Quantum matter without quasiparticles: graphene, random fermion models, and black holes

Discussion Meeting on Current Frontiers in Condensed Matter Research
International Center for Theoretical Sciences, Bengaluru
June 27, 2016

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Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. No quasiparticles

- Quantum criticality near the superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space

Note: Most states with long-range entanglement, like the fractional quantum Hall states, do have quasiparticles
Local thermal equilibration or phase coherence time, $\tau_\varphi$:

- As $T \to 0$, there is an *lower bound* on $\tau_\varphi$ in all many-body quantum systems of order $\hbar/(k_BT)$,

$$\tau_\varphi > C \frac{\hbar}{k_BT},$$

and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, $\tau_\varphi$ is parametrically larger at low $T$; e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$, and in gapped insulators $\tau_\varphi \sim e^\Delta/(k_BT)$ where $\Delta$ is the energy gap.

A bound on quantum chaos:

- The time over which a many-body quantum system becomes chaotic is given by $\tau_L = 1/\lambda_L$, where $\lambda_L$ is the “Lyapunov exponent” determining memory of initial conditions (the “butterfly effect”):

$$D(t) = \langle W(t) V(0) W(t) V(0) \rangle \sim c_0 - \epsilon c_1 e^{\lambda_L t},$$

where we make a (system-dependent) choice to arrange $\epsilon \ll 1$. As $T \to 0$, this Lyapunov time is argued to obey the lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

- Theories holographically dual to Einstein gravity have the shortest possible $\tau_L = \hbar/(2\pi k_B T)$

A.I. Larkin and Y. N. Ovchinnikov, JETP 28, 6 (1969)

S. H. Shenker and D. Stanford, arXiv:1306.0622

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Quantum matter without quasiparticles

\approx fastest possible many-body quantum chaos
Quantum matter without quasiparticles:

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Graphene

Same “Hubbard” model as for ultracold atoms, but for electrons on the honeycomb lattice
Graphene

\[ T(K) \]

-1  -0.5  0  0.5  1

Quantum critical
Dirac liquid

Hole
Fermi liquid

Electron
Fermi liquid

\[ \mu < 0 \]

\[ \mu > 0 \]

\[ n \]

\[ 10^{12}/m^2 \]

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)

Predicted “strange metal” without quasiparticles
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities
Thermal and electrical conductivity with quasiparticles

- Wiedemann-Franz law in a Fermi liquid:

\[
L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{W \cdot \Omega}{K^2}.
\]

Transport in Strange Metals

For a strange metal
with a “relativistic” Hamiltonian,
hydrodynamic, holographic,
and memory function methods yield
Lorentz ratio \( L = \kappa / (T \sigma) \)
\[
L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}
\]

\( Q \rightarrow \) electron density; \( \mathcal{H} \rightarrow \) enthalpy density
\( \sigma_Q \rightarrow \) quantum critical conductivity
\( \tau_{\text{imp}} \rightarrow \) momentum relaxation time from impurities.
Note that for a clean system (\( \tau_{\text{imp}} \rightarrow \infty \) first),
the Lorentz ratio diverges \( L \sim 1/Q^4 \),
as we approach “zero” electron density at the Dirac point.
M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
\[ \sqrt{n} \left( 1 + \lambda \ln \Lambda \sqrt{n} \right) \]

\[ T(K) \]

Graphene

Quantum critical Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

Impurity scattering dominates

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
To minimize disorder, the monolayer graphene samples used in this report are encapsulated in hexagonal boron nitride (hBN) to keep a well-defined temperature profile.

The breakdown of the WF law can be observed.

Temperatures set the experimental window in which the DF and electron-electron scattering rate. These two temperature limits occur when the electron-phonon scattering rate becomes comparable to the minimum conductivity. At the lowest temperatures, the minimum density is limited by disorder (charge puddles). However, above the neutrality point, the chemical potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential.

The thermal conductivity is enhanced and the WF law is violated. Above the minimum density (green) aligns with the temperature axis to the right. Solid black lines correspond to a linear fit of log(\(\kappa(T)\)) with a slope of -0.5 V.

All measurements are performed in a cryostat controlling the temperature and density dependent electrical and thermal conductivity.

