From the SYK model to a theory of the strange metal

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Magnetotransport in a model of a disordered strange metal

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We engineer a microscopic model of two-dimensional conduction electrons locally and randomly scattering off impurity sites which are described by Sachdev-Ye-Kitaev (SYK) models. For a particular choice of the scattering interaction, this model realizes a controlled description of a diffusive marginal-Fermi liquid (MFL) without momentum conservation, which has a linear-in-\(T\) resistivity as \(T\to0\). By tuning the strength of the scattering interaction relative to the bandwidth of the conduction electrons, we can additionally obtain a finite-\(T\) crossover to a fully incoherent regime that also has a linear-in-\(T\) resistivity. We describe the magnetotransport properties of this model.

We then consider a macroscopically disordered sample with domains of such MFLs with varying electron and impurity densities. Using an effective-medium approximation, we obtain a macroscopic electrical resistance that scales linearly in the magnetic field \(B\) applied perpendicular to the plane of the sample, at large \(B\). The resistance also scales linearly in \(T\) at small \(B\), and as \(Tf(B/T)\) at intermediate \(B\). We consider implications for recent experiments reporting linear transverse magnetoresistance in the strange metal phases of the pnictides and cuprates.

I. INTRODUCTION

Essentially all correlated electron high temperature superconductors display an anomalous metallic state at temperatures above the superconducting critical temperature at optimal doping \([1–3]\). This metallic state has a 'strange' linearly-increasing dependence of the resistivity, \(\rho\), on temperature, \(T\); it can also exhibit bad metal behavior with a resistivity much larger than the quantum unit \(\rho_h/e^2\) (in two spatial dimensions) \([4]\). More recently, strange metals have also been demonstrated to have a remarkable linear-in-\(B\) magnetoresistance, with the crossover between the linear-in-\(T\) and linear-in-\(B\) behavior occurring at \(\mu B_B\sim k_B T\) \([5,6]\).

This paper will present a model of a strange metal which exhibits the above linear-in-\(T\) and linear-in-\(B\) behavior. The model builds on a lattice array of 'quantum dots', each of which is described by a Sachdev-Ye-Kitaev (SYK) model of fermions with random all-to-all interactions \([7,8]\). A single SYK site is a 0+1 dimensional non-Fermi liquid in which the imaginary-time (\(\tau\)) fermion Green's function has the low-\(T\)'conformal' form \([7,9–11]\)

\[G(\tau)\sim \sqrt{T} \sin(\pi T \tau) e^{-2\pi E_T \tau}, 0<\tau<1/T, \tag{1.1}\]

where \(E\) is a parameter controlling the particle-hole asymmetry. As was recognized early on \([7]\), such a Green's function implies a 'marginal' susceptibility, with a real part which diverges logarithmically with vanishing frequency (\(\omega\)) or \(T\). Specifically, in the all-to-all limit of the SYK model, vertex corrections are sub-dominant, and

\[(\Delta \tau) = \frac{G(\tau)}{G(\tau)} \quad \text{leads to the spectral density}
\]

\[\text{Im}(\omega) \sim \tanh(\pi \omega T), \tag{1.2}\]

\[Aavishkar Patel\]
The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)
Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

Quantum matter without quasiparticles

Figure: K. Fujita and J. C. Seamus Davis
Quantum matter without quasiparticles

Resistivity $\sim \rho_0 + AT^\alpha$

Superconductivity

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. 74, 3253 – Published 17 April 1995

“Strange”,
“Bad”,
or “Incoherent”,
metal has a resistivity, \( \rho \), which obeys
\[
\rho \sim T,
\]
and
in some cases \( \rho \gg h/e^2 \)
(in two dimensions),
where \( h/e^2 \) is the quantum unit of resistance.
Strange metals just got stranger…

B-linear magnetoresistance!?


