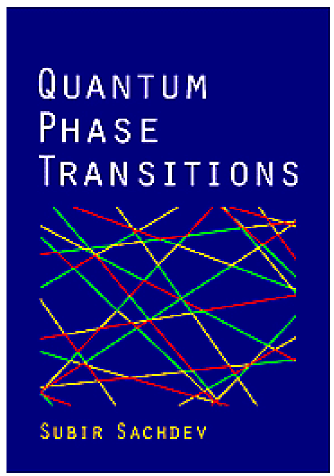
Quantum phase transitions of ultracold atoms

Subir Sachdev

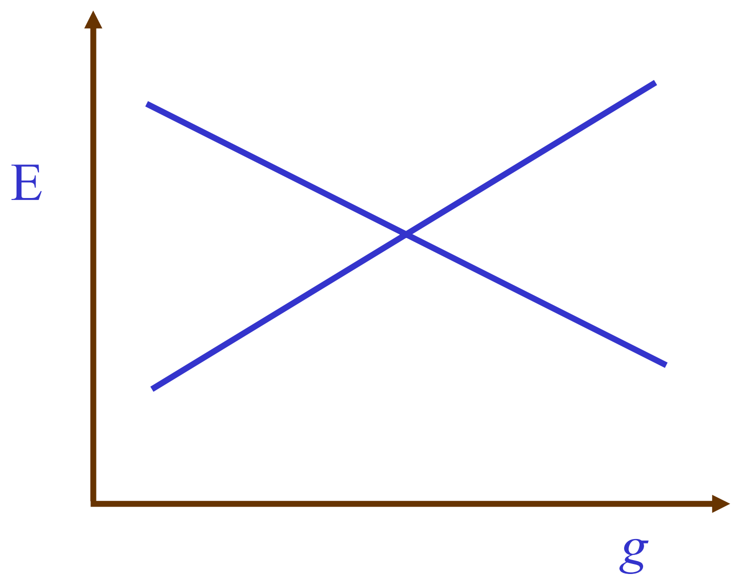


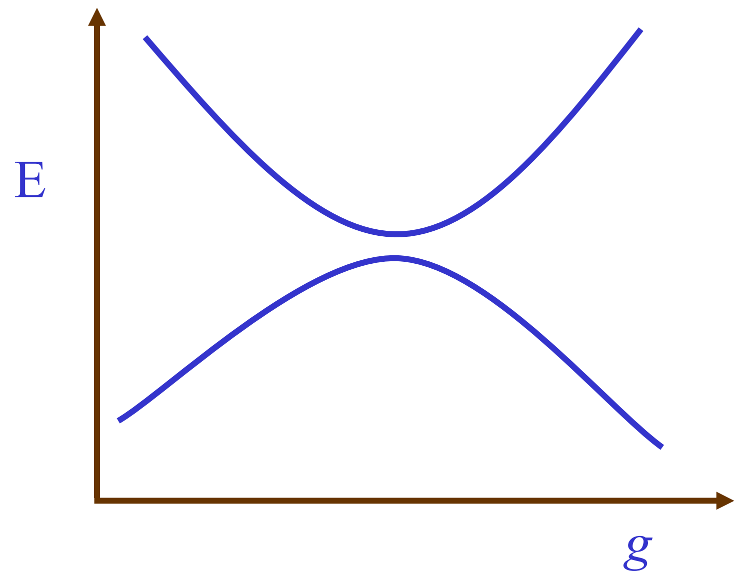
Quantum Phase Transitions
Cambridge University Press (1999)

Transparencies online at
http://pantheon.yale.edu/~subir

What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter *g*





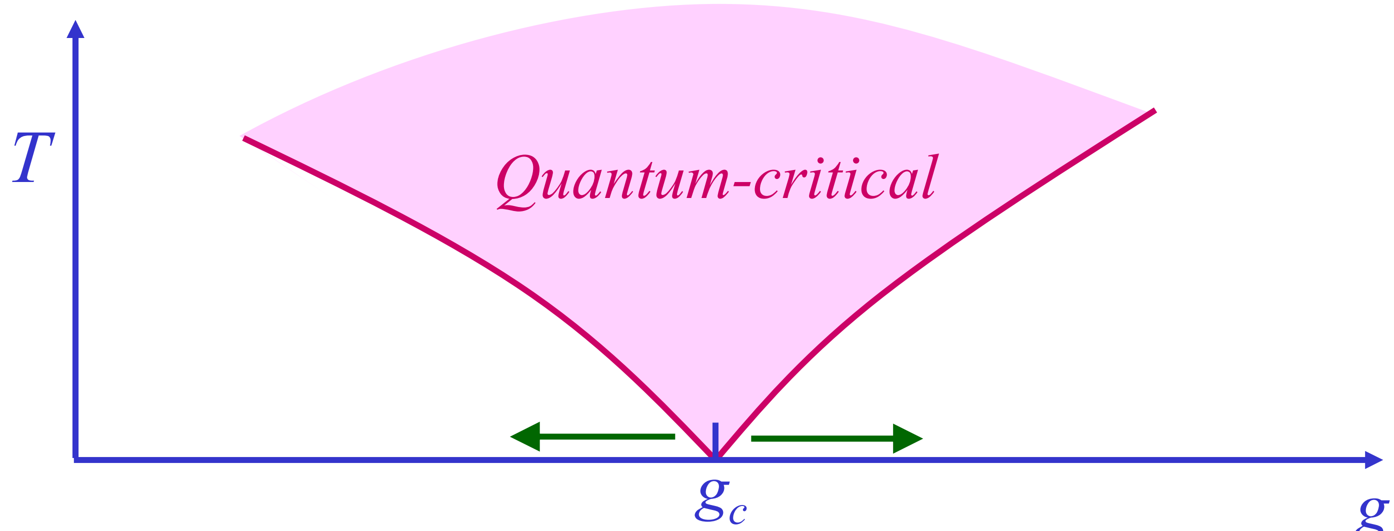
True level crossing:

Usually a *first*-order transition

Avoided level crossing which becomes sharp in the infinite volume limit:

second-order transition

Why study quantum phase transitions ?



• Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

• Critical point is a novel state of matter without quasiparticle excitations

• Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$:
temporal and spatial scale invariance;
characteristic energy scale at other values of g: $\Delta \sim \left|g - g_c\right|^{z\nu}$

Outline

I. The superfluid—Mott-insulator transition

II. Mott insulator in a strong electric field.

S. Sachdev, K. Sengupta, and S. M. Girvin, *Physical Review* B **66**, 075128 (2002).

III. Conclusions

I. The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

M.P.A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev. B* **40**, 546 (1989).

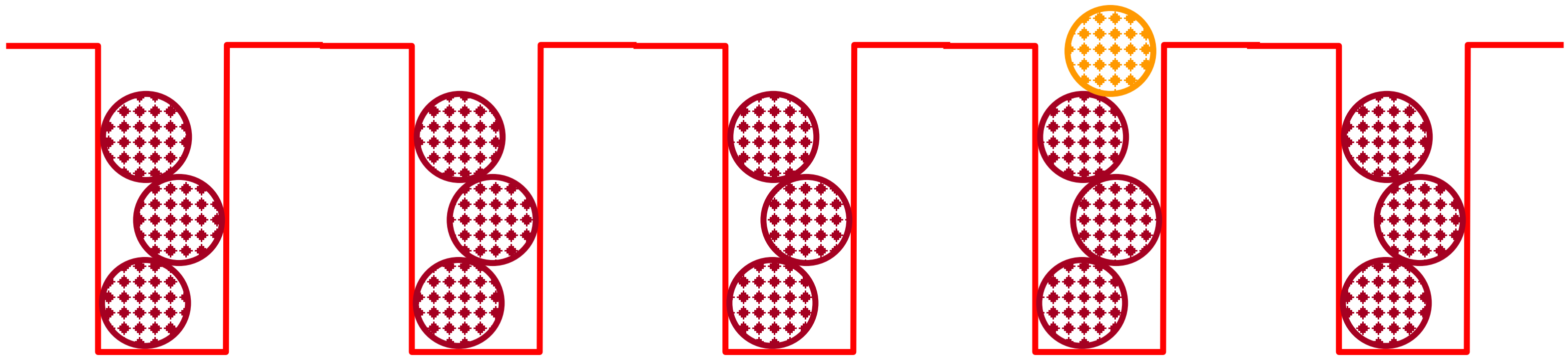
For small U/t, ground state is a superfluid BEC with

superfluid density $\approx$ density of bosons

What is the ground state for large *U*/*t* ?

Typically, the ground state remains a superfluid, but with

superfluid density $\ll$ density of bosons

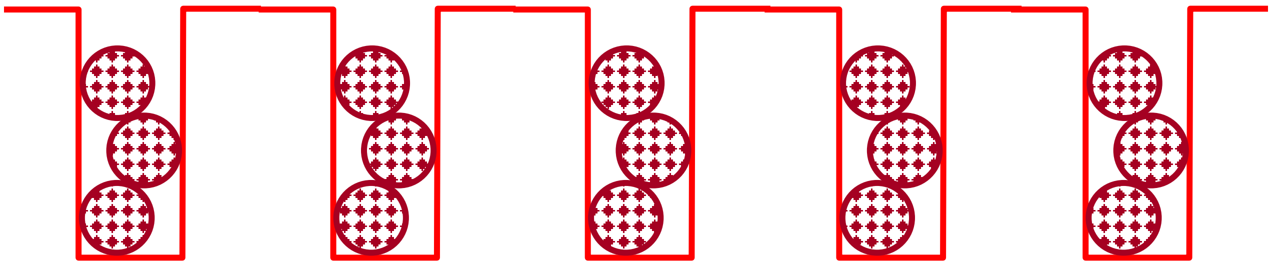


The superfluid density evolves smoothly from large values at small *U*/*t*, to small values at large *U*/*t*, and there is no quantum phase transition at any intermediate value of *U*/*t*.

