Quantum criticality in insulators, metals and superconductors

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Ying Zhang (Maryland)

Colloquium article in *Reviews of Modern Physics*, July 2003, cond-mat/0211005.

Talk online:
Google Sachdev

$T=0$

SDW

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$$

Collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 = 0$

Non-collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 \neq 0$

$$\langle \boldsymbol{S}_j \rangle = 0$$

Pressure,
carrier concentration,....

Quantum critical point

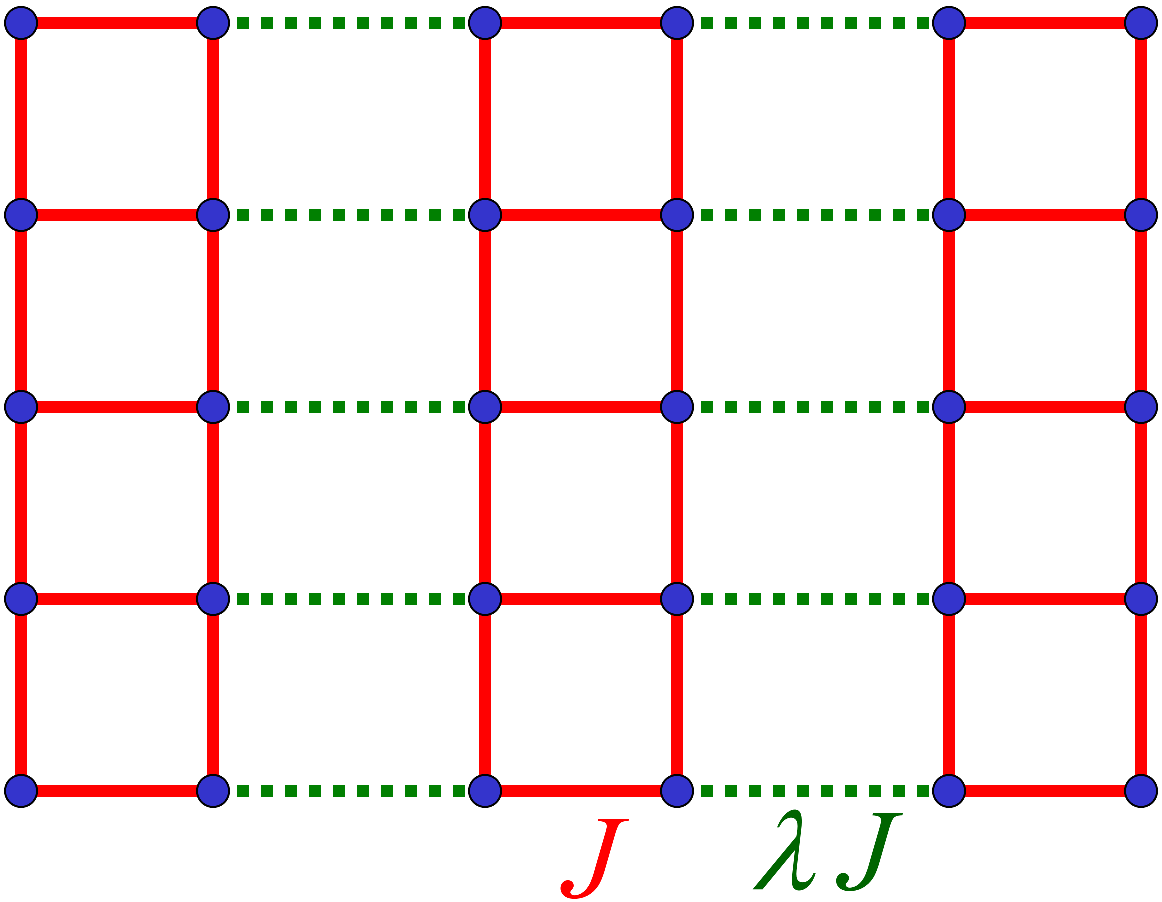
States on both sides of critical point
could be either (A) Insulators
(B) Metals
(C) Superconductors

(A) Insulators

Coupled ladder antiferromagnet

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev*. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev*. B **65**, 014407 (2002).

