

# Quantum phase transition in an atomic Bose gas with Feshbach resonances

M.W.J. Romans (Utrecht)

R.A. Duine (Utrecht)

S. Sachdev (Yale)

H.T.C. Stoof (Utrecht)

cond-mat/0312446



See also L. Radzihovsky, J. Park ,  
P. Weichman, cond-mat/0312237.



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- Can we connect two states by adiabatic evolution of a coupling constant ?

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If not, then the two states are separated by a quantum phase transition(s)

A quantum phase transition must occur if the states have

- distinct “conventional” order parameters/broken symmetry
- distinct “quantum/topological/exotic” order with distinct quantum numbers of excitations

# Fermi gas near a Feshbach resonance

BEC of  
molecules

BCS state of  
paired fermions



detuning

# Fermi gas near a Feshbach resonance

$$\Psi = \text{Antisym} \left[ \prod_{\text{pairs } i, j} \phi_m (r_{i\uparrow} - r_{j\downarrow}) \right]$$

BEC of  
molecules

BCS state of  
paired fermions

detuning

Smooth crossover between two limits;  
Different theories can be judged only by their quantitative accuracy.

# Bose gas near a Feshbach resonance

BEC of  
molecules

BEC of atoms



detuning



# Bose gas near a Feshbach resonance

$$\Psi = \text{Sym} \left[ \prod_{\text{pairs } i, j} \phi_m (r_i - r_j) \right]$$

BEC of  
molecules

$$\Psi = \prod_{\text{atoms } i} \phi_a (r_i)$$

BEC of atoms

detuning

## Bose gas near a Feshbach resonance

$$\Psi = \text{Sym} \left[ \prod_{\text{pairs } i,j} \phi_m(r_i - r_j) \right]$$

BEC of  
molecules

$$\Psi = \prod_{\text{atoms } i} \phi_a(r_i)$$

BEC of atoms

detuning

P. Nozieres and D. Saint James, *J. Physique* **43**, 1133 (1982) found a sharp transition between two superfluids in a variational calculation

## Bose gas near a Feshbach resonance

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We (Romans *et al.* cond-mat/0312446 and Radzihovsky *et al.* cond-mat/0312337) argued for the necessity of a quantum phase transition between two distinct superfluids which are distinguished by an Ising quantum order.

