d-form factor density waves and transport near the nematic quantum critical point

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1. STM observation of $d$-form factor density wave

   Close connection to pseudogap of cuprates;
   Relationship to nematicity

2. Transport near the Ising-nematic quantum critical point

   Domination of “drag” effects, and random “field” perturbations
Outline

1. STM observation of \( d \)-form factor density wave
   
   *Close connection to pseudogap of cuprates;*
   *Relationship to nematicity*

2. Transport near the Ising-nematic quantum critical point
   
   *Domination of “drag” effects, and random “field” perturbations*
Direct phase-sensitive visualization of the $d$-form factor density wave in underdoped cuprates

High temperature superconductors

$YBa_{2}Cu_{3}O_{6+x}$

CuO$_2$ plane

CuO$_2$ plane


R(r,150mV)
Bi2212
UD45K
A density wave with wavelength $\approx 4$ lattice sites?
Charge density wave (CDW) order

\[ \langle c^\dagger_{\alpha}(\mathbf{r}) c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q} \cdot \mathbf{r}} + \text{c.c.} \]
Plot of $P_{ii} = \langle c_{i\alpha}^{\dagger} c_{i\alpha} \rangle$ with

$$P_{ii} = e^{iQ \cdot r_i} + \text{c.c.}$$

with $Q = 2\pi (1/4, 0)$
**Unconventional** density wave (DW): Bose condensation of particle-hole pairs

\[
\langle c^\dagger_\alpha (\mathbf{r}_1) c_\alpha (\mathbf{r}_2) \rangle \\
= \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i \mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}
\]
Unconventional density wave (DW) : Bose condensation of particle-hole pairs

\[
\langle c_\alpha^\dagger (\mathbf{r}_1) c_\alpha (\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i \mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}
\]

Crucial “center-of-mass” co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal
**Unconventional** density wave (DW): Bose condensation of particle-hole pairs

\[
\langle c_\alpha^\dagger (\mathbf{r}_1) c_\alpha (\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i \mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}
\]

Density wave **form factor** (internal particle-hole pair wavefunction)

\[
\mathcal{P}(\mathbf{r}) = \int \frac{d^2 k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i \mathbf{k} \cdot \mathbf{r}}
\]

Time-reversal symmetry requires \( \mathcal{P}(\mathbf{k}) = \mathcal{P}(\mathbf{-k}) \).

We expand (using reflection symmetry for \( \mathbf{Q} \) along axes or diagonals)

\[
\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_s' (\cos k_x + \cos k_y) + \mathcal{P}_d (\cos k_x - \cos k_y)
\]
Plot of $P_{ij} = \langle c_{i\alpha}^{\dagger} c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

\[
P_{ij} = \left[ \int \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}
\]

$\mathcal{P}(\mathbf{k}) = 1$ and $\mathbf{Q} = 2\pi (1/4, 0)$
Unconventional DW order: \(s'-\)form factor

Plot of \(P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle\) for \(i = j\), and \(i, j\) nearest neighbors.

\[
P_{ij} = \left[ \int_k \mathcal{P}(k) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i \mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}
\]

\[
\mathcal{P}(k) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi (1/4, 0)
\]
Current order: $p$-form factor

Plot of $P_{ij} = \langle c^\dagger_{i\alpha} c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

\[
P_{ij} = \left[ \sum_k \mathcal{P}(k) e^{i k \cdot (r_i - r_j)} \right] e^{i Q \cdot (r_i + r_j)/2}
\]

\[
\mathcal{P}(k) = \sin(k_x) - \sin(k_y) \quad \text{and} \quad Q = (\pi, \pi)
\]

This state breaks time-reversal and is also known as “$d$-density wave” (but is $p$-form factor in our notation), and “staggered-flux (SF)”.

(Similar comments apply to “loop” orders of Varma and others.)
This specific $d$-form factor density wave order (with $Q$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).
Unconventional DW order: $d$-form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

$$P_{ij} = \left[ \int \mathcal{P}(k) e^{i k \cdot (r_i - r_j)} \right] e^{i Q \cdot (r_i + r_j)/2} + c.c.$$ 

$$\mathcal{P}(k) = e^{i \phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad Q = 2\pi \left( \frac{1}{4}, 0 \right)$$

Density wave on horizontal bonds has a phase-shift of $\pi$ relative to the wave on vertical bonds.

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$$P_{ij} = \left[ \int \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[ \cos(k_x) - \cos(k_y) \right]$$

and $\mathbf{Q} = 2\pi (0.317, 0)$

Density wave on horizontal bonds has a phase-shift of $\pi$ relative to the wave on vertical bonds.