Fig. 1. Temperature and density dependent electrical and thermal conductivity. (A) Electrical conductivity (blue) as a function of the charge density set by the back gate for different temperatures.

Red dots: data
Blue line: value for \(L = L_0\)

\(\mu\) is locally doped to form electron-hole puddles with finite potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential.
To minimize disorder, the monolayer graphene samples used in this report are encapsulated in hexagonal boron nitride (hBN). At the neutrality point, the residual carrier density (green) is estimated by the intersection of the minimum conductivity with a linear fit of log(\(n\)). All devices used in this study are representative device (see SM for all samples). From this, we estimate the electrical conductivity versus \(n\) representative of the charge carrier density, respectively (see supplementary materials (SM)). All measurements are performed in a cryostat controlling bath temperatures. The residual carrier density at the neutrality point (green) is estimated by the intersection of the minimum conductivity with a linear fit of log(\(n\)). At low temperature and/or high doping (Fig. 1B) using edge technique, electron-phonon coupling becomes \(\sim 80\) K, electron-phonon coupling becomes \(\sim 80\) K, a crossover marked \(\Delta V_g\). Fig. 1A shows the resistance compared to the Wiedemann-Franz law, the breakdown of the WF law can be observed. At the lowest temperatures we find the thermal energy be larger than the local chemical potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential.

\[\Delta V_g (V)\]

\[40\, K\]

\[20\, K\]

\[\text{Red dots: data}\]

\[\text{Blue line: value for } L = L_0\]
Two-terminal to keep a well-defined temperature profile.

Formation of the DF is further complicated by phonon scattering at high temperature which can re-energize momentum by creating additional inelastic scattering.

At the CNP, the residual carrier density (green) is estimated by the intersection of the minimum conductivity with a linear fit of log(\(\text{R} = \text{Elec. Conductivity} \times 4 \text{ e/h} )) away from neutrality (dashed grey lines). Curves have been offset vertically such that the neutrality point.

We employ a back gate voltage to minimize contact resistance. We employ a back gate voltage in order to minimize contact resistance.

All measurements are performed in a cryostat controlling bath temperatures. The residual carrier density at the neutrality point (green) is estimated by the intersection of the minimum conductivity with a linear fit of log(\(\text{R} = \text{Elec. Conductivity} \times 4 \text{ e/h} )) away from neutrality (dashed grey lines). Curves have been offset vertically such that the neutrality point.

Red dots: data

Blue line: value for \( L = L_0 \)
Impurity scattering dominates

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Strange metal in graphene

Wiedemann-Franz violated!
The gate dependence of the Lorentz ratio $L = \kappa / (T\sigma)$

$$L = \frac{\nu_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 \nu_F^2 Q^2 T \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}$$

$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density
$\sigma_Q \rightarrow$ quantum critical conductivity
$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities


J. Crossno et al., Science 351, 1058 (2016)
Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin$^1$, I. Torre$^{2,3}$, R. Krishna Kumar$^{1,4}$, M. Ben Shalom$^{1,5}$, A. Tomadin$^6$, A. Principi$^7$, G. H. Auton$^5$, E. Khestanova$^{1,5}$, K. S. Novoselov$^5$, I. V. Grigorieva$^1$, L. A. Ponomarenko$^{1,4}$, A. K. Geim$^1$, M. Polini$^{3,6}$

Figure 1. Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero $\nu$ (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity $\sigma_{xx}$ and $R_V$ for this device as a function of $n$ induced by applying gate voltage. $I = 0.3$ $\mu$A; $L = 1$ $\mu$m. For more detail, see Supplementary Information.
Quantum matter without quasiparticles:

- Graphene
- Solvable random fermion
- Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
Infinite-range model with quasiparticles

\[ H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ \frac{1}{N} \sum_i c_i^\dagger c_i = Q \]

\[ t_{ij} \] are independent random variables with \( \overline{t_{ij}} = 0 \) and \( \overline{|t_{ij}|^2} = t^2 \)

Fermions occupying the eigenstates of a
\( N \times N \) random matrix
Feynman graph expansion in $t_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)}, \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.
Infinite-range model with quasiparticles

Now add weak interactions

\[
H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell
\]

\(J_{ij;kl}\) are independent random variables with \(\overline{J_{ij;kl}} = 0\) and \(\overline{|J_{ij;kl}|^2} = J^2\). We compute the lifetime of a quasiparticle, \(\tau_\alpha\), in an exact eigenstate \(\psi_\alpha(i)\) of the free particle Hamiltonian with energy \(E_\alpha\). By Fermi’s Golden rule, for \(E_\alpha\) at the Fermi energy

\[
\frac{1}{\tau_\alpha} = \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta)) \delta(E_\alpha + E_\beta - E_\gamma - E_\delta)
\]

\[= \pi^3 J^2 \rho_0^2 T^2 \]

where \(\rho_0\) is the density of states at the Fermi energy.

