Universal low temperature theory of SYK models of strange metals and charged black holes

Subir Sachdev

New Directions in Theoretical Physics 3,
Higgs Centre for Theoretical Physics
University of Edinburgh
January 11, 2019
Wenbo Fu  
Yingfei Gu  
Grigory Tarnopolsky
Main result

SYK model of fermions with random interactions of mean-square-value $J$, with total fermion number $Q$, at temperatures $T \ll J$
**Main result**

SYK model of fermions with random interactions of mean-square-value $J$, with total fermion number $Q$, at temperatures $T \ll J$

and

Charged black holes in 3+1 dimensions of radius $R_h$, with total charge $Q$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in 0 + 1 dimensions
Main result

The common low $T$ path integral is $\mathcal{Z} = \int \mathcal{D}f \mathcal{D}\phi e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left( \frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \text{Sch}\left[\tan(\pi T f(\tau)), \tau\right] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

$\phi$ is a phase conjugate to the charge density with

$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n, \text{ n integer},$$

$\text{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$. The couplings are related to the entropy $S(T, Q)$ and the chemical potential $\mu$ via

$$S(T \to 0, Q) = s_0 + \gamma T, \quad K = \left( \frac{dQ}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{dQ}.$$
Main result

- Not the AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.
- Unlike the AdS/CFT correspondence, both sides of the duality are solvable. This has enabled numerous recent studies of black holes quantum information.
Main result

A. Kitaev (2015)
J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)
J. Engelsoy, T.G. Mertens, and H.Verlinde, JHEP 1607 (2016) 139
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
1. Normal and strange metals
   (a) with quasiparticles (normal): 
       random matrix model
   (b) without quasiparticles (strange): 
       the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian
1. Normal and strange metals
   (a) with quasiparticles (normal):
       random matrix model
   (b) without quasiparticles (strange):
       the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
Quasiparticles are additive excitations: The low-lying excitations of the many-body system can be identified as a set \( \{ n_\alpha \} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha, \beta} F_{\alpha \beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly(\( N \)) numbers.
Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{eq} \sim \frac{\hbar E_F}{(k_B T)^2}, \quad \text{as } T \to 0,$$

where $E_F$ is the Fermi energy.
Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{eq} \sim \frac{\hbar E_F}{(k_B T)^2}$$

as $T \to 0$,

where $E_F$ is the Fermi energy.

This time is much longer than the ‘Planckian time’ $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau_{eq} \gg \frac{\hbar}{k_B T}$$

as $T \to 0$. 

What are quasiparticles?
A simple model of a metal with quasiparticles

Pick a set of random positions
A simple model of a metal with quasiparticles

Place electrons randomly on some sites
A simple model of a metal with quasiparticles

Place electrons randomly on some sites
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

$t_{ij}$ are independent random variables with $\bar{t_{ij}} = 0$ and $|t_{ij}|^2 = t^2$

Fermions occupying the eigenstates of a $N \times N$ random matrix
A simple model of a metal with quasiparticles

Let $\varepsilon_\alpha$ be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest $NQ$ eigenvalues, upto the Fermi energy $E_F$. The single-particle density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$. 

$$\rho(\omega) = -\frac{1}{\pi} \text{Im} G(\omega)$$
A simple model of a metal with quasiparticles

There are $2^N$ many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_{\alpha} \varepsilon_{\alpha},$$

where $n_{\alpha} = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_{\alpha}$ have a level spacing $\sim 1/N$. 

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$
A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|\overline{U_{ij;kl}}|^2 = U^2$. We compute the lifetime of a quasiparticle, $\tau_\alpha$, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamiltonian with energy $\varepsilon_\alpha$. By Fermi’s Golden rule, for $\varepsilon_\alpha$ at the Fermi energy

$$\frac{1}{\tau_\alpha} = \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1-f(\varepsilon_\gamma))(1-f(\varepsilon_\delta)) \delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta)$$

$$= \frac{\pi^3 U^2 \rho_0^2}{4} T^2$$

where $\rho_0$ is the density of states at the Fermi energy, and $f(\varepsilon) = 1/(e^{\varepsilon/T} + 1)$ is the Fermi function.

