Quantum magnetism and criticality

Ribhu Kaul, Yong-Baek Kim, Alexei Kolezhuk, Michael Levin, Subir Sachdev, T. Senthil

Theory of the Nernst effect near the superfluid-insulator transition

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Theory of the Nernst effect near the superfluid-insulator transition
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Outline

1. Quantum “disordering” magnetic order
   Collinear order and confinement

2. $\mathbb{Z}_2$ spin liquids
   Noncollinear order and fractionalization

3. U(1) spin liquids
   Valence bond solid (VBS) order

4. Doped spin liquids
   Superconductors with topological order
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Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in Néel state.
Antiferromagnetic (Neel) order in the insulator

No entanglement of spins
Weaken some bonds to induce spin entanglement in a new quantum phase
SrCu$_2$O$_3$
Ground state is a product of pairs of entangled spins.

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Phase diagram as a function of the ratio of exchange interactions, $\lambda$
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Excitation: $S=1$ \textit{triplon}

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Neutron scattering

Y$_2$BaNiO$_5$

Collision of triplons

\[ = \frac{1}{\sqrt{2}} \left( \left| \uparrow \uparrow \right> - \left| \downarrow \downarrow \right> \right) \]
Collision of triplons

$$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right> - \left| \downarrow \uparrow \right> \right)$$
Collision of triplons

Collision $S$-matrix

$$= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
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$$\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
Neutron scattering linewidth

\[ Y_2\text{BaNiO}_5 \]


Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Quantum critical point with non-local entanglement in spin wavefunction
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

$$S_{\varphi} = \int d^2x d\tau \left[ \frac{1}{2} \left( c^2 (\nabla_x \varphi)^2 + (\partial_\tau \varphi)^2 + s \varphi^2 \right) + u (\varphi^2)^2 \right]$$

Landau-Ginzburg-Wilson Theory
Observation of longitudinal mode in TlCuCl$_3$

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

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Destroy Neel order by perturbations which preserve full square lattice symmetry.

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What is the state with \( \langle \vec{\varphi} \rangle = 0 \)?
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Destroy Neel order by perturbations which preserve full square lattice symmetry e.g. second-neighbor or ring exchange.

What is the state with \( \langle \vec{\varphi} \rangle = 0 \) ?
**LGW theory for quantum criticality**

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian.

$$S_\varphi = \int d^2x d\tau \left[ \frac{1}{2} \left( c^2 (\nabla_x \vec{\varphi})^2 + (\partial_\tau \vec{\varphi})^2 + s \vec{\varphi}^2 \right) + u (\vec{\varphi}^2)^2 \right]$$

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\]

State with no broken symmetries. Fluctuations of \( \vec{\varphi} \) about \( \vec{\varphi} = 0 \) realize a stable \( S = 1 \) quasiparticle with energy \( \varepsilon_k = \sqrt{s + c^2 k^2} \)

\[
\left\langle \vec{\varphi} \right\rangle = 0
\]

\[
\left\langle \vec{\varphi} \right\rangle \neq 0
\]

Néel state

LGW theory for quantum criticality

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However, $S = 1/2$ antiferromagnets on the square lattice have no such state.

$\langle \vec{\varphi} \rangle \neq 0$
Néel state

$S_C$ $S$

$\langle \vec{\varphi} \rangle = 0$
There is no state with a gapped, stable $S=1$ quasiparticle and no broken symmetries
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“Liquid” of valence bonds has fractionalized $S=1/2$ excitations

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\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
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\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Possible theory for fractionalization and topological order

Decompose the Néel order parameter into spinors

\[ \vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha \beta} z_\beta \]

where \( \vec{\sigma} \) are Pauli matrices, and \( z_\alpha \) are complex spinors which carry spin \( S = 1/2 \).
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**Key question:** Can the \( z_\alpha \) become the needed \( S = 1/2 \) excitations of a fractionalized phase?
**Possible theory for fractionalization and topological order**

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**Key question:** Can the \( z_\alpha \) become the needed \( S = 1/2 \) excitations of a fractionalized phase?

Effective theory for spinons must be invariant under the U(1) gauge transformation

\[ z_\alpha \rightarrow e^{i\theta} z_\alpha \]
**Possible theory for fractionalization and topological order**

**Naive expectation:** Low energy spinon theory for “quantum disordering” a Néel state is

\[
S_z = \int d^2 x d\tau \left[ c^2 |(\nabla_x - i A_x)z_\alpha|^2 + |(\partial_\tau - i A_\tau)z_\alpha|^2 + s |z_\alpha|^2 \\
+ u (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]
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\]

Spin liquid state with stable \( S = 1/2 \) \( z_\alpha \) spinons, and a gapless U(1) photon \( A_\mu \) representing the topological order.

\[ \langle z_\alpha \rangle \neq 0 \]

Néel state

\[ \langle z_\alpha \rangle = 0 \]
**Possible theory for fractionalization and topological order**

**Naive expectation:** Low energy spinon theory for “quantum dis-ordering” a Néel state is

\[
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+ u (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]
\]

\[
\langle z_\alpha \rangle = 0
\]

\[
\langle z_\alpha \rangle \neq 0
\]

Néel state

\[S_C \rightarrow S\]

However, **monopoles** in the \( A_\mu \) field will proliferate because of the gap to \( z_\alpha \) excitations, and lead to confinement of \( z_\alpha \).
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$Z_2$ gauge theory for fractionalization and topological order

- *Discrete* gauge theories do have deconfined phases in 2+1 dimensions.
**Z\textsubscript{2} gauge theory for fractionalization and topological order**

- *Discrete* gauge theories do have deconfined phases in 2+1 dimensions.

