Building strange metals from SYK models

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Correlated Electron Systems
Gordon Research Conference
Mount Holyoke College
Quasiparticles are ubiquitous:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are ‘fractions’ of an electron)
What are quasiparticles?

- Quasiparticles are additive excitations:
  The low-lying excitations of the many-body system can be identified as a set \( \{ n_\alpha \} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha, \beta} F_{\alpha \beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly(\( N \)) numbers.
Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

\[ \tau_{eq} \sim \frac{\hbar E_F}{(k_B T)^2}, \quad \text{as } T \to 0, \]

where \( E_F \) is the Fermi energy.

**What are quasiparticles?**
1. Solvable model without quasiparticles
   SYK model of a `quantum island’

2. Lattice models of SYK islands
   Theories of strange metals

3. SYK U(1) gauge theory
   Solvable model with finite density of
   fermions, emergent gauge fields, and disorder
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The SYK model

Place electrons randomly on some sites
Entangle electrons pairwise randomly
Entangle electrons pairwise randomly

The SYK model
The SYK model

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
Entangle electrons pairwise randomly

The SYK model
The SYK model

Entangle electrons pairwise randomly
This describes both a strange metal and a black hole!
The SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;\ell k} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i^\dagger c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ Q = \frac{1}{N} \sum_i c_i^\dagger c_i \]

\( U_{ij;\ell k} \) are independent random variables with \( \overline{U_{ij;\ell k}} = 0 \) and \( |U_{ij;\ell k}|^2 = U^2 \)

\( N \to \infty \) yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

\[
G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)}, \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)
\]

\[
G(\tau = 0^-) = Q.
\]
The SYK model

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\]

\[
G(\tau = 0^-) = Q.
\]

Low frequency analysis shows that the solutions must be gapless and obey

\[
\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}
\]

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $Q$. 

Many-body level spacing \( \sim 2^{-N} = e^{-N \ln 2} \)

Non-quasiparticle excitations with spacing \( \sim e^{-Ns_0} \)

There are \( 2^N \) many body levels with energy \( E \), which do not admit a quasiparticle decomposition. Shown are all values of \( E \) for a single cluster of size \( N = 12 \). The \( T \to 0 \) state has an entropy \( S_{GPS} = Ns_0 \) with

\[
s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848 \ldots < \ln 2
\]

where \( G \) is Catalan’s constant, for the half-filled case \( Q = 1/2 \).

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
The SYK model

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$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where $G$ is Catalan’s constant, for the half-filled case $\mathcal{Q} = 1/2$.

No quasiparticles!

$$E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha, \beta} F_{\alpha \beta} n_\alpha n_\beta + \ldots$$

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$
The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

\[ \tau_{\text{eq}} \sim \frac{\hbar}{k_B T}, \quad \text{as } T \rightarrow 0. \]

A. Georges and O. Parcollet

PRB 59, 5341 (1999)

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)
The SYK model

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- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

\[ \tau_{eq} \sim \frac{\hbar}{k_B T}, \quad \text{as } T \to 0. \]

- Presence of quasiparticles should slow down thermalization, so all quantum systems obey

\[ \tau_{eq} > C \frac{\hbar}{k_B T}, \quad \text{as } T \to 0. \]

Absence of quasiparticles \( \Leftrightarrow \) Fastest possible thermalization

A. Georges and O. Parcollet
PRB 59, 5341 (1999)

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S. Sachdev, Quantum Phase Transitions, Cambridge (1999)
SYK models and black holes

- Black holes have an entropy proportional to their surface area, and a temperature, $T_H = \hbar c^3/(8\pi GM k_B)$.

- Black holes relax to thermal equilibrium in a ‘Planckian’ time $\sim \hbar/(k_B T_H) = 8\pi GM/c^3$.

- Black holes in $d + 1$ spatial dimensions are similar to a quantum system without quasiparticles in $d$ spatial dimensions.
SYK models and black holes

PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

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(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a “small” Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, AdS$_2 \times R^2$ physics of Reissner-Nordström black holes.

