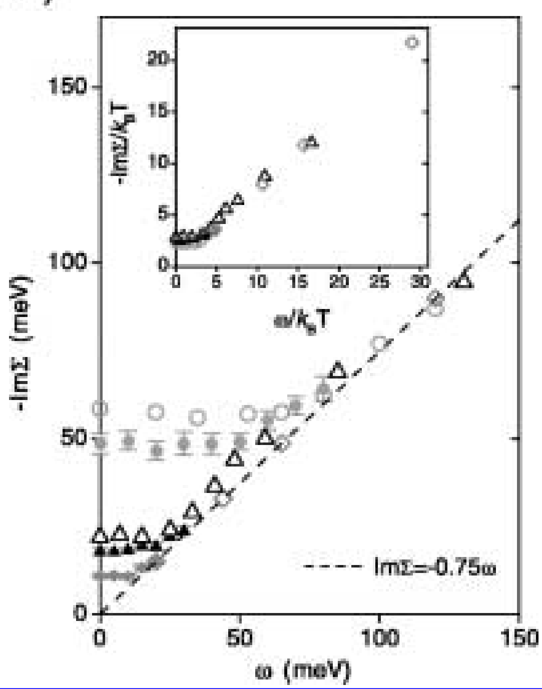
University

Photoemission on BSSCO

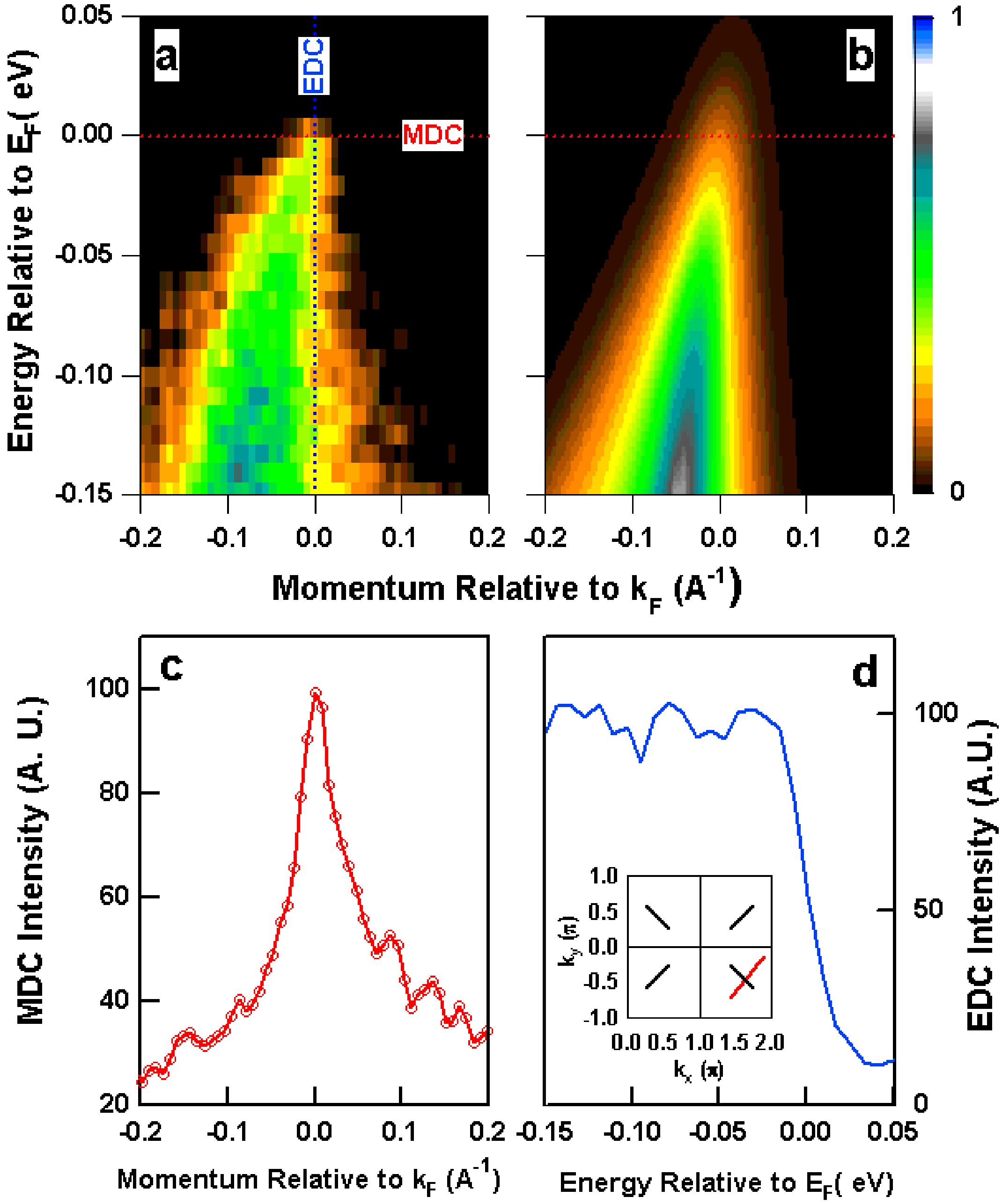
(Valla et al Science **285**, 2110 (1999))