- Find a collective excitation $\Phi$ with the gauge transformation

  $$
  \Phi \rightarrow e^{2i\theta} \Phi
  $$

- Higgs state with $\langle \Phi \rangle \neq 0$ is described by the fractionalized phase of a $Z\textsubscript{2}$ gauge theory in the which the spinons $z_\alpha$ carry $Z\textsubscript{2}$ gauge charges (E. Fradkin and S. Shenker, Phys. Rev. D 19, 3682 (1979)).

Z₂ gauge theory for fractionalization and topological order

- *Discrete* gauge theories do have deconfined phases in 2+1 dimensions.
- Find a collective excitation Φ with the gauge transformation
  \[
  \Phi \rightarrow e^{2i\theta} \Phi
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- Higgs state with \( \langle \Phi \rangle \neq 0 \) is described by the fractionalized phase of a Z₂ gauge theory in the which the spinons \( z_\alpha \) carry Z₂ gauge charges (E. Fradkin and S. Shenker, Phys. Rev. D 19, 3682 (1979)).
- What is Φ in the antiferromagnet? Its physical interpretation becomes clear from its allowed coupling to the spinons:
  \[
  S_{z,\Phi} = \int d^2r d\tau [\lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.}]
  \]
  From this coupling it follows that the states with \( \langle \Phi \rangle \neq 0 \) have coplanar spin correlations.

Collinear magnetic order with $\langle \Phi \rangle = 0$.

A spin density wave:

$$\langle \vec{S}_i \rangle \propto (\cos(K \cdot r_i), \sin(K \cdot r_i), 0)$$

$$K = (\pi, \pi).$$
Coplanar magnetic order with \( \langle \Phi \rangle \neq 0 \).

A spin density wave:

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\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i), \sin(\mathbf{K} \cdot \mathbf{r}_i), 0)
\]

with

\[
\mathbf{K} = (\pi + \langle \Phi \rangle, \pi + \langle \Phi \rangle).
\]

*Experimental realization: CsCuCl\(_3\)*
Phase diagram of gauge theory of spinons

\[ S_z = \int d^2 x \partial^\tau \left[ \left| (\partial_\mu - i A_\mu) z_\alpha \right|^2 + s_1 |z_\alpha|^2 + u \left( |z_\alpha|^2 \right)^2 + \frac{1}{4 e^2} (\epsilon_{\mu
u\lambda} \partial_\nu A_\lambda)^2 \right] \]


Phase diagram of gauge theory of spinons

\[ S_{z,\Phi} = \int d^2xd\tau \left[ \left| (\partial_\mu - iA_\mu)z_\alpha \right|^2 + s_1 |z_\alpha|^2 + u \left| z_\alpha \right|^4 + \frac{1}{4e^2} (\varepsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right. \\
\left. + \left| (\partial_\mu - 2iA_\mu)\Phi \right|^2 + s_2 |\Phi|^2 + \tilde{u} |\Phi|^4 + \lambda \Phi^* \varepsilon_{\alpha\beta} z_\alpha \partial_x z_\beta \right] \]

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Spiral state

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U(1) spin liquid unstable to confinement

Z₂ spin liquid with bosonic spinons \( z_\alpha \)

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Néel state
\[ \langle z_\alpha \rangle \neq 0 \ , \ \langle \Phi \rangle = 0 \]

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\[ Z_2 \text{ spin liquid with bosonic spinons } z_\alpha \]
\[ \langle z_\alpha \rangle = 0 \ , \ \langle \Phi \rangle \neq 0 \]

**Characteristics of $Z_2$ spin liquid**

- Two classes of gapped excitations:
  - Bosonic spinons $z_\alpha$ which carry $Z_2$ gauge charge
  - $Z_2$ vortex associated with $2\pi n$ winding in phase of $\Phi$. This vortex appears as a $\pi$ flux-tubes to spinons

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- Ground state degeneracy is sensitive to topology: a $\Phi$-vortex can be inserted without energy cost in each “hole”: 4-fold degeneracy on a torus.

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- Structure identical to that found later in exactly solvable model: the $Z_2$ toric code (A. Kitaev, quant-ph/9707021).

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- Ground state degeneracy is sensitive to topology: a $\Phi$-vortex can be inserted without energy cost in each “hole”: 4-fold degeneracy on a torus.
- Structure identical to that found later in exactly solvable model: the $Z_2$ toric code (A. Kitaev, quant-ph/9707021).
- Same states (without spinons) and $Z_2$ gauge theories found to describe liquid phases of quantum dimer models (R. Moessner and S. L. Sondhi, Phys. Rev. Lett. 86, 1881 (2001)).