S=1/2 spins on coupled 2-leg ladders



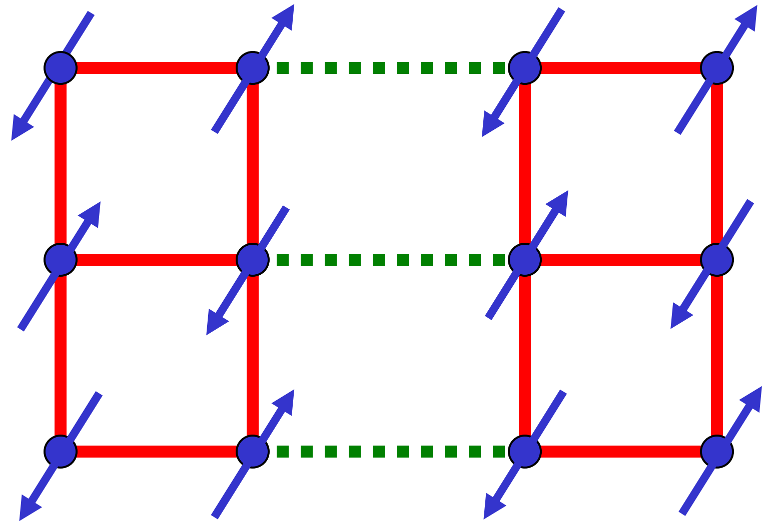
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



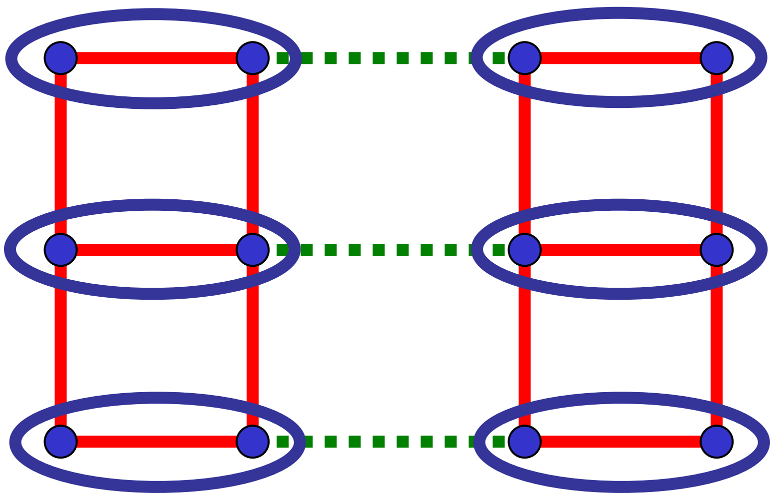
Ground state has long-range collinear magnetic (Neel) order

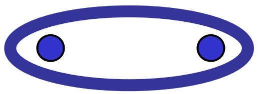
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\quad \varepsilon_p = \sqrt{c_x^{\,2} p_x^{\,2} + c_y^{\,2} p_y^{\,2}}$

