Damping of collective modes and quasiparticles in d-wave superconductors

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Transparencies on-line at http://pantheon.yale.edu/~subir

Review article: cond-mat/0005250
and references therein

Quantum Phase Transitions,
Cambridge University Press

Yale University

Elementary excitations of a d-wave superconductor

(A) *S=0* Cooper pairs, phase fluctuations

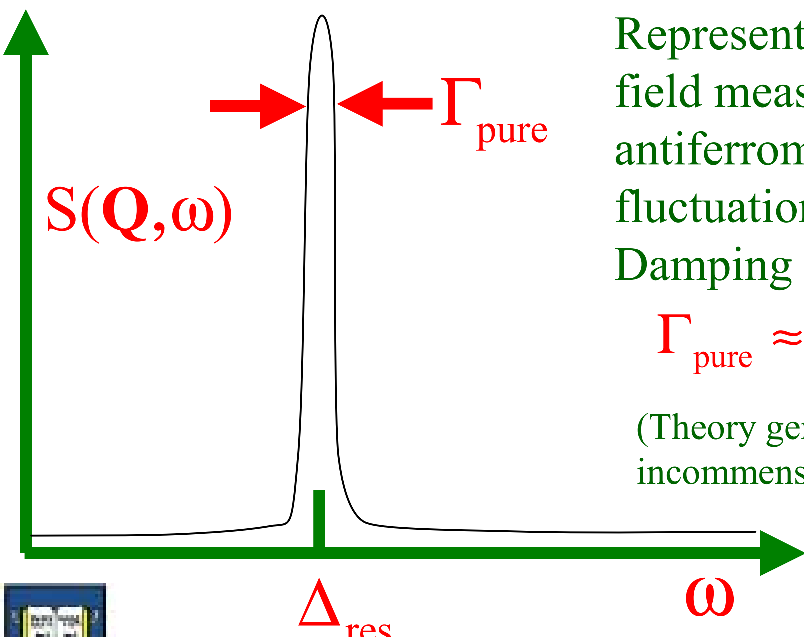
Negligible below T_c except near a T=0 superconductor-insulator transition.

(B) *S=1/2* Fermionic quasiparticles

Ψ_h : strongly paired fermions near $(\pi,0)$, $(0,\pi)$ have an energy gap $\Delta_h \sim$ 30-40 meV

$\Psi_{1,2}$: gapless fermions near the nodes of the superconducting gap at $(\pm K, \pm K)$ with $K = 0.391\pi$

(C) *S=1* Bosonic, resonant collective mode



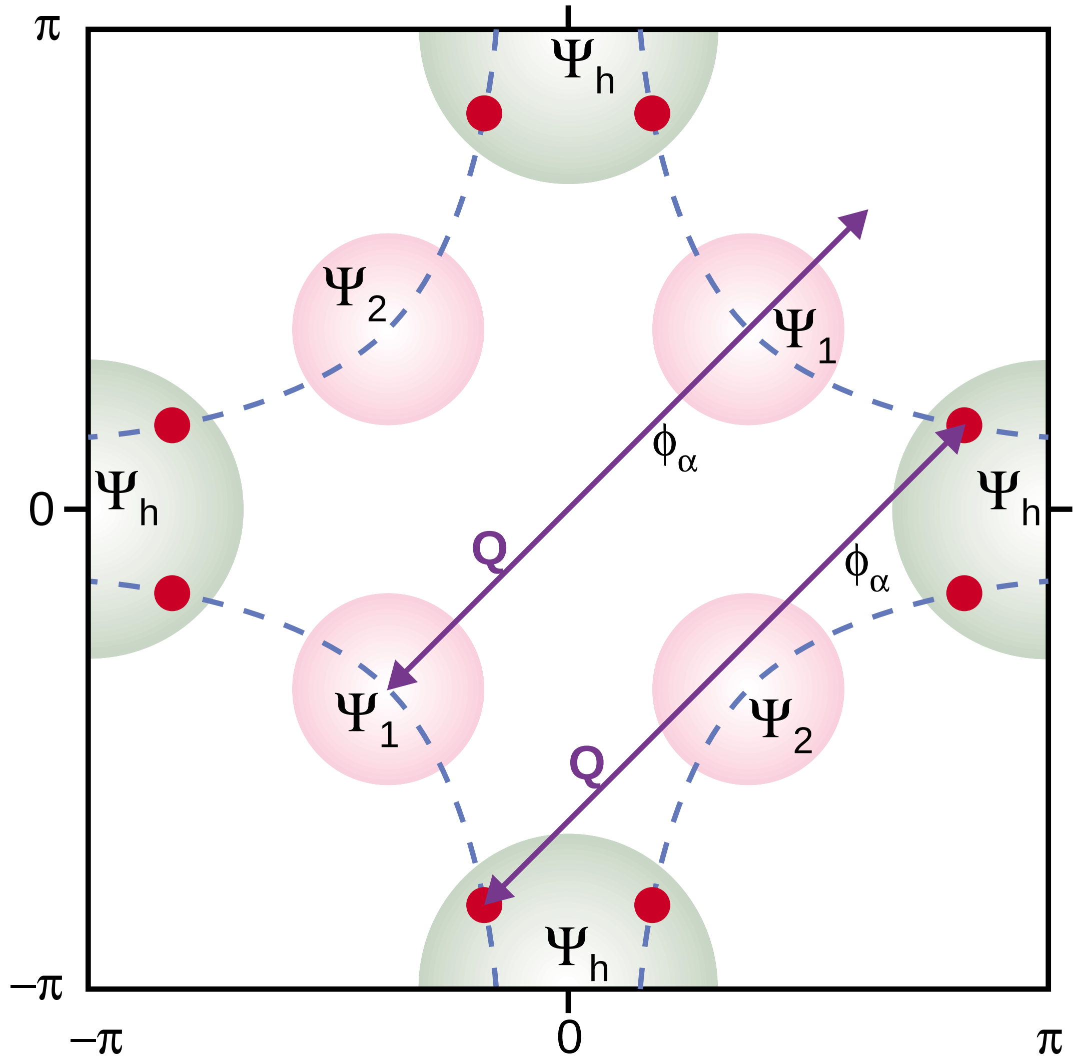
Represented by ϕ_α , a vector field measuring the strength of antiferromagnetic spin fluctuations near $\mathbf{Q} \approx (\pi,\pi)$