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\[
+ |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2 |\Phi|^2 + \tilde{u} |\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_\alpha z_\beta + c.c. \right]
\]

\[
\langle z_\alpha \rangle \neq 0 \ , \langle \Phi \rangle = 0 \quad \text{Néel state}
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U(1) \text{ spin liquid unstable to confinement}
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Z_2 \text{ spin liquid with bosonic spinons } z_\alpha
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**Phase diagram of gauge theory of spinons**

- **Néel state**
  \[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle = 0 \]
- **Spiral state**
  \[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle \neq 0 \]

**U(1) spin liquid unstable to confinement**

- \[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle = 0 \]

**Z\(_2\) spin liquid with bosonic spinons \(z_\alpha\)**

- \[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle \neq 0 \]


**Quantum theory for destruction of Neel order**

Partition function on cubic lattice in spacetime

\[
Z = \prod_a \int d\vec{\varphi}_a \delta (\vec{\varphi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} \right)
\]

LGW theory: weights in partition function are those of a classical ferromagnet at a “temperature” \( g \)

Small \( g \) ⇒ ground state has Neel order with \( \langle \vec{\varphi} \rangle \neq 0 \)

Large \( g \) ⇒ paramagnetic ground state with \( \langle \vec{\varphi} \rangle = 0 \)
Missing ingredient: Spin Berry Phases

$e^{iA/2}$

$A$
Quantum theory for destruction of Neel order

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Quantum theory for destruction of Neel order

Coherent state path integral on cubic lattice in spacetime

\[ Z = \prod_a \int d\vec{\varphi}_a \delta (\vec{\varphi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}} \right) \]

Modulus of weights in partition function: those of a classical ferromagnet at a “temperature” \( g \)

Small \( g \) ⇒ ground state has Neel order with \( \langle \vec{\varphi} \rangle \neq 0 \)

Large \( g \) ⇒ paramagnetic ground state with \( \langle \vec{\varphi} \rangle = 0 \)

Berry phases lead to large cancellations between different time histories
**Quantum theory for destruction of Neel order**

Partition function on cubic lattice

\[ Z = \prod_a \int d\tilde{\varphi}_a \delta (\tilde{\varphi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \tilde{\varphi}_a \cdot \tilde{\varphi}_{a+\mu} + iS_{\text{Berry}} \right) \]

Rewrite partition function in terms of spinors \( z_{a\alpha} \),

with \( \alpha = \uparrow, \downarrow \) and

\[ \tilde{\varphi}_a = z_{a\alpha} \sigma_{\alpha\beta} z_{a\beta} \]

Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\vec{\varphi}_a \delta (\vec{\varphi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + iS_{\text{Berry}} \right) \]

Partition function expressed as a gauge theory of spinor degrees of freedom

\[ Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta \left( \sum_\alpha |z_{a\alpha}|^2 - 1 \right) \times \exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_\alpha \eta_\alpha A_{a\tau} \right) \]

Large $g$ effective action for the $A_{a\mu}$ after integrating $z_{a\mu}$

\[ Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left( \frac{1}{2e^2} \sum \cos (\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) + i \sum \eta_\alpha A_{a\tau} \right) \]

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges $\pm 1$ on two sublattices.

Duality mapping:
The low energy continuum theory is

$$\int d^2r d\tau \left[ \frac{1}{2e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

Decouple this to

$$\int d^2r d\tau \left[ \frac{e^2}{2} J_\mu^2 + iJ_\mu \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda \right]$$

Integrate over $A_\mu$ to obtain constraint $\epsilon_{\mu\nu\lambda} \partial_\nu J_\lambda = 0$. Solve this constraint by $J_\mu = \partial_\mu \chi$ to obtain the dual theory

$$\int d^2r d\tau \left[ \frac{e^2}{2} (\partial_\mu \chi)^2 \right]$$

This theory has a global shift symmetry $\chi \rightarrow \chi + \text{constant}$. This symmetry is spontaneously broken, and the massless $\chi$ particle (i.e. the photon) is the Goldstone boson of this shift symmetry.
Consequences of Berry phases:

- The continuous shift symmetry is an enlargement of the $Z_4$ spatial rotation symmetry of the square lattice. So this spatial rotation symmetry is spontaneously broken in the free photon phase.