# Why are the two superfluids distinct ?

$$\frac{h}{2e}$$

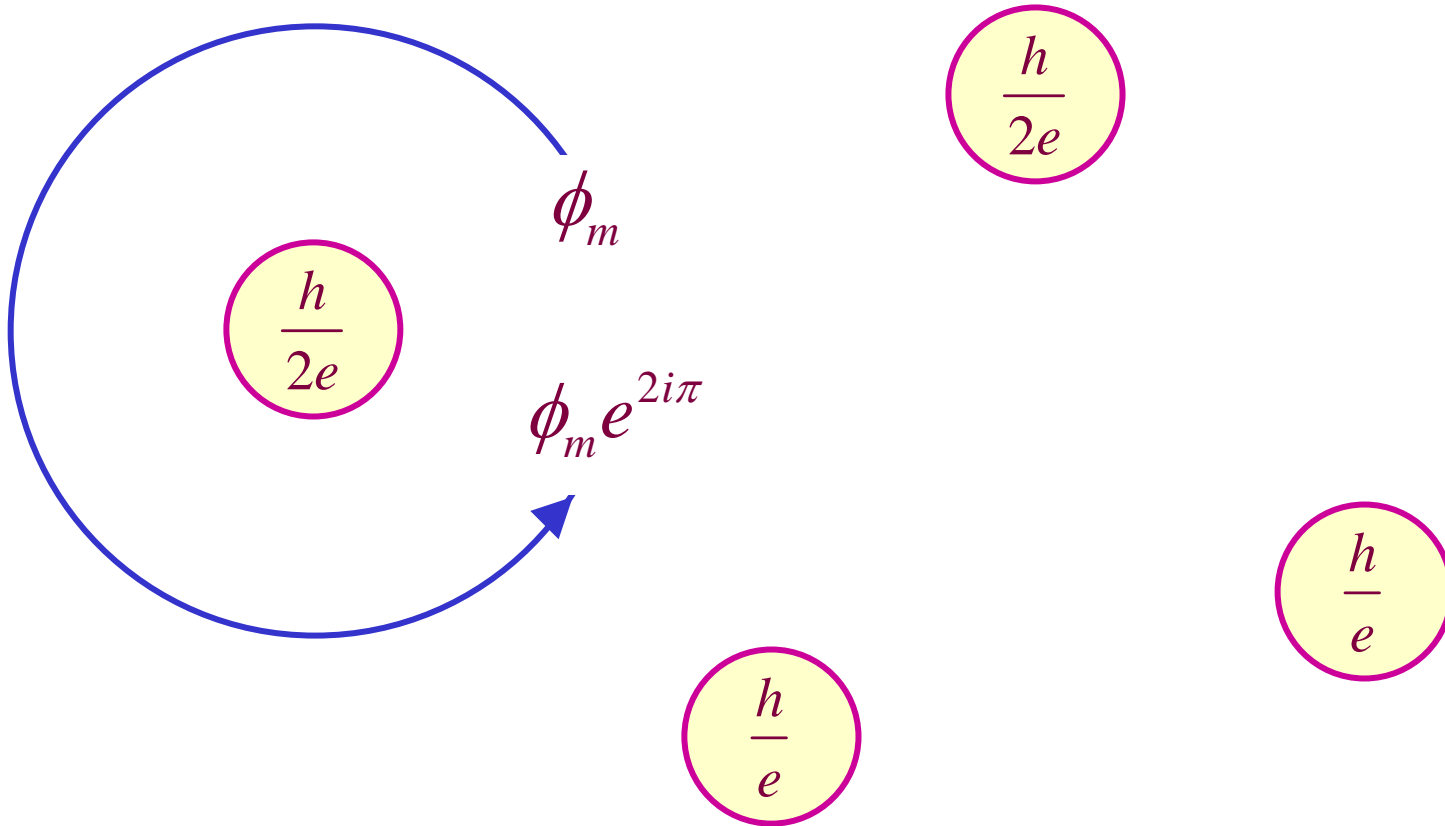
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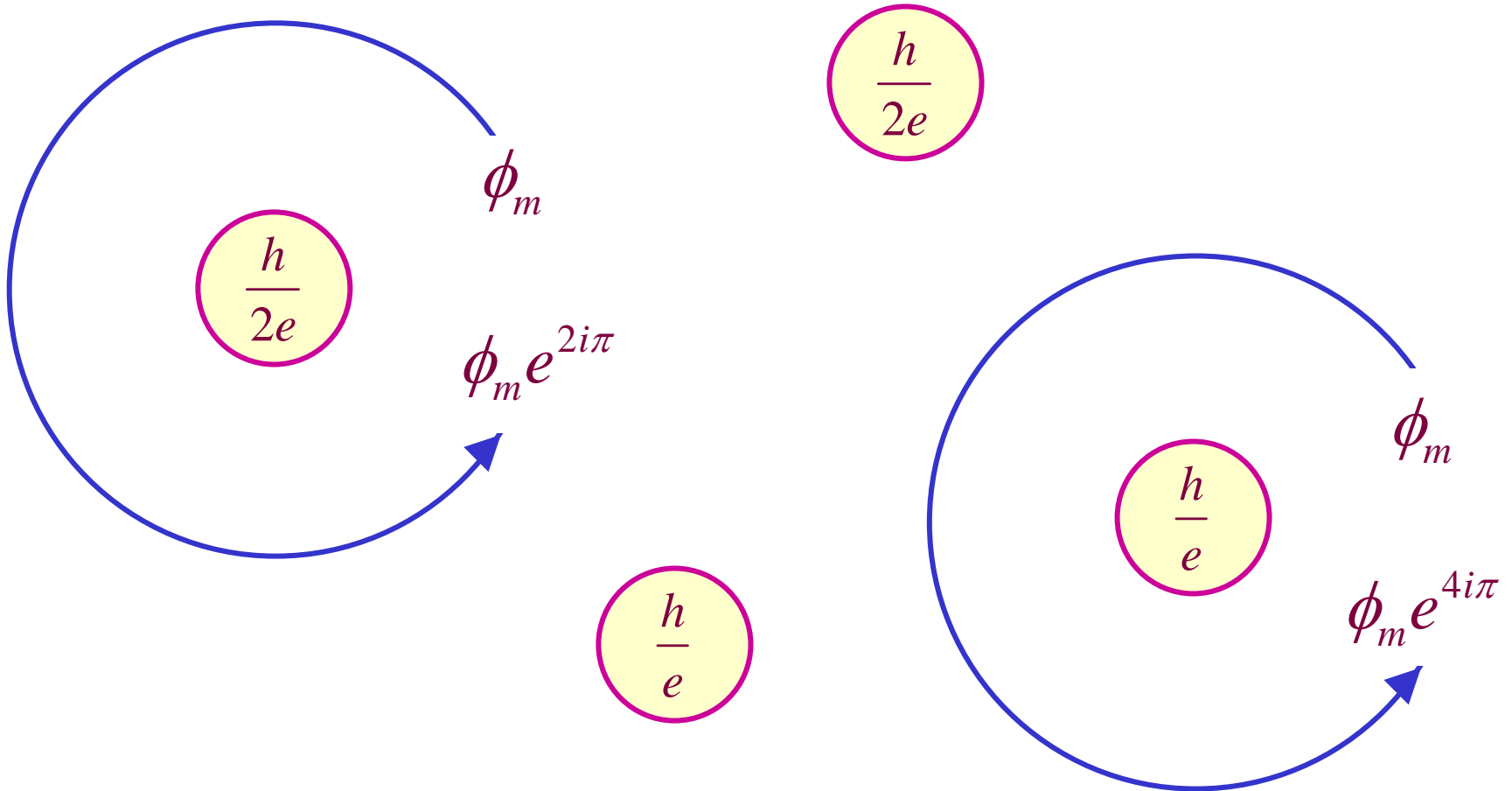
Vortices in a molecular superfluid