Damping is small at *T=0*

$$\Gamma_{pure} \approx 0 \text{ at } T = 0$$

(Theory generalizes to the cases with incommensurate **Q** and $\Gamma_{pure} \neq 0$)

Constraints from momentum conservation



Ψ_h : ***strongly coupled*** to ϕ_α, but do not damp ϕ_α as long as $\Delta_{res} < 2\,\Delta_h$

$\Psi_{1,2}$: ***decoupled*** from ϕ_α

I. Zero temperature broadening of resonant collective mode ϕ_α by impurities: comparison with neutron scattering experiments of Fong *et al* Phys. Rev. Lett. **82**, 1939 (1999)

II. Intrinsic inelastic lifetime of nodal quasiparticles $\Psi_{1,2}$ (Valla *et al* Science **285**, 2110 (1999) and Corson *et al* cond-mat/0003243)

See poster by M. Vojta

Independent low energy quantum field theories for the ϕ_α and the $\Psi_{1,2}$

I. Zero temperature broadening of resonant collective mode by impurities

Effect of *arbitrary localized deformations* ("impurities") of density n_{imp}

Each impurity is characterized by an integer/half-odd-integer S

As $\Delta_{\text{res}} \to 0$

$$\frac{\Gamma_{\text{imp}}}{\Delta_{\text{res}}} = n_{\text{imp}} \left(\frac{\hbar c}{\Delta_{\text{res}}} \right)^2 \left[C_S + O\left(\frac{\Delta_{\text{res}}}{J} \right) \right]$$

Correlation length ξ

$C_S \to$ Universal numbers dependent only on S

$$C_0 = 0 \;;\; C_{1/2} \approx 1$$

Zn impurities in YBCO have S=1/2

"Swiss-cheese" model of quantum impurities (Uemura):

Inverse Q of resonance ~ fractional volume of holes in Swiss cheese.

Analogy with deformation of quantum coherence by a dilute concentration of impurities n_{imp}

Magnetic impurities in a Fermi liquid

Quasiparticle scattering rate

$$\Gamma_{\text{imp}}(\varepsilon) \sim \begin{cases} n_{\text{imp}} J^2 a^{2d} \rho(E_F) & \varepsilon >> T_K \\ \dfrac{n_{\text{imp}}}{\rho(\mathrm{E_F})} & \varepsilon << \mathrm{T_K} \end{cases}$$

Pair-breaking in a non *s*-wave superconductor

Abrikosov-Gorkov pair-breaking parameter

$$\eta = \frac{\Gamma_{\text{imp}}(\Delta_{\text{sc}})}{\Delta_{\text{sc}}}$$

$\Delta_{\text{sc}} \rightarrow$ superconducting pairing energy

As $\Delta_{res} \rightarrow 0$ there is a quantum phase transition to a magnetically ordered state