- The monopole operator

$$V = \exp \left( i \frac{2\pi \chi}{e_0^2} \right)$$

is equivalent to the valence bond solid (VBS) operator $\Psi_{\text{vbs}}$, and $\langle V \rangle \sim \langle \Psi_{\text{vbs}} \rangle \neq 0$
Characterization of VBS state with $\langle \bar{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{vbs} \rangle \neq 0$, where $\Psi_{vbs}$ is the VBS order parameter

$$\Psi_{vbs}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(r_j - r_i)}$$
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$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$

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Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{vbs} \rangle \neq 0$, where $\Psi_{vbs}$ is the **VBS order parameter**

$$\Psi_{vbs}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(r_j - r_i)}$$
Phase diagram of gauge theory of spinons

\[ S_{z,\Phi} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s_1 |z_\alpha|^2 + u (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right. \\
\left. + |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2 |\Phi|^2 + \tilde{u} |\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + c.c. \right] \]

\[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle = 0 \]
Néel state

\[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle = 0 \]
Spiral state

\[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle \neq 0 \]
\[ \langle \Phi \rangle = 0, \langle \Phi \rangle = 0 \]
U(1) spin liquid unstable to confinement

Z\(_2\) spin liquid with bosonic spinons \(z_\alpha\)

Phase diagram of gauge theory of spinons

\[ S_{z,\Phi} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s_1 |z_\alpha|^2 + u (|z_\alpha|^2)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right. \]

\[ + |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2 |\Phi|^2 + \tilde{u} |\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + c.c. \left. \right] + \text{monopoles} + S_{\text{Berry}} \]

\[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle = 0 \]
Néel state

\[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle = 0 \]
U(1) spin liquid unstable to VBS order

\[ \langle z_\alpha \rangle \neq 0, \langle \Phi \rangle \neq 0 \]
Spiral state

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\[ Z_2 \text{ spin liquid with bosonic spinons } z_\alpha \]

Phase diagram of gauge theory of spinons

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S_{z,\Phi} = \int d^2xd\tau \left[ \left| (\partial_\mu - iA_\mu)z_\alpha \right|^2 + s_1 |z_\alpha|^2 + u \left( |z_\alpha|^2 \right)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right.
\]

\[
+ \left| (\partial_\mu - 2iA_\mu)\Phi \right|^2 + s_2 |\Phi|^2 + \tilde{u} |\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \left] + \text{monopoles} + S_{\text{Berry}} \right.
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\langle z_\alpha \rangle \neq 0 , \langle \Phi \rangle = 0
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Néel state

\[
\langle z_\alpha \rangle = 0 , \langle \Phi \rangle = 0
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U(1) spin liquid unstable to VBS order

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\]

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\[ \text{Néel state} \]

\[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle = 0 \]
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\[ \langle z_\alpha \rangle = 0, \langle \Phi \rangle \neq 0 \]

\[ \text{Z}_2 \text{ spin liquid with bosonic spinons } z_\alpha \]

Probability distribution of VBS order $\Psi_{\text{vbs}}$ at quantum critical point

Emergent circular symmetry is evidence for $U(1)$ photon and topological order

Quantum magnetism and criticality

Ribhu Kaul, Yong-Baek Kim, Alexei Kolezhuk,
Michael Levin, Subir Sachdev, T. Senthil

Theory of the Nernst effect near the superfluid-insulator transition

Sean Hartnoll, Pavel Kovtun, Chris Herzog,
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Theory of the Nernst effect near the superfluid-insulator transition

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Outline

1. Quantum “disordering” magnetic order
   - Collinear order and confinement

2. $\mathbb{Z}_2$ spin liquids
   - Noncollinear order and fractionalization

3. $U(1)$ spin liquids
   - Valence bond solid (VBS) order

4. Doped spin liquids
   - Superconductors with topological order
Outline

1. Quantum “disordering” magnetic order
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   Superconductors with topological order
Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,

Algebraic charge liquids and the underdoped cuprates,
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil,
Phase diagram of doped antiferromagnets

La$_2$CuO$_4$

VBS order

Neel order

Hole density $x$

or
Phase diagram of lightly doped antiferromagnet

\[ A = (2\pi)^2 \frac{x}{8} \]

\[ A = (2\pi)^2 \frac{x}{4} \]
Pictorial explanation of factor of 2:

- In the Néel phase, sublattice index is identical to spin index. So for each valley and momentum, degeneracy of the hole state is 2.

- In the VBS state, the sublattice index and the spin index are distinct. So for each valley and momentum, degeneracy of the hole state is 4.
• Begin with the representation of the quantum antiferromagnet as the lattice CP$^1$ model:

$$ S_z = -\frac{1}{g} \sum_{a,\mu} \bar{z}_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} $$

• Write the electron operator at site $r$, $c_\alpha(r)$ in terms of fermionic holon operators $f_\pm$

$$ c_\alpha(r) = \begin{cases} f_+^\dagger(r) z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta} f_-^\dagger(r) z_{r\beta}^* & \text{for } r \text{ on sublattice B} \end{cases} $$

Note that the holons $f_s$ have charge $s$ under the U(1) gauge field $A_\mu$. 
• Choose the dispersion, $\epsilon(\vec{k})$ of the $f_{\pm}$ in momentum space so that its minima are at $(\pm \pi/2, \pm \pi/2)$. To avoid double-counting, these dispersions must be restricted to be within the diamond Brillouin zone.

$$S_f = \int d\tau \sum_{s=\pm} \int \frac{d^2 k}{4\pi^2} f_s^\dagger(\vec{k}) \left( \partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) f_s(\vec{k})$$

• Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$S_t = -t \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.}$$

$$= -t \sum_{\langle rr' \rangle} (f_+^\dagger(r) z_{r\alpha})^\dagger \epsilon_{\alpha\beta} f_-^\dagger(r') \bar{z}_{r\beta}$$

• Complete theory for doped antiferromagnet:

$$S = S_z + S_f + S_t$$
Phase diagram of lightly doped antiferromagnet

$$\mathcal{A} = (2\pi)^2 x / 4$$
Phase diagram of lightly doped antiferromagnet

\[ A = \frac{(2\pi)^2 x}{8} \]

\[ A = \frac{(2\pi)^2 x}{4} \]
A new non-Fermi liquid phase:  
**The holon metal**
An algebraic charge liquid.