**Fermi liquid state:** Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as \(\sim T^{-2}\) at the Fermi level.
SYK model without quasiparticles

To obtain a non-Fermi liquid, we set \( t_{ij} = 0 \):

\[
H_{\text{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i
\]

\[
Q = \frac{1}{N} \sum_i c_i^\dagger c_i
\]

\( H_{\text{SYK}} \) is similar, and has identical properties, to a related model proposed by SY in 1993.

SYK model without quasiparticles

To obtain a non-Fermi liquid, we set \( t_{ij} = 0 \):

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H_{\text{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c^\dagger_i c^\dagger_j c_k c_\ell - \mu \sum_{i} c^\dagger_i c_i
\]

\[
Q = \frac{1}{N} \sum_{i} c^\dagger_i c_i
\]

\( H_{\text{SYK}} \) is similar, and has identical properties, to a related model proposed by SY in 1993.

A fermion can move only by entangling with another fermion: the Hamiltonian has “nothing but entanglement”.

SYK model without quasiparticles

Feynman graph expansion in $J_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2G^2(\tau)G(-\tau)$$

$$G(\tau = 0^-) = Q.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. The ground state is a non-Fermi liquid, with a continuously variable density $Q$.

SYK model without quasiparticles

A better understanding of the above facts can be reached from the perspective of symmetry-protected topological (SPT) phases. As shown recently in Ref. 14, the complex SYK model can be thought of as the boundary of a 1D SPT system in the symmetry class AIII. The periodicity of 4 in $N$ arises from the fact that we need to put 4 chains to gap out the boundary degeneracy without breaking the particle-hole symmetry. In the Majorana SYK case, the symmetric Hamiltonian can be constructed as a symmetric matrix in the Clifford algebra $\mathbb{C}l_{N,1}$, and the Bott periodicity in the real representation of the Clifford algebra gives rise to a $\mathbb{Z}_8$ classification\[14\]. Here, for the complex SYK case, we can similarly construct the Clifford algebra by dividing one complex fermion into two Majorana fermions, and then we will have a periodicity of 4.

A. Green's function

From the above definition of retarded Green's function, we can relate them to the imaginary time Green's function as defined in Eq. (16), $G_R(\tau) = G(i\tau_n + \iota \phi)$. In Fig. 3, we show a comparison between the imaginary part of the Green's function from large $N$ and exact diagonalization. The spectral function from ED is particle-hole symmetric for all $N$, while the bosonic case is qualitatively different from the fermionic case, where $G_F(z) \sim 1/pz$; this inverse square-root behavior also holds in the bosonic case without spin glass order\[1\]. Fig. 10 is our result from ED, with a comparison between $G_B$ with $G_F$. It is evident that the behavior of $G_B$ is qualitatively different from $G_F$, and so an inverse square-root behavior is ruled out. Instead, we can clearly see that, as system size gets larger, $G_B$'s peak value increases much faster than the $G_F$'s peak value. This supports the presence of spin glass order.

Large $N$ solution of equations for $G$ and $\Sigma$ agree well with exact diagonalization of the finite $N$ Hamiltonian.

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.

W. Fu and S. Sachdev, arXiv: 1603.05246
SYK model without quasiparticles

The entropy per site, $S$, has a non-zero limit as $T \to 0$. This is not due to an extensive degeneracy, but due to an energy level spacing $\sim e^{-aN}$ in the entire many-body spectrum all the way down to the ground state. At low $T$ we write

$$S(T \to 0) = S_0 + \gamma T + \ldots$$

where the specific heat is $C = \gamma T$, and $S_0$ obeys

$$\frac{dS_0}{dQ} = 2\pi \mathcal{E},$$

with $\mathcal{E}$ a spectral asymmetry parameter, which is a known function of $Q$. $\mathcal{E}$ fully determines the Green’s function at low $T$ and $\omega$ as a ratio of Gamma functions.