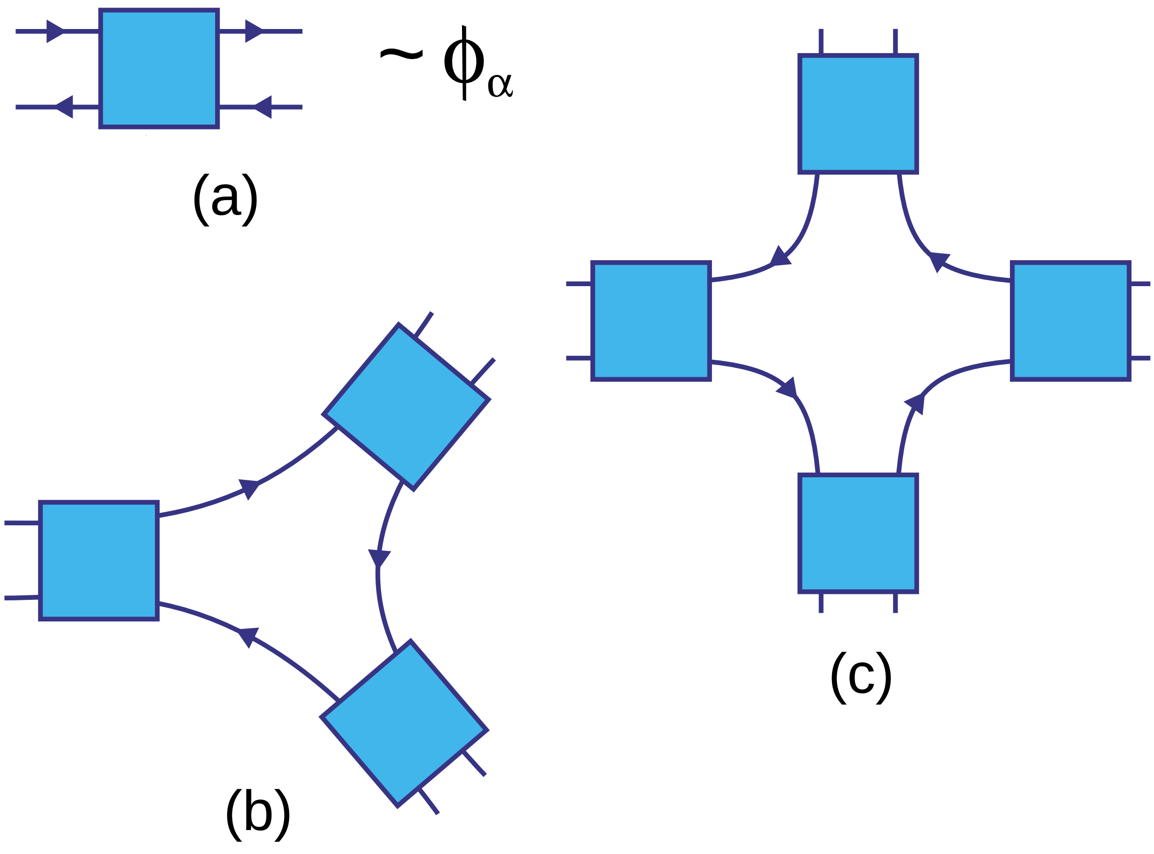
(A) Insulating Neel state (or collinear SDW at wavevector **Q**) ⇔ insulating quantum paramagnet

(B) *d*-wave superconductor with collinear SDW at wavevector **Q** ⇔ *d*-wave superconductor (paramagnet)

Transition (B) is in the same universality class as (A) provided Ψ_h fermions remain gapped at quantum-critical point.

Why appeal to proximity to a quantum phase transition ?

$\phi_\alpha \sim S=1$ bound state in particle-hole channel at the antiferromagnetic wavevector



Quantum field theory of critical point allows systematic treatment of the *strongly relevant* multi-point interactions in (b) and (c).

1. (A) Paramagnetic and Neel ground states in two dimensions --- coupled-ladder antiferromagnet.
   Field theory of quantum phase transition.

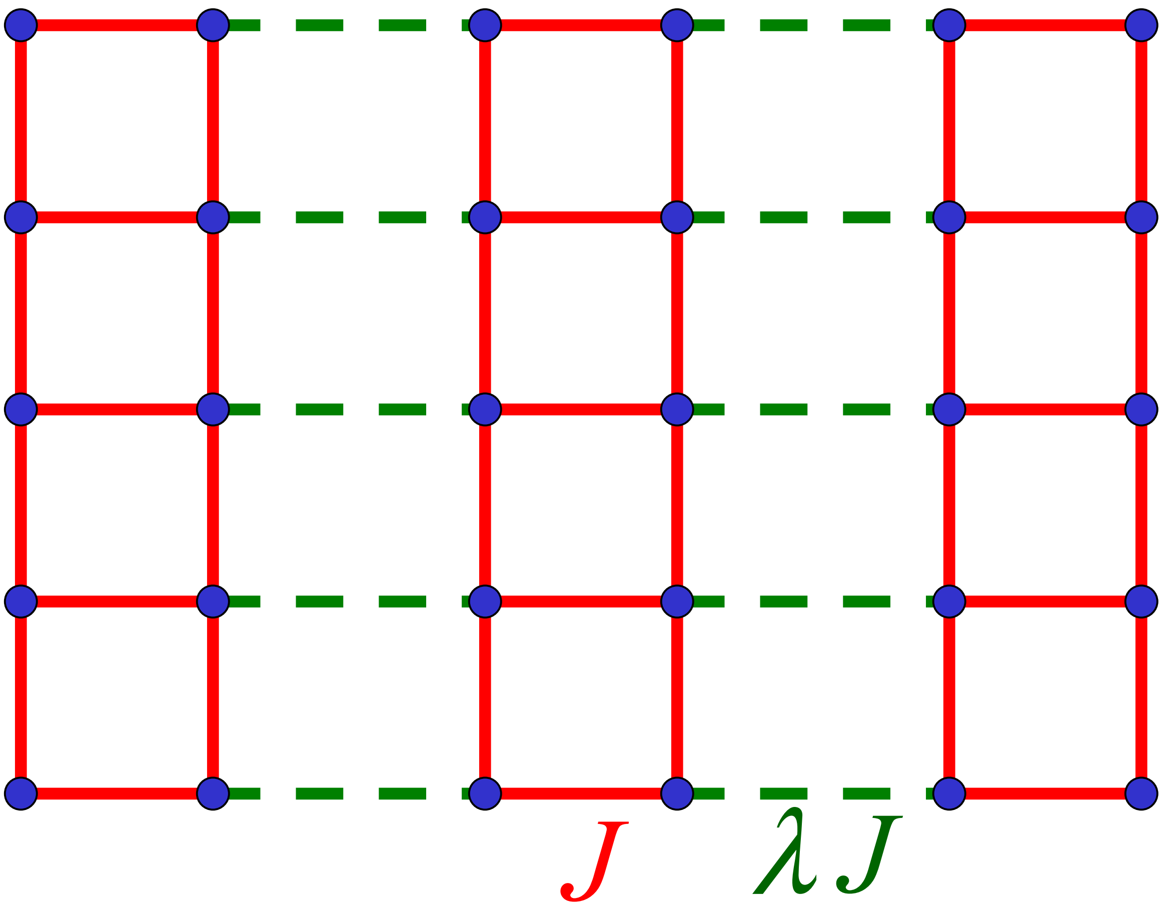
2. Non-magnetic impurities (Zn or Li) in two-dimensional paramagnets.

