Quantum entanglement
and the phases of matter

Girdharilal Mehta Lecture
Harish-Chandra Research Institute, Allahabad
January 13, 2012

sachdev.physics.harvard.edu
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)

S=1/2 spins

Examine ground state as a function of \( \lambda \)
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[
\begin{align*}
\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) 
\end{align*}
\]

At large \( \lambda \) ground state is a “quantum paramagnet” with spins locked in valence bond singlets
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighor spins are “entangled” with each other. Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic (“Néel”) order, and the spins align in a checkerboard pattern.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.

No EPR pairs
\[ \lambda \quad = \quad \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
\[ \lambda_c = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]


\[ \text{Pressure in TlCuCl}_3 \]
An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
TlCuCl$_3$

Quantum paramagnet at ambient pressure
TlCuCl$_3$

Neel order under pressure

\[ \lambda_c = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$ "triplon"
Excitation spectrum in the paramagnetic phase

Spin $S = 1$
"triplon"
Excitation spectrum in the paramagnetic phase

Spin $S = 1$
“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitations of $\text{TlCuCl}_3$ with varying pressure

Excitations of TlCuCl$_3$ with varying pressure


Broken valence bond excitations of the quantum paramagnet
Excitations of TlCuCl$_3$ with varying pressure

Spin waves above the Néel state

Excitations of TlCuCl$_3$ with varying pressure

Longitudinal excitations —similar to the Higgs boson
First observation of the Higgs!

Excitations of TlCuCl$_3$ with varying pressure

“Higgs” particle appears at theoretically predicted energy

Longitudinal excitations –similar to the Higgs boson
First observation of the Higgs!

\[ = \frac{1}{\sqrt{2}} (|↑↓\rangle - |↓↑\rangle) \]
Quantum critical point with non-local entanglement in spin wavefunction

\[ \frac{\lambda}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Tensor network representation of entanglement at quantum critical point

D-dimensional space

depth of entanglement
• Long-range entanglement
• Long-range entanglement

• Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).
• Long-range entanglement

• Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).

• The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a CFT$_3$
• Allows unification of the standard model of particle physics with gravity.

• Low-lying string modes correspond to gauge fields, gravitons, quarks …
• A $D$-brane is a $D$-dimensional surface on which strings can end.
• The low-energy theory on a $D$-brane is an ordinary quantum field theory with no gravity.
A $D$-brane is a $D$-dimensional surface on which strings can end.

The low-energy theory on a $D$-brane is an ordinary quantum field theory with no gravity.

In $D = 2$, we obtain strongly-interacting CFT$_3$s. These are “dual” to string theory on anti-de Sitter space: AdS$_4$. 


Tensor network representation of entanglement at quantum critical point

String theory near a D-brane

Emergent direction of AdS4

D-dimensional space
Tensor network representation of entanglement at quantum critical point

Emergent direction of AdS4

Brian Swingle, arXiv:0905.1317
$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$

**Entanglement entropy** $S_{EE} = -\text{Tr} (\rho_A \ln \rho_A)$
Entanglement entropy

$D$-dimensional space

depth of entanglement
Entanglement entropy

D-dimensional space

A

Emergent direction of AdS4

Draw a surface which intersects the minimal number of links
The entanglement entropy of a region $A$ on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of $A$.

This can be seen both the string and tensor-network pictures.

Brian Swingle, arXiv:0905.1317
Emergent holographic direction

\[ \text{CFT}_{d+1} \]
Quantum matter with long-range entanglement

\[ \text{AdS}_{d+2} \to \mathbb{R}^{d,1}_{\text{Minkowski}} \]

J. McGreevy, arXiv0909.0518
Quantum matter with long-range entanglement
Area measures entanglement entropy

Emergent holographic direction

CFT\textsubscript{d+1}

Quantum matter with long-range entanglement

Minkowski

AdS\textsubscript{d+2}
Quantum critical point with non-local entanglement in spin wavefunction

\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

- Classical spin waves
- Dilute tripon gas
- Quantum critical

Pressure in TlCuCl$_3$
Classical spin waves

Dilute triplon gas

Quantum critical

Short-range entanglement

Neel order

Pressure in TlCuCl$_3$

Classical spin waves

Dilute triplon gas

Quantum critical

Pressure in TlCuCl$_3$


AdS/CFT correspondence at non-zero temperatures

AdS\textsubscript{4}-Schwarzschild black-brane

A 2+1 dimensional system at its quantum critical point
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