Quantum-critical damping of quasi-particles along (1,1)

D. Orgad *et al*, cond-mat/0005457 :
Photoemission on LNSCO



Large ω tail in the fermion spectral function

$$G(k,\omega) \sim \frac{1}{(v_F k - \omega)^{1-\eta_F}}$$

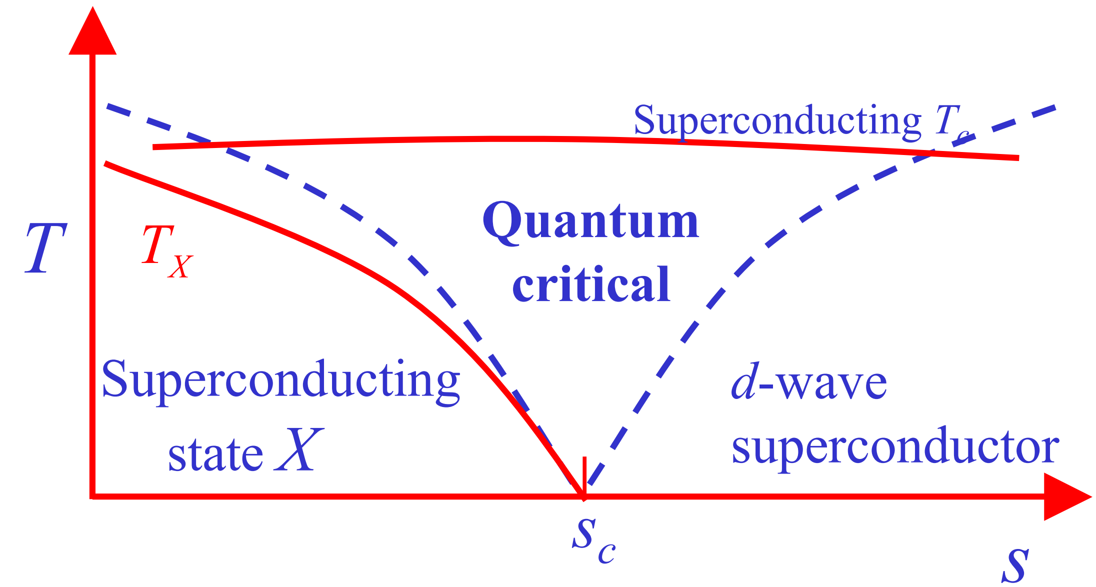
Goal: Classify theories in which, with minimal fine tuning, a *d*-wave superconductor has, for $T \ll T_c$, fermionic quasiparticle spectral functions at the nodal Fermi points with an energy width of order $k_B T$

In a Fermi liquid, spectral width $\sim T^2$

In a BCS d-wave superconductor, spectral width $\sim T^3$

We will find that theories which have a spectral-width of order T also have a large ω tail in nodal quasiparticle spectral function

Proximity to a quantum-critical point



(Crossovers analogous to those near quantum phase transitions in boson models
Weichmann *et al* 1986, Chakravarty *et al* 1989)

Relaxational dynamics in quantum critical region
(Sachdev+Ye, 1992)