(In systems with Galilean invariance and at zero temperature, superfluid density=density of bosons always, independent of the strength of the interactions)

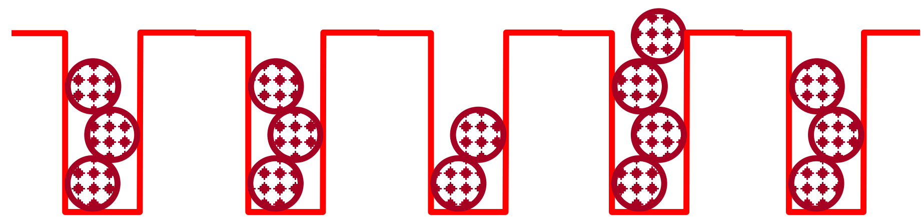
What is the ground state for large U/t ?

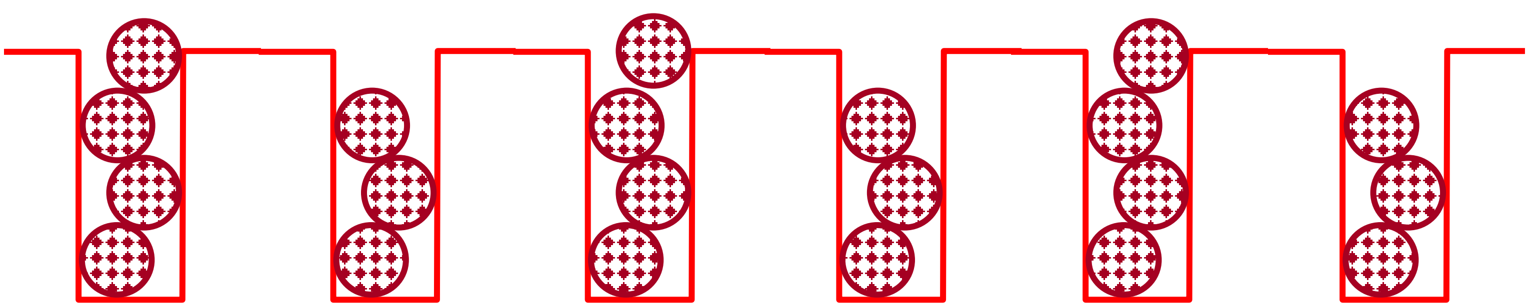
Incompressible, insulating ground states, with zero superfluid density, appear at special commensurate densities



$$\langle n_j \rangle = 3$$

$$-\frac{t}{U}$$

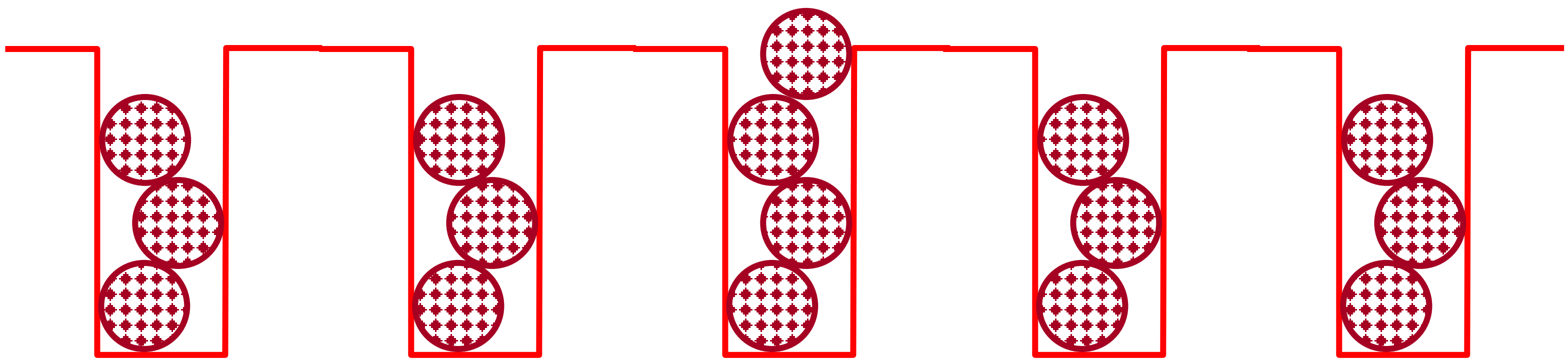




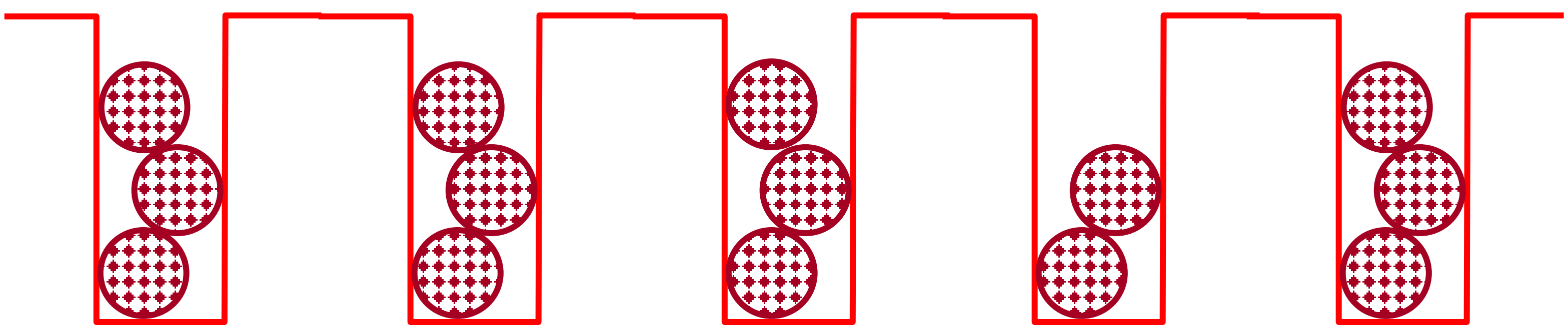
$$\langle n_j \rangle = 7/2$$

Ground state has "density wave" order, which spontaneously breaks lattice symmetries

Excitations of the insulator: infinitely long-lived, finite energy *quasiparticles* and *quasiholes*



Energy of quasi-particles/holes: $\varepsilon_{p,h}(p) = \Delta_{p,h} + \frac{p^2}{2m^*_{p,h}}$

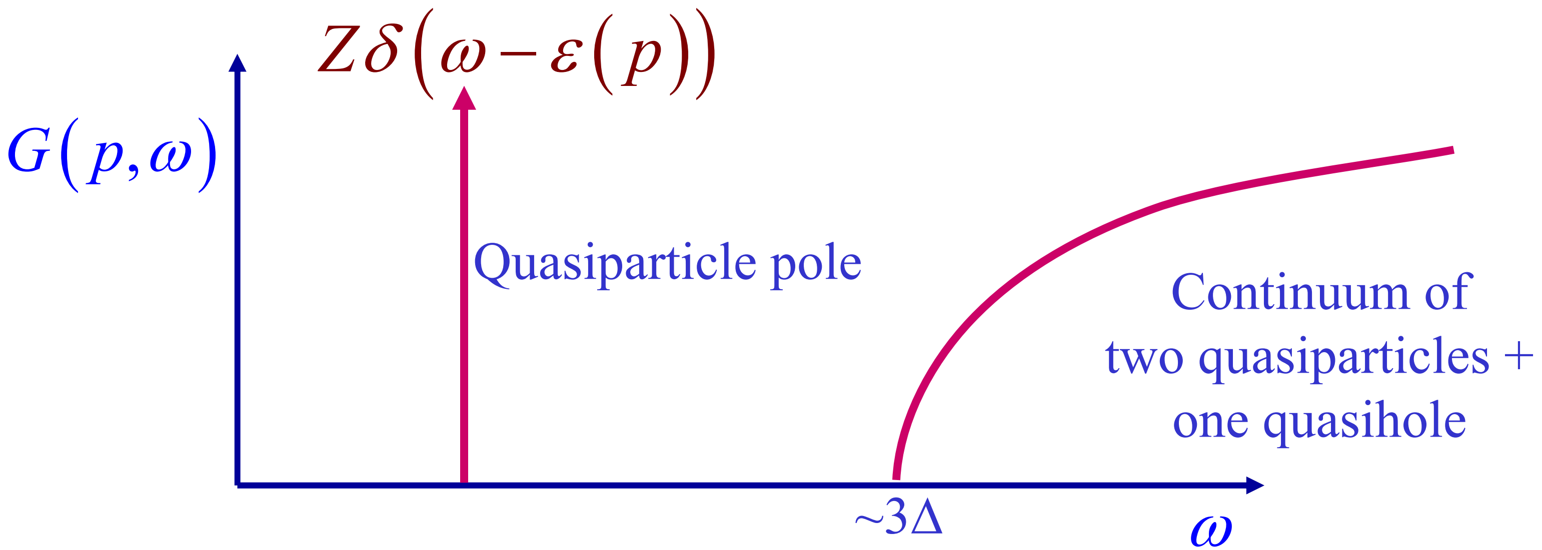


Boson Green's function $G(p,\omega)$:

Insulating ground state

Cross-section to add a boson

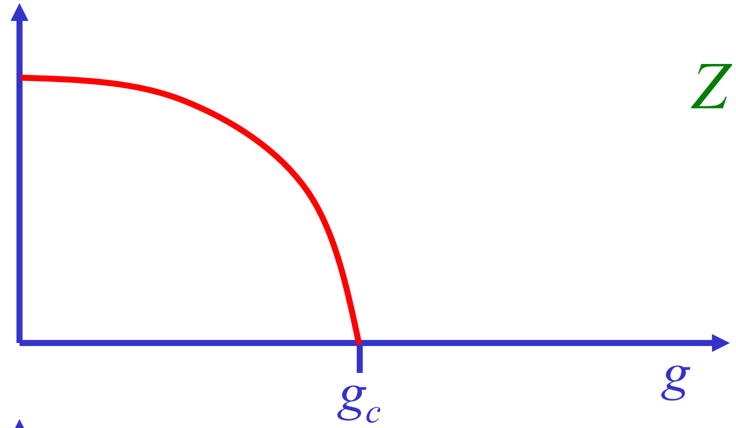
while transferring energy $\hbar\omega$ and momentum p



Similar result for quasi-hole excitations obtained by removing a boson

Entangled states at $g \equiv t/U$ of order unity

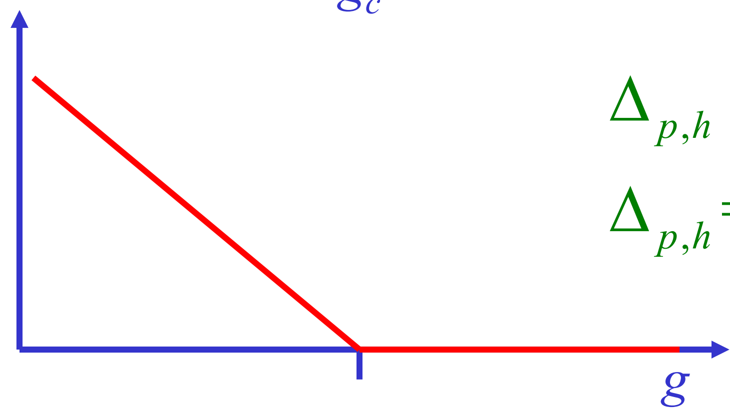
Quasiparticle weight Z



$$Z \sim (g_c - g)^{\eta\nu}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

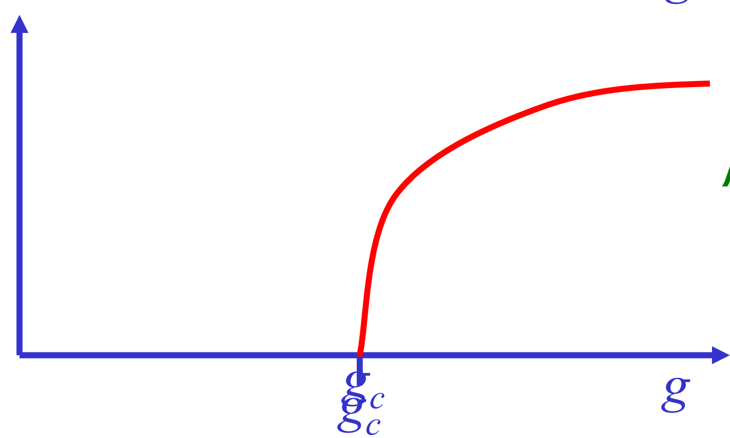
Excitation energy gap Δ



$$\Delta_{p,h} \sim (g_c - g)^{\nu} \text{ for } g < g_c$$

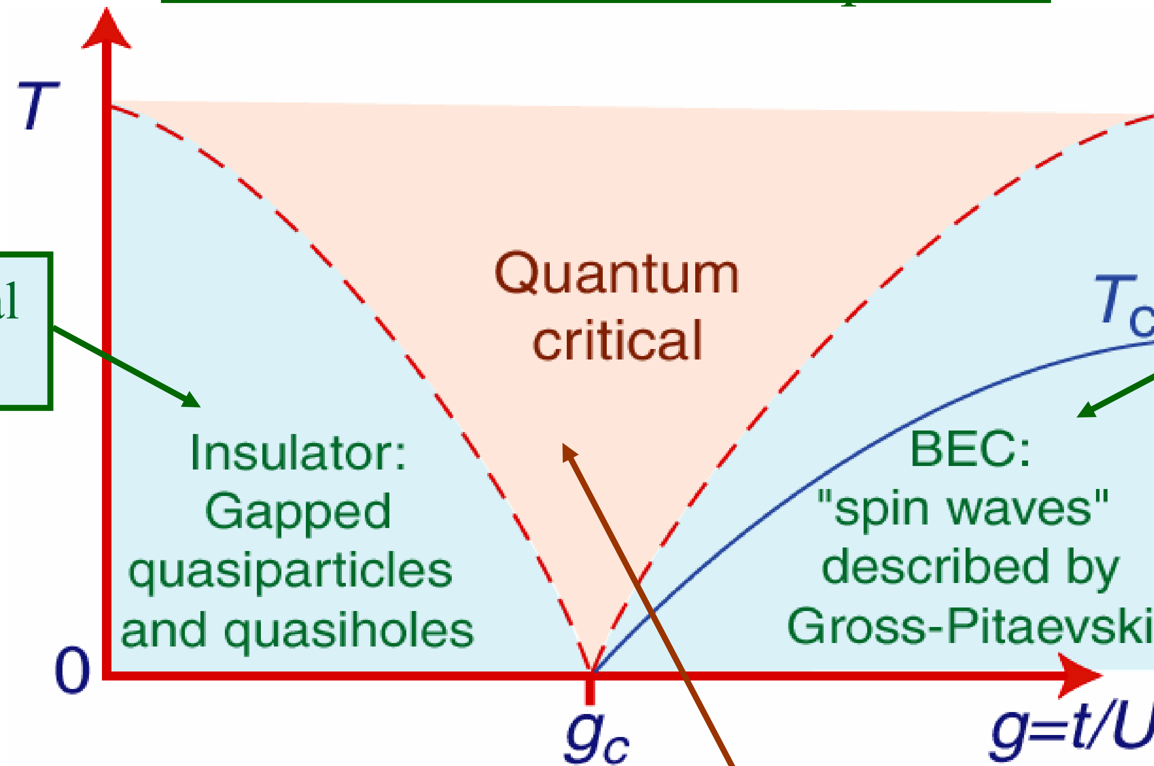
$$\Delta_{p,h} = 0 \text{ for } g > g_c$$

Superfluid density ρ_s



$$\rho_s \sim (g - g_c)^{(d+z-2)\nu}$$

Crossovers at nonzero temperature



Quasiclassical dynamics

Quasiclassical dynamics

Relaxational dynamics ("Bose molasses") with phase coherence/relaxation time τ_φ given by

$$\frac{1}{\tau_\varphi} = (\text{Universal number})\frac{k_B T}{\hbar} \quad (1\mu\text{K} = 20.9\text{kHz})$$

S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
K. Damle and S. Sachdev *Phys. Rev*. B **56**, 8714 (1997).

$$\text{Conductivity (in d=2)} = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \qquad \Sigma \rightarrow \text{universal function}$$

M.P.A. Fisher, G. Girvin, and G. Grinstein, *Phys. Rev. Lett.* **64**, 587 (1990).
K. Damle and S. Sachdev *Phys. Rev.* B **56**, 8714 (1997).

<u>**Outline**</u>

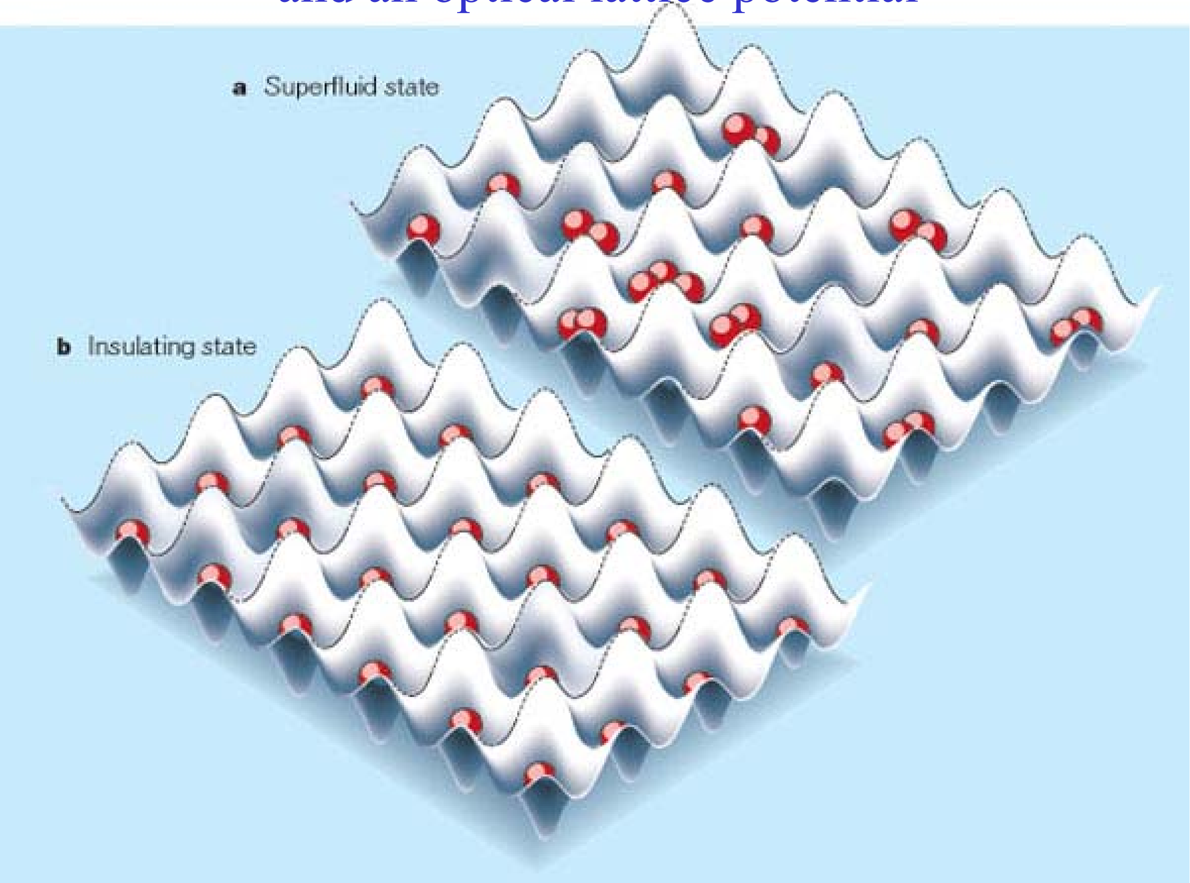
I. The superfluid—Mott-insulator transition

II. Mott insulator in a strong electric field

S. Sachdev, K. Sengupta, and S. M. Girvin, *Physical Review* B **66**, 075128 (2002).

III. Conclusions

Superfluid-insulator transition of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Related earlier work by C. Orzel, A.K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, *Science* **291**, 2386 (2001).