$\boxed{\lambda \text{ close to } 0}$

Weakly coupled ladders



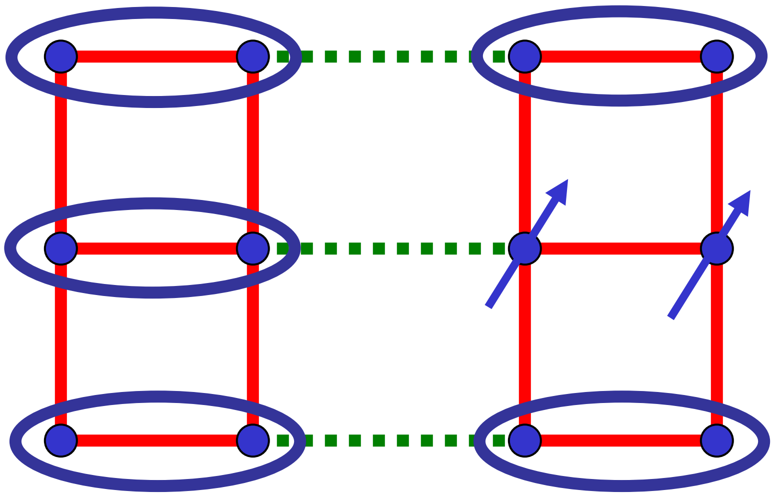

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

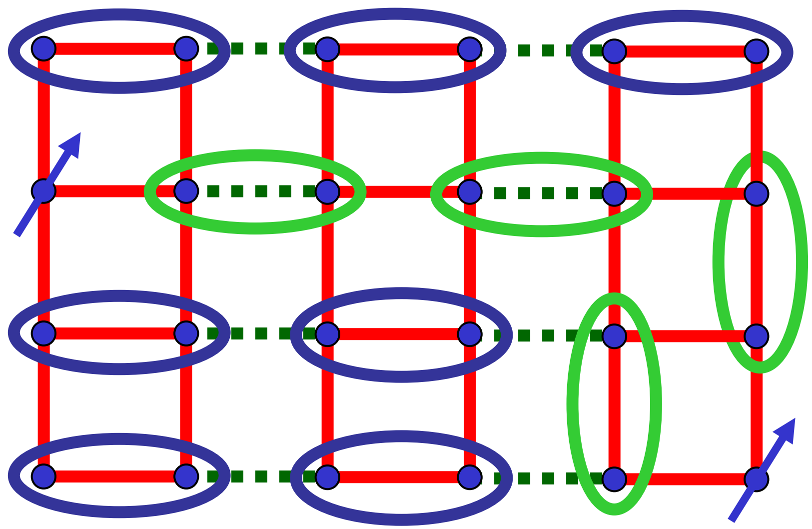