P. Giraldo-Gallo et. al., arXiv:1705.05806
Strange metals just got stranger…
Scaling between B and T !?

\[ \rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma \]

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  
The low-lying excitations of the many-body system can be identified as a set \( \{n_\alpha\} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly(\( N \)) numbers.
• Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

\[ \tau_{eq} \sim \frac{\hbar E_F}{(k_B T)^2}, \quad \text{as } T \to 0, \]

where \( E_F \) is the Fermi energy.
A simple model of a metal with quasiparticles

Pick a set of random positions
A simple model of a metal with quasiparticles

Place electrons randomly on some sites
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

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\[ H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ \frac{1}{N} \sum_i c_i^\dagger c_i = Q \]

\( t_{ij} \) are independent random variables with \( \overline{t_{ij}} = 0 \) and \( |t_{ij}|^2 = t^2 \)

Fermions occupying the eigenstates of a \( N \times N \) random matrix
A simple model of a metal with quasiparticles

Let $\varepsilon_\alpha$ be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest $NQ$ eigenvalues, upto the Fermi energy $E_F$. The density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$. 
A simple model of a metal with quasiparticles

There are $2^N$ many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_\alpha$ have a level spacing $\sim 1/N$. 

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$
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Quasiparticle excitations with spacing $\sim 1/N$
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites
Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

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Entangle electrons pairwise randomly
Entangle electrons pairwise randomly

The SYK model
The SYK model

Entangle electrons pairwise randomly
This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_{i}^\dagger c_{j}^\dagger c_{k} c_{\ell} - \mu \sum_{i} c_{i}^\dagger c_{i}$$

$$c_{i} c_{j} + c_{j} c_{i} = 0 \quad , \quad c_{i} c_{j}^\dagger + c_{j} c_{i}^\dagger = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_{i} c_{i}^\dagger c_{i}$$

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|U_{ij;kl}|^{2} = U^{2}$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

There are $2^N$ many body levels with energy $E$, which do not admit a quasiparticle decomposition. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = Ns_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848 \ldots < \ln 2$$

where $G$ is Catalan’s constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

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No quasiparticles!

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \ldots$$

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
Feynman graph expansion in $J_{ij...}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = Q.$$ 

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. The ground state is a non-Fermi liquid, with a continuously variable density $Q$. 

The SYK model

- Low energy, many-body density of states
  \[ \rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E - E_0)N\gamma}) \]

(for Majorana model)

A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
D. Stanford and E. Witten, 1703.04612
A. M. Garcia-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694
The SYK model

- Low energy, many-body density of states
  \[ \rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E - E_0)}N\gamma) \]

- Low temperature entropy
  \[ S = Ns_0 + N\gamma T + \ldots \]

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818
The SYK model

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  \[ \rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)} N \gamma) \]

- Low temperature entropy \( S = N s_0 + N \gamma T + \ldots \). 

- \( T = 0 \) fermion Green’s function is incoherent: \( G(\tau) \sim \tau^{-1/2} \) at large \( \tau \). (Fermi liquids with quasiparticles have the coherent: \( G(\tau) \sim 1/\tau \))

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- \( T > 0 \) Green’s function has conformal invariance
  \[ G \sim e^{-2\pi\varepsilon T\tau}(T/\sin(\pi k_B T\tau/\hbar))^{1/2}; \]
  \( \varepsilon \) measures particle-hole asymmetry.

A. Georges and O. Parcollet PRB 59, 5341 (1999)
S. Sachdev, PRX, 5, 041025 (2015)
The SYK model

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  \[ \rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0) N \gamma}) \]

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- \( T > 0 \) Green’s function has conformal invariance
  \[ G \sim e^{-2 \pi \mathcal{E} T \tau} (T/\sin(\pi k_B T \tau / \hbar))^{1/2}; \]
  \( \mathcal{E} \) measures particle-hole asymmetry.

