Quantum phase transition in an atomic Bose gas with Feshbach resonances

M.W.J. Romans (Utrecht)
R.A. Duine (Utrecht)
S. Sachdev (Yale)
H.T.C. Stoof (Utrecht)

cond-mat/0312446

See also L. Radzihovsky, J. Park, P. Weichman, cond-mat/0312237.
Exotic order in an atomic Bose gas with Feshbach resonances

M.W.J. Romans (Utrecht)
R.A. Duine (Utrecht)
S. Sachdev (Yale)
H.T.C. Stoof (Utrecht)

cond-mat/0312446

See also L. Radzihovsky, J. Park , P. Weichman, cond-mat/0312237.
• When are two quantum states (of an infinite system) really distinct?
• Can we connect two states by adiabatic evolution of a coupling constant?
• When are two quantum states (of an infinite system) really distinct?
• Can we connect two states by adiabatic evolution of a coupling constant?

If not, then the two states are separated by a quantum phase transition(s)
• When are two quantum states (of an infinite system) really distinct?
• Can we connect two states by adiabatic evolution of a coupling constant?

If not, then the two states are separated by a quantum phase transition(s)

A quantum phase transition must occur if the states have

• distinct “conventional” order parameters/broken symmetry
• distinct “quantum/topological/exotic” order with distinct quantum numbers of excitations
Fermi gas near a Feshbach resonance

BEC of molecules

BCS state of paired fermions

detuning
Fermi gas near a Feshbach resonance

\[ \Psi = \text{Antisym} \prod_{\text{pairs } i,j} \phi_m \left( r_i^{\uparrow} - r_j^{\downarrow} \right) \]

BEC of molecules

BCS state of paired fermions

detuning

Smooth crossover between two limits;
Different theories can be judged only by their quantitative accuracy.
Bose gas near a Feshbach resonance

BEC of molecules

BEC of atoms

detuning
Bose gas near a Feshbach resonance

\[ \Psi = \text{Sym} \left[ \prod_{\text{pairs } i,j} \phi_m (r_i - r_j) \right] \]

\[ \Psi = \prod_{\text{atoms } i} \phi_a (r_i) \]

BEC of molecules

BEC of atoms

detuning
Bose gas near a Feshbach resonance

\[ \Psi = \text{Sym} \left[ \prod_{\text{pairs } i,j} \phi_m(r_i - r_j) \right] \]

\[ \Psi = \prod_{\text{atoms } i} \phi_a(r_i) \]

**Bose gas near a Feshbach resonance**

\[ \Psi = \text{Sym} \left[ \prod_{\text{pairs } i,j} \phi_m \left( r_i - r_j \right) \right] \]

\[ \Psi = \prod_{\text{atoms } i} \phi_a \left( r_i \right) \]


We (Romans *et al*. cond-mat/0312446 and Radzihovsky *et al*. cond-mat/0312337) argued for the necessity of a quantum phase transition between two distinct superfluids which are distinguished by an Ising quantum order.
Why are the two superfluids distinct?

Vortices in a molecular superfluid
Why are the two superfluids distinct?

\[ \phi_m \]
\[ \frac{h}{2e} \]
\[ \phi_m e^{2i\pi} \]
\[ \frac{h}{e} \]

Vortices in a molecular superfluid
Why are the two superfluids distinct?

Vortices in a molecular superfluid
Why are the two superfluids distinct?

Vortices in an atomic superfluid
Why are the two superfluids distinct?

Vortices in an atomic superfluid
Why are the two superfluids distinct?

Vortices in an atomic superfluid
Why are the two superfluids distinct?

Half vortices are confined in pairs by a "branch cut" which costs a finite energy per unit length.

Vortices in an atomic superfluid
Bose gas near a Feshbach resonance

Half-vortices
free

Half-vortices
confined

BEC of molecules

BEC of atoms

Confinement-deconfinement transition in the 3d Ising universality class at $T>0$
Bose gas near a Feshbach resonance

\[ H = \int dx \psi_m^\dagger(x) \left[ -\frac{\hbar^2 \nabla^2}{4m} + \delta - 2\mu \right] \psi_m(x) + \int dx \psi_a^\dagger(x) \left[ -\frac{\hbar^2 \nabla^2}{2m} - \mu + \frac{T_{bg}}{2} \psi_a^\dagger(x) \psi_a(x) \right] \psi_a(x) \\
+ \int dx g [\psi_m^\dagger(x) \psi_a(x) \psi_a(x) + \psi_a(x) \psi_a^\dagger(x) \psi_m(x)] \\
+ \int dx \frac{T_{mm}}{2} \psi_m^\dagger(x) \psi_m(x) \psi_m(x) \psi_m(x) + \int dx T_{am} \psi_m^\dagger(x) \psi_a^\dagger(x) \psi_a(x) \psi_m(x) \]

\[ \eta^2 = \frac{g^4 m^3}{4\pi^2 \hbar^6} \]

Note: transition occurs at negative detuning
Bose gas near a Feshbach resonance

Detecting the Ising transition:

• Observation of vortices
• Non-analytic increase in atomic density
• “Critical slowing down” : strong damping of collective modes and decay of atomic density fluctuations into critical fluctuations