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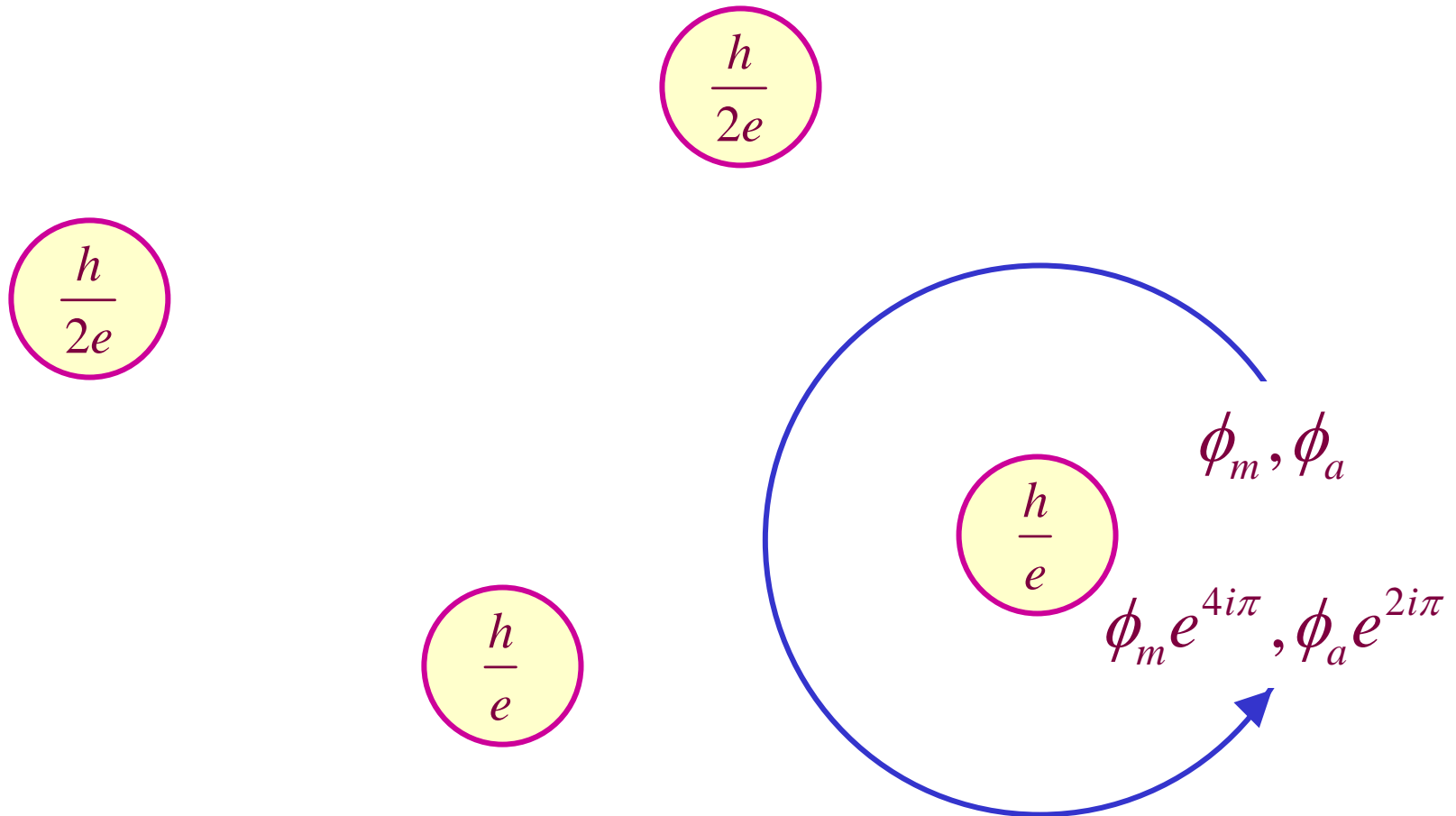
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Vortices in an atomic superfluid