λ close to 0

Excitations



Excitation: S=1 ***exciton***

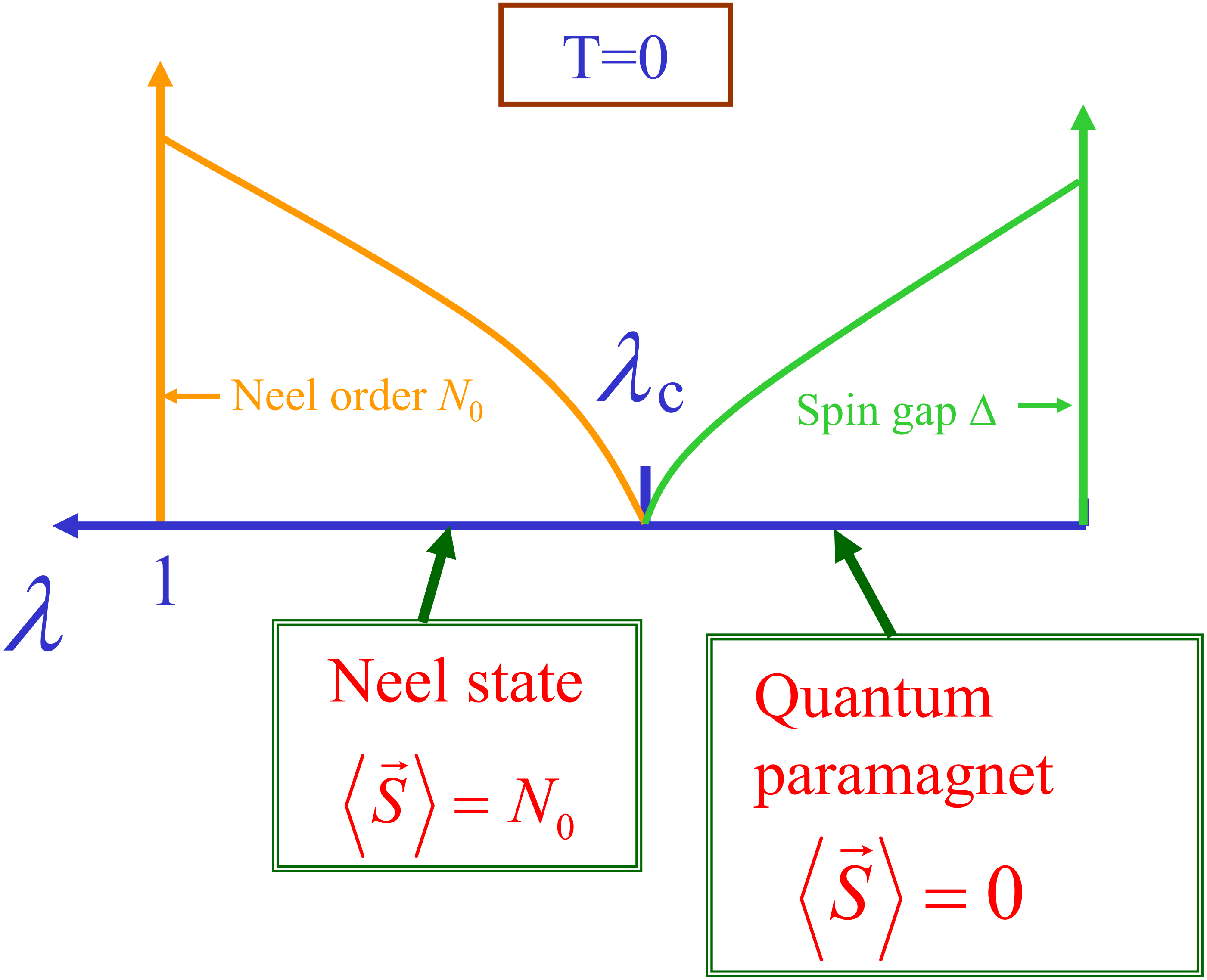
(spin collective mode, "triplon")



S=1/2 spinons are ***confined***
by a linear potential.