**Fermi liquid state:** Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.
1. Normal and strange metals
   (a) with quasiparticles (normal):
   random matrix model
   (b) without quasiparticles (strange):
   the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
“Strange”,

“Bad”,

or “Incoherent”,

metal found ubiquitously at temperatures $T > T_c$ (the superconducting critical temperature) has a resistivity, $\rho$, which obeys

$$\rho \sim T,$$

and

in some cases $\rho \gg \hbar/e^2$

(in two dimensions), where $\hbar/e^2$ is the quantum unit of resistance.
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen molecule:

\[
\begin{align*}
\text{Hydrogen molecule:} & = \begin{array}{ccc}
\bullet & \bullet & \bullet \\
\uparrow & \downarrow & \uparrow \\
\end{array} \\
& = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\end{align*}
\]
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away.
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The SYK model

Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites
Place electrons randomly on some sites
Entangle electrons pairwise randomly

The SYK model
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
The SYK model

This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell}^{N} U_{ij;k\ell} c_\ell^{\dagger} c_j^{\dagger} c_k c_\ell - \mu \sum_i c_i^{\dagger} c_i \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j c_i^{\dagger} = \delta_{ij} \]

\[ Q = \frac{1}{N} \sum_i c_i^{\dagger} c_i \]

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $|U_{ij;k\ell}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

The SYK model

The large $N$ limit is given by the sum of “melon” Feynman graphs

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

- Solution of the melon diagrams yields a Green’s function at $T = 0$ of the form

$$G(\tau) \sim \begin{cases} 
-\tau^{-2\Delta} & \tau > 0 \\
e^{-2\pi \mathcal{E} (\tau)^{-2\Delta}} & \tau < 0
\end{cases} , \quad T = 0$$

where $\Delta = 1/4$ is the fermion scaling dimension. The \textit{particle-hole asymmetry} is determined by the parameter $\mathcal{E}$ which universally depends only upon $Q$.

- At $T > 0$ this has the conformal form

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{2\Delta} , \quad 0 < \tau < 1/T .$$

S. Sachdev and J. Ye, PRL \textbf{70}, 3339 (1993)
Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

There are $2^N$ many body levels with energy $E$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = Ns_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi \mathcal{E}.$$ 

At $Q = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where $G$ is Catalan’s constant.

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

\[ \tau_{eq} \sim \frac{\hbar}{k_B T}, \quad \text{as } T \to 0. \]

Established by solution of Schwinger-Keldysh equations for a quench.

A. Georges and O. Parcollet
PRB 59, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)
The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

\[ \tau_{eq} \sim \frac{\hbar}{k_BT} , \quad \text{as } T \to 0. \]

Established by solution of Schwinger-Keldysh equations for a quench.

- Presence of quasiparticles should slow down thermalization, so all quantum systems obey

\[ \tau_{eq} > C \frac{\hbar}{k_BT} , \quad \text{as } T \to 0. \]

Absence of quasiparticles \(\Leftrightarrow\) Fastest possible thermalization


A. Georges and O. Parcollet
PRB 59, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)
Coupled SYK Islands

SYK quantum islands of electrons with random or regular hopping between them.

\[ H = \sum_{x} \sum_{i < j, k < l} U_{ijkl,x} c_{i,x}^\dagger c_{j,x} c_{k,x} c_{l,x} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x} c_{i,x'} c_{j,x'} \]

\[ |U_{ijkl}|^2 = \frac{2U^2}{N^3} \quad |t_{ij,xx'}|^2 = \frac{t_0^2}{N}. \]

Pengfei Zhang, PRB 96, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX 8, 021049 (2018)

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Coupled SYK Islands

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Coupled SYK Islands

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( T < E_c \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[
\rho = \frac{\hbar}{e^2} \left[ c_1 + c_2 \left( \frac{T}{E_c} \right)^2 \right]
\]

\[
s \sim s_0 \left( \frac{T}{E_c} \right)
\]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( E_c < T < U \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho \sim \frac{\hbar}{e^2} \left( \frac{T}{E_c} \right), \quad s = s_0 \]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
1. Normal and strange metals
   (a) with quasiparticles (*normal*):
      random matrix model
   (b) without quasiparticles (*strange*):
      the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
• Black holes have an entropy and a temperature, $T_H$

• The entropy is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds.
The ring-down is predicted by General Relativity to happen in a time
$$\frac{8\pi GM}{c^3} \sim 8 \text{ milliseconds.}$$
Curiously this happens to equal
$$\frac{\hbar}{k_B T_H},$$
so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!
• Black holes have an entropy and a temperature, $T_H$

• The entropy is proportional to their surface area.