3. Application to (B) d-wave superconductors.
   Comparison with, and predictions for, expts

1. Paramagnetic and Neel states in insulators

(Katoh and Imada;
Tworzydlo, Osman, van Duin and Zaanen)

S=1/2 spins on coupled 2-leg ladders



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

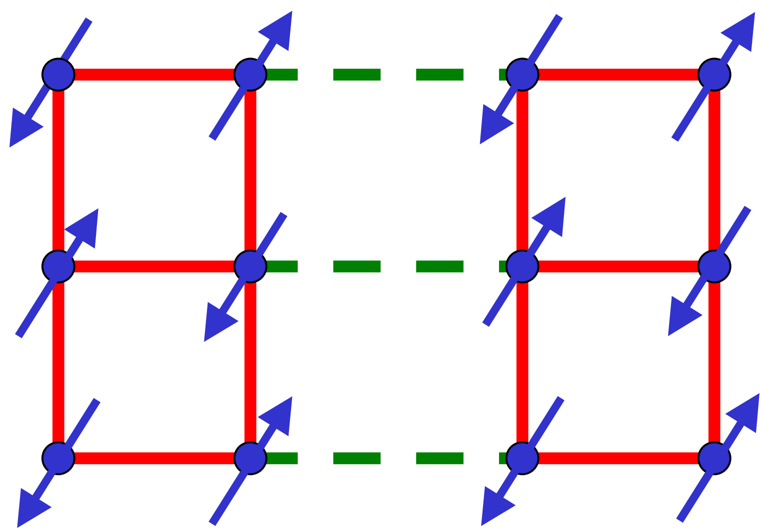
Follow ground state as a function of λ

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



Ground state has long-range magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

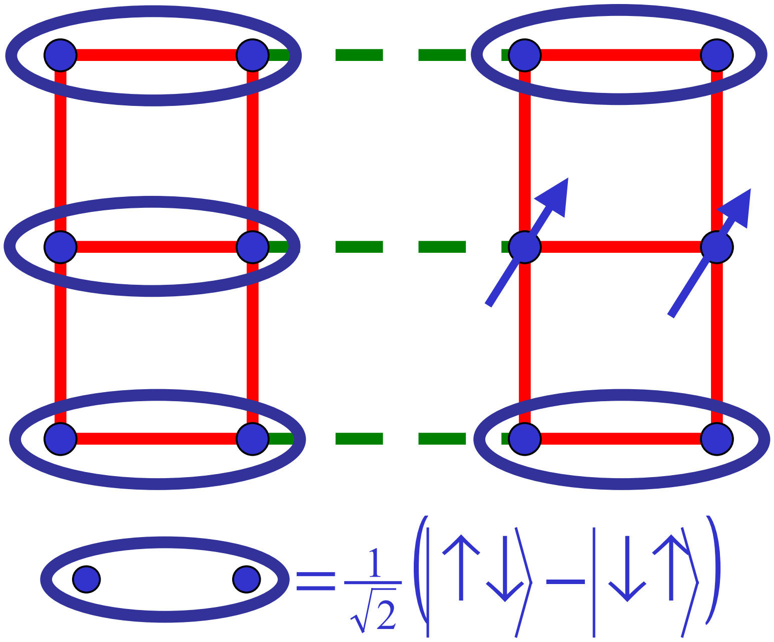
Excitations: 2 spin waves

Quasiclassical wave dynamics at low T

(Chakravarty et al, 1989; Tyc et al, 1989)