- The last property indicates \( \tau_{eq} \sim \hbar/(k_B T) \), and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803
Quantum matter without quasiparticles:

- If there are no quasiparticles, then

\[ E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha, \beta} F_{\alpha \beta} n_\alpha n_\beta + \ldots \]
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\[ \tau_{eq} = \# \frac{\hbar}{k_B T} \]

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• If there are no quasiparticles, then

\[ \tau_{\text{eq}} = \# \frac{\hbar}{k_B T} \]

• Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as \( T \to 0 \)

\[ \tau_{\text{eq}} > C \frac{\hbar}{k_B T} \]

– In Fermi liquids \( \tau_{\text{eq}} \sim 1/T^2 \), and so the bound is obeyed as \( T \to 0 \).
– This bound rules out quantum systems with e.g. \( \tau_{\text{eq}} \sim \hbar/(Jk_B T)^{1/2} \).
– There is no bound in classical mechanics (\( \hbar \to 0 \)). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

The basic features can be determined by a simple power-counting. Considering for simplicity scale metallic behaviors appear, resembling the previously mentioned physics, here as we vary the temperature, two distinctive cluster interaction of strength most-studied SYK.

Prominent systems like the high-T$_c$ SU$_2$0 linear in temperature resistivity in the incoherent regime, and a Lorentz ratio. Commonly a highly renormalized heavy Fermi liquid occurs below a small coherence scale, while at higher temperatures a strongly correlated metal built from Sachdev-Ye-Kitaev models (µ) generated and does not require infinite dimensions. We obtained in a double limit of infinite dimension and large collet and Georges[28], who studied a variant SYK model obtained further results on the thermal conductivity and an incoherent metal regime and the resistivity[30, 31] in this crossover. This also obtain further results on the thermal conductivity.

\[ H = \sum_x \sum_{i<j, k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx'\rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'} \]

\[ |U_{ijkl}|^2 = \frac{2U^2}{N^3} \]
\[ |t_{ij,xx'}|^2 = \frac{t_0^2}{N}. \]
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models
Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]
Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( E_c < T < U \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho \sim \frac{\hbar}{e^2} \left( \frac{T}{E_c} \right), \quad s = s_0 \]
This scaling collapse is verified by direct numerical calculation of the scaling form (B2) for the Kadowaki-Woods ratio, verifying interference from closed loops. At low energy, 

\[ E_c \sim \frac{t_0^2}{U} \]

For \( E_c < T < U \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho \sim \frac{h}{e^2} \left( \frac{T}{E_c} \right) \quad \text{and} \quad s = s_0 \]
Low ‘coherence’ scale

\[ E_C \sim \frac{t_0^2}{U} \]

For \( T < E_c \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho = \frac{h}{e^2} \left[ c_1 + c_2 \left( \frac{T}{E_C} \right)^2 \right] \]

\[ s \sim s_0 \left( \frac{T}{E_C} \right) \]
Can we build a bridge between the 0-dimensional SYK model and a more conventional FS-based system?

\[
H = -t \sum_{i=1}^{M} \langle r r' \rangle_{i=1} (c_r^\dagger c_r + \text{h.c.}) - \mu_c \sum_{i=1}^{M} c_r^\dagger c_r - \mu \sum_{i=1}^{N} f_r^\dagger f_r \\
+ \frac{1}{NM^{1/2}} \sum_{r; i, j=1}^{N} \sum_{k, l=1}^{M} g_{ijkl} f_r^\dagger f_j f_k c_{r} c_{l} + \frac{1}{N^{3/2}} \sum_{r; i, j, k, l=1}^{N} J_{ijkl}^r f_r^\dagger f_j f_k f_l.
\]

A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, to appear...

See also: D. Ben-Zion and J. McGreevy, arXiv: 1711.02686
Infecting a Fermi liquid and making it SYK

\[ \Sigma(\tau - \tau') = -J^2 G^2(\tau - \tau')G(\tau' - \tau) - \frac{M}{N} g^2 G(\tau - \tau')G^c(\tau - \tau')G^c(\tau' - \tau), \]

\[ G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}, \quad (f \text{ electrons}) \]

\[ \Sigma^c(\tau - \tau') = -g^2 G^c(\tau - \tau')G(\tau - \tau')G(\tau' - \tau), \]

\[ G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)} \cdot (c \text{ electrons}) \]

Exactly solvable in the large \( N,M \) limits!

