Quantum Criticality and Black Holes

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Three foci of modern physics

Quantum phase transitions
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Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)
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Quantum phase transitions

Black holes
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Black holes

Bekenstein and Hawking originated the quantum theory, which has found fruition in string theory
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Quantum phase transitions

Hydrodynamics

Black holes
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Quantum phase transitions

Hydrodynamics
Universal description of fluids based upon conservation laws and positivity of entropy production

Black holes
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- Quantum phase transitions
- Hydrodynamics
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- Hydrodynamics
- Canonical problem in condensed matter: transport properties of a correlated electron system
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Hydrodynamics

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New insights and results from detour unifies disparate fields of physics
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Hydrodynamics

Black holes
An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

TlCuCl$_3$ at ambient pressure

Sharp spin 1 particle excitation above an energy gap (spin gap)

Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in Néel state.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Pressure in TICuCl$_3$
Quantum critical point with non-local entanglement in spin wavefunction

\[
\frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)
\]
Excitation spectrum in the paramagnetic phase
Excitation spectrum in the paramagnetic phase
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TlCuCl$_3$ at ambient pressure

Sharp spin 1 particle excitation above an energy gap (spin gap)

Excitation spectrum in the Néel phase
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Description using Landau-Ginzburg field theory

\[ S = \int d^2 r d\tau \left[ (\partial_\tau \varphi)^2 + c^2 (\nabla_r \varphi)^2 + (\lambda - \lambda_c) \varphi^2 + u (\varphi^2)^2 \right] \]
Excitation spectrum in the paramagnetic phase

$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$

$\lambda > \lambda_c$

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

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Spin \( S = 1 \) “triplon”
Excitation spectrum in the paramagnetic phase

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Excitation spectrum in the Néel phase
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Spin waves
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Excitation spectrum in the Néel phase

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\[ \lambda < \lambda_c \]
Excitation spectrum in the Néel phase

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle.
Observation of $3 \rightarrow 2$ low energy modes, emergence of new Higgs particle in the Néel phase, and vanishing of Néel temperature at the quantum critical point.

Prediction of quantum field theory

Energy of “Higgs” particle  \[\sqrt{2}\]

Energy of triplon

\[V(\bar{\phi}) = (\lambda - \lambda_c)\bar{\phi}^2 + u(\bar{\phi}^2)^2\]

\[\lambda = 1.07 \text{ kbar}\]

\[T = 1.85 \text{ K}\]

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Black holes
Superfluid-insulator transition

Indium Oxide films

Superfluid-insulator transition

Ultracold $^87\text{Rb}$ atoms - bosons

The image depicts a phase diagram with axes labeled $T$ (temperature) and $g$ (possibly a coupling constant). The diagram illustrates a quantum critical point $g_c$ where the system transitions between superfluid and insulator states. The $T_{KT}$ line separates the superfluid region from the quantum critical region, and the $g$ axis extends to the right with a note indicating $g_c$. The diagram highlights the transition between different quantum phases.
Classical vortices and wave oscillations of the condensate

Dilute Boltzmann/Landau gas of particle and holes

Superfluid

Quantum critical

Insulator
CFT at $T>0$

Quantum critical

Superfluid

Insulator

$T_{KT}$
Quantum critical transport

Quantum "perfect fluid" with shortest possible relaxation time, $\tau_R$

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

Quantum critical transport

Transport co-efficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

\[ \sigma = \frac{e^2}{h} \times [\text{Universal constant } O(1) ] \]

Quantum critical transport

Transport coefficients not determined by collision rate, but by universal constants of nature

Momentum transport

\[
\eta \equiv \frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}} = \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1) ]
\]

Superfluid-insulator transition

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Black holes
Black Holes

Objects so massive that light is gravitationally bound to them.
Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Entropy of a black hole $S = \frac{k_B A}{4\ell_P^2}$

where $A$ is the area of the horizon, and

$\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$
Black Hole Thermodynamics

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Horizon temperature: $4\pi k_B T = \frac{\hbar^2}{2M \ell_P^2}$
AdS/CFT correspondence
The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

Maldacena, Gubser, Klebanov, Polyakov, Witten
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space

A 2+1 dimensional system at its quantum critical point
AdS/CFT correspondence
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$3+1$ dimensional AdS space