Note that $S_0$ and $\mathcal{E}$ involve low-lying states, while $Q$ depends upon all states, and details of the UV structure.

J. Maldacena and D. Stanford, arXiv:1604.07818
Infinite-range (SYK) model without quasiparticles

After integrating the fermions, the partition function can be written as a path integral with an action $S$ analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det \left[ \delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2) \right]$$

$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[ G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2) \right]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[ f'(\sigma_1)f'(\sigma_2) \right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[ f'(\sigma_1)f'(\sigma_2) \right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB 59, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX 5, 041025 (2015)
Infinite-range (SYK) model without quasiparticles

Let us write the large $N$ saddle point solutions of $S$ as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}, \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$ 

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$ 

So the (approximate) reparametrization symmetry is spontaneously broken.

**Reparametrization zero mode**

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4}G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for $\Sigma$) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768
Infinite-range (SYK) model without quasiparticles

However the effective action must vanish for SL(2,R) transformations because $G_s, \Sigma_s$ are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$NS_{\text{eff}} = - \frac{N \gamma}{4\pi^2} \int d\tau \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left( \frac{f''}{f'} \right)^2 ,$$

where the specific heat, $C = \gamma T$.

The Schwarzian effective action implies that the SYK model saturates the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768
Infinite-range (SYK) model without quasiparticles

The Schwarzian describes fluctuations of the energy operator with scaling dimension $h = 2$.
Apart from the energy operator associated with the Schwarzian, there are an infinite number of other scalar operators with irrational scaling dimensions given by the roots of

$$\tan\left(\frac{\pi(2h - 1)}{4}\right) = \frac{1 - 2h}{3}$$

$$\Rightarrow \quad h = 3.77354\ldots, 5.67946\ldots, 7.63197\ldots, 9.60396\ldots, \ldots$$
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Holographic Metals and the Fractionalized Fermi Liquid

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(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a “small” Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, AdS$_2 \times R^2$ physics of Reissner-Nordström black holes.
SYK and AdS$_2$

The non-zero $T \to 0$ entropy density, $S_0$, matches the Bekenstein-Hawking-Wald entropy density of extremal AdS$_2$ horizons, and the dependence of the fermion Green’s function on $\omega$, $T$, and $\mathcal{E}$, matches that of a Dirac fermion in AdS$_2$ (as computed by T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)).

More recently, it was noted that the relation $dS_0/dQ = 2\pi \mathcal{E}$ also matches between SYK and gravity, where $\mathcal{E}$, the electric field on the horizon, also determines the spectral asymmetry of the Dirac fermion.

S. Sachdev, PRL 105, 151602 (2010)

S. Sachdev, PRX 5, 041025 (2015)
The same Schwarzian effective action describes low energy fluctuations on the boundary theory of gravity theories with AdS$_2$ near-horizon geometries (including the AdS-Reissner-Nordstrom solution of Einstein-Maxwell theory in 4 space-time dimensions). And the co-efficient of the Schwarzian, $N\gamma/4\pi^2$, determines the specific heat $C = \gamma T$. 

SYK and AdS$_2$

\[
\text{AdS}_2 \times R^2 \\
\text{ds}^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \\
\text{Gauge field: } A = (E/\zeta)dt
\]

The Schwarzian effective action implies that both the SYK model and the AdS$_2$ theories \textit{saturate} the lower bound on the Lyapunov time

\[
\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}.
\]

Quantum matter without quasiparticles:

- Solvable random fermion
  Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene
**Entangled quantum matter without quasiparticles**

- No quasiparticle excitations

- Shortest possible “phase coherence” time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order $\frac{\hbar}{k_B T}$

- Experiments on graphene agree well with predictions of a theory of a nearly relativistic quantum liquid without quasiparticles.

- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time. Its properties have some similarities to non-rational, large central charge CFT2s.

- Remarkable match between SYK and quantum gravity of black holes with AdS$_2$ horizons, including a $\text{SL}(2,\mathbb{R})$-invariant Schwarzian effective action for thermal energy fluctuations.