Entanglement, holography, and the quantum phases of matter

Fermilab, November 7, 2012

Subir Sachdev

Lecture at the 100th anniversary Solvay conference, Theory of the Quantum World
arXiv:1203.4565
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.

**Band insulators**

An even number of electrons per unit cell.
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

Metals

An odd number of electrons per unit cell
Sommerfeld-Bloch theory of metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states.
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:

\textit{many-particle}
\textit{quantum entanglement}
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen molecule:
Quantum Entanglement: quantum superposition with more than one particle.
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
\[ |\Psi\rangle \Rightarrow \text{Ground state of entire system,} \]
\[ \rho = |\Psi\rangle \langle \Psi| \]
\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

**Entanglement entropy**

\[ S_E = -\text{Tr} \left( \rho_A \ln \rho_A \right) \]
\[ |\Psi\rangle \Rightarrow \text{Ground state of entire system,} \]
\[ \rho = |\Psi\rangle \langle \Psi| \]

Take \[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B) \]

Then \[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]
\[ = \frac{1}{2} (|\uparrow\rangle_A \langle \uparrow|_A + |\downarrow\rangle_A \langle \downarrow|_A) \]

**Entanglement entropy** \[ S_E = -\text{Tr} (\rho_A \ln \rho_A) \]
\[ = \ln 2 \]
“Complex entangled” states of quantum matter,
not adiabatically connected to independent particle states

Gapped quantum matter
Spin liquids, quantum Hall states

Conformal quantum matter
Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter
Strange metals in high temperature superconductors, Bose metals
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“Complex entangled” states of quantum matter in $d$ spatial dimensions

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Entanglement entropy of a band insulator

Band insulators

An even number of electrons per unit cell
Entanglement entropy of a band insulator

\[ S_E = aP - b\exp(-cP) \]

where \( P \) is the surface area (perimeter) of the boundary between A and B.
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle) \]

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\[
\begin{array}{c}
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\end{array}
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\[ \text{vertex } = \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right) \]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \bullet \bullet = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration
Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration
Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Entanglement in the $\mathbb{Z}_2$ spin liquid

Sum over closed loops: only an even number of links cross the boundary between A and B

\[ S_E = aP - \ln(2) \]

where \( P \) is the surface area (perimeter) of the boundary between A and B.

Entanglement in the $\mathbb{Z}_2$ spin liquid

\[ S_E = aP - \ln(4) \]

where $P$ is the surface area (perimeter) of the boundary between A and B.

Entanglement in the $\mathbb{Z}_2$ spin liquid

\[ S_E = aP - \ln(2) \]

where $P$ is the surface area (perimeter) of the boundary between A and B.

Mott insulator: Kagome antiferromagnet

Strong numerical evidence for a $\mathbb{Z}_2$ spin liquid


Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

Mott insulator: Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012

ZnCu$_3$(OH)$_6$Cl$_2$ (also called Herbertsmithite)
“Complex entangled” states of quantum matter in \( d \) spatial dimensions

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Superfluid-insulator transition


Ultracold $^{87}$Rb atoms - bosons
\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) ; \quad n_i \equiv b_i^\dagger b_i \]
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Tensor network representation of entanglement at quantum critical point

Entanglement entropy obeys $S_E = aP - \gamma$, where $\gamma$ is a shape-dependent universal number associated with the CFT3.
Tensor network representation of entanglement at quantum critical point

$d$-dimensional space

depth of entanglement

Tensor network representation of entanglement at quantum critical point

$A$

d-dimensional space

depth of entanglement

Brian Swingle, arXiv:0905.1317
Tensor network representation of entanglement at quantum critical point

$d$-dimensional space

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317
Key idea: Implement $r$ as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.
For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta t, \quad ds \rightarrow ds$$
This gives the unique metric
\[ ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2) \]

This is the metric of anti-de Sitter space $\text{AdS}_{d+2}$. 
AdS/CFT correspondence

AdS$_4$

$R^{2,1}$

Minkowski

CFT$_3$

$x_i$

$r$
AdS/CFT correspondence

$\text{AdS}_4$

$\mathbb{R}^{2,1}$

Minkowski

CFT3

Friday, November 9, 12
Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside” : i.e. the region is surrounded by an imaginary horizon.