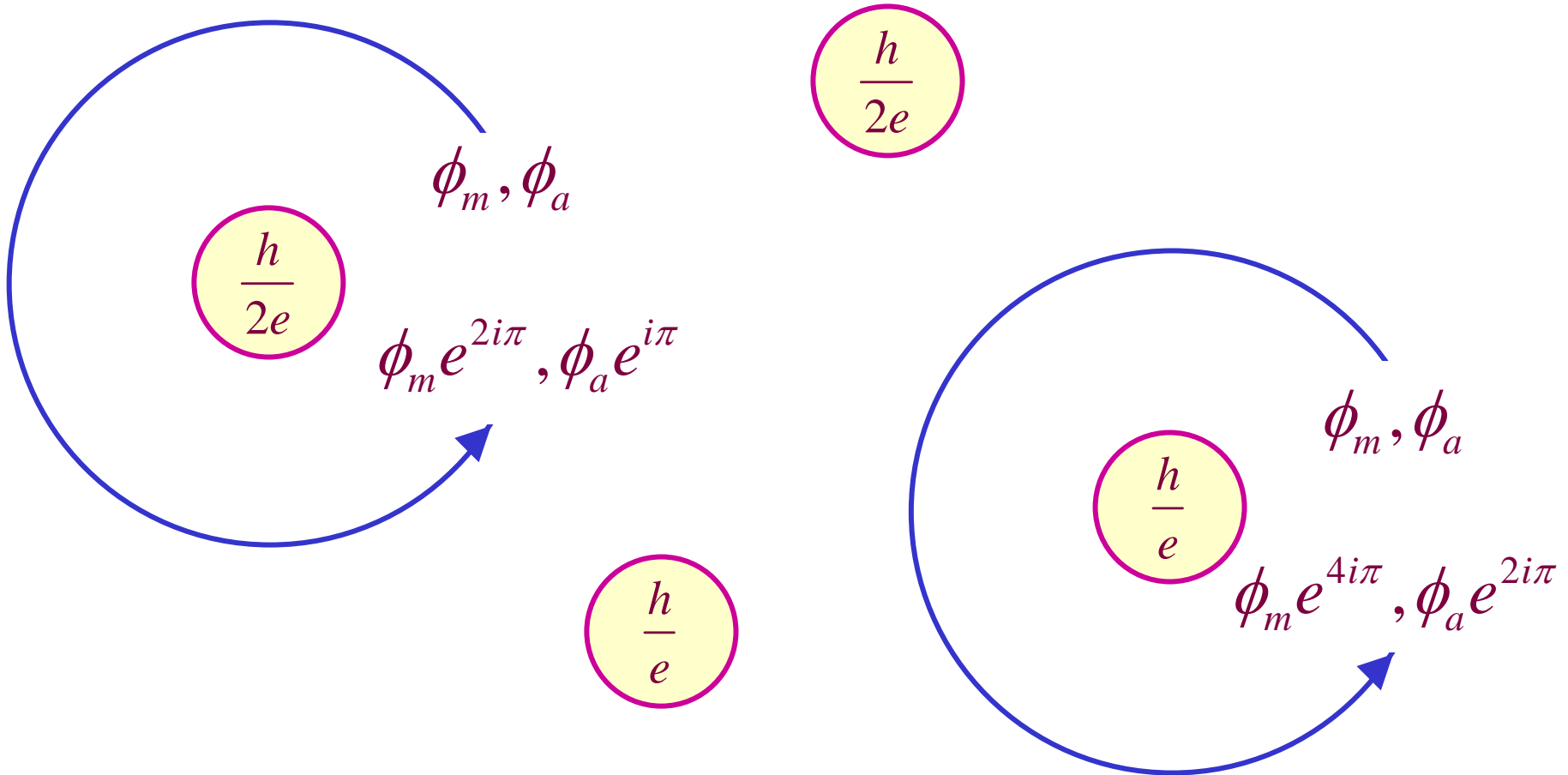
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Vortices in an atomic superfluid