- Ignore compactness in $A_\mu$ and Berry phase term.
- Neutral spinons $z_\alpha$ are gapped.
- Charge $e$ fermions $f_s$ form Fermi surfaces and carry charges $s = \pm 1$ under the U(1) gauge field $A_\mu$.
- Quasi-long range order in a variety of VBS and pairing correlations.

Area of each Fermi pocket,
\[ A = (2\pi)^2 x/4. \]

The Fermi pocket will show sharp magnetoresistance oscillations, but it is invisible to photoemission.
Quantum oscillations and the Fermi surface in an underdoped high $T_c$ superconductor (ortho-II ordered YBa$_2$Cu$_3$O$_{6.5}$).

Holon pairing leading to \( d \)-wave superconductivity

First consider holon pairing in the Neel state, where holon=hole.

This was studied in V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, *Physica C* **227**, 267 (1994); V. I. Belincher *et al.*, *Phys. Rev. B* **51**, 6076 (1995). They found \( p \)-wave pairing of holons, induced by spin-wave exchange from the sublattice mixing term \( S_t \). This corresponds to \( d \)-wave pairing of physical electrons.
Holon pairing leading to $d$-wave superconductivity
Holon pairing leading to $d$-wave superconductivity
Holon pairing leading to $d$-wave superconductivity

We assume the same pairing holds across a transition involving loss of long-range Néel order. The resulting phase is another algebraic charge liquid - the holon superconductor. This superconductor has gapped spinons with no electrical charge, and spinless, nodal Bogoliubov-Dirac quasiparticles. The superconductivity does not gap the U(1) gauge field $A_\mu$, because the Cooper pairs are gauge neutral.
Low energy theory of holon superconductor

4 two-component Dirac quasiparticles coupled to a U(1) gauge field

\[
S_{\text{holon superconductor}} = \int d\tau d^2r \left[ \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right.
\]

\[
+ \sum_{i=1}^{4} \psi_i^\dagger \left( D_\tau - iv_F (\partial_x - iA_x) \tau^x - iv_F (\partial_y - iA_y) \tau^y \right) \psi_i \right]
\]
Low energy theory of holon superconductor

External vector potential $\vec{A}$ couples as

$$\mathcal{H}_A = \vec{j} \cdot \vec{A}$$

where

$$j_x = v_F \left( \psi_3^\dagger \psi_3 - \psi_1^\dagger \psi_1 \right), \quad j_x = v_F \left( \psi_4^\dagger \psi_4 - \psi_2^\dagger \psi_2 \right)$$

are conserve charges of $S_{\text{holon superconductor}}$.

**Fundamental property:** The superfluid density, $\rho_s$, has the following $x$ and $T$ dependence:

$$\rho_s(x, T) = cx - \mathcal{R}k_B T$$

where $c$ is a non-universal constant and $\mathcal{R}$ is a universal constant obtained in a $1/N$ expansion ($N = 4$ is the number of Dirac fermions):

$$\mathcal{R} = 0.4412 + \frac{0.307}{N} + \ldots$$
Conclusions

1. Theory for $\mathbb{Z}_2$ and U(1) spin liquids in quantum antiferromagnets, and evidence for their realization in model spin systems.

2. **Algebraic charge liquids** appear naturally upon adding fermionic carriers to spin liquids with bosonic spinons. These are conducting states with topological order.

3. The holon metal/superconductor, obtained by doping a Neel-ordered insulator, matches several observed characteristics of the underdoped cuprates.
Quantum magnetism and criticality

Ribhu Kaul, Yong-Baek Kim, Alexei Kolezhuk,
Michael Levin, Subir Sachdev, T. Senthil

Theory of the Nernst effect near
the superfluid-insulator transition

Sean Hartnoll, Pavel Kovtun, Chris Herzog,
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Outline

1. Superfluid-insulator transition
   *Integer and fractional filling*

2. Quantum-critical transport
   *Collisionless-to-hydrodynamic crossover of CFT3s*

3. SYM3 with $\mathcal{N} = 8$ supersymmetry

4. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*
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Ultracold $^{87}$Rb atoms - bosons

The insulator:
Excitations of the insulator:

Particles $\sim \psi^\dagger$

Holes $\sim \psi$
Excitations of the insulator:

Particles $\sim \psi^\dagger$

Holes $\sim \psi$

Density of particles $=$ density of holes $\Rightarrow$

"relativistic" field theory for $\psi$:

$$S = \int d^2r d\tau \left[ |\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