Bekenstein-Hawking entropy of AdS$_2$ horizon
at $T = 0 \leftrightarrow Ns_0$ entropy of SYK model

Black hole horizon

Einstein-Maxwell theory + cosmological constant
1. Solvable model without quasiparticles
   SYK model of a `quantum island’

2. Lattice models of SYK islands
   Theories of strange metals

3. SYK $U(1)$ gauge theory
   Solvable model with finite density of fermions, emergent gauge fields, and disorder
Coupled SYK Islands
SYK quantum islands of electrons with random hopping between them.

\[ H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} \]

\[ + \sum_{\langle xx'\rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x',x'} \]

\[ |U_{ijkl}|^2 = \frac{2U^2}{N^3} \quad |t_{ij,xx}|^2 = t^2_0/N. \]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
The basic features can be determined by a simple power-counting. Considering for simplicity implies strong interactions, and focus on the correlated regime and thermal conductivities completely governed by di-fermion "pair hopping" interactions. They obtained electrical with majorana fermion modes with random all-to-all four-fermion studied SYK during puzzle at the heart of condensed matter physics. Com-

\[ H = \sum_{x} \sum_{i<j,k<l} U_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij} c_i^\dagger c_j c_{i'} c_{j'} \]

Coupled SYK Islands
Can also use non-random t, and the same U on all “islands”.

Pengfei Zhang, PRB 96, 205138 (2017)

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Coupled SYK Islands

Low ‘coherence’ scale

\[
E_c \sim \frac{t_0^2}{U}
\]

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Coupled SYK Islands

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( T < E_c \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[
\rho = \frac{h}{e^2} \left[ c_1 + c_2 \left( \frac{T}{E_c} \right)^2 \right]
\]

\[
s \sim s_0 \left( \frac{T}{E_c} \right)
\]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
Coupled SYK Islands

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( E_c < T < U \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho \sim \frac{h}{e^2} \left( \frac{T}{E_c} \right), \quad s = s_0 \]

See also A. Georges and O. Parcollet PRB 59, 5341 (1999)
SYK-Kondo lattice models

Mobile electrons ($c$) interacting with SYK quantum islands ($f$) with random exchange interactions.
Mobile electrons ($c$) interacting with SYK quantum islands ($f$) with non-random exchange interactions.
SYK-Kondo lattice models

\[ \frac{\rho}{h/e^2} \]

\[ T/T_0 \]

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178
SYK-Kondo lattice models

\[ \frac{\rho}{\hbar/e^2} \]

\[ \frac{\rho}{\hbar/e^2} = c_1 \frac{h}{e^2} \left( \frac{T}{T_0} \right) \]

Incoherent metal

Negligible magnetoresistance

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178
SYK-Kondo lattice models

\[
\frac{\rho}{h/e^2} = c_2 \frac{h}{e^2} \left( \frac{T}{T_0} \right)
\]

Linear-in-$B$ magnetoresistance with $B/T$ scaling in the presence of mesoscopic disorder.

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178
Resistivity \( \sim \rho_0 + AT^\alpha \)

Strange Metal

Superconductivity

\( \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \)


Linear-in-$B$ magnetoresistance with $B/T$ scaling

\[
\rho(H, T) - \rho(0, 0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma
\]


See talk by James Analytis
SYK-Kondo lattice models

\[ \frac{\rho}{\frac{h}{e^2}} \]

\[ \frac{T}{T_0} \]

\[ 1 \]

\[ 1 \]

Marginal Fermi liquid

However, this is a marginal Fermi liquid with a small Fermi surface

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178
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3. SYK U(1) gauge theory
   Solvable model with finite density of
   fermions, emergent gauge fields, and disorder
Anomalous Criticality in the Electrical Resistivity of La$_{2-x}$Sr$_x$CuO$_4$

R. A. Cooper,$^1$ Y. Wang,$^1$ B. Vignolle,$^2$ O. J. Lipscombe,$^1$ S. M. Hayden,$^1$ Y. Tanabe,$^3$ T. Adachi,$^3$
Y. Koike,$^3$ M. Nohara,$^4$* H. Takagi,$^4$ Cyril Proust,$^2$ N. E. Hussey$^1$†

