What is a “phase transition”? 

A change in the collective properties of a macroscopic number of atoms
What is a “quantum phase transition”?

Change in the nature of entanglement in a macroscopic quantum system.
Entanglement

Hydrogen atom:

Hydrogen molecule:

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Superposition of two electron states leads to non-local correlations between spins
Outline

*Quantum phase transitions*

1. Spin ordering in “Han purple”

2. Entanglement at the critical point: physical consequences at non-zero temperatures
   (a) Double-layer antiferromagnet
   (b) Superfluid-insulator transition
   (c) Hydrodynamics via mapping to quantum theory of black holes.

3. Entanglement of valence bonds

4. Conclusions
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Chinese Terracotta warriors (479-221 BC)

Han Purple – BaCuSi$_2$O$_6$

Each Cu$^{2+}$ has a single free electron spin

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Each Cu$^{2+}$ has a single free electron spin.

Vary the ratio $J/J'$.  

Vary the ratio $J/J'$
Vary the ratio $J/J'$
Temperature, $T$

Vary the ratio $J/J'$

Spin wave
Vary the ratio $J/J'$
Temperature, $T$

Vary the ratio $J/J'$

Neel

Spin gap

"Triplet magnon"

Vary the ratio $J/J'$
Vary the ratio $J/J'$

Temperature, $T$

Neel

Spin gap
Temperature, $T$

Vary the ratio $J/J'$

Vary the ratio $J/J'$

Temperature, $T$

**Quantum Criticality**

Thermal excitations interact via a universal S matrix.

Vary the ratio $J/J'$

Temperature, $T$

Decoherence time $= \frac{\hbar}{k_B T}$

Vary the ratio $J/J'$

Quantum critical transport

Spin diffusion constant

\[ D_s = \Theta \frac{c^2}{k_B T} \]

where \( \Theta \) is a universal number

Temperature, \( T \)

Vary the ratio \( J/J' \)

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Trap for ultracold $^{87}\text{Rb}$ atoms
The Bose-Einstein condensate in a periodic potential

$$|G\rangle = \left| \begin{array}{c} \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big|\end{array}\right| + \ldots 27 \text{ terms}$$


Lowest energy state for many atoms

$$|\text{BEC}\rangle = |G\rangle |G\rangle |G\rangle$$

$$= \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \left| \begin{array}{c} \big| \big| O \big| \big| O \big| \big|\end{array}\right| + \ldots 27 \text{ terms}$$

Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)
By tuning repulsive interactions between the atoms, states with multiple atoms in a potential well can be suppressed. The lowest energy state is then a \textit{Mott insulator} – it has negligible number fluctuations, and atoms cannot “flow”
Velocity distribution of $^{87}$Rb atoms

Velocity distribution of $^{87}$Rb atoms

Non-zero temperature phase diagram

Depth of periodic potential
Non-zero temperature phase diagram

Dynamics of the classical Gross-Pitaevski equation

Superfluid

Quantum critical

Insulator

Depth of periodic potential
Non-zero temperature phase diagram

Dilute Boltzmann gas of particle and holes

Superfluid

Insulator

Depth of periodic potential
Non-zero temperature phase diagram

No wave or quasiparticle description

Superfluid

Insulator

Depth of periodic potential
Resistivity of Bi films

Conductivity $\sigma$

$\sigma_{\text{Superconductor}} (T \to 0) = \infty$

$\sigma_{\text{Insulator}} (T \to 0) = 0$

$\sigma_{\text{Quantum critical point}} (T \to 0) \approx \frac{4e^2}{h}$


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.
Non-zero temperature phase diagram

Depth of periodic potential
Non-zero temperature phase diagram

Collisionless-to hydrodynamic crossover of a conformal field theory (CFT)

Superfluid

Insulator

Depth of periodic potential

Non-zero temperature phase diagram

Needed: Cold atom experiments in this regime

Collisionless-to hydrodynamic crossover of a conformal field theory (CFT)

Superfluid

Insulator

Depth of periodic potential

Maldacena’s AdS/CFT correspondence relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.
Maldacena’s AdS/CFT correspondence relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.
Hydrodynamics of a conformal field theory (CFT)

Waves of gauge fields in a curved background
The scattering cross-section of the thermal excitations is universal and so transport coefficients are universally determined by $k_BT$.