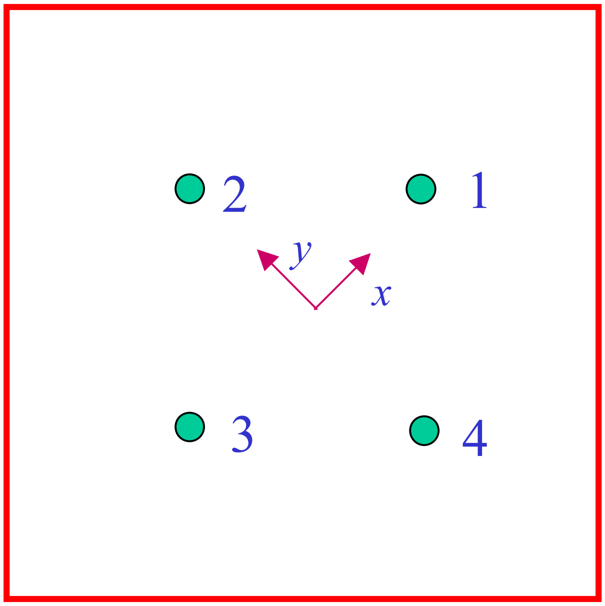
$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \Phi\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$

Nodal quasiparticle Green's function
$k \rightarrow$ wavevector separation from node

Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Nodal quasi-particles should be part of the critical-field theory.
3. Critical field theory should not be free – required to obtain damping in the scaling limit.

Low energy fermionic excitations of a *d*-wave superconductor



Gapless Fermi Points in a *d*-wave superconductor at wavevectors $(\pm K, \pm K)$

$K = 0.391\pi$

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^* \\ f_{1\downarrow} \\ -f_{3\uparrow}^* \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^* \\ f_{2\downarrow} \\ -f_{4\uparrow}^* \end{pmatrix}$$

$$\begin{aligned} S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_1 \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_2 . \end{aligned}$$

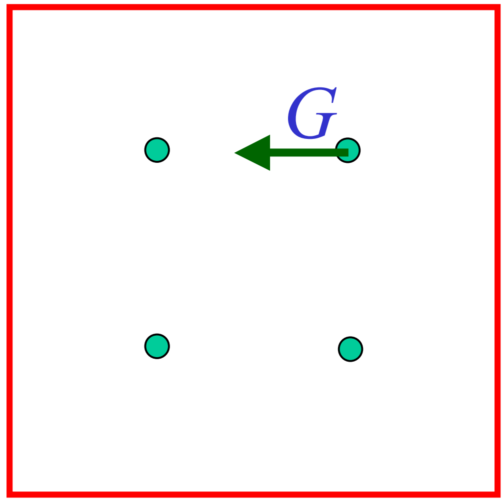
τ^x, τ^z are Pauli matrices in Nambu space

A spin-singlet, fermion bilinear, zero momentum order parameter for X is preferred.

e.g. An order parameter with momentum G:
Charge density-wave order

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$

If $G \neq 2K$ fermions are not part of the critical theory



Action for quantum fluctuations of order parameter

$$S_\Phi = \int d^2x d\tau \Big[|\partial_\tau \Phi_x|^2 + |\partial_\tau \Phi_y|^2 + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + s\left(|\Phi_x|^2 + |\Phi_y|^2\right) + \frac{u_0}{2}\left(|\Phi_x|^4 + |\Phi_y|^4\right) + v_0 |\Phi_x|^2 |\Phi_y|^2 \Big]$$

Coupling to fermions $\sim \lambda \int d^d x d\tau |\Phi_a|^2 \Psi^\dagger \tau^z \Psi$
and λ is irrelevant at the critical point

$$\mathrm{Im}\Sigma \sim T^{2d+1-2/\nu}$$

$$\sim T^{(\text{between 2 and 3})} \text{ for } 2/3 < \nu < 1$$

For *G=2K*, charge-density-wave (or spin-density wave) ordering in *X* satisfies conditions 1,2,3.

However, this is probably an additional fine-tuning requirement.

<u>The staggered flux state</u>:

The order parameter has *G*=(π,π). However, even for *G=2K*, fermions couple to the order parameter by a gradient coupling and are not part of the critical theory.

Order parameter for X should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c^{\dagger}_{k\alpha} c_{k\alpha} \rangle \text{ (excitonic order)}$$

Complete group-theoretic classification

X has $d_{x^2-y^2}$ pairing plus

(A) is pairing

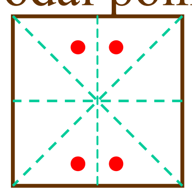
(B) id_{xy} pairing

→ fermion spectrum fully gapped

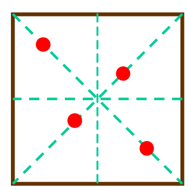
(C) ig pairing

Nodal points

(D) s pairing

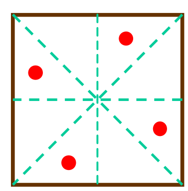


(E) d_{xy} excitons

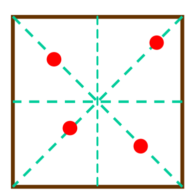


(D), (E) → superconducting nematics

(F) d_{xy} pairing



(G) p excitons



Quantum field theory for critical point

Ising order parameter ϕ (except for case (G))

$$S_\phi = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s}{2}\phi^2 + \frac{u}{24}\phi^4 \right]$$