The presence or absence of a quantum critical point and its location in the phase diagram of high-magnetic fields to expose the normal state superconductors may also be governed by proximal critical systems.
From the resistivity, they determined the value of the number $\alpha$ defined by

$$\rho(T) = \rho_0 + \alpha \frac{\hbar}{2e^2} \left( \frac{T}{T_F} \right)$$

where $T_F = (\pi \hbar^2 / k_B)(n/m^*)$ and $m^*$ is determined from the specific heat. This expression is obtained from the Drude form $\rho = m^* / (ne^2\tau)$ and $\hbar/\tau = \alpha k_B T$. 

Universal $T$-linear resistivity and Planckian limit in overdoped cuprates

A. Legros$^{1,2}$, S. Benhabib$^3$, W. Tabis$^{3,4}$, F. Laliberté$^1$, M. Dion$^1$, M. Lizaire$^1$,
B. Vignolle$^3$, D. Vignolles$^3$, H. Raffy$^5$, Z. Z. Li$^5$, P. Auban-Senzier$^5$,
N. Doiron-Leyraud$^1$, P. Fournier$^{1,6}$, D. Colson$^2$, L. Taillefer$^{1,6}$, and C. Proust$^{3,6}$

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4 AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, Al. Mickiewicza 30, 30-059 Krakow, Poland
5 Laboratoire de Physique des Solides, Université Paris-Sud, Université Paris-Saclay, CNRS UMR 8502, Orsay 91405, France
6 Canadian Institute for Advanced Research, Toronto, Ontario M5G 1Z8, Canada
Universal \( T \)-linear resistivity and Planckian limit in overdoped cuprates

A. Legros\(^{1,2} \), S. Benhabib\(^{3} \), W. Tabis\(^{3,4} \), F. Laliberté\(^{1} \), M. Dion\(^{1} \), M. Lizaire\(^{1} \), B. Vignolle\(^{3} \), D. Vignolles\(^{3} \), H. Raffy\(^{5} \), Z. Z. Li\(^{5} \), P. Auban-Senzier\(^{5} \), N. Doiron-Leyraud\(^{1} \), P. Fournier\(^{1,6} \), D. Colson\(^{2} \), L. Taillefer\(^{1,6} \), and C. Proust\(^{3,6} \)

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Material} & n & m^* & A_1/d & h/(2e^2T_F) & \alpha \\
\hline
\text{Bi2212} & p = 0.23 & 6.8 & 8.4 \pm 1.6 & 8.0 \pm 0.9 & 7.4 \pm 1.4 & 1.1 \pm 0.3 \\
\text{Bi2201} & p \sim 0.4 & 3.5 & 7 \pm 1.5 & 8 \pm 2 & 8 \pm 2 & 1.0 \pm 0.4 \\
\text{LSCO} & p = 0.26 & 7.8 & 9.8 \pm 1.7 & 8.2 \pm 1.0 & 8.9 \pm 1.8 & 0.9 \pm 0.3 \\
\text{Nd-LSCO} & p = 0.24 & 7.9 & 12 \pm 4 & 7.4 \pm 0.8 & 10.6 \pm 3.7 & 0.7 \pm 0.4 \\
\text{PCCO} & x = 0.17 & 8.8 & 2.4 \pm 0.1 & 1.7 \pm 0.3 & 2.1 \pm 0.1 & 0.8 \pm 0.2 \\
\text{LCCO} & x = 0.15 & 9.0 & 3.0 \pm 0.3 & 3.0 \pm 0.45 & 2.6 \pm 0.3 & 1.2 \pm 0.3 \\
\text{TMTSF} & P = 11 \text{ kbar} & 1.4 & 1.15 \pm 0.2 & 2.8 \pm 0.3 & 2.8 \pm 0.4 & 1.0 \pm 0.3 \\
\hline
\end{array}
\]