Insulator $\Leftrightarrow \langle \psi \rangle = 0$

Superfluid $\Leftrightarrow \langle \psi \rangle \neq 0$
\[ S = \int d^2 r d\tau \left[ |\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right] \]
Conformal field theory:
Wilson-Fisher fixed point

Superfluid
\[ \langle \psi \rangle \neq 0 \]
\[ \sigma = \infty \]

Insulator
\[ \langle \psi \rangle = 0 \]
\[ \sigma = 0 \]

\[ S = \int d^2 r d\tau \left[ \left| \partial_\tau \psi \right|^2 + c^2 \left| \vec{\nabla} \psi \right|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right] \]
Superfluid-insulator transition at fractional filling $f$

Possible continuous superfluid-insulator transition is described by a more complex CFT

Superfluid-insulator transition at fractional filling $f$

Possible continuous superfluid-insulator transition is described by a more complex CFT

Dope the antiferomagnets with charge carriers of density $x$ by applying a chemical potential $\mu$. 

\[ Ca_{1.90}Na_{0.10}CuO_2Cl_2 \]

\[ Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y \]
Superconductor
Scanning tunnelling microscopy
STM studies of the underdoped superconductor

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

$a_0 = 3.9\,\text{Å}$

$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

$a_0 = 5.4\,\text{Å}$
Topograph

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

Intense Tunneling-Asymmetry (TA) variation are highly similar

$\text{dI/dV Spectra}$

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$

$\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$

Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$  $\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$

Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$  

$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_{2}O_y$

Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$  \hspace{2cm} $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

Indistinguishable bond-centered TA contrast with disperse 4$a_0$-wide nanodomains

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

$Ca_{1.88}Na_{0.12}CuO_2Cl_2$, 4 K

R map (150 mV)

$4a_0$

12 nm

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

$Ca_{1.88}Na_{0.12}CuO_2Cl_2$, 4 K

Evidence for VBS order - a valence bond supersolid

Use coupling $g$ to induce a transition to a VBS insulator.

Superconductor

Insulator $x=1/8$
Proposed generalized phase diagram

Superconductor

Insulator $x=1/8$
Outline

1. Superfluid-insulator transition
   *Integer and fractional filling*

2. Quantum-critical transport
   *Collisionless-to-hydrodynamic crossover of CFT3s*

3. SYM3 with $\mathcal{N} = 8$ supersymmetry

4. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*
1. Superfluid-insulator transition
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Conformal field theory:

Wilson-Fisher fixed point

Superfluid
\[ \langle \psi \rangle \neq 0 \]
\[ \sigma = \infty \]

Insulator
\[ \langle \psi \rangle = 0 \]
\[ \sigma = 0 \]

\[ S = \int d^2 r d\tau \left[ |\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s |\psi|^2 + \frac{u}{2} |\psi|^4 \right] \]
The diagram illustrates a phase diagram with two axes: $T$ and $g$. The phase transitions are denoted by $T_{KT}$, the critical temperature, and $g_c$, the critical coupling. The regions are labeled as Superfluid, Quantum critical, and Insulator.
Wave oscillations of the condensate (classical Gross-Pitaevski equation)
Insulator Superfluid

Quantum critical

Dilute Boltzmann gas of particle and holes

$T_{KT}$

Superfluid

Insulator

$T_c$
CFT at $T>0$

Quantum critical

$T_{KT}$

Superfluid

Insulator
Resistivity of Bi films

Conductivity $\sigma$

$$
\sigma_{\text{Superconductor}}(T \to 0) = \infty \\
\sigma_{\text{Insulator}}(T \to 0) = 0 \\
\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}
$$


---

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.
Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For all CFT2s, at all $\hbar \omega/k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{vk^2}{v^2k^2 - \omega^2} ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where $K$ is a universal number characterizing the CFT2 (the level number), and $v$ is the velocity of “light”.
Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For all CFT3s, at $\hbar \omega \gg k_B T$

$$\chi(k, \omega) = \frac{4e^2}{\hbar} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} ; \quad \sigma(\omega) = \frac{4e^2}{\hbar} K$$

where $K$ is a universal number characterizing the CFT3, and $v$ is the velocity of “light”.
Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

However, for all CFT3s, at $\hbar \omega \ll k_B T$, we have the Einstein relation

$$\chi(k, \omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} ; \quad \sigma(\omega) = 4e^2 D \chi_c = \frac{4e^2}{\hbar} \Theta_1 \Theta_2$$

where the compressibility, $\chi_c$, and the diffusion constant $D$ obey

$$\chi = \frac{k_B T}{(h\nu)^2} \Theta_1 ; \quad D = \frac{h\nu^2}{k_B T} \Theta_2$$

with $\Theta_1$ and $\Theta_2$ universal numbers characteristic of the CFT3

Density correlations in CFTs at $T>0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar \omega \gg k_B T$, to hydrodynamic behavior for $\hbar \omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} 
\frac{4e^2}{h} K & , \quad \hbar \omega \gg k_B T \\
\frac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar \omega \ll k_B T 
\end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

Outline

1. Superfluid-insulator transition
   *Integer and fractional filling*

2. Quantum-critical transport
   *Collisionless-to-hydrodynamic crossover of CFT3s*

3. SYM3 with $\mathcal{N} = 8$ supersymmetry

4. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*
Outline

1. Superfluid-insulator transition
   Integer and fractional filling

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SU($N$) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, $g$, which flows to a strong-coupling fixed point $g = g^*$ in the infrared.