λ close to 0

Weakly coupled ladders



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

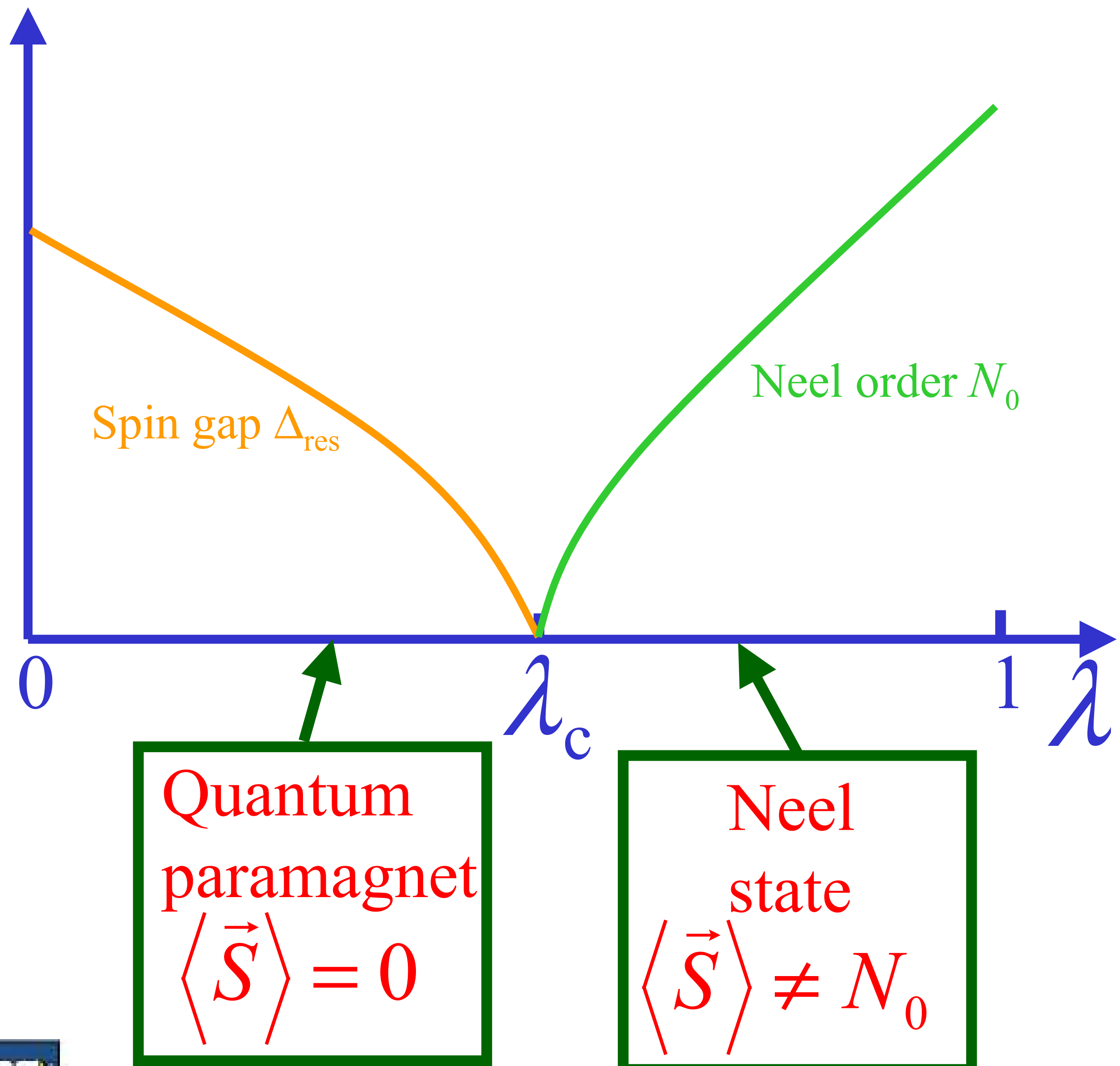
Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

Excitation: S=1, ϕ_α particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta_{res} + \frac{c^2 k^2}{2\Delta_{res}}$$

$\Delta_{res} \rightarrow$ Spin gap


Spin gap Δ_{res}
Neel order N_0
0
λ_c
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Nearly-critical paramagnets

λ is close to λ_c

Quantum field theory:

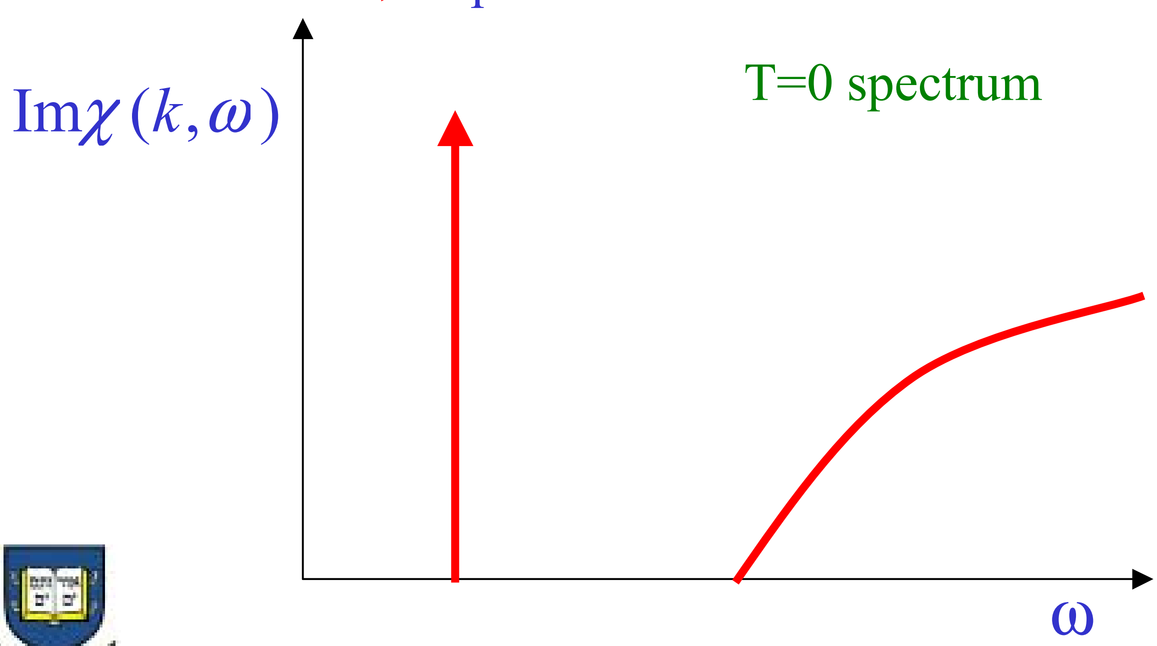
$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$	→	$\lambda < \lambda_c$
$r < 0$	→	$\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)

→ spin-1 collective mode



Coupling *g* approaches fixed-point value under renormalization group flow: beta function (ε = 3-*d*) :

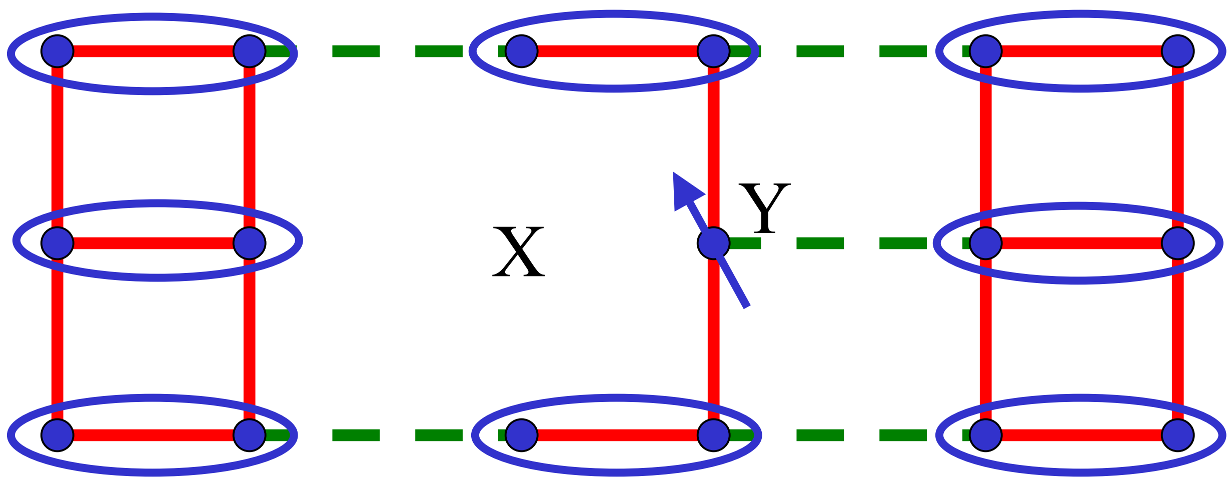
$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

Only relevant perturbation – ***r***
strength is measured by the spin gap Δ

Δ_{res} and *c* completely determine entire spectrum of quasi-particle peak and multiparticle continua, the S matrices for scattering between the excitations, and $T > 0$ modifications.

2. Quantum impurities in nearly-critical paramagnets

Make *any* localized deformation of antiferromagnet; e.g. remove a spin



Susceptibility $\chi = A\chi_b + \chi_{imp}$

(A = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta_{\text{res}}}{\hbar^2 c^2 \pi}\right) e^{-\Delta_{\text{res}}/k_B T} \; ; \; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

For a general impurity χ_{imp} *defines* the value of S

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\text{imp}} = \int d\tau \left[iSA_\alpha(n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau) \phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $S_b + S_{\text{imp}}$

Recall -

$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Coupling γ approaches *also* approaches a fixed-point value under the renormalization group flow

Beta function:

(Sengupta, 97
Sachdev+Ye, 93
Smith+Si 99)

$$\beta(\gamma) = -\frac{\epsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \frac{\pi^2}{3}\left(S(S+1) - \frac{1}{3}\right) g\gamma^3 + \mathcal{O}\left((\gamma, \sqrt{g})^7\right)$$

No new relevant perturbations on the boundary;
All other boundary perturbations are irrelevant –

e.g. $\lambda \int d\tau \phi_\alpha^2 (x=0, \tau)$

(This is the simplest allowed boundary perturbation for S=0 – its irrelevance implies $C_0 = 0$)

Δ_{res} and c completely determine spin dynamics near an impurity –

No new parameters are necessary !

