Quantum criticality and gauge-gravity duality

Flato Lectures, Ben Gurion University, March 10, 2011

Talk online: sachdev.physics.harvard.edu
Outline

1. Coupled dimer antiferromagnets
   Quantum criticality and conformal field theories

2. The AdS/CFT correspondence
   Quantum criticality and black holes

3. Quantum transport and Einstein-Maxwell theory on AdS$_4$

4. Compressible quantum matter
   Fermi surfaces
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Quantum antiferromagnets

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ [S_{i\alpha}, S_{j\beta}] = i\delta_{ij} \epsilon_{\alpha\beta\gamma} S_{i\gamma} \]

\[ \alpha = x, y, z \]

Spin \( S = 1/2 \), \( \vec{S}_i^2 = S(S + 1) \)
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in Néel state.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right) \]
\[ \ell = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

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Spin $S = 1$
“triplon”
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves

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Excitation spectrum in the Néel phase

Spin waves
Observation of $3 \rightarrow 2$ low energy modes, emergence of new Higgs particle in the Néel phase.

Quantum critical point with non-local entanglement in spin wavefunction

\[ \lambda_c = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Conformal field theory in 2+1 dimensions (CFT3)

\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
\[ Z = \int D\phi(r, \tau) \exp(-S) \]

\[ S = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla_r \phi)^2 + (\lambda - \lambda_c) \phi^2 + u (\phi^2)^2 \right] \]

Classical spin waves

Quantum critical

Dilute triplon gas

Pressure in TlCuCl$_3$

Classical spin waves

Dilute triplon gas

CFT3 at $T>0$

Quantum critical

Neel order

Pressure in TlCuCl$_3$

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Classical spin waves

Dilute triplon gas

Quantum critical Neel order

CFT3 at $T > 0$

Strongly coupled dynamics and transport with no particle/wave interpretation, and relaxation thermal equilibration times are universally proportional to $\hbar/k_B T$

Neel order

Pressure in TlCuCl$_3$

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Field theories in $D$ spacetime dimensions are characterized by couplings $g$ which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where $u$ is the energy scale. The RG equation is local in energy scale, i.e. the RHS does not depend upon $u$. 
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**Key idea:** Implement $u$ as an extra dimension, and map to a local theory in $D + 1$ dimensions.
At the RG fixed point, $\beta(g) = 0$, the $D$ dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b , \quad u \rightarrow b u$$
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This is an invariance of the metric of the theory in $D + 1$ dimensions. The unique solution is

$$ds^2 = \left( \frac{u}{L} \right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2 / u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space $\text{AdS}_{D+1}$, and $L$ is the AdS radius.
Figure 1: The extra (‘radial’) dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter \( z \). The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

Maldacena, Gubser, Klebanov, Polyakov, Witten
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3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality
**AdS/CFT correspondence**

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Kovtun, Policastro, Son

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Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

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Strong coupling problem:
General solution of spin and magneto-thermo-electric transport in quantum critical region.


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Classical spin waves

Dilute triplon gas

Quantum critical

CFT3 at $T>0$

Pressure in TlCuCl$_3$
Quantum critical transport

Quantum "perfect fluid" with shortest possible relaxation time, $\tau_R$

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

Quantum critical transport

Transport co-efficients not determined by collision rate, but by universal constants of nature

Spin/charge conductivity

\[ \sigma = \frac{Q^2}{\hbar} \times [\text{Universal constant } \mathcal{O}(1) ] \]

(Q is the quantum of spin/charge)

Quantum critical transport

Transport co-efficients not determined by collision rate, but by universal constants of nature

Momentum transport

\[ \eta \equiv \frac{\text{viscosity}}{s} = \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)] \]
Structure of conductivity for complex frequencies
Structure of conductivity for complex frequencies

\[ \omega = i2\pi nk_B T/\hbar, \]

\( n \) integer:

computable in perturbative analysis of conformal field theory about free field theory
Structure of conductivity for complex frequencies

\[ \omega \ll k_B T / \hbar, \]

hydrodynamic regime: requires computation in dual gravity theory
Structure of conductivity for complex frequencies

$\omega \ll k_B T/\hbar$, hydrodynamic regime: requires computation in dual gravity theory
Boltzmann theory of quantum critical transport

\[ \sigma = \frac{Q^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \rightarrow \text{a universal function} \]

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Collisionless "critical" dissipation

Boltzmann theory of quantum critical transport

\[ \sigma = \frac{Q^2}{h} \Sigma \left( \frac{\hbar \omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function} \]

Collision-dominated hydrodynamics

AdS$_4$ theory of strongly interacting “perfect fluids”

To leading order in a gradient expansion, an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamicstion on AdS$_4$

\[ S_{EM} = \frac{1}{g_4^2} \int d^4 x \sqrt{-g} \left[ -\frac{1}{4} F_{ab} F^{ab} \right]. \]

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\]

This theory is self-dual under $F_{ab} \rightarrow \epsilon_{abcd} F^{cd}$, and this leads to some artifacts in the properties of the CFT3

AdS$_4$ theory of strongly interacting “perfect fluids”

To leading order in a gradient expansion, an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$

We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only one dimensionless constant $\gamma$ ($L$ is the radius of AdS$_4$):

\[ S = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right], \]

where $C_{abcd}$ is the Weyl curvature tensor.