Energy dispersion away from wavevector $\vec{K}$

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

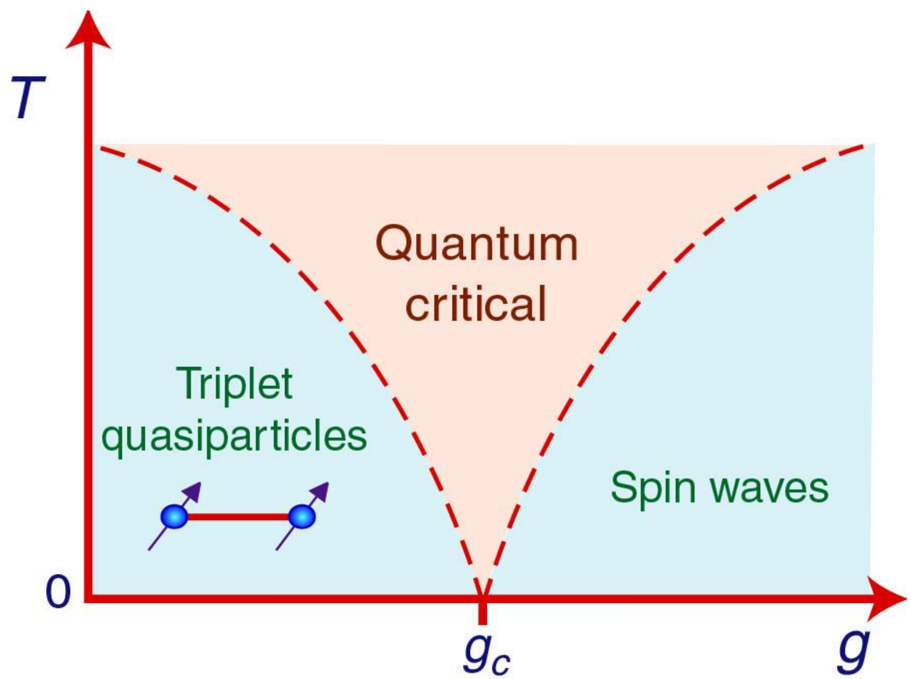

T=0
Neel order N_0
λ_c
Spin gap Δ
λ
1
Neel state
$\langle \vec{S} \rangle = N_0$
Quantum paramagnet
$\langle \vec{S} \rangle = 0$

Field theory for quantum criticality

λ close to λ_c : use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + \left(\lambda_c - \lambda\right)\phi_\alpha^2 \right) + \frac{u}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

$\phi_\alpha \rightarrow$ 3-component antiferromagnetic order parameter



Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega / k_B T$

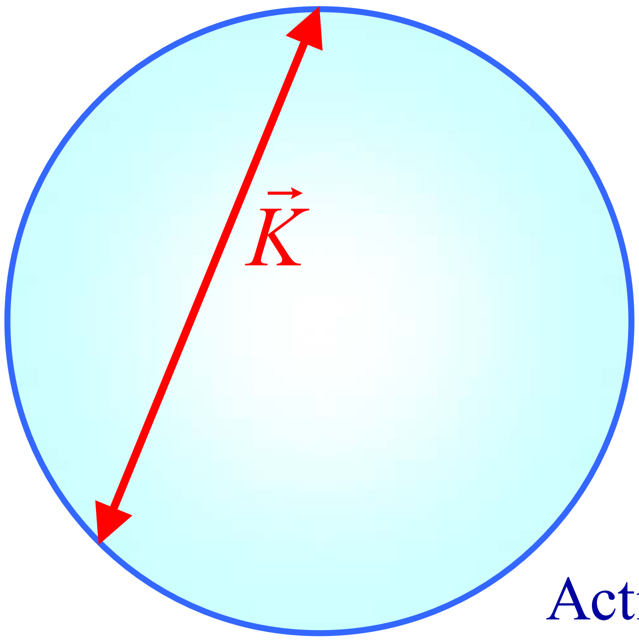
$$\chi(\omega, T) = T^\eta g\left(\hbar\omega / k_B T\right)$$

Exciton scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

(B) Metals

Spin density wave order in the presence of a Fermi surface



Low energy "paramagnon" excitations near the Fermi surface

Damping by fermionic quasiparticles leads to

Action: $$S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left| \vec{\phi}(q,\omega) \right|^2 \left(q^2 + |\omega| + \Gamma(\delta, T) \right)$$

M. T. Beal-Monod and K. Maki, *Phys. Rev. Lett.* **34**, 1461 (1975); J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).