This specific $d$-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. 111, 027202 (2013).
Unconventional DW order: $(d + s)$-form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[ 0.2 + \cos(k_x) - \cos(k_y) \right] \quad \text{and} \quad Q = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of $\pi$ relative to the wave on vertical bonds.

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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)]$$

and

$$\mathbf{Q} = 2\pi (1/4, 0)$$

A DW with $d$-form factor predominant is nearly degenerate with $d$-wave SC near the quantum-critical point for antiferromagnetism in a metal.


See also Y. Wang and A. V. Chubukov, arXiv:1401.0712;
See also


Note that these are identical images.

Broad \((0,Q)\) and \((Q,0)\) DW Features

Re \(O_x(q)\)

Im \(O_x(q)\)

Re \(O_y(q)\)

Im \(O_y(q)\)

For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.

Phase-sensitive measurement of the $d$-form factor of density wave order
Complex value of $O_x$ at a pixel

Phase-sensitive measurement of the $d$-form factor of density wave order
Phase-sensitive measurement of the $d$-form factor of density wave order

Complex value of $O_x$ at a pixel

Complex value of $O_y$ at same pixel

Complex value of $O_x$ at a pixel

Complex value of $O_y$ at same pixel

Phase-sensitive measurement of the $d$-form factor of density wave order

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Phase-sensitive measurement of the $d$-form factor of density wave order.

\[ \left| \frac{O_x(q,e) - O_y(q,e)}{2} \right|^2 \]

Spectral weight inside the broken circle

*d*-form Factor Predominant at Pseudogap Energy
$d$-form Factor Predominant at Pseudogap Energy

$|O_x(q,e) - O_y(q,e)|/2|^2$

Spectral weight inside the broken circle
$d$-form Factor Predominant at Pseudogap Energy

\[ \left| (O_x(q,e) - O_y(q,e)) / 2 \right|^2 \]

Spectral weight inside the broken circle
\[ \left| \frac{O_x(q,e) - O_y(q,e)}{2} \right|^2 \]

Spectral weight inside the broken circle
\[ \left| \frac{O_x(q,e) - O_y(q,e)}{2} \right|^2 \]

Spectral weight inside the broken circle
$d$-form Factor Predominant at Pseudogap Energy

$\left| \left( O_x(q,e) - O_y(q,e) \right)/2 \right|^2$

Spectral weight inside the broken circle
See also


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This specific $d$-form factor density wave order (with $Q$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. 111, 027202 (2013).
d form-factor density wave has unidirectional domains
dFF-DW Unidirectional Domains

$Z(r, 150\text{mV})$
\[
\frac{|O_y(r,q=Q_x)| - |O_x(r,q=Q_y)|}{|O_y(r,q=Q_x)| + |O_x(r,q=Q_y)|}
\]

Primary DW direction Orange : // (1,0), Blue : // (0,1)

dFF-DW Unidirectional Domains
dFF-DW Unidirectional Domains

\[
\frac{|O_y(r,q=Q_x)| - |O_x(r,q=Q_y)|}{|O_y(r,q=Q_x)| + |O_x(r,q=Q_y)|}
\]

Primary DW direction Orange : // (1,0), Blue : // (0,1)
Phases resolved Visualization of $d$-form factor DW in Cuprates

Doping dependence of density wave
Doping dependence of density wave

Doping dependence of nematicity

Science 344, 612 (2014)
Doping dependence of density wave

Doping dependence of nematicity

Nematicity and pseudogap tied to spin density wave fluctuations at lower doping?
Doping Dependence of DW Wavevector

\[ q_1^* \]

\[ q_5^* \]

**References**

Phase-resolved Visualization of $d$-form factor DW in Cuprates

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Quantum criticality of Ising-nematic ordering in a metal

A metal with a **Fermi surface** with full square lattice symmetry
Quantum criticality of Ising-nematic ordering in a metal

or

\[ \langle \phi \rangle \neq 0 \]

\[ \langle \phi \rangle = 0 \]

Pomeranchuk instability as a function of coupling \( \lambda \)
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

$T_{I-n}$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

$\lambda_c$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

Only at higher energies; at the lowest energy bosonic $\phi$ fluctuations are strongly coupled to fermionic excitations near Fermi surface.
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[
S_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]
\]

\[
S_c = \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger (\partial_\tau + \varepsilon_k) c_{k\alpha}
\]

\[
S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q (\cos k_x - \cos k_y) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha}
\]
Quantum criticality of Ising-nematic ordering in a metal