Finite density of impurities n_{imp}

Relevant perturbation – strength determined by only energy scale that is linear in n_{imp} and contains only bulk parameters

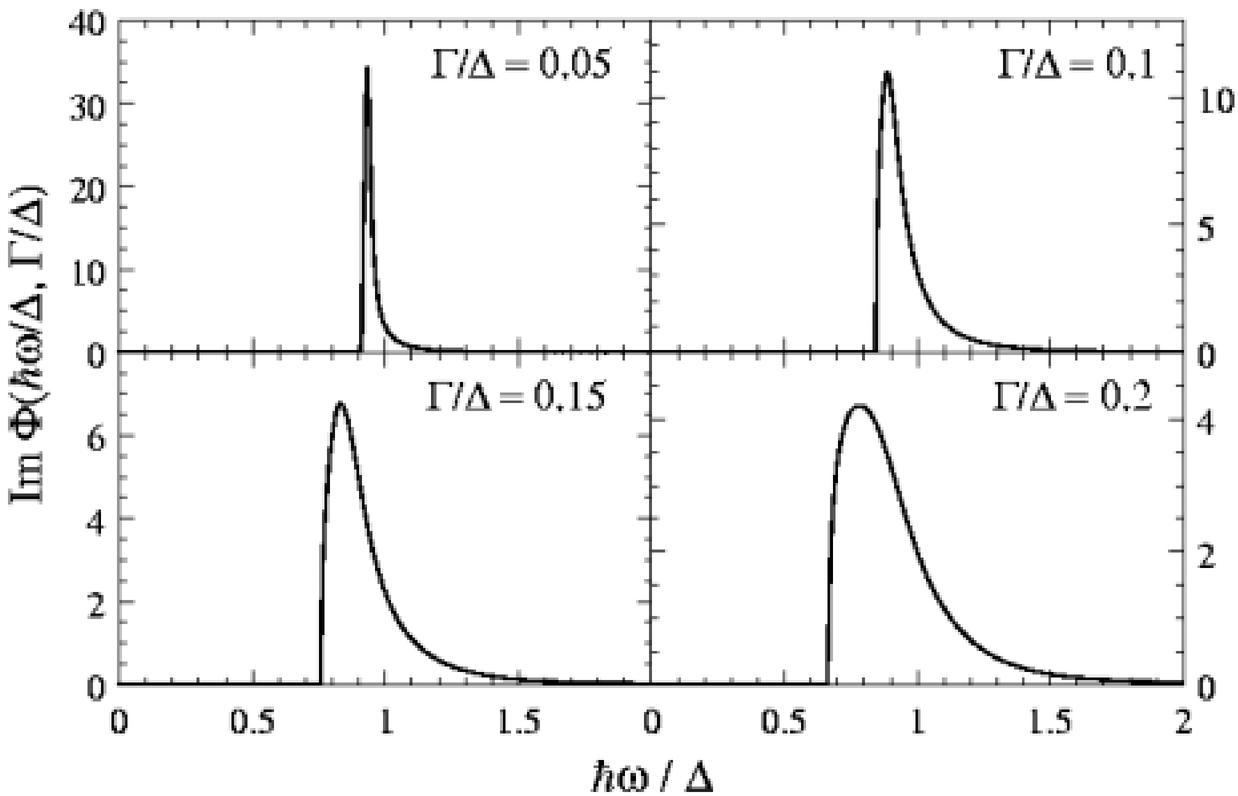
$$\Gamma \equiv \frac{n_{imp} (\hbar c)^2}{\Delta_{res}}$$

Fate of collective mode peak

Without impurities $\chi(G,\omega) = \frac{A}{\Delta_{res}^2 - \omega^2}$

With impurities $\chi(G,\omega) = \frac{A}{\Delta_{res}^2} \Phi\left(\frac{\hbar\omega}{\Delta_{res}}, \frac{\Gamma}{\Delta_{res}}\right)$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