A 2+1 dimensional system at its quantum critical point
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Provides successful description of many properties of quantum critical points at non-zero temperatures
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
The cuprate superconductors
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order
Hole-doped

Electron-doped

Superconductor

Bose condensate of pairs of electrons

Short-range entanglement

La$_{2-x}$Sr$_x$CuO$_4$
Electron-doped cuprate superconductors
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

\[ \rho \sim \rho_0 + AT^n \]
Electron-doped cuprate superconductors

Resistivity \( \sim \rho_0 + AT^n \)

Ordinary metal (Fermi liquid)
Short-range entanglement

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity
\[ \rho \sim \rho_0 + AT^n \]
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity
\[ \sim \rho_0 + AT^n \]
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity
\[ \sim \rho_0 + AT^n \]
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity
\[ \sim \rho_0 + AT^n \]
Iron pnictides: a new class of high temperature superconductors

Temperature-doping phase diagram of the iron pnictides:

\[ \text{Resistivity} \sim \rho_0 + AT^\alpha \]


Temperature-doping phase diagram of the iron pnictides:

\[ \text{Resistivity } \sim \rho_0 + AT^\alpha \]

Temperature-doping phase diagram of the iron pnictides:

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Resistivity $\sim \rho_0 + AT^\alpha$


Temperature-doping phase diagram of the iron pnictides:

\[ \text{Resistivity} \sim \rho_0 + AT^\alpha \]

Temperature-doping phase diagram of the iron pnictides:

\( T_{SDW} \)

\( T_{0} \)

BaFe\(_2\)(As\(_{1-x}\)Px)\(_2\)

Strange Metal

\( T_{c} \)

Resistivity \( \sim \rho_0 + AT^\alpha \)


*Physical Review B* 81, 184519 (2010)
Temperature-doping phase diagram of the iron pnictides:

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]

Resistivity \( \rho \) behaves as \( \rho \sim \rho_0 + AT^\alpha \).


Temperature-pressure phase diagram of an organic superconductor

Temperature-pressure phase diagram of an heavy-fermion superconductor

Ordinary metals (Fermi liquids)

Metal with “large” Fermi surface

Momenta with electron states occupied

Momenta with electron states empty
Fermi surface + antiferromagnetism

The electron spin polarization obeys

\[ \langle \vec{S}(r, \tau) \rangle = \varphi(r, \tau)e^{iK \cdot r} \]

where \( K \) is the ordering wavevector.
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

AF $\langle \phi \rangle \neq 0$

Metal with “large” Fermi surface

$\langle \phi \rangle = 0$

Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

Friday, January 13, 2012
Quantum oscillations

$\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$

T. Helm, M. V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross,
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

Metal with “large” Fermi surface

AF $\langle \varphi \rangle \neq 0$

$s$

$\langle \varphi \rangle = 0$
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

AF \( \langle \varphi \rangle \neq 0 \)

\( \langle \varphi \rangle = 0 \)

Metal with “large” Fermi surface

\( s \)
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

AF \langle \phi \rangle \neq 0

Metal with “large” Fermi surface

Quantum critical point realizes a strongly-coupled “strange metal” with long-range entanglement!
AF metal with “small” Fermi pockets

Fermi liquid with “large” Fermi surface
AF metal with “small” Fermi pockets

High temperature Superconductor ✓

Fermi liquid with “large” Fermi surface

AF metal with “small” Fermi pockets

Fermi liquid with “large” Fermi surface

High temperature Superconductor ✓

Strange Metal with long-range entanglement

High temperature Superconductor

AF metal with “small” Fermi pockets

Fermi liquid with “large” Fermi surface

Challenge to string theory:

Describe quantum critical points and phases of metallic systems
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

Yes
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals?

Yes, lots of them, with many “strange” properties!
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement?

Yes, a very select subset has the proper logarithmic violation of the area law of entanglement!!

These are now being studied intensively........

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.
Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets.
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.
Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”