• They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.
Zooming into the near-horizon region of a charged black hole, zooming at low temperature, yields a quantum theory in one space ($\vec{x}$) and one time dimension ($\zeta$).
Charged black holes

\[ S_{EM} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( \mathcal{R} + \frac{6}{L^2} - \frac{L^2}{g_F^2} F^2 \right) \]

Solutions of \( S_{EM} \) have metric and gauge field \((F = dA)\)

\[
d s^2 = -V(r) dt^2 + r^2 d\Omega_2^2 + \frac{dr^2}{V(r)} \quad , \quad A = \mu \left( 1 - \frac{r_0}{r} \right) dt
\]

\[
V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^2} - \frac{M}{r}.
\]

where \( d\Omega_2^2 \) is the metric of the 2-sphere. All parameters of the solution, and the thermodynamics are determined in terms of the chemical potential \( \mu \), and the Hawking temperature of horizon, \( T \).

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD 60, 064018 (1999)
Charged black holes

In the $T \to 0$ limit, at fixed $\mu$, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of $R_h$

- In the near-horizon region, we change co-ordinates from $r$ to $\zeta$ so that

$$r - R_h = \frac{R_2^2}{\zeta}, \quad R_2 = \frac{LR_h}{\sqrt{6R_h^2 + L^2}}.$$

Then the near-horizon metric becomes $\text{AdS}_2 \times S_2$, with

$$ds^2 = R_2^2 \left[ \frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega^2_2, \quad A = \frac{\mathcal{E}}{\zeta} dt.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = g_F R_h \frac{\sqrt{3R_h^2 + L^2}}{6R_h^2 + L^2}.$$
Charged black holes

Black hole horizon of radius $R_h$ and entropy $s_0$

\[ ds^2 = R_h^2 (d\zeta^2 - dt^2)/\zeta^2 + R_h^2 d\Omega_2^2 \]

Gauge field: $A = (\mathcal{E}/\zeta) dt$

The entropy $s_0$, the charge $Q$, and the dimensionless electric field $\mathcal{E}$ obey

\[ \frac{ds_0}{dQ} = 2\pi \mathcal{E} \]
A probe fermion has a near-horizon Green’s function at $T = 0$ of the form

$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi \mathcal{E}} (-\tau)^{-2\Delta} & \tau < 0 \\ \end{cases}, \quad T = 0$$

where $\Delta$ is the fermion scaling dimension. The particle-hole asymmetry is determined by the dimensionless electric field $\mathcal{E}$ at the surface of the black hole.

At $T > 0$ this has the conformal form

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{2\Delta}, \quad 0 < \tau < 1/T.$$
1. Normal and strange metals
   (a) with quasiparticles (**normal**):
      random matrix model
   (b) without quasiparticles (**strange**):
      the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian
The SYK model

The theory of fluctuations about the large $N$ saddle point are expressed in terms of a bilocal time Green’s function, $G(\tau_1, \tau_2)$. This theory is invariant at low energies under a time reparameterization $f$, and an emergent gauge transformation $g$

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{G}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
The SYK model

The large $N$ saddle point is

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}.$$

The saddle point will be invariant under a reparametrization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\tilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for $\Sigma$). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.
The effective theory of time reparameterizations $f(\tau)$ broken down to SL(2,R), and gauge transformations $g(\tau) = e^{i\phi(\tau)}$ is

$$
S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau f)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \left\{ \tan(\pi T f(\tau)), \tau \right\},
$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings $K$, $\gamma$, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$
\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left( \frac{g''}{g'} \right)^2.
$$

Specifically, an argument constraining the effective action at $T = 0$ is

$$
S_{\text{eff}} \left[ f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,
$$

and this is origin of the Schwarzian.
Near-extremal black hole of radius $R$ and $T \ll 1/R$ from 4D Einstein-Maxwell

$\Longrightarrow$ 2D Einstein-Maxwell with a scalar field (fluctuations of black hole radius)

$\Longrightarrow$ Absence of 2D quantum gravity fluctuations: reduction to (0+1)D

$\Longrightarrow$ (0 + 1)D theory is precisely the Schwarzian theory of the SYK model.

\[
S_{4D} = \int d^4 x \sqrt{-\hat{g}} \left( \hat{R} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),
\]

\[
\text{AdS}_2 \times S^2
\]

\[
ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2
\]

\[
\text{Gauge field: } A = (\mathcal{E}/\zeta) dt
\]

\[
\zeta = \infty
\]

\[
\zeta
\]

SYK models and black holes

- Reparameterization and gauge invariance are defining properties of Einstein-Maxwell theory.

- In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

\[ ds^2 = \frac{(d\tau^2 + d\zeta^2)}{\zeta^2} \]

\[ \tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1. \]
Quantum matter without quasiparticles

- Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.
Quantum matter without quasiparticles

- Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.

- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.
Quantum matter without quasiparticles

- Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.

- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS$_2$ horizons
Quantum matter without quasiparticles

- Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.

- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS$_2$ horizons

- Lattices of SYK islands have led to a partial understanding of strange metals.