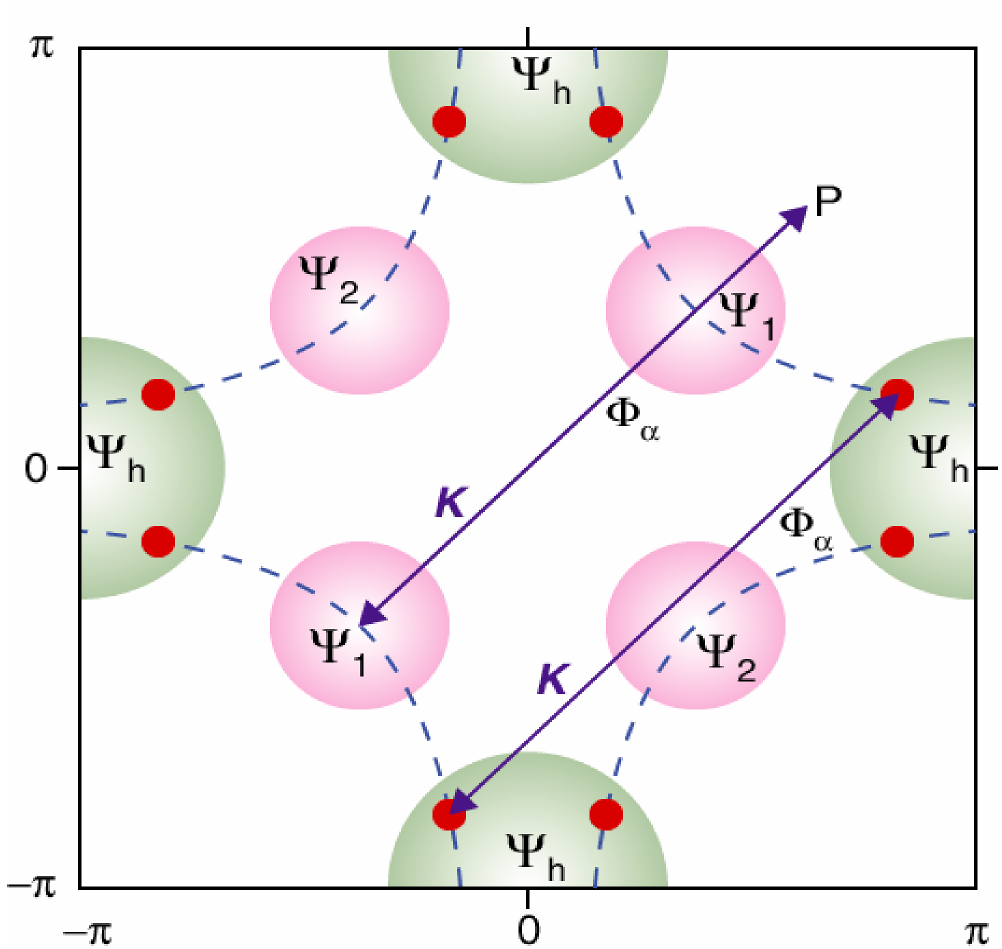
Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal *corrections* to scaling, generated by $\vec{\phi}^4$ term.

J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985);
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985); A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

(C) Superconductors

Co-existence of superconductivity and spin-density wave order



If $\vec{K}$ does not exactly connect two nodal points, critical theory is as in an insulator

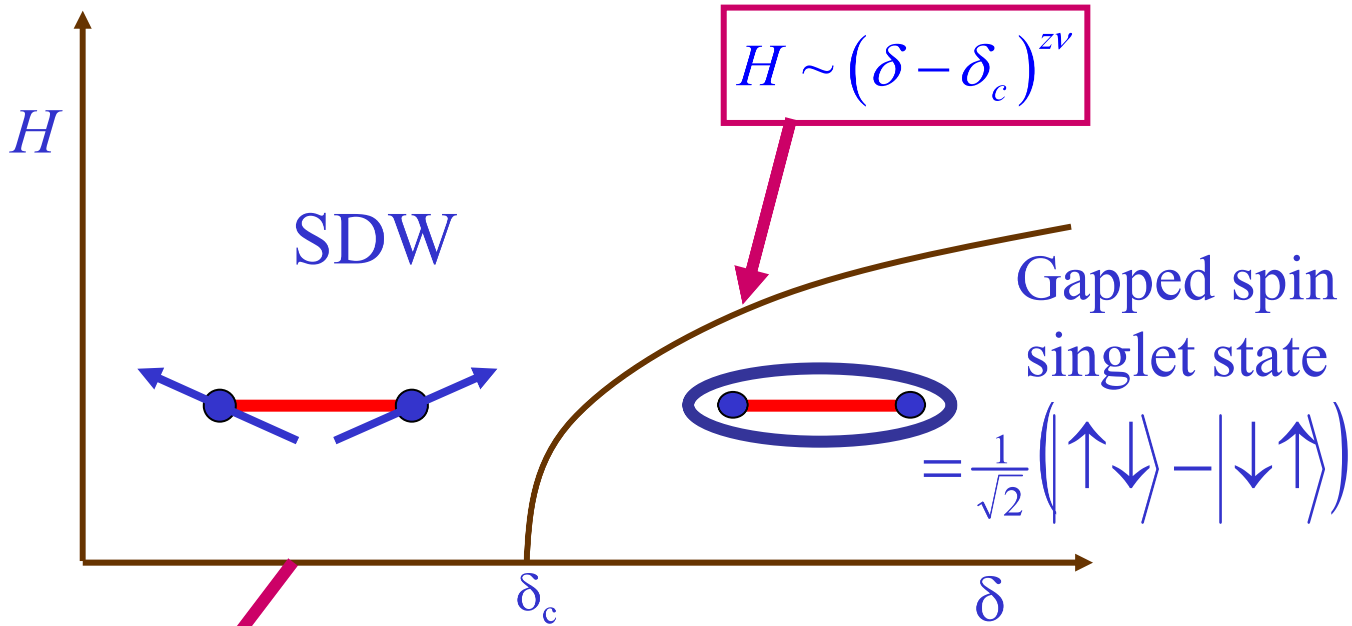
Otherwise, new theory of coupled excitons and nodal quasiparticles

