A simple model of many-particle entanglement: how it describes black holes and superconductors

New Horizons in Physics-IPA50
Commemorating 50 years of Indian Physics Association
APS-IPA Joint Lecture
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Talk online: sachdev.physics.harvard.edu
In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.
Quantum entanglement
Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ |\uparrow\rangle \]

Hydrogen molecule:

\[ = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle) \]
Quantum Entanglement: quantum superposition with more than one particle
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Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away.
Quantum entanglement
Quantum entanglement

Black holes
LOFAR LBA Sky Survey showing 25000 supermassive black holes on 4% of the northern sky. Obtained by 52 radio telescopes across Europe.

de Gasperin et al. (2021)
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)

\( G \) Newton’s constant, \( c \) velocity of light, \( M \) mass of black hole
Black Holes

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Horizon radius $R = \frac{2GM}{c^2}$

$G$ Newton’s constant, $c$ velocity of light, $M$ mass of black hole

For $M = \text{earth’s mass}$, $R \approx 9 \text{ mm}$!
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$.
- The entropy, $S_{BH}$ is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
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All many-body quantum systems (without quantum gravity) have an entropy proportional to their volume !?!?

J. D. Bekenstein, PRD 7, 2333 (1973)
Thermodynamics of quantum black holes:

\[ \int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Einstein gravity}}^{(d)} \right) \]

Quantum gravity: a summation over all possible configurations of spacetime, each weighted by a factor which is the exponential of (the ‘action’ of Einstein gravity)/(Planck’s constant)
Thermodynamics of quantum black holes:

\begin{align*}
\int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Einstein gravity}}^{(d)}[g_{\mu\nu}] \right) \\
= \exp(S_{BH}) \times \left( \ldots \right)
\end{align*}

Gibbons, Hawking (1977)

Quantum gravity: a summation over all possible configurations of spacetime, each weighted by a factor which is the exponential of (the ‘action’ of Einstein gravity)/(Planck’s constant)
Holography and duality

Thermodynamics of quantum black holes:

\[ \int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Einstein gravity}}^{(d)}[g_{\mu\nu}] \right) \]

\[ = \exp(S_{BH}) \times \left( \text{Many body quantum theory in } d - 1 \text{ dimensions without gravity} \right) \]

Black holes are represented as a `hologram' by a quantum many-body system in one lower dimension.

**Duality:** a `change of variables' between the many-particle configurations and the metric of spacetime

Susskind, Maldacena…..
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**Duality:** a `change of variables’ between the many-particle configurations and the metric of spacetime.
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away 0.1 seconds later!
The ring-down time \( \frac{8\pi GM}{c^3} \sim 8 \) milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals \( \frac{\hbar}{k_B T_H} \), where \( \hbar \) Planck’s constant, \( k_B \) Boltzmann’s constant.
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• Black holes have an entropy and a temperature, $T_H$

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• They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$. 
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Quantum Black holes

- Black holes have an entropy and a temperature, \( T_H \).
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time \( \sim \frac{\hbar}{k_B T_H} \).

The hologram of a black hole in \( d \) dimensions is a quantum many-particle system in \((d - 1)\) dimensions which relaxes to thermal equilibrium in a Planckian time \( \sim \frac{\hbar}{k_B T} \).
Quantum entanglement

Black holes

A simple many-particle (SYK) model
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle. The existence of quasiparticles implies limited many-particle entanglement.

R.D. Mattuck
Current flow with quasiparticles

Flowing quasiparticles scatter off each other in a typical scattering time $\tau$

This time is much longer than a limiting ‘Planckian time’ $\frac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Place electrons randomly on some sites

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Place electrons randomly on some sites

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Place electrons randomly on some sites

Sachdev, Ye (1993); Kitaev (2015)
Entangle electrons pairwise randomly
The SYK model

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Complex multi-particle entanglement in the SYK model leads to a state without ‘quasiparticle’ excitations; i.e. multiple excitations cannot be built by composing an elementary set of ‘quasiparticle’ excitations.
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Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \frac{\hbar}{k_B T}$. 
Quantum entanglement

Black holes

A simple many-particle (SYK) model

Hologram?
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

(Similar considerations also apply to rapidly rotating black holes, Moitra, Sake, Trivedi, Vishal (2019))
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

(Similar considerations also apply to rapidly rotating black holes, Moitra, Sake, Trivedi, Vishal (2019))

Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space ($\vec{x}$) and one time dimension.
SYK model and charged black holes

Horizon
Area yields Bekenstein-Hawking entropy $S_{BH}$

$\text{AdS}_2 \times S^2$

Boundary graviton

$\vec{x}$

$\zeta$
SYK model and charged black holes

\( \text{AdS}_2 \times S^2 \)

