Quantum Criticality and Black Holes

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Talk online at http://sachdev.physics.harvard.edu
Quantum Entanglement

Hydrogen atom:

Hydrogen molecule:

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

Superposition of two electron states leads to non-local correlations between spins
Quantum Phase Transition

Change in the nature of entanglement in a macroscopic quantum system.

Familiar phase transitions, such as water boiling to steam, also involve macroscopic changes, but in thermal motion.
Quantum Criticality

The complex and non-local entanglement at the critical point between two quantum phases
Outline

1. Entanglement of spins
   *Experiments on antiferromagnetic insulators*

2. Black Hole Thermodynamics
   *Connections to quantum criticality*

3. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*
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The cuprate superconductors

Na-CCOC
- Cu
- Ca/Na
- O
- Cl

Temperature

hole concentration

AFI
PG
ECG
dSC

~2-3 %
~5-10 %
~15 %
Antiferromagnetic (Neel) order in the insulator

No entanglement of spins
Antiferromagnetic (Neel) order in the insulator

Excitations: 2 spin waves (Goldstone modes)
Weaken some bonds to induce spin entanglement in a new quantum phase.
Ground state is a product of pairs of entangled spins.

\[ = \frac{1}{\sqrt{2}} (\uparrow \downarrow \rangle - \downarrow \uparrow \rangle) \]
Excitations: 3 $S=1$ triplons

$$= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
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$= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Quantum critical point with non-local entanglement in spin wavefunction
$\text{TlCuCl}_3$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Pressure in TlCuCl$_3$
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K.

Observation of $3 \rightarrow 2$ low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j ; \quad \vec{S}_i \rightarrow \text{spin operator with } S = 1/2 \]
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]

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Why should we care about the entanglement at an isolated critical point in the parameter space?
Quantum criticality

Conformal field theory (CFT) at $T > 0$

Temperature, $T$

Neel VBS

$K/J$
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Black Holes

Objects so massive that light is gravitationally bound to them.
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The region inside the black hole horizon is causally disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole \( S = \frac{k_B A}{4 \ell_P^2} \)

where \( A \) is the area of the horizon, and

\[ \ell_P = \sqrt{\frac{G \hbar}{c^3}} \] is the Planck length.

The Second Law: \( dA \geq 0 \)
Black Hole Thermodynamics

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Horizon temperature: \[4\pi k_B T = \frac{\hbar^2}{2M \ell_P^2}\]
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
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Black hole temperature = temperature of quantum criticality

Strominger, Vafa
AdS/CFT correspondence

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Black hole entropy

= entropy of quantum criticality in 2+1 dimensions

Strominger, Vafa
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Dynamics of quantum criticality = waves in curved gravitational background

Maldacena
AdS/CFT correspondence

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"Friction" of quantum critical dynamics = black hole absorption rates

Son
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Dope the antiferomagnets with charge carriers of density $x$ by applying a chemical potential $\mu$.

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

$a_0 = 3.9\text{Å}$

$a_0 = 5.4\text{Å}$
Superconductor
Superconductor

Scanning tunnelling microscopy
STM studies of the underdoped superconductor

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\[ a_0 = 3.9\text{Å} \]

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Topograph

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$

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Intense Tunneling-Asymmetry (TA) variation are highly similar

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Tunneling Asymmetry (TA)-map at \( E = 150 \text{meV} \)

\[ \text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2 \quad \text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y \]

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Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K

R map (150 mV)

12 nm

$4a_0$

Evidence for a predicted valence bond supersolid

Superconductor

Scanning tunnelling microscopy
Use coupling $g$ to induce a transition to a VBS insulator.
Use coupling $g$ to induce a transition to a VBS insulator.
Proposed generalized phase diagram

Superconductor

Insulator $x=1/8$
Nernst measurements

Superconductor

Insulator \( x = 1/8 \)
Nernst experiment
Nernst measurements
Non-zero temperature phase diagram

- VBS Supersolid
- Superfluid
- Quantum critical
- VBS Insulator

Coulomb interactions
Non-zero temperature phase diagram

VBS Supersolid

Quantum-critical dynamics of vortices in a magnetic field, at generic density, and with impurities

Superfluid

VBS Insulator

Coulomb interactions
To the CFT of the quantum critical point, we add

- A chemical potential $\mu$
- A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

A precise correspondence is found between general hydrodynamics of vortices near quantum critical points and solvable models of black holes with electric and magnetic charges

In the hydrodynamic regime, $\hbar \omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu \nu}$,

\[
F^{\mu \nu} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & B \\
0 & -B & 0
\end{pmatrix},
\]

- $T^{\mu \nu}$, the stress energy tensor,
- $\rho$, the local number density,
- $P$, the local pressure,
- $\sigma_Q$, a universal conductivity, which is the single transport co-efficient.

- $J^\mu$, the current,
- $\varepsilon$, the local energy density,
- $u^\mu$, the local velocity, and

The dependence of $\varepsilon, P, \sigma_Q$ on $T$ and $v$ follows from simple scaling arguments.
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

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\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu
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\[ \partial_\mu J^\mu = 0 \]
\[ \partial_\mu T^\mu_\nu = F^\mu_\nu J_\nu \]
\[ T^\mu_\nu = (\varepsilon + P)u^\mu u^\nu + Pg^\mu_\nu \]
\[ J^\mu = \rho u^\mu \]

Constitutive relations which follow from Lorentz transformation to moving frame

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\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu \]
\[ T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} \]
\[ J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left( -\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \]

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient.

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[
\begin{align*}
\partial_\mu J^\mu & = 0 \\
\partial_\mu T^{\mu\nu} & = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} (\delta_\nu^{\mu} + u_\mu u_\nu) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} & = (\varepsilon + P) u_\mu u_\nu + P g^{\mu\nu} \\
J^\mu & = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u_\mu u_\nu) \left[ (\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\end{align*}
\]

Momentum relaxation from impurities

Only input parameters

\[ \hbar \nu = 47 \text{ meV } \text{Å} \]

\[ \tau_{\text{imp}} \approx 10^{-12} \text{ s} \]


Output

\[ \omega_c = 6.2 \text{GHz} \cdot \frac{B}{1 \text{T}} \left( \frac{35\text{K}}{T} \right)^3 \]
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THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

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Conclusions

- Quantum phase transitions in antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.