Detection method

Trap is released and atoms expand to a distance far larger than original trap dimension

$$\psi(\boldsymbol{R},T) = \exp\left(i\frac{m\boldsymbol{R}^2}{2\hbar T}\right)\psi(\boldsymbol{0},0) \approx \exp\left(i\frac{m\boldsymbol{R}_0^2}{2\hbar T} + i\frac{m\boldsymbol{R}_0\cdot\boldsymbol{r}}{\hbar T}\right)\psi(\boldsymbol{0},0)$$

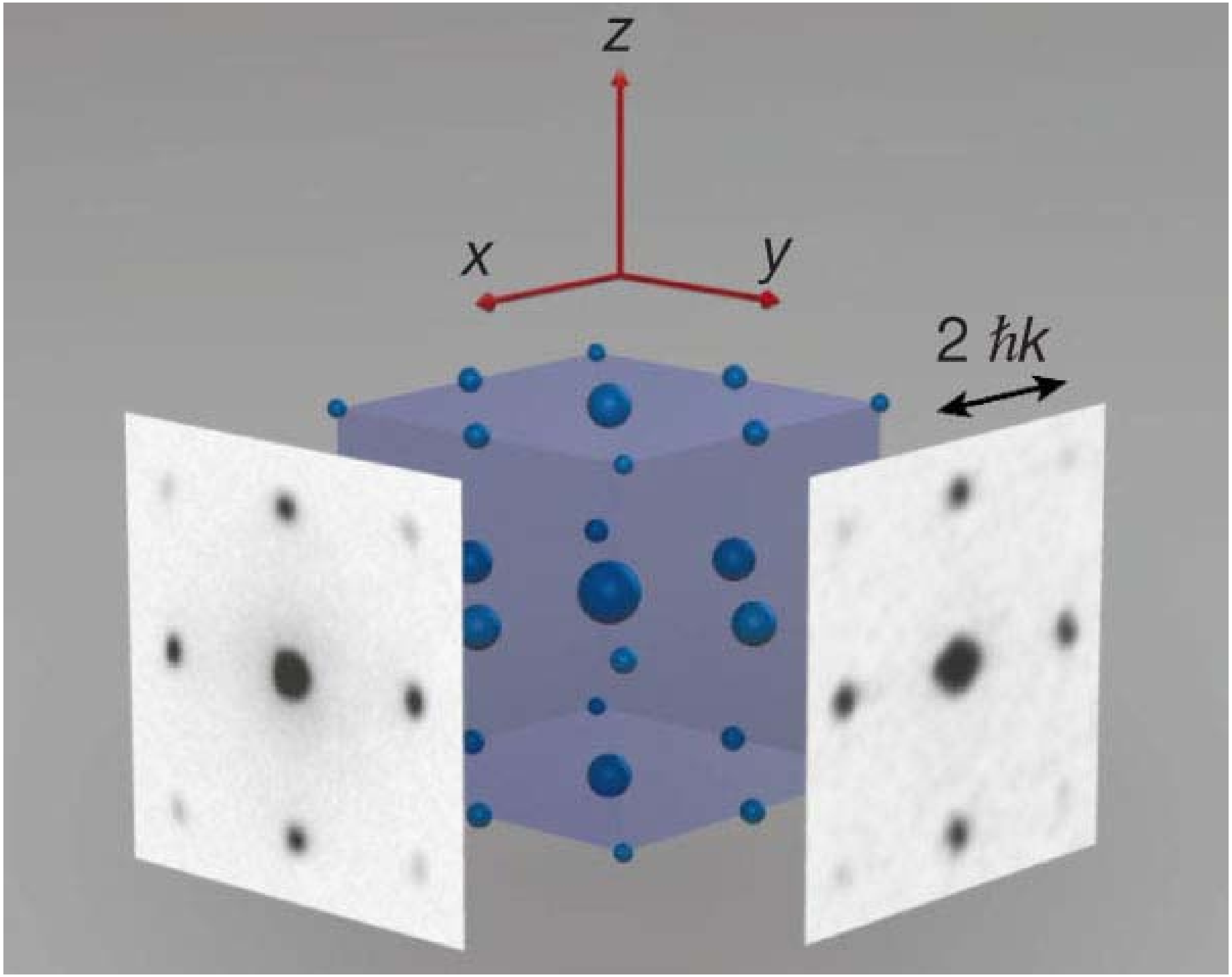
where $\boldsymbol{R} = \boldsymbol{R}_0 + \boldsymbol{r}$, with $\boldsymbol{R}_0$ = the expansion distance, and $\boldsymbol{r}$ = position within trap

In tight-binding model of lattice bosons b_i,

$$\text{detection probability} \propto \sum_{i,j}\left\langle b_i^\dagger b_j\right\rangle \exp\left(i\boldsymbol{q}\cdot\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right)\right) \text{ with } \boldsymbol{q} = \frac{m\boldsymbol{R}_0}{\hbar T}$$

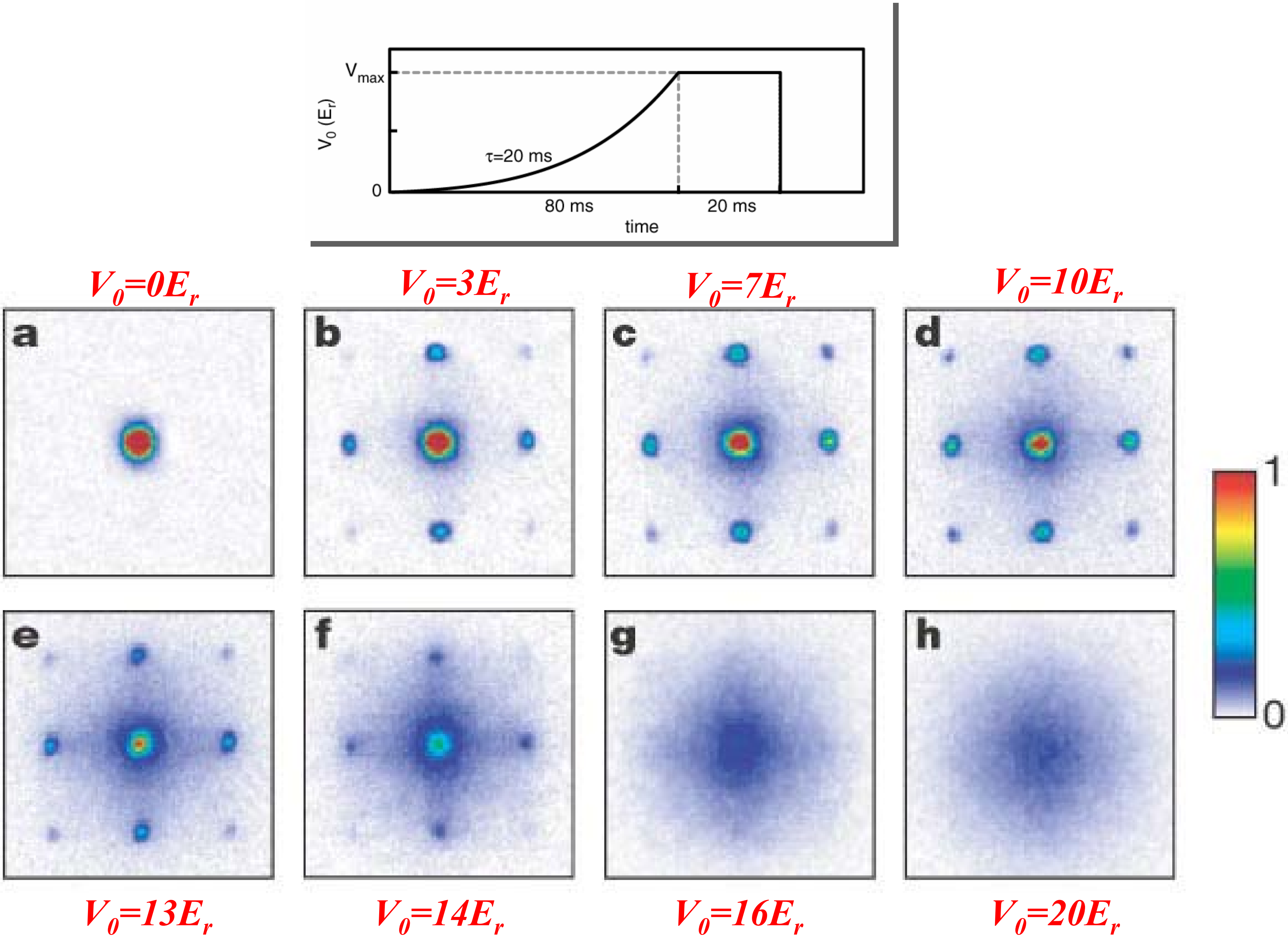
Measurement of momentum distribution function