- Low-\( T \) phase: \( c \) electrons form a Marginal Fermi-liquid (MFL), \( f \) electrons are local SYK models

\[ \Sigma^c(i\omega_n) = \frac{ig^2 \nu(0) T}{2J \cosh^{1/2}(2\pi \mathcal{E}) \pi^{3/2}} \left( \frac{\omega_n}{T} \ln \left( \frac{2\pi T e^{\gamma \mathcal{E} - 1}}{J} \right) + \frac{\omega_n}{T} \psi \left( \frac{\omega_n}{2\pi T} \right) + \pi \right), \]

\[ \Sigma^c(i\omega_n) \to \frac{ig^2 \nu(0)}{2J \cosh^{1/2}(2\pi \mathcal{E}) \pi^{3/2}} \omega_n \ln \left( \frac{|\omega_n| e^{\gamma \mathcal{E} - 1}}{J} \right), \quad |\omega_n| \gg T \quad (\nu(0) \sim 1/t) \]
Infecting a Fermi liquid and making it SYK

- High-\(T\) phase: \(c\) electrons form an “incoherent metal” (IM), with local Green’s function, and no notion of momentum; \(f\) electrons remain local SYK models

\[
G_c(\tau) = -\frac{C_c}{\sqrt{1 + e^{-4\pi\mathcal{E}_c}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi\mathcal{E}_c T \tau}, \quad G(\tau) = -\frac{C}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi\mathcal{E} T \tau}, \quad 0 \leq \tau < \beta
\]

\[
C = \cosh^{1/4}(2\pi\mathcal{E}) \frac{\pi^{1/4}}{J^{1/2}} \left( 1 - \frac{M}{N} \frac{\Lambda \nu(0)}{2\pi} \frac{\cosh(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)} \right)^{1/4}, \quad C_c = \frac{\cosh^{1/2}(2\pi\mathcal{E}) \Lambda^{1/2} \nu^{1/2}(0)}{2^{1/2} C g},
\]

\((\Lambda \sim t, \nu(0) \sim 1/t)\)
Linear-in-\(T\) resistivity

*Both* the MFL and the IM are not translationally-invariant and have linear-in-\(T\) resistivities!

\[
\sigma_0^{\text{MFL}} = 0.120251 \times MT^{-1} J \times \left( \frac{v_F^2}{g^2} \right) \cosh^{1/2}(2\pi \mathcal{E}). \quad (v_F \sim t)
\]

\[
\sigma_0^{\text{IM}} = \left( \frac{\pi^{1/2}}{8} \right) \times MT^{-1} J \times \left( \frac{\Lambda}{\nu(0)g^2} \right) \frac{\cosh^{1/2}(2\pi \mathcal{E})}{\cosh(2\pi \mathcal{E}_c)}.
\]

[Can be obtained straightforwardly from Kubo formula in the large-\(N,M\) limits]

The IM is also a “Bad metal” with \(\sigma_0^{\text{IM}} \ll 1\)
Magnetotransport: Marginal-Fermi liquid

- Thanks to large $N,M$, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields...

\[(\mathcal{B} = eBa^2/\hbar) \text{ (i.e. flux per unit cell)}\]

\[
\begin{align*}
\sigma_{L}^{\text{MFL}} &= M \frac{v_F^2\nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{-\text{Im}[\Sigma^c_R(E_1)]}{\text{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2 \mathcal{B}^2}, \\
\sigma_{H}^{\text{MFL}} &= -M \frac{v_F^2\nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{(v_F/(2k_F))\mathcal{B}}{\text{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2 \mathcal{B}^2}.
\end{align*}
\]

\[
\begin{align*}
\sigma_{L}^{\text{MFL}} &\sim T^{-1}s_L((v_F/k_F)(\mathcal{B}/T)), & \sigma_{H}^{\text{MFL}} &\sim -\mathcal{B}T^{-2}s_H((v_F/k_F)(\mathcal{B}/T)).
\end{align*}
\]

\[
s_{L,H}(x \to \infty) \propto 1/x^2, \quad s_{L,H}(x \to 0) \propto x^0.
\]

Scaling between magnetic field and temperature in **orbital** magnetotransport!
Macroscopic magnetotransport in the MFL

- Let us consider the MFL with additional **macroscopic** disorder (charge puddles etc.)