*Stability and causality constraints restrict* $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
AdS$_4$ theory of strongly interacting “perfect fluids”

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Note: exact determination of the transport co-efficient of an interacting many-body quantum system!

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AdS$_4$ theory of strongly interacting “perfect fluids”

Note: exact determination of the transport co-efficient of an interacting many-body quantum system!
Also note: no diffusion and hydrodynamics for CFT2 and AdS$_3$.

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
AdS$_4$ theory of strongly interacting “perfect fluids”

- Stability constraints on the effective theory allow only a limited $\omega$-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations.

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
AdS$_4$ theory of strongly interacting “perfect fluids”

The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
AdS\(_4\) theory of strongly interacting “perfect fluids”

\[ F_{ab} \rightarrow \epsilon_{abcd}F^{cd} \] duality of theory on AdS\(_4\)
maps onto particle-vortex duality of CFT\(_3\)

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443
Frequency dependency of integer quantum Hall effect

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui,

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Graphene

Semi-metal with massless Dirac fermions

Brillouin zone
Turn on a chemical potential on a CFT
Turn on a chemical potential on a CFT

Electron

Fermi surface

\[ \mu > 0 \]
Compressible quantum matter

- Consider a continuum quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$. 
Compressible quantum matter

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- We are interested in zero temperature phases where \( \langle Q \rangle \) varies smoothly as a function of any external parameter \( \mu \) (the “chemical potential”). For simplicity, we assume \( \mu \) couples linearly to \( Q \).
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We will also restrict our attention to phases where this global U(1) symmetry is not spontaneously broken, and translational symmetry is preserved.
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There are only a few established examples of such phases in condensed matter physics. However, they appear naturally as duals of gravitational theories, and we want to interpret them in the gauge theory.
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All known examples of such phases have a Fermi Surface

(even in systems with only bosons in the Hamiltonian)
The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $Q$.

\[ G_{\text{fermion}}^{-1}(k = k_F, \omega = 0) = 0. \]

Luttinger relation: The total “volume (area)” $A$ enclosed by Fermi surfaces of fermions carrying charge $Q$ is equal to $\langle Q \rangle$. This is a key constraint which allows extrapolation from weak to strong coupling.
• $\text{U}(N) \times \text{U}(N)$ gauge field.

• $4N^2$ Weyl fermions carrying fundamental charges of $\text{U}(N) \times \text{U}(N) \times \text{SU}(4)_R$.

• $4N^2$ complex bosons carrying fundamental charges of $\text{U}(N) \times \text{U}(N) \times \text{SU}(4)_R$.

• $\mathcal{N} = 6$ supersymmetry
Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

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- $\mathcal{N} = 6$ supersymmetry

- Add a chemical potential $\mu$ coupling to a global $SU(4)_R$ charge $Q$.

Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance
Phases of ABJM-like theories

Fermi liquid (FL) of gauge-neutral particles, $c$, which carry 2 units of $Q$ charge.

$$2A_c = \langle Q \rangle$$

Fermi liquid (FL) of gauge-neutral particles

Gauge theory is in confining phase

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**Phases of ABJM-like theories**

Fractionalized Fermi liquid (FL*)

Gauge theory is in deconfined phase

Fermi surface of gauge charged particles, $f$, which quench gauge forces and lead to deconfinement

$$2A_c + 2A_f = \langle Q \rangle$$

Fractionalized Fermi liquid (FL*)

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Phases of ABJM-like theories

Fractionalized Fermi liquid (FL*)

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\[ 2A_c + 2A_f = \langle Q \rangle \]
Phases of ABJM-like theories

Fermi surface of gauge charged particles, $f$, which quench gauge forces and lead to deconfinement.

Fractionalized Fermi liquid (FL*)

Gauge theory is in deconfined phase

$$2A_c + 2A_f = \langle Q \rangle$$

Claim: this is the phase underlying recent holographic theories of compressible metallic states. However, a number of artifacts appear in the classical gravity approximation.
Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry

The CFT is dual to a gravity theory on AdS$_4 \times S^7$
Gauge-gravity duality

- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry
- Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry)

The CFT is dual to a gravity theory on AdS$_4 \times S^7$
- In the Einstein-Maxwell theory, the chemical potential leads at $T=0$ to an extremal Reissner-Nordstrom black hole in the AdS$_4$ spacetime.
Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry
Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry)

The CFT is dual to a gravity theory on $\text{AdS}_4 \times S^7$
In the Einstein-Maxwell theory, the chemical potential leads at $T=0$ to an extremal Reissner-Nordstrom black hole in the AdS$_4$ spacetime.
The RN black hole describes compressible quantum matter with Fermi surfaces.

Compressible quantum matter is characterized by Fermi surfaces.

Summary
Compressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)
Summary

Compressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Fermi liquids are everywhere.
Compressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Fermi liquids are everywhere.

There is evidence that FL* phases have been recently been observed in some intermetallic compounds. The FL* and related phases are attractive candidates for “strange metals” in the higher temperature superconductors
Compressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Gauge-gravity duality is a very promising approach to solving strong-coupling problems associated with FL*-like phases.

FL* and related phases are attractive candidates for "strange metals" in the higher temperature superconductors.
Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle picture.
Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density.