- The CFT3 describing this fixed point resembles “critical spin liquid” theories.

- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\text{AdS}_4 \times S_7$.

- The CFT3 has a global $\text{SO}(8)$ $R$ symmetry, and correlators of the $\text{SO}(8)$ charge density can be computed exactly in the large $N$ limit, even at $T > 0$. 
Collisionless to hydrodynamic crossover of SYM3

$\text{Im} \chi(k, \omega)/k^2$

$\text{Im} \frac{K}{\sqrt{k^2 - \omega^2}}$

$\frac{3k}{4\pi T} = 1.0, 2.0, 3.0, 4.0$

Collisionless to hydrodynamic crossover of SYM3

$\text{Im} \chi(k, \omega) / k^2$

$\frac{3k}{4\pi T} = 0.2, 0.5, 1.0$

$\text{Im} \frac{D\chi_c}{Dk^2 - i\omega}$

Universal constants of SYM3

\[
\begin{align*}
\chi_c &= \frac{k_B T}{(h\nu)^2} \Theta_1 \\
D &= \frac{h\nu^2}{k_B T} \Theta_2 \\
\sigma(\omega) &= \begin{cases} 
\frac{4e^2}{\hbar} K, & \hbar \omega \gg k_B T \\
\frac{4e^2}{\hbar} \Theta_1 \Theta_2, & \hbar \omega \ll k_B T
\end{cases}
\end{align*}
\]

\[
\begin{align*}
K &= \frac{\sqrt{2} N^{3/2}}{3} \\
\Theta_1 &= \frac{8\pi^2 \sqrt{2} N^{3/2}}{9} \\
\Theta_2 &= \frac{3}{8\pi^2}
\end{align*}
\]

C. Herzog, JHEP 0212, 026 (2002)
Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$.

- This is traced to a four-dimensional electromagnetic self-duality of the theory on AdS$_4$. In the large $N$ limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS$_4$.

- This special property is not expected for generic CFT3s.

- Open question: Does $K = \Theta_1 \Theta_2$ hold beyond the $N \to \infty$ limit? In other words, does this “self-duality” survive in the full M theory.
Outline

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1. Superfluid-insulator transition
   \it{Integer and fractional filling}

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4. Nernst effect in the cuprate superconductors
   \it{Quantum criticality and dyonic black holes}
Dope the antiferromagnets with charge carriers of density $x$ by applying a chemical potential $\mu$. 

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

$a_0 = 3.9\text{Å}$

$a_0 = 5.4\text{Å}$
Superconductor
Nernst measurements

Superconductor
Nernst experiment
Use coupling $g$ to induce a transition to a VBS insulator.
Nernst measurements

$T$

$g$

$\mu$

Superconductor

Insulator $x=1/8$
For experimental applications, we must move away from the ideal CFT

- A chemical potential $\mu$
- A magnetic field $B$

In the gravity dual theory, these perturbations correspond to electric and magnetic charges on the black hole

e.g.

$$S = \int d^2 r d\tau \left[ (\partial_\tau - \mu)\psi^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$
In the hydrodynamic regime, $\hbar \omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.

The variables entering the hydrodynamic theory are

- the external magnetic field $F_{\mu\nu}$,

\[
F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},
\]

- $T_{\mu\nu}$, the stress energy tensor,

- $\rho$, the local number density,

- $P$, the local pressure,

- $\sigma_Q$, a universal conductivity, which is the \textbf{single transport co-efficient}.

The dependence of $\varepsilon$, $P$, $\sigma_Q$ on $T$ and $v$ follows from simple scaling arguments.
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

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\[
\partial_\mu J^\mu = 0 \\
\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu
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\[ \partial_\mu J^\mu = 0 \]

\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu \]

\[ T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} \]

\[ J^\mu = \rho u^\mu \]

Constitutive relations which follow from Lorentz transformation to moving frame

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[
\begin{align*}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (\partial_\nu \mu + F_\nu^\lambda u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\end{align*}
\]

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient.