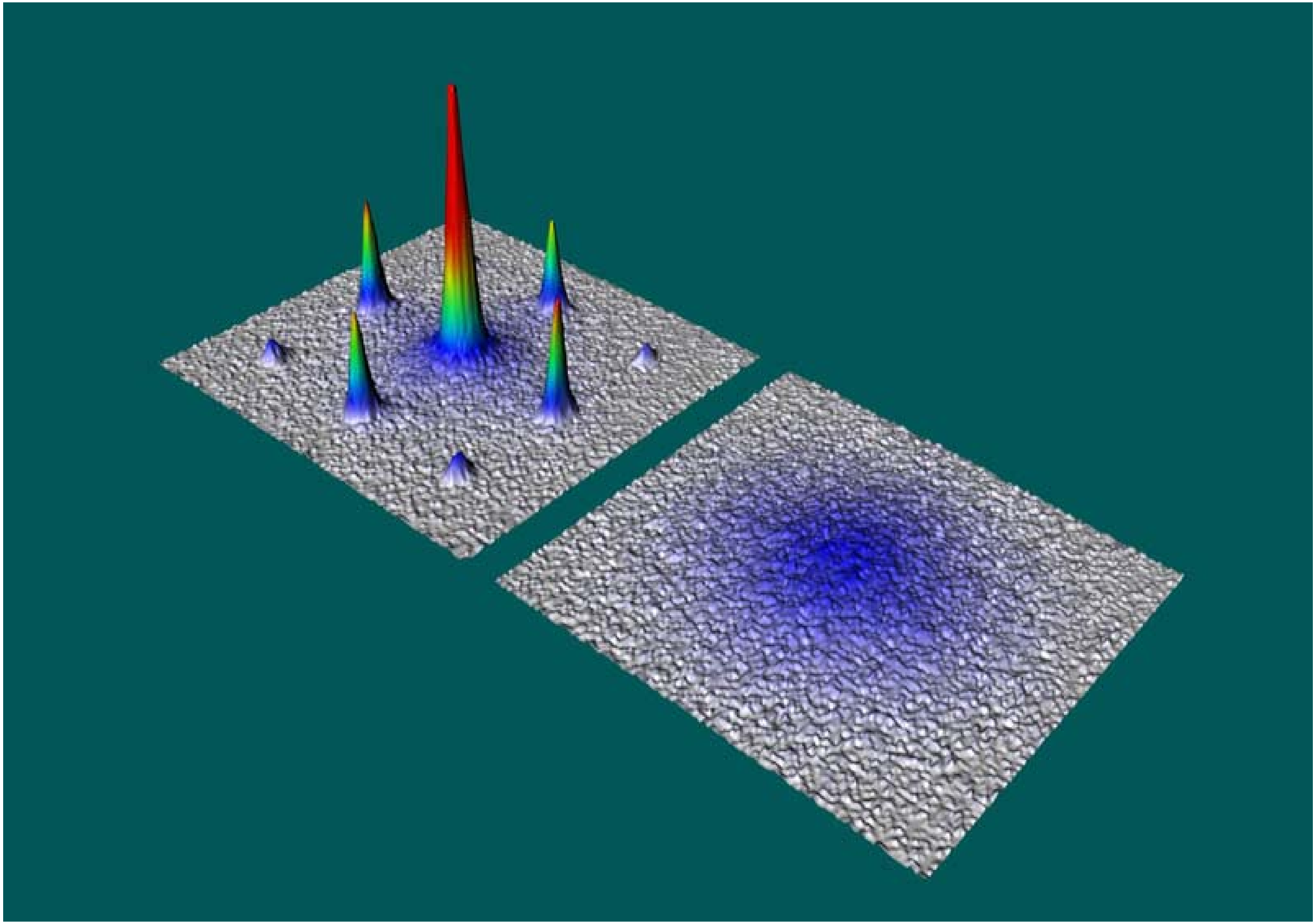
Superfluid state



Schematic three-dimensional interference pattern with measured absorption images taken along two orthogonal directions. The absorption images were obtained after ballistic expansion from a lattice with a potential depth of $V_0 = 10\ E_r$ and a time of flight of 15 ms.

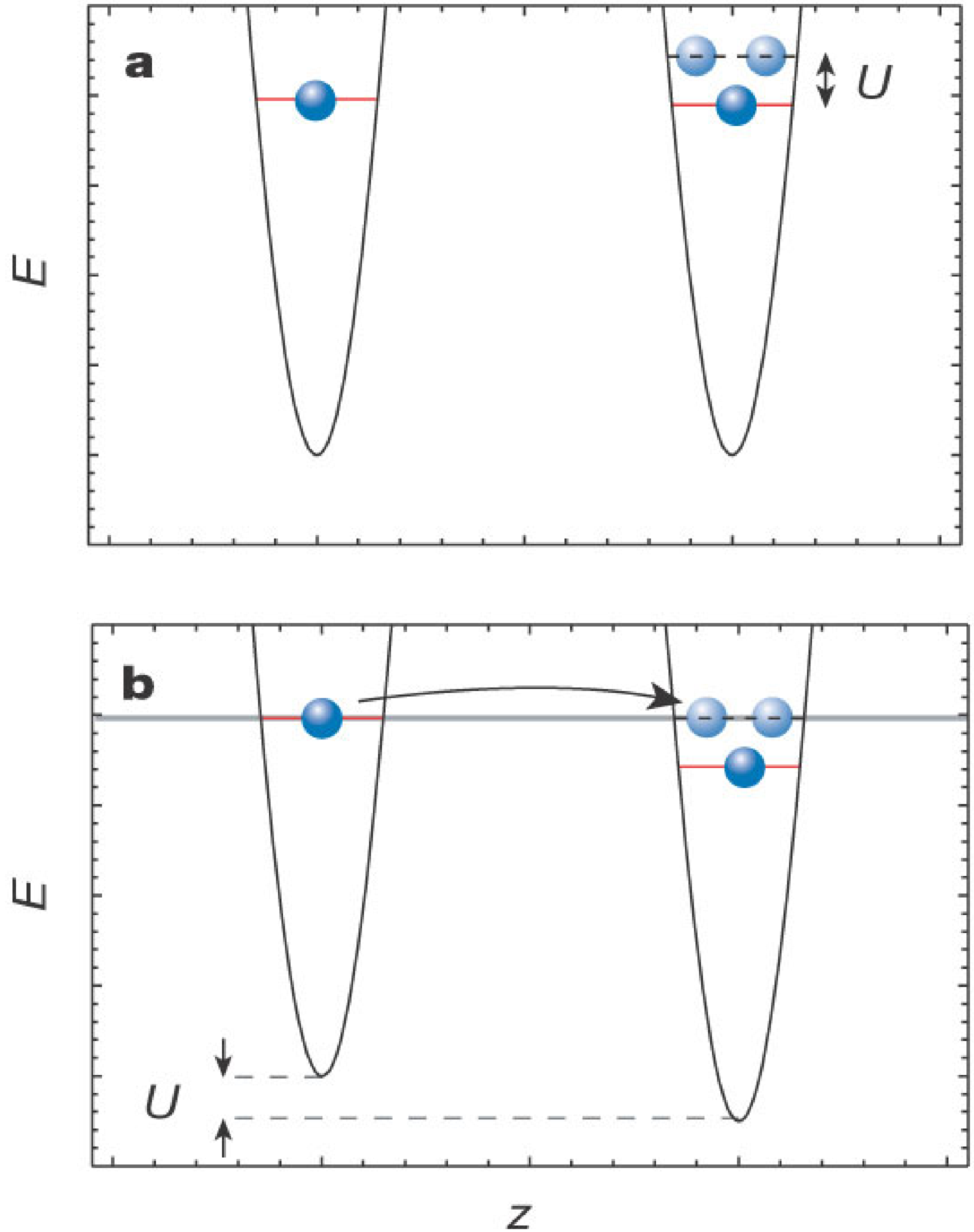
Superfluid-insulator transition

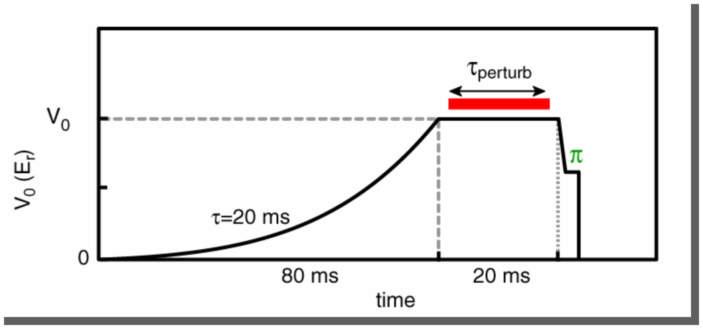


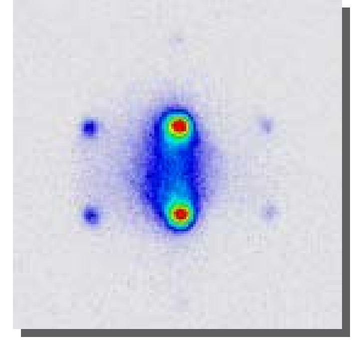


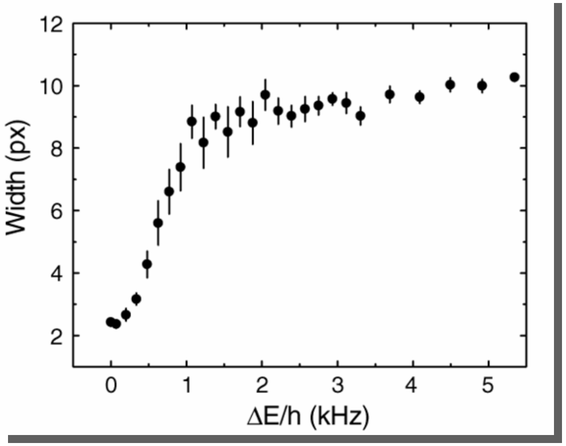
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Applying an “electric” field to the Mott insulator