![Diagram of macroscopic magnetotransport](image)

*Figure: N. Ramakrishnan et. al., arXiv: 1703.05478*

- No macroscopic momentum, so equations describing charge transport are just

  \[ \nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla \Phi(\mathbf{x}). \n\]

- Very weak thermoelectricity for large FS, so charge effectively decoupled from heat transport.
Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Physical picture

- Current path length increases linearly with $B$ at large $B$ due to local Hall effect, which causes the global resistance to increase linearly with $B$ at large $B$. 

Figure 3 Visualization of currents and voltages at large magnetic field in a $10 \times 10$ random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in $H$. 

Solvable toy model: two-component disorder

- Two types of domains $a, b$ with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly

\[
\left( I + \frac{\sigma^a - \sigma^e}{2\sigma^e_L} \right)^{\!-1} \cdot (\sigma^a - \sigma^e) + \left( I + \frac{\sigma^b - \sigma^e}{2\sigma^e_L} \right)^{\!-1} \cdot (\sigma^b - \sigma^e) = 0.
\]

\[
\rho_L^e \equiv \frac{\sigma_L^e}{\sigma_L^{e2} + \sigma_H^{e2}} = \frac{\sqrt{(B/m)^2 \left( \gamma_a \sigma_{0a}^{\text{MFL}} - \gamma_b \sigma_{0b}^{\text{MFL}} \right)^2 + \gamma_a^2 \gamma_b^2 \left( \sigma_{0a}^{\text{MFL}} + \sigma_{0b}^{\text{MFL}} \right)^2}}{\gamma_a \gamma_b (\sigma_{0a}^{\text{MFL}} + \sigma_{0b}^{\text{MFL}})^{\!1/2} \left( \sigma_{0a}^{\text{MFL}} + \sigma_{0b}^{\text{MFL}} \right)},
\]

\[
\rho_H^e \equiv -\frac{\sigma_H^e/B}{\sigma_L^{e2} + \sigma_H^{e2}} = \frac{\gamma_a + \gamma_b}{m \gamma_a \gamma_b (\sigma_{0a}^{\text{MFL}} + \sigma_{0b}^{\text{MFL}})} \cdot (m = k_F/v_F \sim 1/t)
\]

\[
\gamma_{a,b} \sim T \quad \text{(i.e. effective transport scattering rates)}
\]

\[
\rho_L^e \sim \sqrt{c_1 T^2 + c_2 B^2}
\]

Scaling between $B$ and $T$ at microscopic orbital level has been transferred to global MR!
Scaling between $B$ and $T$

\[ \frac{\Delta \rho_L^e}{\rho_L^e} \]

\[ n_b / n_a = 0.8 \]
\[ \gamma_b / \gamma_a = 0.8 \]
\[ \gamma_a = 0.1 k_B T \]

\[ B \approx 50 \text{T} \ (a = 3.82 \text{A}) \]
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states:
  \[ E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \ldots \]

- Thermalization and many-body chaos in the shortest possible time of order \( \hbar/(k_B T) \).

- These are also characteristics of black holes in quantum gravity.
Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-\(T\) resistance, with a magnetoresistance which scales as \(B \sim T\).

Macroscopic disorder then leads to linear-in-\(B\) magnetoresistance, and a combined dependence which scales as \(\sim \sqrt{B^2 + T^2}\).

Higher temperatures lead to an incoherent metal with a local Green’s function and a linear-in-\(T\) resistance, but negligible magnetoresistance.