Coupling to nodal fermions

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda\phi \left(\Psi_1^\dagger M_1 \Psi_1 + \Psi_2^\dagger M_2 \Psi_2 \right) \right].$$

(A) $M_1 = \tau^y$; $M_2 = \tau^y$

(B) $M_1 = \tau^y$; $M_2 = -\tau^y$

(C) λ=0, so fermions are not critical

(D) $M_1 = \tau^x$; $M_2 = \tau^x$

(E) $M_1 = \tau^z$; $M_2 = -\tau^z$

(F) $M_1 = \tau^x$; $M_2 = -\tau^x$

(G) $M_1 = 1$; $M_2 = 1$ but ϕ has 2 components

Main results

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with

$\lambda = \lambda^* \neq 0$ and $u = u^* \neq 0$

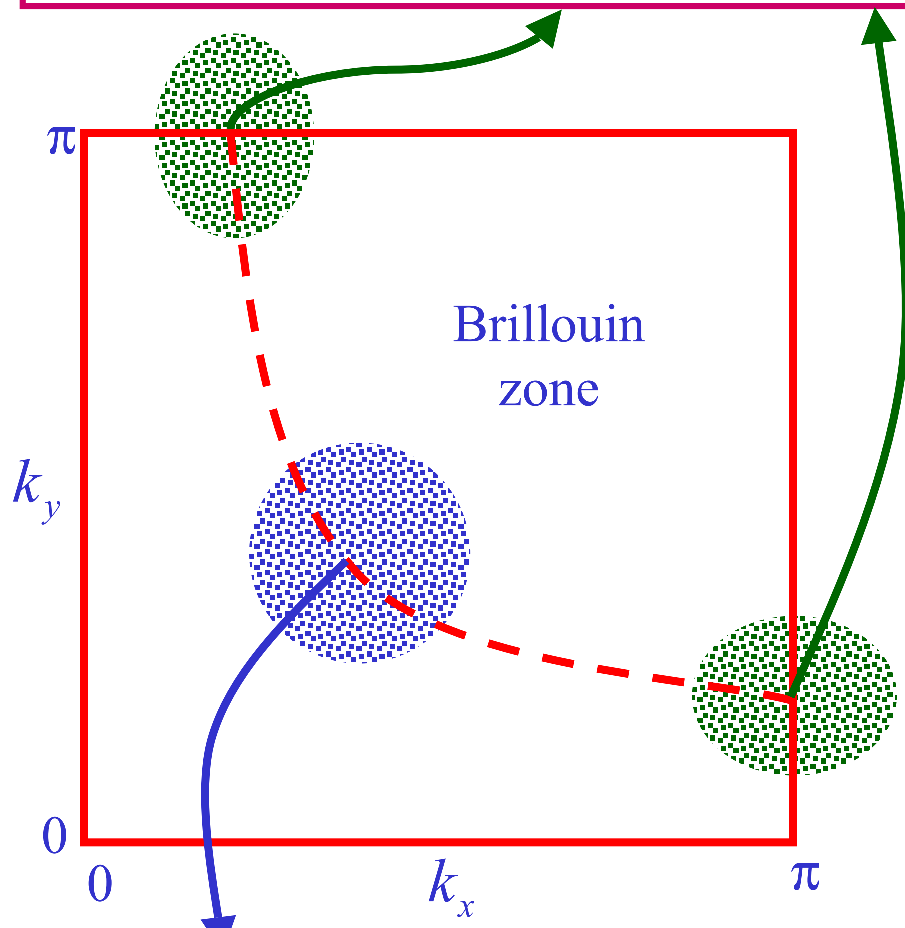
Only cases (A) and (B) satisfy conditions 1,2,3

d_{xy} pairing vanishes along the (1,0),(0,1) directions, and so only case (B) does not strongly scatter the anti-nodal quasiparticles

Gapped quasiparticles:

Below T_c : negligible damping

Above T_c: damping from strong coupling to superconducting phase and SDW fluctuations.