Boundary graviton

1+1 spacetime dimensions

3+1 spacetime dimensions
SYK model and charged black holes

Quantum gravity can be exactly solved in this region!
Thermodynamics of charged quantum black holes

\[ \int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Einstein–Maxwell theory}}^{(3+1)} \left[ g_{\mu\nu} \right] \right) \quad T \to 0, \]

\[ \approx \int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Gravity of AdS}_2 \text{ and boundary}}^{(1+1)} \left[ g_{\mu\nu} \right] \right) \]
SYK model and charged black holes

Thermodynamics of charged quantum black holes

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\[ \approx \int \mathcal{D}g_{\mu\nu} \exp \left( -\frac{1}{\hbar} S_{\text{Gravity of AdS}_2 \ \text{and boundary}}^{(1+1)}[g_{\mu\nu}] \right) \]

\[ = \exp \left( S_{BH} \right) \times \exp \left( -\frac{1}{T} \times \text{Free energy of SYK model} \right) \]

The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. The near-horizon 1+1D-gravity theory is precisely that of the low T limit of the SYK models.
Quantum entanglement

Charged black holes

Low temperatures

A simple many-particle (SYK) model

Quantum gravity in 1+1 dimensions
Complex multi-particle entanglement leads to quantum systems without quasiparticle excitations.

Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \frac{\hbar}{k_B T}$. 
Quantum entanglement

Charged black holes

A simple many-particle (SYK) model

Copper-based superconductors
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
High temperature superconductors

CuO$_2$ plane

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Insulating antiferromagnet
Antiferromagnet doped with hole density $p$
Real-space view

$p$ mobile holes in a background of fluctuating spins
$p$ mobile holes in a background of fluctuating spins

Baskaran, Anderson (1988)
$p$ mobile holes in a background of fluctuating spins

Baskaran, Anderson (1988)
Real-space view

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Baskaran, Anderson (1988)
Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.

\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T \]
Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.

\[
\frac{1}{\tau} = \frac{1}{\tau_{\text{iso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T
\]
Maria Tikhanovskaya

Grigory Tarnopolsky

Haoyu Guo

arXiv:2010.09742

arXiv:2012.14449
Random $t$-$J$ model
Random $t$-$J$ model
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Random $t$-$J$ model
Random $t-J$ model
Random $t$-$J$ model
Random $t-J$ model
Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar \omega$
Near criticality, the model is predicted to exhibit SYK behavior of finite systems. The spectral function for the random model is suitable for describing behavior of low-frequency humps for dopings shown for several values of doping in Fig. 4. Plotted is the dynamic spin susceptibility, which is a measure of the probability to flip an electron spin while absorbing energy $\hbar \omega$. The graph shows the spectral weight obtained by rescaling the solution near the critical point at low frequencies. The spectral function shows agreement with the large-$N$ prediction for $p < p_c \approx 0.4$.

Spin glass order is found below a critical temperature, and the spin glass contribution to $G_m$ is non-zero. Our analysis gives a large-$N$ model in the thermodynamic limit, whereas the spectral weight, shown in Fig. 4, remains non-zero. Our analysis gives a large-$M$ model in the holographic dual, while preserving total integrated weight for $\delta E$. Away from $\delta T = 0$, the residual SYK result and a low-frequency hump are observed. A large-$N$ model is obtained from the SYK result and a low-frequency hump. A large-$M$ model is obtained from the SYK result and a low-frequency hump. A large-$N$ model is obtained from the SYK result and a low-frequency hump. A large-$M$ model is obtained from the SYK result and a low-frequency hump. A large-$N$ model is obtained from the SYK result and a low-frequency hump. A large-$M$ model is obtained from the SYK result and a low-frequency hump. A large-$N$ model is obtained from the SYK result and a low-frequency hump.
Dynamic spin susceptibility
Probability to flip an electron spin while absorbing energy $\hbar \omega$

Spin susceptibility and other properties match those of an ordinary metal $p > p_c$
Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar \omega$

Critical spin susceptibility matches the SYK model!

Planckian dissipation in time $\sim \hbar/(k_B T)$, and frequency dependence $\sim \text{sgn}(\omega) [1 - C \gamma |\omega| + \ldots]$ matches contribution of boundary graviton.
Dynamic spin susceptibility

Probability to flip an electron spin while absorbing energy $\hbar \omega$

SYK criticality can be understood by the 
fractionalization of the electron into ‘partons’
carrying its spin and charge.
These partons obey an SYK-like model

D. G. Joshi,
Chenyuan Li,
G. Tarnopolsky,
A. Georges, and
S. Sachdev,
PRX 10, 021033 (2020)
Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Copper-based superconductors
Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

2D quantum gravity

Copper-based superconductors
Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

SYK criticality of partons near $p = p_c$

Copper-based superconductors
Complex multi-particle entanglement leads to quantum systems without quasiparticle excitations.

Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \frac{\hbar}{k_B T}$. 