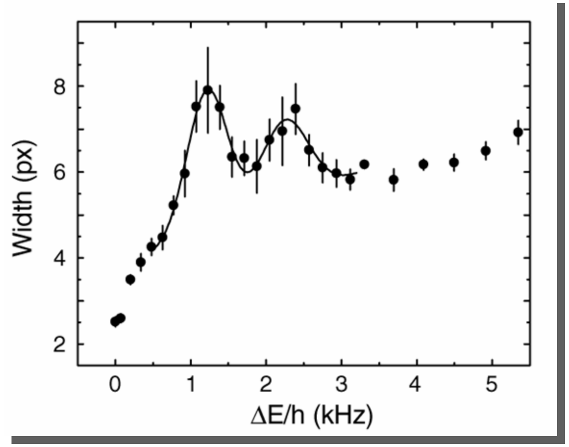






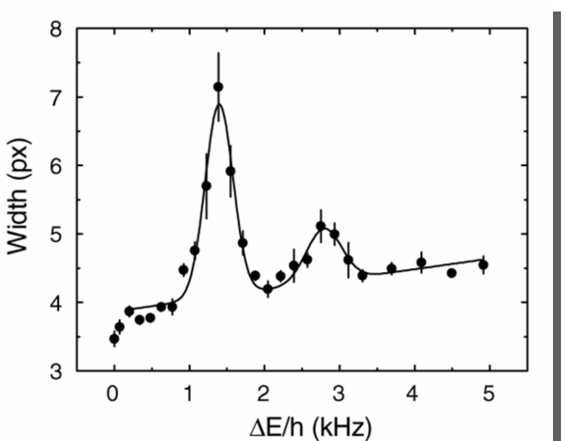


V_0=10 E_{recoil} $\tau_{perturb}$ = 2 ms

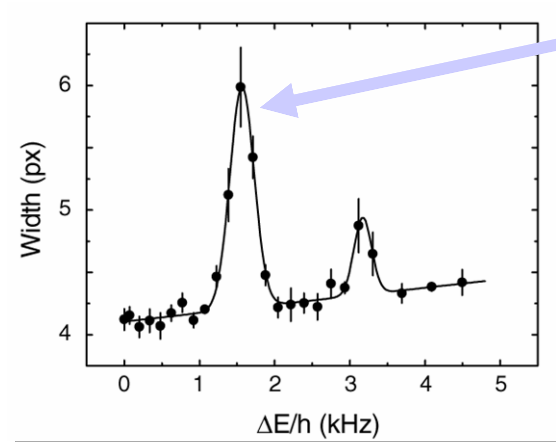


V_0= 13 E_{recoil} $\tau_{perturb}$ = 4 ms

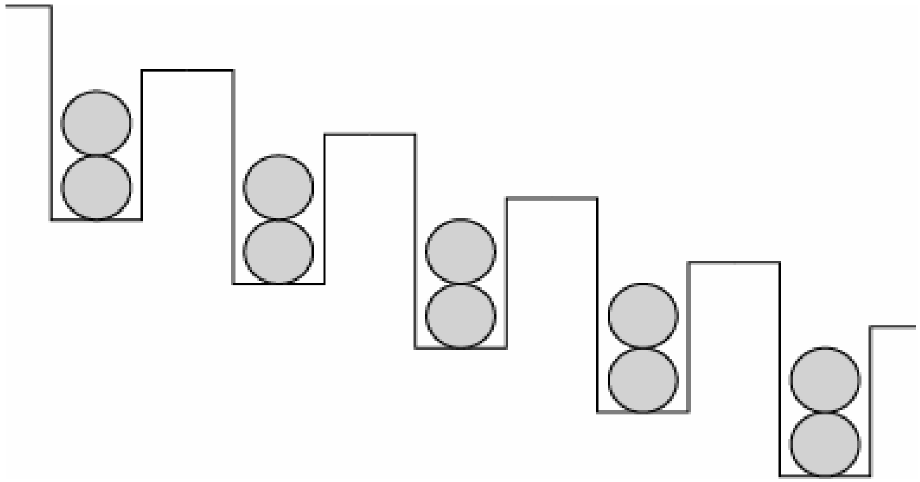
What is the quantum state here ?



V_0= 16 E_{recoil} $\tau_{perturb}$ = 9 ms



V_0= 20 E_{recoil} $\tau_{perturb}$ = 20 ms

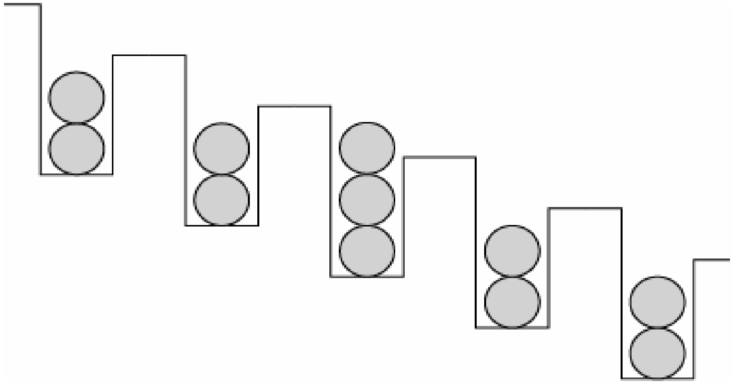


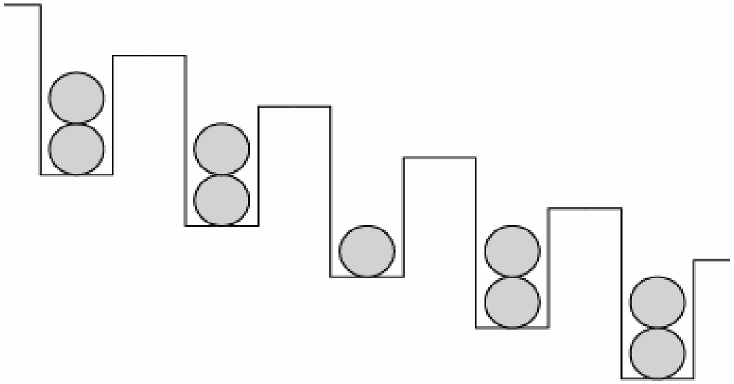
$$H = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

$$n_i = b_i^\dagger b_i$$

$$\left|U - E\right|, t \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator





Effective Hamiltonian for a quasiparticle in one dimension (similar for a quasihole):

$$H_{\text{eff}} = -\sum_j \left[3t\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j \right) + Ejb_j^\dagger b_j \right]$$

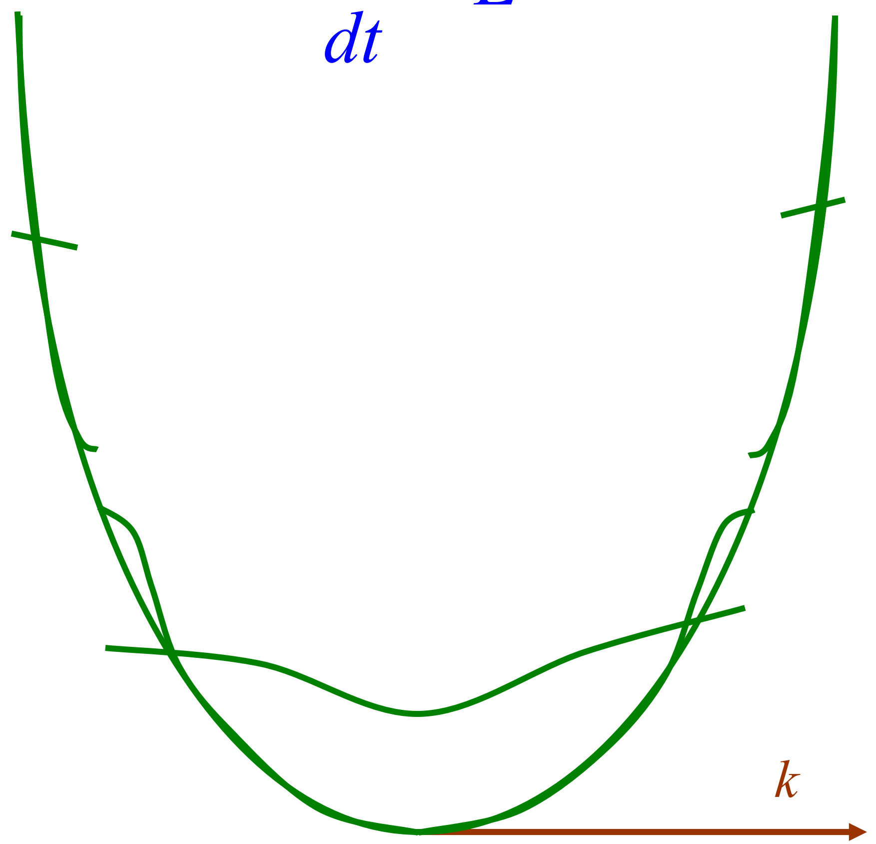
Exact eigenvalues $\varepsilon_m = Em \quad ; \quad m = -\infty \cdots \infty$

Exact eigenvectors $\psi_m\left(j\right) = J_{j-m}\left(6t / E\right)$

All charged excitations are strongly localized in the plane perpendicular electric field.
Wavefunction is periodic in time, with period *h/E* (Bloch oscillations)
Quasiparticles and quasiholes are not accelerated out to infinity