The entropy of this region is bounded by its surface area (Bekenstein-Hawking-’t Hooft-Susskind)

**AdS/CFT correspondence**

AdS$_4$ \quad R$_{2,1}$

Minkowski

Minimal surface area measures entanglement entropy

CFT3

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.
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Brian Swingle, arXiv:0905.1317
Entanglement entropy

$A$-dimensional space

Emergent direction of AdS$_{d+2}$

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317
AdS/CFT correspondence

Computation of minimal surface area yields

\[ S_E = aP - \gamma, \]

where \( \gamma \) is a shape-dependent universal number.

Many-particle quantum entanglement

Holography

Quantum critical points of atoms and electrons
Many-particle quantum entanglement

Holography

CFT3

Quantum critical points of atoms and electrons
Many-particle quantum entanglement

Holography and string theory

Quantum critical points of atoms and electrons

Black holes
Many-particle quantum entanglement

Holography and string theory

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Black holes
String theory

- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...
• A $D$-brane is a $d$-dimensional surface on which strings can end.
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The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.
• A $D$-brane is a $d$-dimensional surface on which strings can end.

• The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.

• In $d = 2$, we obtain strongly-interacting CFT3s. These are "dual" to string theory on anti-de Sitter space: AdS4.
Tensor network representation of entanglement at quantum critical point

Emergent direction of AdS4

Brian Swingle, arXiv:0905.1317
String theory near a D-brane

Emergent direction of AdS4

$d$-dimensional space
Many-particle quantum entanglement

Holography and string theory

Quantum critical points of atoms and electrons

Black holes
\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) \ ; \ n_i \equiv b_i^\dagger b_i \]

Quantum critical point described by a CFT3
\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) ; \quad n_i \equiv b_i^\dagger b_i \]

Quantum critical point described by a CFT3
The Hamiltonian is given by:

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) ; \quad n_i \equiv b_i^\dagger b_i \]

In the context of the diagram:

- The region below the line \( T_{KT} \) is labeled as Superfluid.
- The region above the line \( T_{KT} \) is labeled as Insulator.
- The point \( g_c \) is marked as the Quantum critical point described by a CFT3.
- The region labeled as CFT3 at \( T > 0 \) is highlighted.

Key physical quantities:

- \( g = \frac{U}{t} \)
\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) ; \quad n_i \equiv b_i^\dagger b_i \]

t = \frac{U}{t} 

CFT3 at \( T > 0 \)

Needed: Accurate theory of quantum critical dynamics

Quantum critical point described by a CFT3

\( T_{KT} \)

Superfluid

Insulator

\( g = \frac{U}{t} \)
String theory at non-zero temperatures

A 2+1 dimensional system at its quantum critical point
A 2+1 dimensional system at its quantum critical point

A “horizon”, similar to the surface of a black hole!

String theory at non-zero temperatures
Objects so massive that light is gravitationally bound to them.
Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon

![Diagram showing entangled states across a black hole horizon]
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy).
String theory at non-zero temperatures

A “horizon”, whose temperature and entropy equal those of the quantum critical point

A 2+1 dimensional system at its quantum critical point
String theory at non-zero temperatures

A 2+1 dimensional system at its quantum critical point

Friction of quantum criticality = waves falling into black brane

A “horizon”, whose temperature and entropy equal those of the quantum critical point
A 2+1 dimensional system at its quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

A “horizon”, whose temperature and entropy equal those of the quantum critical point
AdS$_4$ theory of charge transport in a CFT3

\[ \gamma = \frac{1}{12} \]

\[ \gamma = 0 \]

\[ \gamma = -\frac{1}{12} \]


D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247
AdS$_4$ theory of charge transport in a CFT3

\[\frac{\sigma(\omega)}{\sigma(\infty)}\]

\[\gamma = \frac{1}{12}\]

\[\gamma = 0\]

\[\gamma = -\frac{1}{12}\]

Effective theory on AdS$_4$ obtained by a gradient expansion; all parameters fixed by "OPE data" from CFT3.

\[S_{EM} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right],\]

where \(C_{abcd}\) is the Weyl curvature tensor.