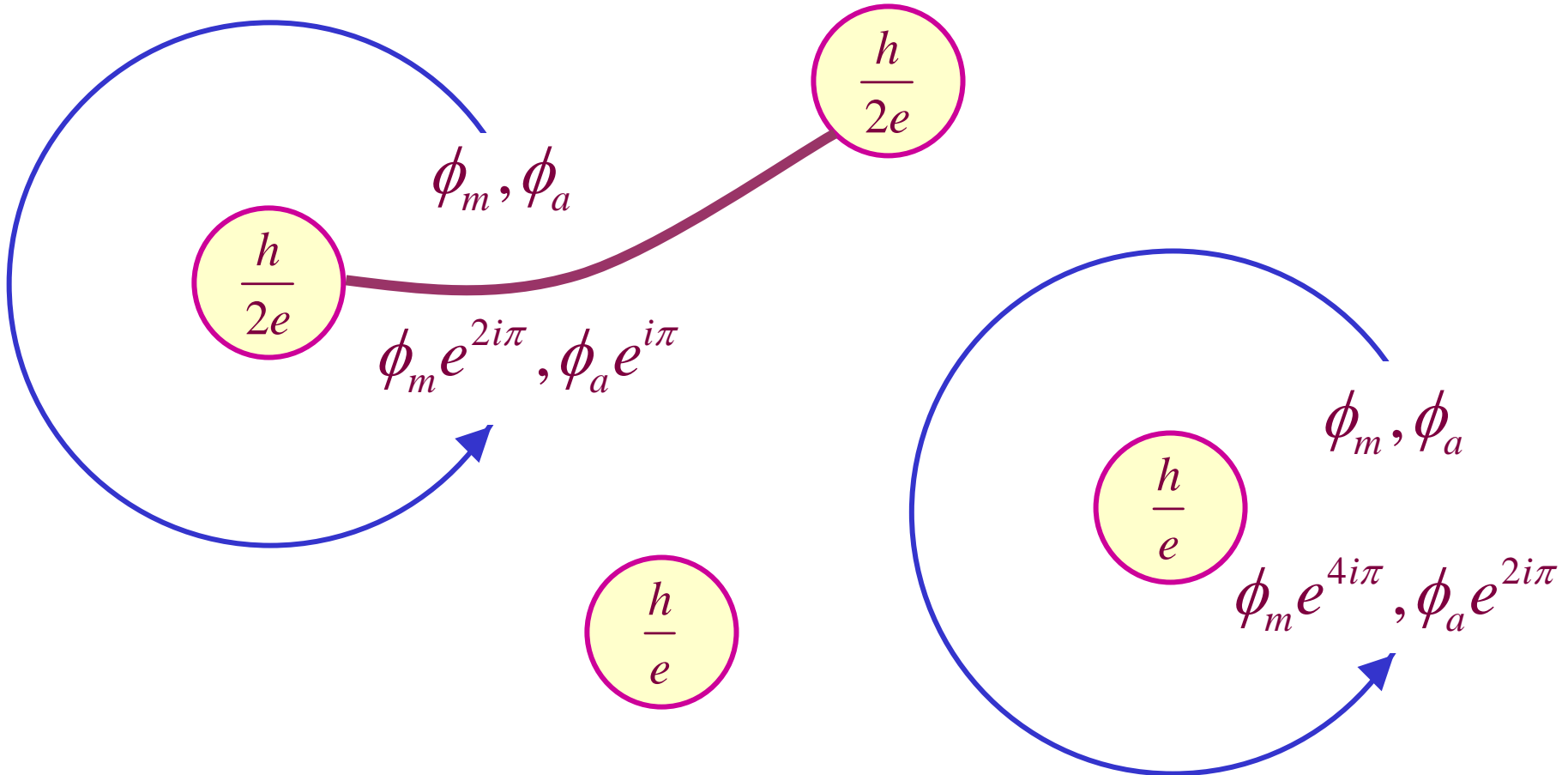
# Why are the two superfluids distinct ?



