1. “Conventional” phases of matter
   *Metals, insulators, superconductors*

2. Emergent gauge fields and topology
   *Spin liquids with Rydberg atoms*

3. Strange metals
   *SYK model and emergent gravity*
1. “Conventional” phases of matter
   *Metals, insulators, superconductors*

2. Emergent gauge fields and topology
   *Spin liquids with Rydberg atoms*

3. Strange metals
   *SYK model and emergent gravity*
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states
Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals
Current flow with quasiparticles

Flowing quasiparticles scatter off each other in a typical scattering time \( \tau \)

This time is much longer than a limiting 'Planckian time' \( \frac{\hbar}{k_B T} \).

The long scattering time implies that quasiparticles are well-defined.
Band insulators

An even number of electrons per unit cell
Metals
Electrons pair, and the pairs undergo Bose-Einstein condensation
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
1. “Conventional” phases of matter
   - Metals, insulators, superconductors

2. Emergent gauge fields and topology
   - Spin liquids with Rydberg atoms

3. Strange metals
   - SYK model and emergent gravity
Mott insulator: Triangular lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighbor model has non-collinear Neel order
Mott insulator: Triangular lattice antiferromagnet

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Nearest-neighbor model has non-collinear Neel order
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\[ \frac{1}{\sqrt{2}} \left( |\uparrow\uparrow\rangle - |\downarrow\uparrow\rangle \right) \]

\[ |G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle \]

\( \mathcal{D} \rightarrow \text{dimer covering of lattice} \)

Mott insulator: Triangular lattice antiferromagnet

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

\[ |G\rangle = \sum_{D} c_{D} |D\rangle \]

\( D \rightarrow \) dimer covering of lattice

Mott insulator: Triangular lattice antiferromagnet

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\( \mathcal{D} \rightarrow \) dimer covering of lattice

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Mott insulator: Triangular lattice antiferromagnet

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Mott insulator: Triangular lattice antiferromagnet

non-collinear Néel state

$Z_2$ spin liquid with neutral $S = 1/2$ spinons and vison excitations

Excitations of the $\mathbb{Z}_2$ Spin liquid

**Spinon: $S_z = 1/2$**

e (boson) or $\epsilon$ (fermion) particle

$$\frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right)$$
Excitations of the $Z_2$ Spin liquid

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Excitations of the $\mathbb{Z}_2$ Spin liquid

A vison $m$ (boson) particle

$$|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line

$$= \frac{1}{\sqrt{2}} \left(|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle\right)$$
Excitations of the $\mathbb{Z}_2$ Spin liquid

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$$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right> - \left| \downarrow \uparrow \right> \right)$$

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$$= \frac{1}{\sqrt{2}} \left( |↑↓\rangle - |↓↑\rangle \right)$$

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Excitations of the $\mathbb{Z}_2$ Spin liquid

A vison

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$$\psi = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line
Excitations of the $Z_2$ Spin liquid

- A spinon adiabatically transported around a vison picks up a phase factor of $-1$: spinons and visons are mutual semions.

- A bound state of a spinon and a vison picks up a phase factor of $-1$ when exchanged with another bound state of a spinon and a vison:
  
  - The $\epsilon$ spinon (fermion) is a bound state of the $e$ spinon (boson) and a vison ($\epsilon = e \times m$).
  
  - The $e$ spinon (boson) is a bound state of the $\epsilon$ spinon (fermion) and a vison ($e = \epsilon \times m$).
Ground state degeneracy on the torus

Place insulator on a torus:
Obtain a degenerate orthogonal state by modifying the wavefunction on a “branch-cut” encircling the torus.

Place insulator on a torus:

Ground state degeneracy on the torus
Ground state degeneracy on the torus

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Place insulator on a torus:

Number of dimers crossing “branch-cut” is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB 36, 7187 (1987)
Ground state degeneracy on the torus

\[ = \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2} \]

Place insulator on a torus:

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D.J. Thouless, PRB 36, 7187 (1987)
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\[ \frac{2}{\sqrt{2}} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \]

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Ground state degeneracy on the torus

\[ \frac{\left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)}{\sqrt{2}} \]

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Place insulator on a torus:

D.J. Thouless, PRB 36, 7187 (1987)
Rydberg atoms

Optical tweezer (traps atom)

Excited state (large principle quantum number)

Ground state

\[ H_{\text{Ryd}} = \sum_i \left[ \frac{\Omega}{2} (|g\rangle\langle r| + |r\rangle\langle g|)_i - \Delta |r\rangle\langle r| \right] + \sum_{(i,j)} V_{|i-j|} \left( |r\rangle\langle r|_i \otimes |r\rangle\langle r|_j \right) \]

Ground state

\[ V_{|i-j|} \sim |i-j|^{-6} \]

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms
Rydberg atoms on the square lattice: theory

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PRL 124, 103601 (2020)
Rydberg atoms on the square lattice: experiment