Stability and causality constraints restrict \(|\gamma| < 1/12|\).

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Strange metals in high temperature superconductors, Bose metals
Resistivity \( \sim \rho_0 + AT^n \)

\( k_F^d \sim Q \), the fermion density
Fermi liquid

- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$. 
FL
Fermi liquid

- $k_F^d \sim Q$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$. 
\( k_F^d \sim Q \), the fermion density

- Sharp fermionic excitations near Fermi surface with \( \omega \sim |q|^z \), and \( z = 1 \).

- Entropy density \( S \sim T^{(d-\theta)/z} \) with violation of hyperscaling exponent \( \theta = d - 1 \).

- Entanglement entropy
  \[ S_E \sim k_F^{d-1} P \ln P. \]

Resistivity \( \sim \rho_0 + AT^n \)
A *Bose metal*: a compressible phase of bosons which breaks no symmetries.
Bosons with correlated hopping

\[ H = -t \sum_{\langle i,j \rangle} b_{i}^{\dagger} b_{j} + \frac{U}{2} \sum_{i} n_{i}(n_{i} - 1) + w \sum_{ijkl \in \square} b_{i}^{\dagger} b_{k}^{\dagger} b_{j} b_{\ell} \]

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• **Bose metal**: the boson, $b$, fractionalizes into (say) 2 fermions, $f_1$ and $f_2$ ("quarks"), each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "hidden".

$$Q = b^\dagger b$$

$$A_f = \langle Q \rangle$$


S. Sachdev, to appear
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\[
\begin{align*}
&b \rightarrow f_1 f_2 \\
&\text{Gauge invariance:} \\
&f_1(x) \rightarrow f_1(x)e^{i\theta(x)}, \\
&f_2(x) \rightarrow f_2(x)e^{-i\theta(x)}
\end{align*}
\]

S. Sachdev, to appear
In particle physics: Quarks and gauge fields are “fundamental”, and two quarks can bind to form a bosonic meson.
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In condensed matter: The lattice boson is “fundamental”, but it can fractionalize into fermionic quarks and emergent gauge fields.
• $k_F^d \sim Q$, the fermion density

• Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$. 

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NFL Bose metal

• Hidden Fermi surface with $k_F^d \sim Q$. 

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FL Fermi liquid
**Fermi Liquid (FL)**

- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

**Bose Metal (NFL)**

- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

---


**FL Fermi liquid**

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- **NFL Bose metal**
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\begin{itemize}
\item $k_F^d \sim Q$, the fermion density
\item Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
\item Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
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\end{itemize}

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\item Hidden Fermi surface with $k_F^d \sim Q$.
\item Diffuse fermionic excitations with $z = \frac{3}{2}$ to three loops.
\item $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
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\end{itemize}
Logarithmic violation of “area law”: \( S_E \propto (k_F P) \ln(k_F P) \)

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A.

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Logarithmic violation of “area law”: $S_E \propto (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum $k_F$, where $P$ is the perimeter of region A. The coefficient is independent of the shape of A.

Holography
Consider a metric which transforms under rescaling as

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds.$$

Recall: conformal matter has $\theta = 0, \ z = 1$, and the metric is anti-de Sitter
Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The value \( \theta = d - 1 \) reproduces all the essential characteristics of the entropy and entanglement entropy of a Bose metal.


Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The null-energy condition of gravity yields \( z \geq 1 + \theta/d \). In \( d = 2 \), this corresponds to \( z \geq 3/2 \) (recall: field theory yields \( z = 3/2 \)).
Holographic theory of a Bose metal

Hidden Fermi surfaces of "quarks"

\[ r = 1 \]

Electric flux

\[ \mathcal{E}_r = Q \]

Fully fractionalized state has all the electric flux exiting to the horizon at \( r = \infty \)
Conclusions

Realizations of many-particle entanglement:
$\mathbb{Z}_2$ spin liquids and conformal quantum critical points
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”