Nodal quasiparticles:

Below T_c : damping from fluctuations to $d_{x^2-y^2} + id_{xy}$ order

Above T_c: same mechanism applies as long as quantum-critical length < superconducting phase coherence length. Quasiparticles do not couple to phase or SDW fluctuations.

Speculation

Near optimal doping we have $s>s_c$ for a state X with $d_{x^2-y^2}+id_{xy}$ order.

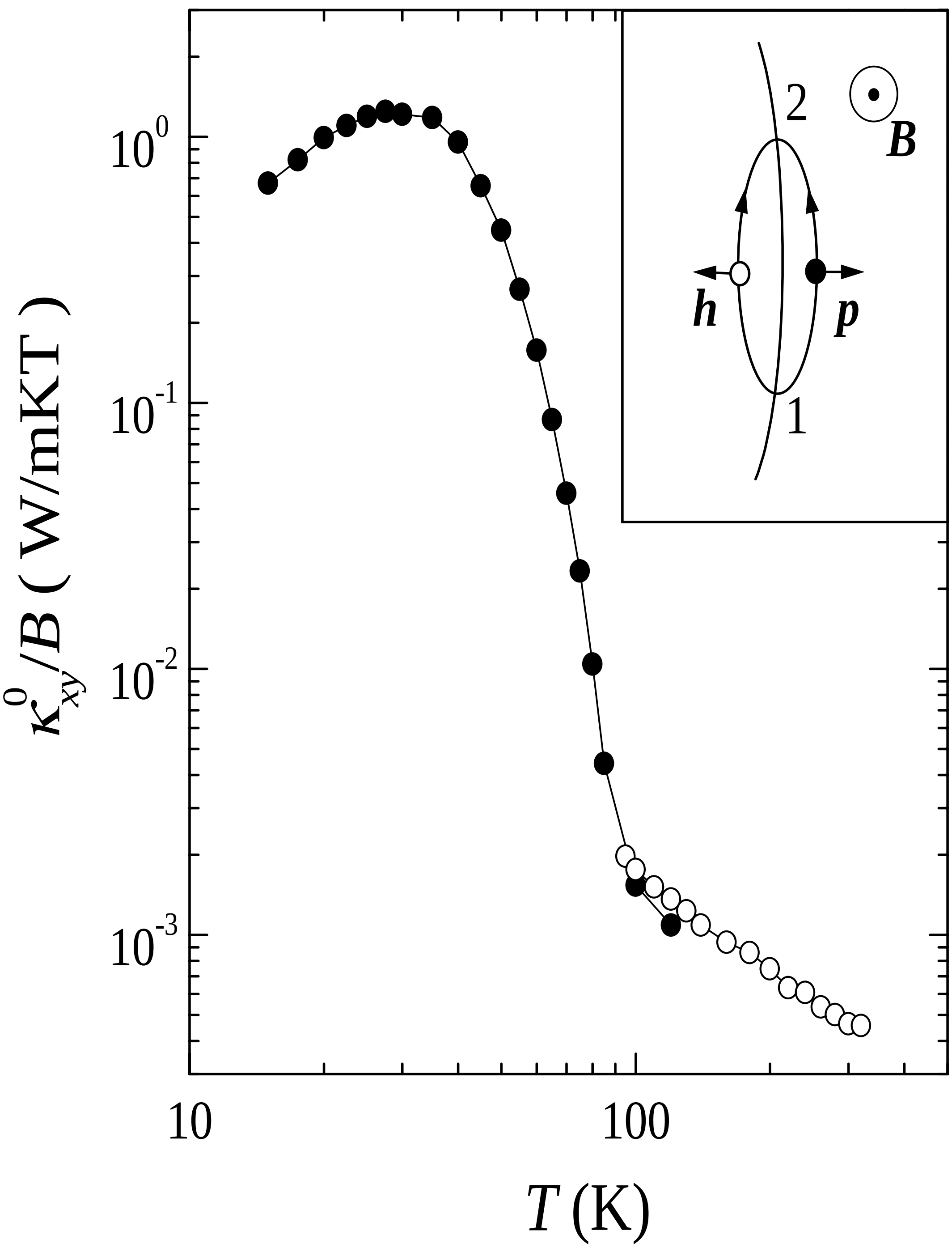
Such a state has a non-zero κ_{xy} for $T<T_X$ even in zero external field (Laughlin, Senthil *et al)*

So for $s>s_c$ we can expect large enhancement of κ_{xy} in the quantum-critical region.

