Quantum matter
and
gauge-gravity duality

HRI, Allahabad
January 12, 2012

Subir Sachdev
1. The superfluid-insulator quantum phase transition

   A. Field theory

   B. Holography

2. Compressible quantum liquids

   A. Field theory

   B. Holography
1. The superfluid-insulator quantum phase transition
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2. Compressible quantum liquids
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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$. 

- Describe zero temperature phases where $d Q/d\mu = 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H \rightarrow H - \mu Q$. 

- Compressible systems must be gapless. 

- “Relativistic” quantum critical systems are compressible in $d = 1$, but not for $d > 1$. 

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Compressible systems must be gapless.

“Relativistic” quantum critical systems are compressible in $d = 1$, but not for $d > 1$. 
One compressible state is the **solid** (or “Wigner crystal” or “stripe”). This state breaks translational symmetry.
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $Q$.

Condensate of fermion pairs
Graphene
The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**.
The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**.

- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.
The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**

- **Luttinger relation**: The total “volume (area)” $A$ enclosed by the Fermi surface is equal to $\langle Q \rangle$. 

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The Fermi Liquid (FL)
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Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green’s function $G_f$ has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

\[ \mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms} \]

\[ \mathcal{A} = \langle f_\sigma^\dagger f_\sigma \rangle = \langle Q_\sigma \rangle \]

\[ G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2} \]
The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field $A_\mu$.

\[
\mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - i A_\tau - \frac{(\nabla - i A)^2}{2m} - \mu \right) f_\sigma
\]

\[
= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
\]
**The Non-Fermi Liquid (NFL)**

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field $A_\mu$.

- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.


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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.

Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.

The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

\[ G^{-1} = q_1 - \eta F(\omega/qz) \]

where \( q_x = k_x - k_F \), \( q_y = k_y \), and \( q = q_x + q_y/(2k_F) \), and \( \eta \) and \( z \) are anomalous exponents.

To three-loop order, we find \( \eta = 0 \) and \( z = \frac{3}{2} \).

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One-loop order: \( G_f^{-1} \sim v_F q + i\omega^{2/3} \)

\[ A = \langle f_\sigma^\dagger f_\sigma \rangle = \langle Q_\sigma \rangle \]

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Key question:

How do we detect the “hidden Fermi surfaces” of fermions with gauge charges in the non-Fermi liquid phases?

These are not directly visible in the gauge-invariant fermion correlations computable via holography.
How do we detect the “hidden Fermi surfaces” of fermions with gauge charges in the non-Fermi liquid phases?

Compute entanglement entropy

Entanglement entropy of Fermi surfaces

\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

Entanglement entropy \( S_{EE} = -\text{Tr} (\rho_A \ln \rho_A) \)
Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum $k_F$, where $P$ is the perimeter of region A with an arbitrary smooth shape.

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Non-Fermi liquids have, at most, the “1/12” prefactor modified.

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Consider the following (most) general metric for the holographic theory

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + \frac{r^{2\theta/(d-\theta)}}{d-\theta} dr^2 + d\boldsymbol{x}_i^2 \right) \]
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\]

This metric transforms under rescaling as

\[
    x_i \rightarrow \zeta x_i \\
    t \rightarrow \zeta^z t \\
    ds \rightarrow \zeta^{\theta/d} ds.
\]

This identifies \( z \) as the dynamic critical exponent (\( z = 1 \) for “relativistic” quantum critical points).
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This identifies \( z \) as the dynamic critical exponent \((z = 1 \text{ for “relativistic” quantum critical points})\).

What is \( \theta \)? \((\theta = 0 \text{ for “relativistic” quantum critical points})\).
At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, $S$, is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$.
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The entropy density, $S$, is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$

Under rescaling $r \rightarrow \zeta^{(d-\theta)/d}r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/\zeta}$$

So $\theta$ is the “violation of hyperscaling” exponent.
A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent $z$. So we expect compressible quantum states to have an effective dimension $d - \theta$ with

$$\theta = d - 1$$
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

Entanglement entropy of the metric

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + \frac{r^{2\theta/(d-\theta)}}{d-\theta} dr^2 + dx_i^2 \right) \]