For experimental applications, we must move away from the ideal CFT

- A chemical potential $\mu$

- A magnetic field $B$

In the gravity dual theory, these perturbations correspond to electric and magnetic charges on the black hole

\[
S = \int d^2r d\tau \left[ \left| (\partial_\tau - \mu)\psi \right|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]
\]

\[\nabla \times \vec{A} = B\]
For experimental applications, we must move away from the ideal CFT

- A chemical potential $\mu$
- A magnetic field $B$
- An impurity scattering rate $1/\tau_{\text{imp}}$ (its $T$ dependence follows from scaling arguments)

\[
S = \int d^2r dt \left[ \left( \partial_t - \mu \right) \psi^2 + v^2 \left( \nabla - iA \right) \psi^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]
\]

$\nabla \times \vec{A} = B$ , $\overline{V(r)} = 0$ , $\overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r-r')$
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[
\partial_\mu J^\mu = 0
\]

\[
\partial_\mu T^{\mu\nu} = \frac{1}{\tau_{\text{imp}}} (\delta_\nu^{\mu} + u^{\mu} u_\nu) T^{\nu\gamma} u_\gamma
\]

\[
T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + Pg^{\mu\nu}
\]

\[
J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\]

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

\[ \omega_c = \frac{2eB \rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2v^2}{c^2(\varepsilon + P)} \]

Longitudinal conductivity

\[ \sigma_{xx} = \sigma_Q \left[ \frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] . \]

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**Longitudinal conductivity**

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\]

\[
= \sigma_Q + \frac{4e^2\rho^2v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0
\]

From these relations, we obtained results for the transport coefficients, expressed in terms of a “cyclotron” frequency and damping:

\[
\omega_c = \frac{2eB \rho v^2}{c(\epsilon + P)}, \quad \gamma = \sigma Q \frac{B^2 v^2}{c^2(\epsilon + P)}
\]

Hall conductivity

\[
\sigma_{xy} = -\frac{2e\rho c}{B} \left[ \frac{\omega_c^2 - 2i\gamma \omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right]
\]

\[
= B \left[ \frac{4e^2 v^2}{\sigma Q (\epsilon + P)(1/\tau_{\text{imp}} - i\omega)} + \frac{8e^3 \rho^3 v^4}{(\epsilon + P)^2(1/\tau_{\text{imp}} - i\omega)^2} \right] \quad \text{as } B \to 0
\]

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

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\]

**Thermal conductivity**

\[
\kappa_{xx} = \sigma_Q \left( \frac{k_B^2 T}{4e^2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 \left[ \frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{imp})}{(\omega_c^2/\gamma + 1/\tau_{imp})^2 + \omega_c^2} \right]
\]

\[
= \frac{1}{\sigma_Q} k_B^2 T \left( \frac{c(\varepsilon + P)}{k_B TB} \right)^2 \left[ \frac{\gamma(\omega_c^2/\gamma + 1/\tau_{imp})}{(\omega_c^2/\gamma + 1/\tau_{imp})^2 + \omega_c^2} \right]
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**Thermal conductivity**

$$\kappa_{xx} = \sigma_Q \left( \frac{k_B^2 T}{4e^2} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 \left[ \frac{\omega_c^2/\gamma}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right]$$

$$= \frac{1}{\sigma_Q} k_B^2 T \left( \frac{c(\varepsilon + P)}{k_B T B} \right)^2 \quad \rightarrow 1 \text{ as } \rho \rightarrow 0$$

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

\[ \omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2v^2}{c^2(\varepsilon + P)} \]

Nernst signal

\[ e_N = \left( \frac{k_B}{2e} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right) \left[ \frac{\omega_c/\tau_{imp}}{\left( \omega_c^2/\gamma + 1/\tau_{imp} \right)^2 + \omega_c^2} \right] \]

\[ \frac{k_B}{2e} = 43.086 \mu V/K \]

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

\[
\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2v^2}{c^2(\varepsilon + P)}
\]

Transverse thermoelectric co-efficient

\[
\left( \frac{h}{2e k_B} \right) \alpha_{xy} = \Phi_s \overline{B} (k_B T)^2 \left( \frac{2\pi \tau_{\text{imp}}}{\hbar} \right)^2 \frac{\bar{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3 \hbar/2\pi \tau_{\text{imp}}}{\Phi_{\varepsilon+P}(k_B T)^6 + \overline{B}^2 \bar{\rho}^2 (2\pi \tau_{\text{imp}}/\hbar)^2},
\]

where

\[
B = \overline{B} \phi_0/(\hbar v)^2 \quad ; \quad \rho = \bar{\rho}/(\hbar v)^2.
\]

LSCO - Experiments

N. P. Ong et al.
Only input parameters

\( \hbar \nu = 47 \text{ meV} \cdot \text{Å} \)

\( \tau_{\text{imp}} \approx 10^{-12} \text{ s} \)

Output

\[ \omega_c = 6.2 \text{GHz} \cdot \frac{B}{1 \text{T}} \left( \frac{35 \text{K}}{T} \right)^3 \]

LSCO - Theory

Only input parameters

\[ \hbar \nu = 47 \text{ meV} \ \text{Å} \]

\[ \tau_{\text{imp}} \approx 10^{-12} \ \text{s} \]


Output

\[ \omega_c = 6.2 \text{GHz} \cdot \frac{B}{1 \text{T}} \left( \frac{35 \text{K}}{T} \right)^3 \]
To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential $\mu$
- A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in precise accord with all hydrodynamic results presented earlier.

THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
Conclusions

- Condensed matter systems realize several interesting CFT3s.
- Collisionless-to-hydrodynamic crossover in CFT3s at T>0.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.