Semiclassical picture

$$\frac{dk}{dt} = E$$

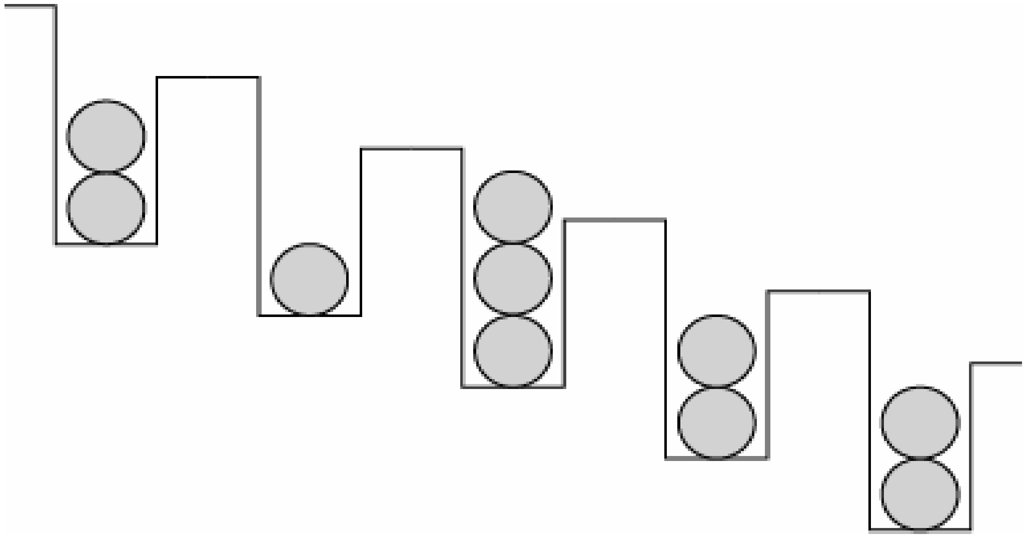


Free particle: is accelerated to infinity

In a weak periodic potential, escape to infinity occurs via Zener tunneling across band gaps

Experimental situation: Strong potential, for which there is negligible Zener tunneling, and the particle undergoes Bloch oscillations

Important neutral excitations (in one dimension)



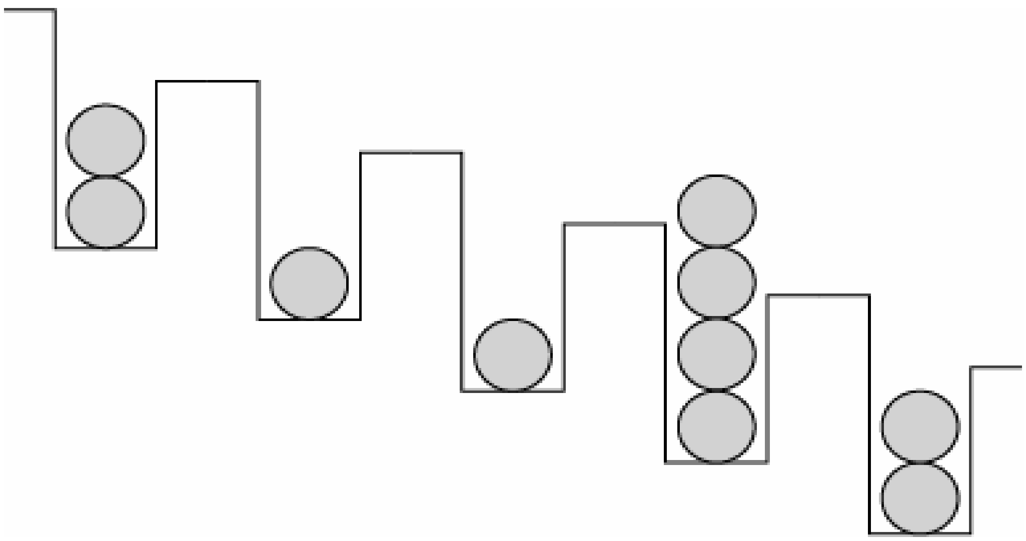
Nearest-neighbor dipole

Nearest-neighbor dipoles

Creating dipoles on nearest-neighbor links creates a state with relative energy *U-2E ;* such states are *not* part of the resonant manifold

Dipoles can appear resonantly on non-nearest-neighbor links.
Within resonant manifold, dipoles have infinite on-link and nearest-link repulsion

A non-dipole state



State has energy *3(U-E)* but is connected to resonant state by a matrix element smaller than t^2/U

State is not part of resonant manifold

Hamiltonian for resonant dipole states (in one dimension)

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}t\sum_\ell \left(d_\ell^\dagger + d_\ell\right) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $d_\ell^\dagger d_\ell \le 1 \quad ; \quad d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of *(U-E)/t*

Note: there is no explicit dipole hopping term.

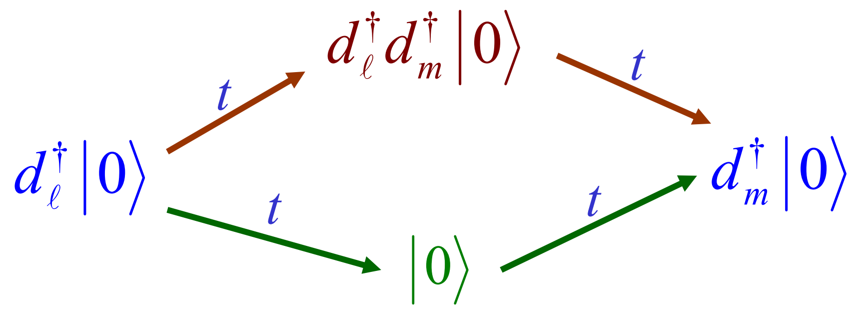
However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak electric fields: *(U-E)* $\gg t$

Ground state is dipole vacuum (Mott insulator) $|0\rangle$

First excited levels: single dipole states $d_\ell^\dagger |0\rangle$

Effective hopping between dipole states



If both processes are permitted, they exactly cancel each other.
The top processes is blocked when ℓ, m are nearest neighbors

$\Rightarrow$ A nearest-neighbor dipole hopping term $\sim \dfrac{t^2}{U-E}$ is generated

Strong electric fields: $(E\text{-}U) \gg t$

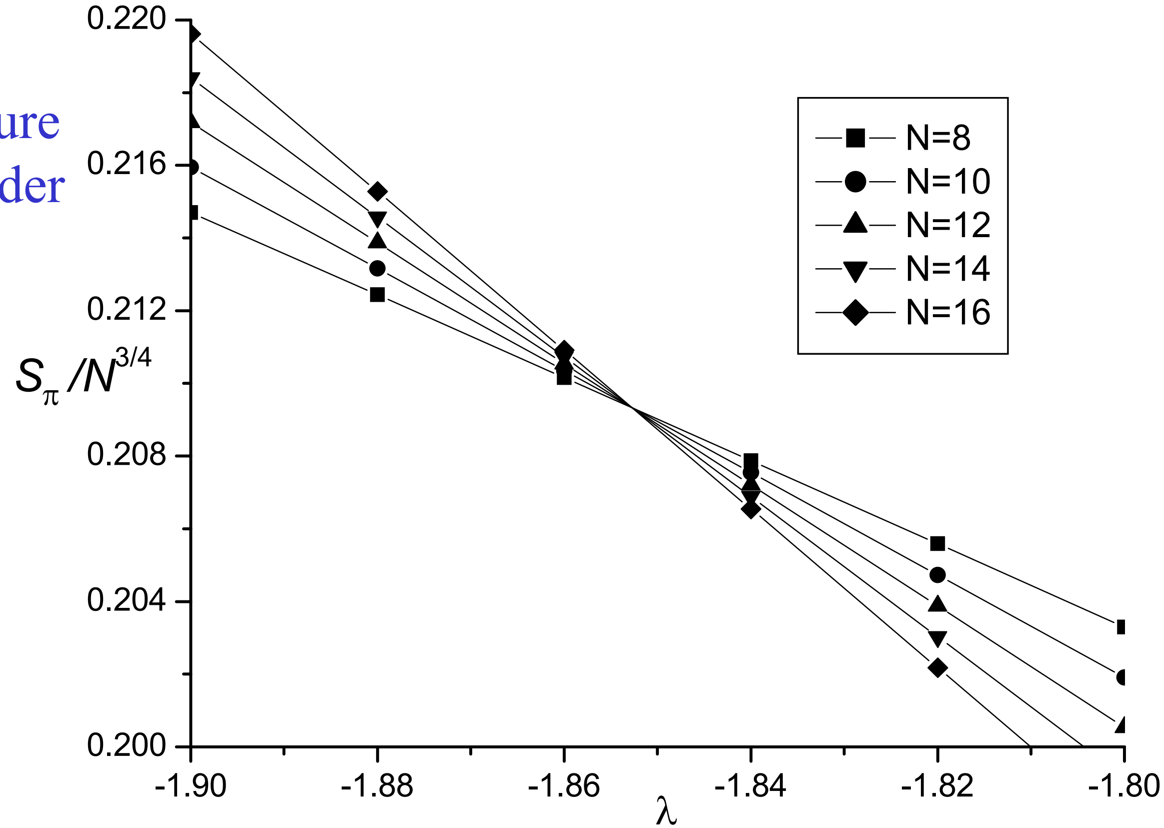
Ground state has maximal dipole number.