Black hole temperature $=$ temperature of quantum criticality

Quantum criticality in 2+1 dimensions

Maldacena, Gubser, Klebanov, Polyakov, Witten
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Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten
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Kovtun, Policastro, Son
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2. Hydrodynamics
3. Black holes

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New insights and results from detour unifies disparate fields of physics
Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics

Uses physical model but strong-coupling makes explicit solution difficult
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   *Yields hydrodynamic relations which apply to general classes of quantum critical systems. First exact numerical results for transport co-efficients (for supersymmetric systems).*
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Find perfect agreement between 1. and 2. In some cases, results were obtained by 2. earlier !!
Applications:

1. Magneto-thermo-electric transport near the superconductor-insulator transition and in graphene
   *Hydrodynamic cyclotron resonance*
   *Nernst effect*

2. Quark-gluon plasma
   *Low viscosity fluid*

3. Fermi gas at unitarity
   *Non-relativistic AdS/CFT*
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The cuprate superconductors
The cuprate superconductors

Proximity to an insulator at 12.5% hole concentration in the “underdoped” regime
Underdoped Cuprates

Superconductor

Insulator $x=1/8$
Underdoped Cuprates

STM observations of VBS modulations by Y. Kohsaka et al., Nature 454, 1072 (2008)

Insulator $x = 1/8$
Underdoped Cuprates
Underdoped Cuprates

Thermoelectric measurements

Superconductor

Insulator $x=1/8$

CFT?
Underdoped Cuprates

Thermoelectric measurements

Superconductor

Insulator $x = 1/8$

CFT?
Hydrodynamic cyclotron resonance at a frequency

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and with width

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

where $B = \text{magnetic field}$, $\rho = \text{charge density away from density of CFT}$, $\varepsilon = \text{energy density}$, $P = \text{pressure}$, $v = \text{velocity of “light” in CFT}$, and $\sigma_Q e^2/h$ is the universal conductivity of the CFT.
“Wiedemann-Franz”-like relation for thermal conductivity, \( \kappa \) at \( B = 0 \)

\[
\kappa = \sigma_Q \left( \frac{k_B^2 T}{e^*^2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2.
\]

At \( B \neq 0 \) and \( \rho = 0 \) we have a “Wiedemann-Franz” relation for “vortices”

\[
\kappa = \frac{1}{\sigma_Q} k_B^2 T \left( \frac{\nu(\varepsilon + P)}{k_B T \rho} \right)^2.
\]

Nernst experiment
Nernst signal (transverse thermoelectric response)

\[ e_N = \left( \frac{k_B}{e^*} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right) \left[ \frac{\omega_c/\tau_{\text{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \]

where \( \tau_{\text{imp}} \) is the momentum relaxation time due to impurities or umklapp scattering.

LSCO Experiments

$B$ and $T$ dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, $\tau_{\text{imp}}$ and $v$. 
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Similar results apply to electronic transport in graphene, where the relativistic Dirac spectrum of the electrons leads to analogies with the hydrodynamics of CFTs. We have made specific quantitative predictions for THz experiments on graphene at room temperature in the presence of a modest applied magnetic field.
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Au+Au collisions at RHIC

Quark-gluon plasma can be described as "quantum critical QCD"
Phases of nuclear matter
S=1/2 Fermi gas at a Feshbach resonance
= detuning from Feshbach resonance
RG fixed point described by a “non-relativistic” CFT

= detuning from Feshbach resonance
CFT is dual to quantum gravity models on AdS space. Explicit solutions of such gravity models with supersymmetry have been obtained.

\[ \frac{\eta}{s} \equiv \text{viscosity} / \text{entropy density} \]

Ultracold $^6$Li gas at Feshbach resonance

\[ \frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}} \]

- Ultracold $^6\text{Li}$ gas at Feshbach resonance
- Quark gluon plasma

References:

\[ \eta \equiv \frac{\text{viscosity}}{s} = \frac{\text{entropy density}}{s} \]

Ultracold \(^6\text{Li}\) gas at Feshbach resonance

Quark gluon plasma

Supersymmetric black hole theory

\[ \eta \equiv \frac{\text{viscosity}}{\text{entropy density}} \]

Ultracold $^6$Li gas at Feshbach resonance

Quark gluon plasma

Supersymmetric black hole theory


A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.