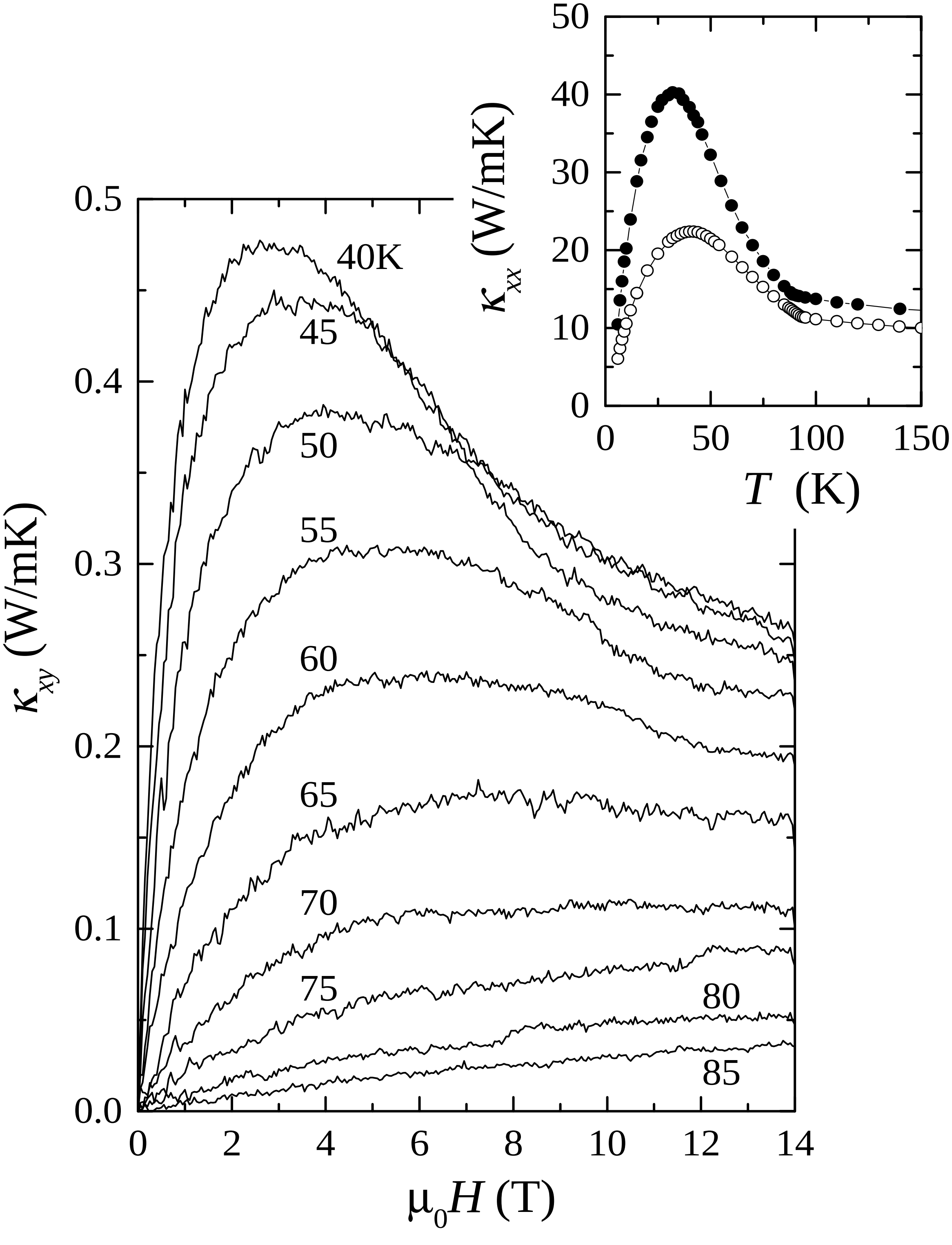
Enhancement is suppressed upon crossover out of quantum-criticality into an ordinary *d*-wave superconductor at low temperatures.

Experiments of Zhang *et al* (cond-mat/0008140) suggest that this crossover happens ~ 28 K.

<u>Prediction</u>: the quasi-particle spectral linewidths should also stop being ~ T at temperatures ~ 28 K



Zhang *et al* (cond-mat/0008140)



Zhang *et al* (cond-mat/0008140)