Two-fold degeneracy associated with Ising density wave order:

$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \qquad or \qquad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$

Ising quantum critical point at *E-U=1.08 t*

Equal-time structure factor for Ising order parameter



Hamiltonian for resonant states in higher dimensions

$p^\dagger_{\ell,n} \Rightarrow$ Creates quasiparticle in column ℓ and transverse position n

$h^\dagger_{\ell,n} \Rightarrow$ Creates quasihole in column ℓ and transverse position n

$$H_{ph} = -\sqrt{6}t\sum_{\ell,n}\left(p_{\ell+1,n}h_{\ell,n} + p^\dagger_{\ell+1,n}h^\dagger_{\ell,n}\right)$$

$$+\frac{(U-E)}{2}\sum_{\ell,n}\left(p^\dagger_{\ell,n}p_{\ell,n} + h^\dagger_{\ell,n}h_{\ell,n}\right)$$

$$-t\sum_{\ell,\langle nm\rangle}\left(2h^\dagger_{\ell,n}h_{\ell,m} + 3p^\dagger_{\ell,n}p_{\ell,m} + \text{H.c.}\right)$$

Terms as in one dimension

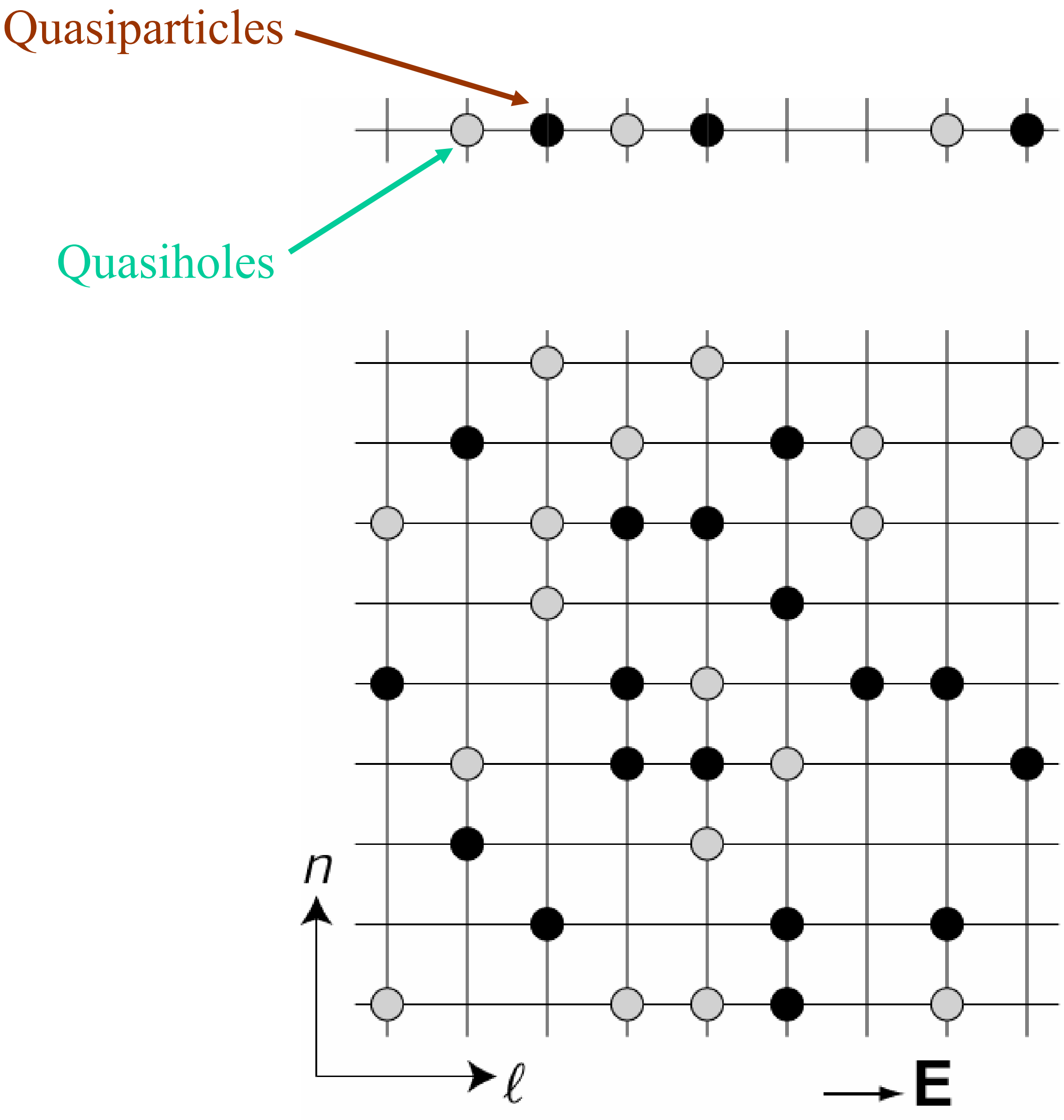
Transverse hopping

$$p^\dagger_{\ell,n}p_{\ell,n} \le 1 \quad ; \quad h^\dagger_{\ell,n}h_{\ell,n} \le 1 \quad ; \quad p^\dagger_{\ell,n}p_{\ell,n}h^\dagger_{\ell,n}h_{\ell,n} = 0$$

Constraints

New possibility: superfluidity in transverse direction (a smectic)

Resonant states in higher dimensions

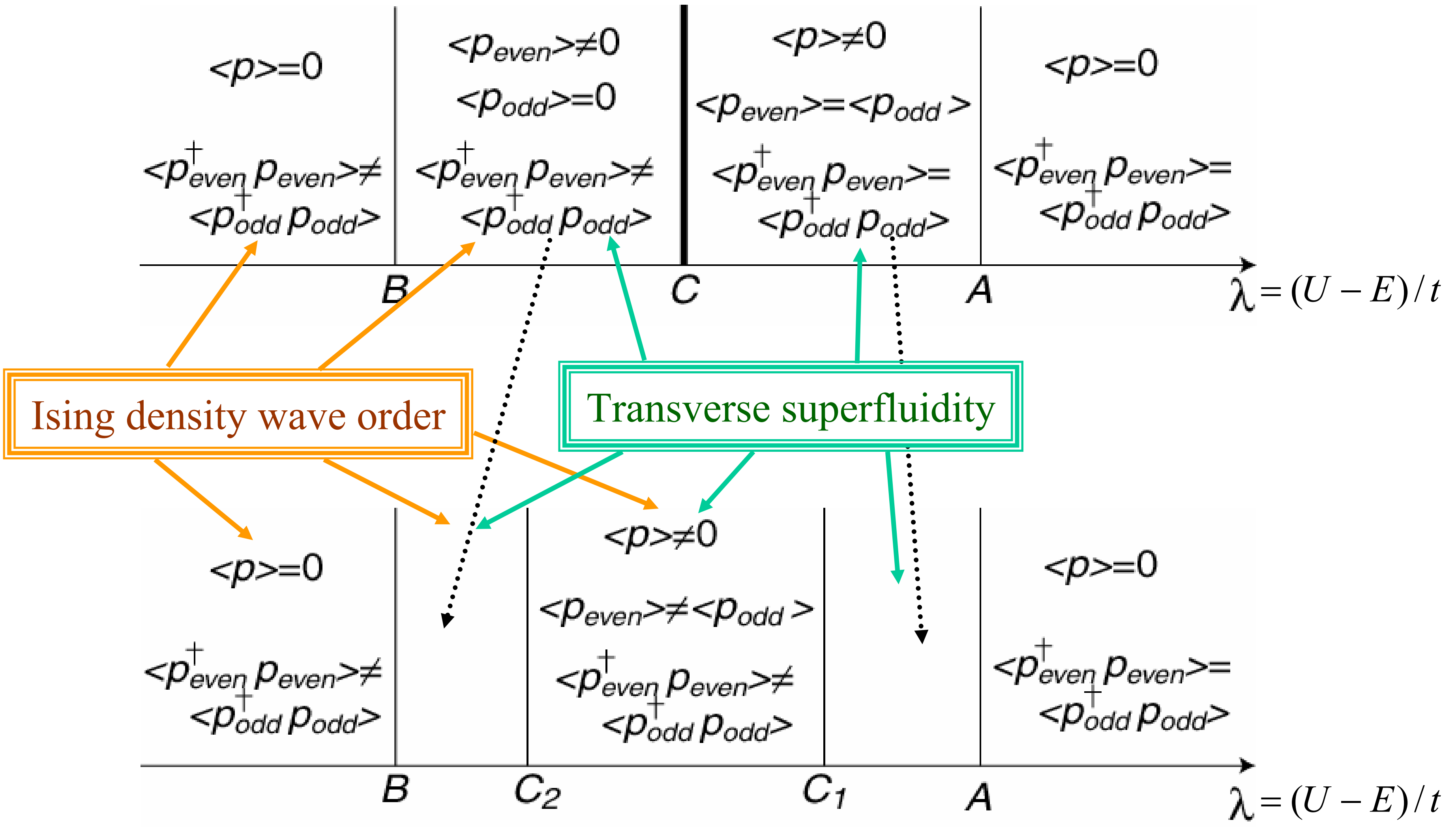


Dipole states in one dimension

Quasiparticles and quasiholes can move resonantly in the transverse directions in higher dimensions.

Constraint: number of quasiparticles in any column = number of quasiholes in column to its left.

Possible phase diagrams in higher dimensions



Implications for experiments

•Observed resonant response is due to gapless spectrum near quantum critical point(s).

•Transverse superfluidity (smectic order) can be detected by looking for “Bragg lines” in momentum distribution function---bosons are phase coherent in the transverse direction.

•Present experiments are insensitive to Ising density wave order. Future experiments could introduce a phase-locked subharmonic standing wave at half the wave vector of the optical lattice---this would couple linearly to the Ising order parameter.

Conclusions

I. Study of quantum phase transitions offers a controlled and systematic method of understanding many-body systems in a region of strong entanglement.

II. Atomic gases offer many exciting opportunities to study quantum phase transitions because of ease by which system parameters can be continuously tuned.