First observation of Ising quantum phase transition in 2+1 dimensions
Rydberg atoms on site-kagome lattice: theory

(a) PBC

(b) Stripe: $\delta = 2.2$, $R_b = 1.2$

(c) Nematic: $\delta = 3.3$, $R_b = 1.7$

(d) Staggered: $\delta = 3.3$, $R_b = 2.1$

(e) $S_{\gamma N}$

Rydberg atoms on site-kagome lattice: theory

(a) PBC

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(d) Staggered: $\delta = 3.3$, $R_b = 2.1$

(e) $R_b/\alpha$

Probing Topological Spin Liquids on a Programmable Quantum Simulator


Rydberg atoms on the link-kagome lattice: experiment
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High temperature superconductors
Strange Metal

Temperature (K)

AF insulator

$T_N$

$T^*$

Superconductor

$p_c$

$T_c$

Hole doping, $\rho$
Table 1 | Slope of $T$-linear resistivity and Planckian limit in seven materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>$p$</th>
<th>$n$ $(10^{27} \text{m}^{-3})$</th>
<th>$m^*$ $(m_0)$</th>
<th>$\Lambda_1/d$ $(\Omega/\text{K})$</th>
<th>$h/(2e^2T_F)$ $(\Omega/\text{K})$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi2212</td>
<td>$p = 0.23$</td>
<td>6.8</td>
<td>8.4 ± 1.6</td>
<td>8.0 ± 0.9</td>
<td>7.4 ± 1.4</td>
<td>1.1 ± 0.3</td>
</tr>
<tr>
<td>Bi2201</td>
<td>$p \approx 0.4$</td>
<td>3.5</td>
<td>7 ± 1.5</td>
<td>8 ± 2</td>
<td>8 ± 2</td>
<td>1.0 ± 0.4</td>
</tr>
<tr>
<td>LSCO</td>
<td>$p = 0.26$</td>
<td>7.8</td>
<td>9.8 ± 1.7</td>
<td>8.2 ± 1.0</td>
<td>8.9 ± 1.8</td>
<td>0.9 ± 0.3</td>
</tr>
<tr>
<td>Nd-LSCO</td>
<td>$p = 0.24$</td>
<td>7.9</td>
<td>12 ± 4</td>
<td>7.4 ± 0.8</td>
<td>10.6 ± 3.7</td>
<td>0.7 ± 0.4</td>
</tr>
<tr>
<td>PCCO</td>
<td>$x = 0.17$</td>
<td>8.8</td>
<td>2.4 ± 0.1</td>
<td>1.7 ± 0.3</td>
<td>2.1 ± 0.1</td>
<td>0.8 ± 0.2</td>
</tr>
<tr>
<td>LCCO</td>
<td>$x = 0.15$</td>
<td>9.0</td>
<td>3.0 ± 0.3</td>
<td>3.0 ± 0.45</td>
<td>2.6 ± 0.3</td>
<td>1.2 ± 0.3</td>
</tr>
<tr>
<td>TMTSF</td>
<td>$P = 11 \text{kbar}$</td>
<td>1.4</td>
<td>1.15 ± 0.2</td>
<td>2.8 ± 0.3</td>
<td>2.8 ± 0.4</td>
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</tr>
</tbody>
</table>

Electron scattering time $\tau$ in 7 different strange metals

$$\frac{1}{\tau} = \alpha \frac{k_BT}{\hbar}$$

Current flow without quasiparticles
The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

In one set of variables, it describes certain *strange metals*

In a *dual* set of variables it describes certain *black holes*

Sachdev, Ye (1993)

The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Place electrons randomly on some sites

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Entangle electrons pairwise randomly

Sachdev, Ye (1993); Kitaev (2015)
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Electron scattering time $\tau$ in the SYK model

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

Entangle electrons pairwise randomly
The SYK model has a scale-invariant entanglement structure: i.e. electrons are entangled at all distance and time scales.

In one set of variables, it describes certain *strange metals*.

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Sachdev, Ye (1993)  
Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \frac{2GM}{c^2}$

$G$ Newton’s constant, $c$ velocity of light, $M$ mass of black hole
For $M =$ earth’s mass, $R \approx 9 \text{ mm}$!
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

![Diagram showing quantum entanglement across a black hole horizon.](image-url)
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature.
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)
Black holes have an entropy and a temperature, $T_H$

The entropy is proportional to their surface area.

They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Black holes are represented as a `hologram' by a quantum many-body system in one lower dimension.

**Duality:** a `change of variables' between the many-particle configurations and the metric of spacetime

Susskind, Maldacena…..
Black holes have an entropy and a temperature, $T_H$

- The entropy is proportional to their surface area.

- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

The hologram of a black hole in $d$ dimensions is a quantum many-particle system in $(d - 1)$ dimensions which relaxes to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T)$.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

The near-horizon geometry of a charged black hole is one-dimensional ($\zeta$).
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018); Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)
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