**Vortices in an atomic superfluid**

# Why are the two superfluids distinct ?

Half vortices are confined in pairs by a ``branch cut'' which costs a finite energy per unit length



**Vortices in an atomic superfluid**

# Bose gas near a Feshbach resonance

Half-vortices  
free

Half-vortices  
confined

BEC of  
molecules

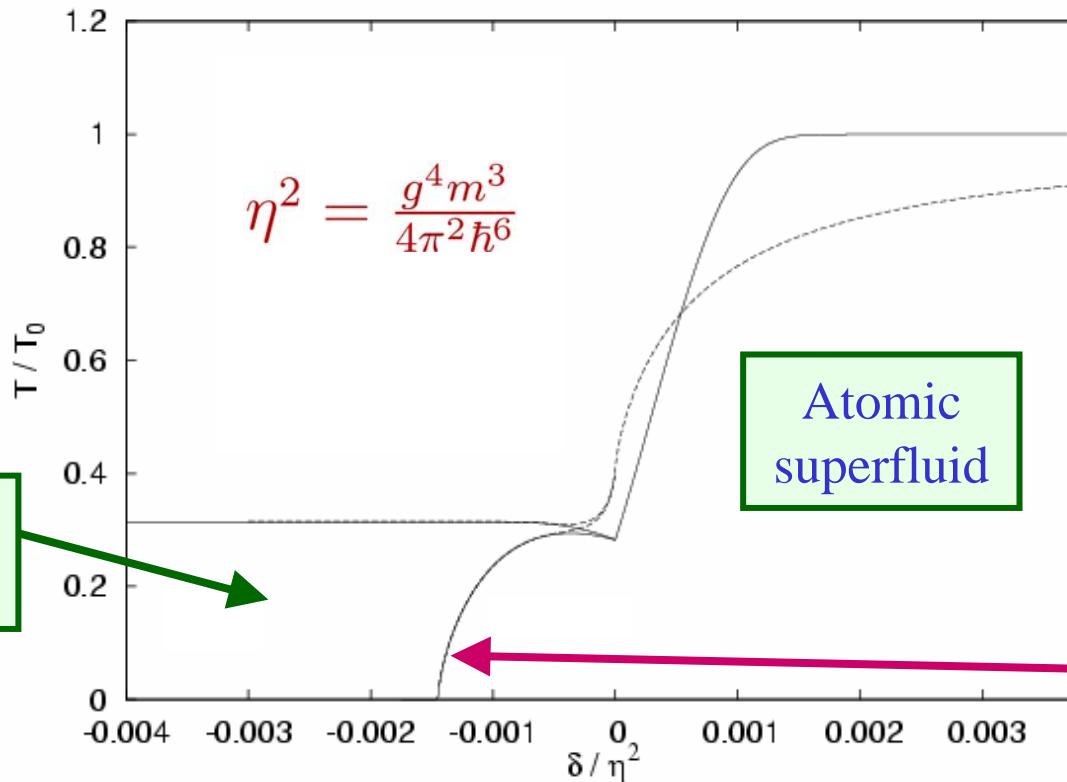
BEC of atoms

detuning

Confinement-deconfinement transition in  
the 3d Ising universality class at  $T > 0$

# Bose gas near a Feshbach resonance

$$\begin{aligned}
 H = & \int dx \psi_m^\dagger(x) \left[ -\frac{\hbar^2 \nabla^2}{4m} + \delta - 2\mu \right] \psi_m(x) + \int dx \psi_a^\dagger(x) \left[ -\frac{\hbar^2 \nabla^2}{2m} - \mu + \frac{T_{\text{bg}}}{2} \psi_a^\dagger(x) \psi_a(x) \right] \psi_a(x) \\
 & + \int dx g \left[ \psi_m^\dagger(x) \psi_a(x) \psi_a(x) + \psi_a^\dagger(x) \psi_a^\dagger(x) \psi_m(x) \right] \\
 & + \int dx \frac{T_{\text{mm}}}{2} \psi_m^\dagger(x) \psi_m^\dagger(x) \psi_m(x) \psi_m(x) + \int dx T_{\text{am}} \psi_m^\dagger(x) \psi_a^\dagger(x) \psi_a(x) \psi_m(x)
 \end{aligned}$$



Note: transition occurs at negative detuning

Molecular superfluid

Atomic superfluid

Ising transition

## Bose gas near a Feshbach resonance

Detecting the Ising transition:

- Observation of vortices
- Non-analytic increase in atomic density
- “Critical slowing down” : strong damping of collective modes and decay of atomic density fluctuations into critical fluctuations