Non-Fermi liquids
vs.
Strange Metals
vs.
Bad Metals

Gordon Research Conference, Mt. Holyoke, June 29, 2022
Subir Sachdev

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
arXiv: 2109.05037, Reviews of Modern Physics
Quantum criticality of Ising-nematic ordering in a metal

Pomeranchuk instability as a function of coupling $\lambda$

or

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$
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Properties of a non-Fermi liquid:

- No quasiparticle excitations.
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- However, there is a sharp Fermi surface in momentum space, and its enclosed volume obeys the Luttinger relation.
- Relaxational and dissipative phenomena (and OTOCs) are controlled by a ‘Planckian’ time $\sim \hbar/(k_BT)$, which is independent of the energy scale of the interactions.

$$G(\omega, k) = \frac{1}{\omega - \varepsilon(k) + iT^{2/3}F\left(\frac{\hbar\omega}{k_BT}\right)}$$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

Quantum critical

$\langle \phi \rangle = 0$

$\langle \phi \rangle \neq 0$

Fermi liquid

Strongly-coupled "non-Fermi liquid" metal with no quasiparticles

$T_{I-n}$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

- Strongly-coupled "non-Fermi liquid" metal with no quasiparticles
- Fermi liquid
- Strange Metal ??

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Strange Metal ??

No.
FIG. 2 Measurement of the diffusion constant (a) and compressibility ((a)-inset) for a gas of ultra-cold 6Li atoms in an optical lattice, realizing a two-dimensional Fermi-Hubbard model with $U/t'\approx 5$ at a density $n_0^{\prime}$. (b) Reconstructed ‘resistivity’ using Einstein-Sutherland relation. Grey horizontal dashed line represents the estimated MIR value. Theoretical calculations using DMFT (in green) and the finite-T Lanczos method (in blue) are shown; the band representation indicates estimated error bars. Adapted from (Brown et al., 2019).

\[ A = \frac{d\rho}{dT} \]

LSCO: Giraldo-Gallo et al. 2018

MATBG: Jaoui et al. 2021

Strange metals
Strange metals

Properties of a strange metal:

- Resitivity \( \rho(T) = \rho_0 + AT + \ldots \) as \( T \to 0 \) and \( \rho(T) < h/e^2 \) (in \( d = 2 \)). Metals with \( \rho(T) > h/e^2 \) are bad metals.
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- Optical conductivity

$$\sigma(\omega) = \frac{K}{\tau(\omega)} - i \frac{m^*(\omega)}{m}; \quad \frac{1}{\tau(\omega)} = T G \left( \frac{\hbar \omega}{k_B T} \right)$$


B. Michon…A. Georges, arXiv:2205.04030
Fermi surface coupled to a critical boson

Occupied states
\( \varepsilon(k) < 0 \)

Empty states
\( \varepsilon(k) > 0 \)

a critical boson \( \phi \)

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order...
Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the $\phi$ bosons.

Analogous to electron-phonon scattering in metals, where we have "Bloch's law":

$$\rho \sim T \times T^{-5/3}$$

"Bloch's law" for the non-Fermi liquid yields:

$$\rho \sim T^{-4/3}$$

(after including the $(1 - \cos \theta)$ factor in transport rate).

However, Bloch's law ignores conservation of total momentum, or phonon drag.
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- “Bloch’s law” for the non-Fermi liquid in $d = 2$ yields $\rho(T) \sim T^{4/3}$ (after including ‘$(1 - \cos \theta)$’ factor in transport rate).
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**PHONON DRAG**

Peierls$^{28}$ pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$. This behavior has yet to be observed.

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Transport in non-Fermi liquids:

We cannot separate the momenta carried by the fermions and the bosons, because neither of them exists at low energies! We must treat the combined system together (extreme drag), and conservation of momentum in the low energy theory implies $\rho(T) = 0$. This first became clear from holographic and hydrodynamic approaches, and has now been confirmed in complete diagrammatic/Boltzmann computations.
Strange metals

- Occupied states $\varepsilon(k) < 0$
- Empty states $\varepsilon(k) > 0$

$\psi$

$a$ critical boson $\phi$

- Nematic order
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$+ \text{“something else....”}$
Strange metals

- **Umklapp**

- **$t$-$J$ model in large dimensions, with random $J_{ij}$**

- **Order parameter with current-like symmetry**

- **Spatial disorder in fermion-boson Yukawa coupling**

- **Interference of disordered ‘diffusons’ and critical boson**
  Tsz Chun Wu, Yunxiang Liao, and Matthew S. Foster, arXiv:2206.01762