The area law is obeyed for

\[ \theta \leq d - 1 \]
Entanglement entropy of the metric

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d/(d-1)}(d-\theta)} + \frac{r^{2\theta/(d-\theta)}d^2 + dx_i^2}{r^{2d/(d-1)}(d-\theta)} \right) \]

For \( \theta = d - 1 \), the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

\[ S_E = \Xi \frac{Q^{(d-1)/d}}{\Sigma} \ln \left( \frac{Q^{(d-1)/d}}{\Sigma} \right). \]

- \( \Sigma \) is the \((d - 1)\)-dimensional surface area of entangling region (in \( d = 2 \), \( \Sigma = P \) is the perimeter). Note \( S_E \) is otherwise independent of the shape of the entangling region, unlike other gapless systems. This is a characteristic property of a Fermi surface.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573
Entanglement entropy of the metric

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + \frac{r^{2\theta/(d-\theta)}dr^2 + dx_i^2}{(d-\theta)} \right) \]

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- The dependence of the entanglement entropy on the boundary charge density, \( Q \), is computed by realizing the metric in an Einstein-Maxwell-dilaton theory.

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- \( \Xi \) is a dimensionless constant which is **independent** of \( Q \) and of any property of the entangling region.

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\[ S_E = \Xi Q^{(d-1)/d} \Sigma \ln \left( Q^{(d-1)/d} \Sigma \right). \]

- The metric has a complicated dependence on \( Q \), but \( S_E \) is just proportional to \( Q^{(d-1)/d} \). Many UV details are irrelevant, and \( S_E \) flows to the universal \( Q \) dependence in the IR. By Luttinger’s relation \( Q \sim k_F^d \), and so the prefactor is the area of the Fermi surface, as expected from field theory.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573
Inequalities

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]

The area law of entanglement entropy is obeyed for

\[ \theta \leq d - 1. \]

The “null energy condition” of the gravity theory yields

\[ z \geq 1 + \frac{\theta}{d}. \]

Remarkably, for \( d = 2, \theta = d - 1 \) and \( z = 1 + \theta/d \), we obtain \( z = 3/2 \), the same value associated with the field theory.
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Holographic theory of a non-Fermi liquid (NFL)
Holographic theory of a fractionalized-Fermi liquid (FL*)

\[ \mathcal{E}_r = Q - Q_{\text{mesino}} \]

\[ \mathcal{E}_r = Q \]
Holographic theory of a Fermi liquid (FL)

$\mathcal{E}_r = 0$

$\mathcal{E}_r = Q$
Holographic theory of a Fermi liquid (FL)

Gauss Law in the bulk
\[ \mathcal{E}_r = 0 \]

⇔

Luttinger theorem on the boundary
\[ \mathcal{E}_r = Q \]

Gauss Law in the bulk
⇔
Luttinger theorem on the boundary
**Theory of a non-Fermi liquid (NFL)**

### Field theory

- A gauge-dependent Fermi surface of overdamped gapless fermions.

### Holography

- Fermi surface is hidden.

---

**Theory of a non-Fermi liquid (NFL)**

- **Thermal entropy density**
  \[
  S \sim T^{1/z}
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  in \( d = 2 \), where \( z \) is the dynamic critical exponent.

- **Logarithmic violation of area law of entanglement entropy**,
  with prefactor proportional to the product of \( Q (d-1)/d \) and the boundary area of the entangling region.

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**Theory of a non-Fermi liquid (NFL)**

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  in all \( d \) for hyperscaling violation exponent \( \theta = d - 1 \), and \( z \) the dynamic critical exponent.

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Strongly interacting non-Fermi liquid states (with “hidden” Fermi surfaces of gauge-dependent particles) are realized by holographic theories with dynamic critical exponent $z$, and violation of hyperscaling exponent $\theta = d-1$.
Strongly interacting non-Fermi liquid states (with “hidden” Fermi surfaces of gauge-dependent particles) are realized by holographic theories with dynamic critical exponent $z$, and violation of hyperscaling exponent $\theta = d-1$.

The entanglement entropy of this theory has all the properties expected for a Fermi surface enclosing the volume expected by the Luttinger relation.
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The thermal entropy also has the properties expected by the presence such a Fermi surface.
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The thermal entropy also has the properties expected by the presence such a Fermi surface.

“Visible” Fermi surfaces of gauge-neutral particles are realized in a confining metric.