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Field theory of bosonic order parameter
Quantum criticality of Ising-nematic ordering in a metal

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Electrons with a Fermi surface: \( \varepsilon_k = -2t(\cos k_x + \cos k_y) - \mu \ldots \)
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[ S_\phi = \int d^2rd\tau ( (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - 1) |\phi|^2 ) \]

\[ S_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha} \]

\[ S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_\mathbf{q} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha} \]

“Yukawa” coupling between bosons and fermions
**Boltzmann view of electrical transport:**

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic $\phi$ fluctuations.
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic $\phi$ fluctuations.

- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$. 

Quantum criticality of Ising-nematic ordering in a metal

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• “Bloch’s law” for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$. 
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

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- “Bloch’s law” for the Ising-nematic critical point yields \( \rho(T) \sim T^{4/3} \).

However, this ignores “phonon drag”

PHONON DRAG

Peierls\(^{28}\) pointed out a way in which the low temperature resistivity might decline more rapidly than \( T^5 \).

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations $\phi$ fluctuations.
- Analogous to electron-phonon scattering in metals, where we have "Bloch’s law": a resistivity $\rho(T) \sim T^5$.
- "Bloch’s law" for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

However, this ignores “phonon drag”

PHONON DRAG

Peierls$^{28}$ pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$. This behavior has yet to be observed.

Rates of Momentum Flow

Electrons

SLOW

Phonons
Rates of Momentum Flow

Electrons → Phonons

\[ \text{SLOW} \]

Phonons → Defects

\[ \text{FAST} \]
Rates of Momentum Flow

Electrons

Phonons

Process controlling resistivity

Defects
Rates of Momentum Flow

Electrons \quad \text{FAST} \quad \text{Nematic boson } \phi
Rates of Momentum Flow

Electrons

LESS FAST

Nematic boson $\phi$

FAST
Electrons

Rates of Momentum Flow

LESS FAST

Electrons

FAST

Nematic boson $\phi$

SLOW

Defects
Rates of Momentum Flow

Electrons

LESS FAST

FAST

EVEN SLOWER

Defects

Nematic boson $\phi$

Defects

SLOW
Rates of Momentum Flow

Electrons

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Nematic boson $\phi$

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EVEN SLOWER

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Process controlling resistivity

SLOW
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[ S_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2(\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right] \]

\[ S_c = \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger (\partial_\tau + \varepsilon_k) c_{k\alpha} \]

\[ S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q \left( \cos k_x - \cos k_y \right) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha} \]
Quantum criticality of Ising-nematic ordering in a metal

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\[ S_c = \sum_{\alpha=1}^{N_f} \int d^2r d\tau c_\alpha^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} + \frac{\nabla^4}{2m'} + \ldots - \mu \right) c_\alpha \]

\[ S_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[ c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha \right\} \right. \]

\[ + \left. \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha^\dagger \right\} c_\alpha \right] \]

This continuum theory has strong electron–\( \phi \) scattering, and no quasi-particle excitations. But it has a conserved momentum \( \mathbf{P} \), and \( \chi_{J,\mathbf{P}} \neq 0 \) (“phonon drag”), and so the resistivity \( \rho(T) = 0 \).
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between $J_\mu$ and $T_{\mu\nu}$!

The most-probable state with a non-zero current $\mathbf{J}$ has a non-zero momentum $\mathbf{P}$ (and vice versa).

At non-zero density, $\mathbf{J}$ “drags” $\mathbf{P}$. 

$\mathbf{J}$

$\mathbf{P}$
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between $J_\mu$ and $T_{\mu\nu}$.

The most-probable state with a non-zero current $J$ has a non-zero momentum $P$ (and vice versa).

At non-zero density, $J$ “drags” $P$.

The resistivity of this metal is not determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by any excitation, whether neutral or charged, or fermionic or bosonic.
Quantum criticality of Ising-nematic ordering in a metal

**Transport without quasiparticles:**

- Focus on the interplay between $J_\mu$ and $T_{\mu\nu}$!