Slope of \( T \)-linear resistivity vs Planckian limit in seven materials.
Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero,
PRX 8, 021048 (2018)

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev,
PNAS 115, E3665 (2018)

See talk by Antoine Georges and poster by Mathias Scheurer
Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, PRX 8, 021048 (2018)


See talk by Antoine Georges and poster by Mathias Scheurer

Optimal doping critical point is associated with vanishing of the Higgs condensate. Overdoped regime is described by (a large Fermi surface of) electrically-charged fermions coupled to an emergent SU(2) gauge field in the presence of disorder

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB 80, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB 91, 115123 (2015)
Fermions with random hopping coupled to a fluctuating $U(1)$ gauge field

\[
H = -\frac{1}{(MN)^{1/2}} \sum_{ij=1}^{N} \sum_{\alpha=1}^{M} \left[ t_{ij}^{\alpha\beta} e^{iA_{ij}} f_{i\alpha} f_{j\beta} + (MN)^{1/2} \mu \delta_{ij} f_{i\alpha} f_{i\alpha} \right]
\]

\[
\ll t_{ij}^{\alpha\beta} t_{ji}^{\beta\alpha} \gg = \ll |t_{ij}^{\alpha\beta}|^2 \gg = t^2, \quad A_{ji} = -A_{ij}.
\]
Fermions with random hopping coupled to a fluctuating U(1) gauge field

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

$$\Pi(i\Omega_m) = 2t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n)G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}.$$
Fermions with random hopping coupled to a fluctuating U(1) gauge field

General low energy solution

\[ G(\tau > 0) = -\frac{C(\mathcal{E})}{t^{1-x}\tau^{1-x}}, \quad G(\tau < 0) = \frac{C(\mathcal{E})e^{-2\pi\mathcal{E}}}{t^{1-x}|\tau|^{1-x}}. \]

where \( \mathcal{E} \) is a parameter universally related to the filling fraction \( (\mathcal{E} = 0 \text{ at half-filling}) \). The exponent \( x \) is the solution to

\[
\frac{(1/x - 2)(\cosh(2\pi\mathcal{E}) - \cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{2M}{N}.
\]
Fermions with random hopping coupled to a fluctuating $U(1)$ gauge field

Resistivity $\rho \sim \frac{h}{e^2} \left( \frac{T}{t} \right)^{2x}$

Disordered strange metal as $T \to 0$ with all electrons contributing to transport.
The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

\[ \tau_{eq} \sim \frac{\hbar}{k_B T} , \quad \text{as } T \to 0. \]

- Presence of quasiparticles should slow down thermalization, so all quantum systems obey

\[ \tau_{eq} > C \frac{\hbar}{k_B T} , \quad \text{as } T \to 0. \]

Absence of quasiparticles \( \Leftrightarrow \) Fastest possible thermalization

A. Georges and O. Parcollet
PRB 59, 5341 (1999)

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)

S. Sachdev, Quantum Phase Transitions, Cambridge (1999)
Conclusions

- Solvable model without quasiparticles: SYK model of a ‘quantum island’

- Lattice models of SYK islands: Bad metal behavior with $\gamma \sim (T/E_c)(h/e^2)$ for $T > E_c$, and Fermi liquid behavior for $T < E_c$.

- SYK-Kondo lattice models: Bad metal behavior with $\gamma \sim (T/T_0)(h/e^2)$ for $T > T_0$, and marginal Fermi liquid (MFL) behavior for $T < T_0$ with $\gamma \sim (T/T_0)(h/e^2)$. MFL regime has small Fermi surface, and magnetoresistance $B/T$ scaling (with mesoscopic disorder).

- SYK U(1) gauge theory: solvable model with finite density of fermions, emergent gauge fields, and disorder. Strange metal behavior with $\gamma \sim (T/t)^2x(h/e^2)$ as $T \rightarrow 0$, with all electrons mobile.
Conclusions

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  \[\rho \sim (T/t)^{2x}(h/e^2)\] as \(T \to 0\), with \textit{all} electrons mobile.