Predictions: Half-width of line $\approx \Gamma$

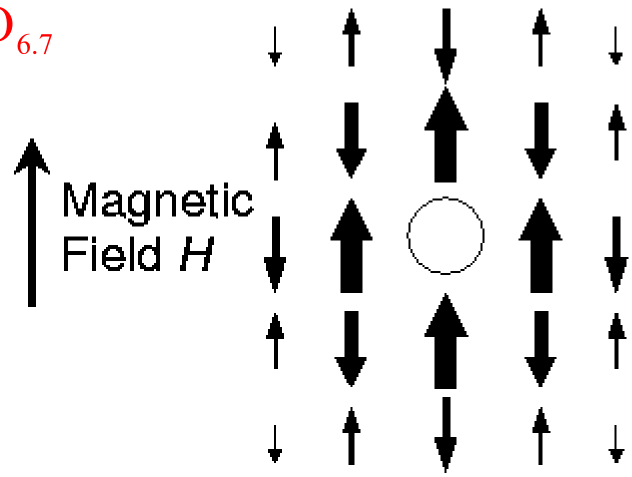
Universal asymmetric lineshape

3. Application to d-wave superconductors (YBCO)

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

Pepin and Lee: Modeled Zn impurity as a potential scatterer in the unitarity limit, and obtained quasi-bound states at the Fermi level.
Our approach: Each bound state captures only one electron and this yields a Berry phase of S=1/2; residual potential scattering of quasiparticles is not in the unitarity limit.

Additional low-energy spin fluctuations in a *d*-wave superconductor

Nodal quasiparticles $\Psi_{1,2}$

There is a Kondo coupling between moment around impurity and Ψ: $J_K S\, n_\alpha \Psi^* \sigma^\alpha \Psi$

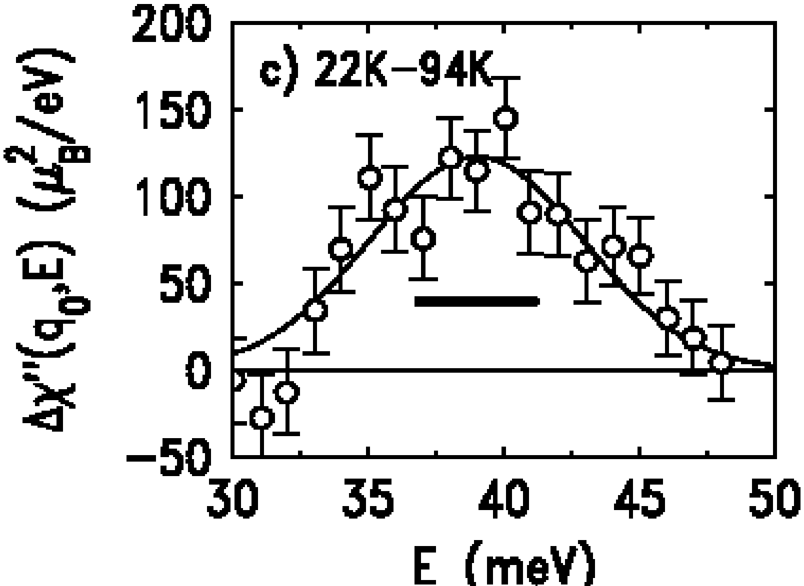
However, because density of states vanishes linearly at the Fermi level, there is no Kondo screening for any finite J_K (below a finite J_K) with (without) particle-hole symmetry

(Withoff+Fradkin, Chen+Jayaprakash, Buxton+Ingersent)

See poster by A. Polkovnikov

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



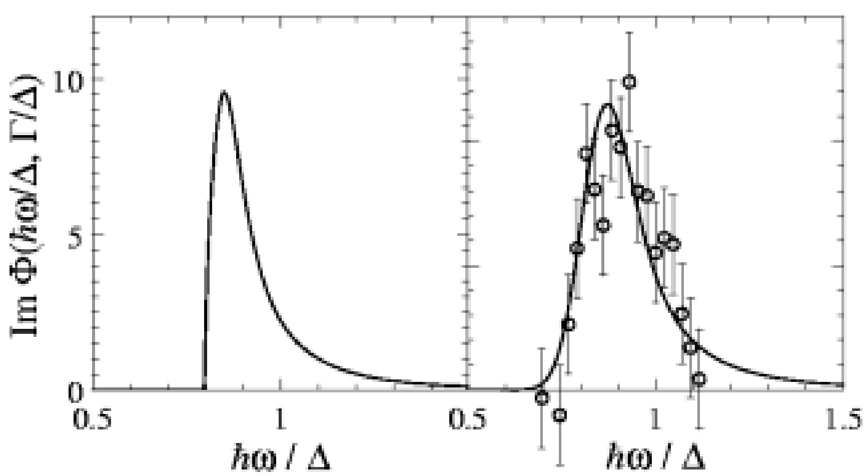
$$n_{\text{imp}} = 0.005$$

$$\Delta_{\text{res}} = 40\ \text{meV}$$

$$\hbar c = 0.2\ \text{eV}$$

$$\Rightarrow \quad \Gamma = 5\ \text{meV},\ \Gamma/\Delta_{\text{res}} = 0.125$$

Quoted half-width = 4.25 meV



Conclusions:

1. Universal *T=0* damping of S=1 collective mode by non-magnetic impurities.

   Linewidth: $$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta_{\text{res}}}$$

   independent of impurity parameters.

2. New interacting boundary conformal field theory in 2+1 dimensions

3. Universal irrational spin near the impurity at the critical point.