The dominant momentum loss occurs via the scattering of the neutral bosonic $\phi$ excitations off random fields. This is good news for the AdS/CMT approaches, which do not capture the Fermi surface of most of the charged carriers.
dFF-DW Unidirectional Domains

\[ \frac{|O_y(r,q=Q_x)| - |O_x(r,q=Q_y)|}{|O_y(r,q=Q_x)| + |O_x(r,q=Q_y)|} \]

Primary DW direction Orange : // (1,0), Blue : // (0,1)
Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[ S_{\text{dis}} = \int d^2rd\tau \left[ V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi \right], \]

\[ V(\mathbf{r}) = 0 \quad \text{;} \quad V(\mathbf{r})V(\mathbf{r}') = V_0^2 \delta(\mathbf{r} - \mathbf{r}'), \]
\[ h(\mathbf{r}) = 0 \quad \text{;} \quad h(\mathbf{r})h(\mathbf{r}') = h_0^2 \delta(\mathbf{r} - \mathbf{r}'), \]

we use the memory-function approach to obtain the resistivity for current along angle \( \vartheta \)

\[ \rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D^R_{\phi}(\omega, \mathbf{k})}{\omega} \right). \]
Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$S_{\text{dis}} = \int d^2 r d\tau \left[ V(r) c^\dagger c + h(r) \phi \right] ,$$

$$\overline{V(r)} = 0 \quad ; \quad \overline{V(r)V(r')} = V_0^2 \delta(r - r') ,$$

$$\overline{h(r)} = 0 \quad ; \quad \overline{h(r)h(r')} = h_0^2 \delta(r - r') ,$$

we use the memory-function approach to obtain the resistivity for current along angle $\vartheta$

$$\rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi_{c\dagger c}^R(\omega, k)}{\omega} + h_0^2 \frac{\text{Im} D^R_\phi(\omega, k)}{\omega} \right) .$$

Fermi surface term: Obtain $T$-dependent corrections to residual resistivity similar to earlier work


Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$S_{\text{dis}} = \int d^2r d\tau \left[ V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi \right],$$

$$\langle V(\mathbf{r}) \rangle = 0 ; \quad \langle V(\mathbf{r}) V(\mathbf{r'}) \rangle = V_0^2 \delta(\mathbf{r} - \mathbf{r'}),$$

$$\langle h(\mathbf{r}) \rangle = 0 ; \quad \langle h(\mathbf{r}) h(\mathbf{r'}) \rangle = h_0^2 \delta(\mathbf{r} - \mathbf{r'}),$$

we use the memory-function approach to obtain the resistivity for current along angle $\vartheta$

$$\rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi_{c\dagger c}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D_{\phi}^R(\omega, \mathbf{k})}{\omega} \right).$$

**Bosonic term: Dominant contribution:**

$$\rho(T) \sim h_0^2 T^{(d-z+\eta)/z}$$

Crosses over from the “relativistic” form ($z = 1, \eta \approx 0$) with $\rho(T) \sim h_0^2 T$ at higher $T$, to the “Landau-damped” form ($z = 3, \eta = 0$) with $\rho(T) \sim h_0^2 \left( T \ln(1/T) \right)^{-1/2}$ at lower $T$ (subtle corrections to scaling specific to this field theory).

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[
S_{\text{dis}} = \int d^2r d\tau \left[ V(r) c^\dagger c + h(r) \phi \right],
\]

\[
\overline{V(r)} = 0 \quad ; \quad \overline{V(r)V(r')} = V_0^2 \delta(r - r'),
\]

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Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

Resistivity from random-field disorder

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

\[ \rho(T) \sim h_0^2 T \] in region with ‘relativistic’ criticality of \( \phi \), with dynamic critical exponent \( z = 1 \).

Obtained by “memory function” \textit{and} by holography.

A. Lucas, S. Sachdev, and K. Schalm, \texttt{arXiv:1401.7933}
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

\[ \rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2} \]

in region with Landau-damped criticality of \( \phi \),
with dynamic critical exponent \( z = 3 \).

\[ \rho(T) \sim h_0^2 T \]

in region with 'relativistic' criticality of \( \phi \),
with dynamic critical exponent \( z = 1 \).

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Resistivity from random-field disorder


Pre-empted by SC

Conclusions

- *d*-form factor density wave observed in STM studies of numerous cuprates. Close spectroscopic link to pseudogap at low temperatures, and to quantum oscillations at high fields. Fluctuating order captures many aspects of pseudogap at higher temperatures.

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*d*-form factor density wave observed in STM studies of numerous cuprates. Close spectroscopic link to pseudogap at low temperatures, and to quantum oscillations at high fields. Fluctuating order captures many aspects of pseudogap at higher temperatures.


The resistivity of strongly-coupled metals is not determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by any excitation, whether neutral or charged, or fermionic or bosonic.