**Charge diffusion constant**

$$D_c = \Theta \frac{c^2}{k_BT}$$

**Conductivity**

$$\sigma = \Theta \frac{4e^2}{h}$$

For the (unique) CFT with a SU($N$) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant

\[ D_s = \frac{3}{4\pi} \frac{c^2}{k_B T} \]

Spin conductivity

\[ \sigma = \frac{N^{3/2}}{3\sqrt{2\pi}} \]

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, hep-th/0701036
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Resonance in benzene leads to a symmetric configuration of valence bonds

(F. Kekulé, L. Pauling)
Resonance in benzene leads to a symmetric configuration of valence bonds

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Resonance in benzene leads to a symmetric configuration of valence bonds

(F. Kekulé, L. Pauling)
Temperature-doping phase diagram of the cuprate superconductors
Antiferromagnetic (Neel) order in the insulator

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j; \quad \vec{S}_i \Rightarrow \text{spin operator with } S=1/2 \]
Induce formation of valence bonds by e.g. ring-exchange interactions

\[ H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum \text{4-spin exchange} \]

A. W. Sandvik, cond-mat/0611343
As in $\text{H}_2$ and benzene, each electron wants to pair up with another electron and form a valence bond

$$\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$
\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle \right) \]
\[
\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]
\[
\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]
\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Entangled liquid of valence bonds (Resonating valence bonds – RVB)

$$\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Valence bond solid (VBS)

Valence bond solid (VBS)

\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

Valence bond solid (VBS)

More possibilities for entanglement with nearby states

\[
\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
\]


Valence bond solid (VBS)
More possibilities for entanglement with nearby states

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More possibilities for entanglement with nearby states

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N. Read and S. Sachdev, 

Valence bond solid (VBS)
More possibilities for entanglement with nearby states

\[ \frac{1}{\sqrt{2}} (|↑↓⟩ - |↓↑⟩) \]

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More possibilities for entanglement with nearby states

\[ \frac{1}{\sqrt{2}} \left( |↑↓⟩ - |↓↑⟩ \right) \]

N. Read and S. Sachdev, 
Excitations of the RVB liquid

\[ \frac{1}{\sqrt{2}} \left( |↑↓⟩ - |↓↑⟩ \right) \]
Excitations of the RVB liquid

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
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\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Excitations of the RVB liquid

\[
\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
\]
Excitations of the RVB liquid

Electron *fractionalization*:
Excitations carry spin $S=1/2$ but no charge

$$ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$
Excitations of the VBS

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Excitations of the VBS

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\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
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Excitations of the VBS

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\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]
Excitations of the VBS

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Excitations of the VBS

Free spins are unable to move apart:
no fractionalization, but confinement

$$\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
Phase diagram of square lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_\square \text{4-spin exchange} \]

A. W. Sandvik, cond-mat/0611343
Phase diagram of square lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{4-spin exchange} \]

Phase diagram of square lattice antiferromagnet

RVB physics appears at the quantum critical point which has fractionalized excitations: “deconfined criticality”

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{4-spin exchange} \]

Second-order critical point described by

\[ S_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + r |z_\alpha|^2 + \frac{\mu}{2} \left( |z_\alpha|^2 \right)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right] \]

at its critical point \( r = r_c \), where \( z_\alpha \) are the neutral \( S = 1/2 \) spinons and \( A_\mu \) is a non-compact \( U(1) \) gauge field.

Quantum criticality of fractionalized excitations
Phases of nuclear matter
Observation of a valence bond solid (VBS)

$X[Pd(dmit)_2]_2$

One free electron spin on each vertex of a triangular lattice

Observation of a valence bond solid (VBS)

Pressure-temperature phase diagram of ETMe$_3$P[Pd(dmit)$_2$]$_2$

Y. Shimizu et al. cond-mat/0612545
Temperature-doping phase diagram of the cuprate superconductors
Temperature-doping phase diagram of the cuprate superconductors

Deconfined quantum critical point (DQCP)

Neel order

VBS order
Temperature-doping phase diagram of the cuprate superconductors

Neel order

Neel order + $d$-wave superconductivity

DQCP

“Superconducting algebraic holon liquid”

$d$-wave superconductivity

Hole concentration

R.K. Kaul, Y.-B. Kim, S. Sachdev and T. Senthil, to appear
Temperature-doping phase diagram of the cuprate superconductors

Quantum critical phases with enhanced VBS correlations

Neel order

Neel order + $d$-wave superconductivity

DQCP

“Superconducting algebraic holon liquid”

$d$-wave superconductivity

Hole concentration

R.K. Kaul, Y.-B. Kim, S. Sachdev and T. Senthil, to appear
Temperature-doping phase diagram of the cuprate superconductors

STM in zero field
“Glassy” Valence Bond Solid (VBS)

Temperature-doping phase diagram of the cuprate superconductors

“Glassy” Valence Bond Solid (VBS)
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Conclusions

• Studies of new materials and trapped ultracold atoms are yielding new quantum phases, with novel forms of quantum entanglement.

• Some materials are of technological importance: e.g. high temperature superconductors.

• Real-world studies on the entanglement of large numbers of qubits: insights may be important for quantum cryptography and quantum computing.

• Tabletop “laboratories for the entire universe”: quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.