L. Balents, M.P.A. Fisher, C. Nayak, *Int. J. Mod. Phys.* B **12**, 1033 (1998).

Effect of an applied magnetic field

(A) Insulators

Zeeman term: only effect in coupled ladder system



Elastic scattering intensity

$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

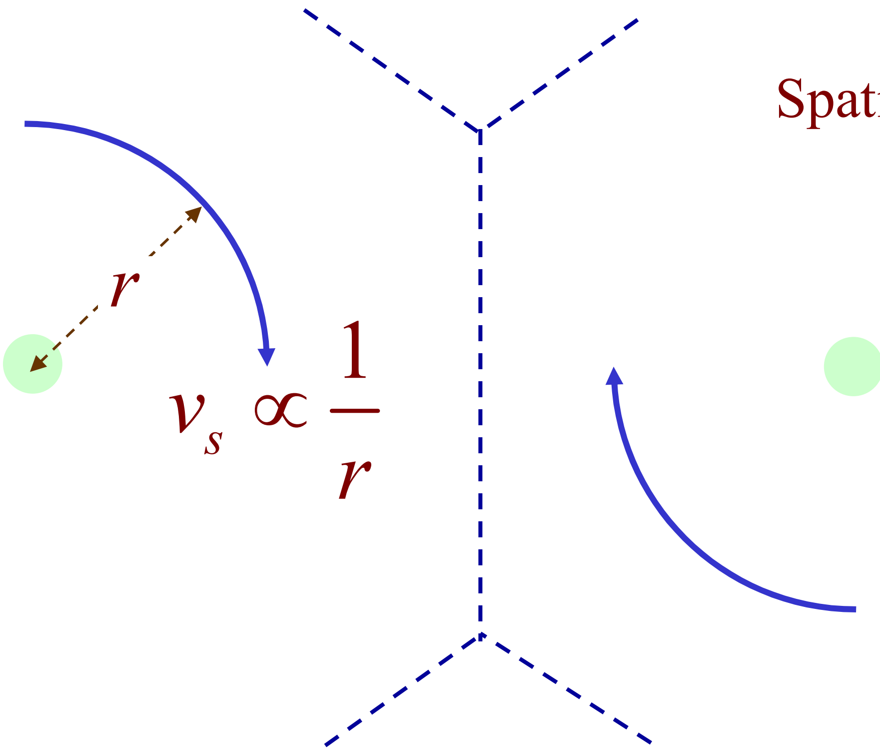
Effect is negligible over experimental field scales

(B) Metals

Weak effects (shifts in phase boundary) of order H^2 at small H.

(First order) transitions involving changes in Fermi surface topology at large H

(C) Superconductors



Spatially averaged superflow kinetic energy

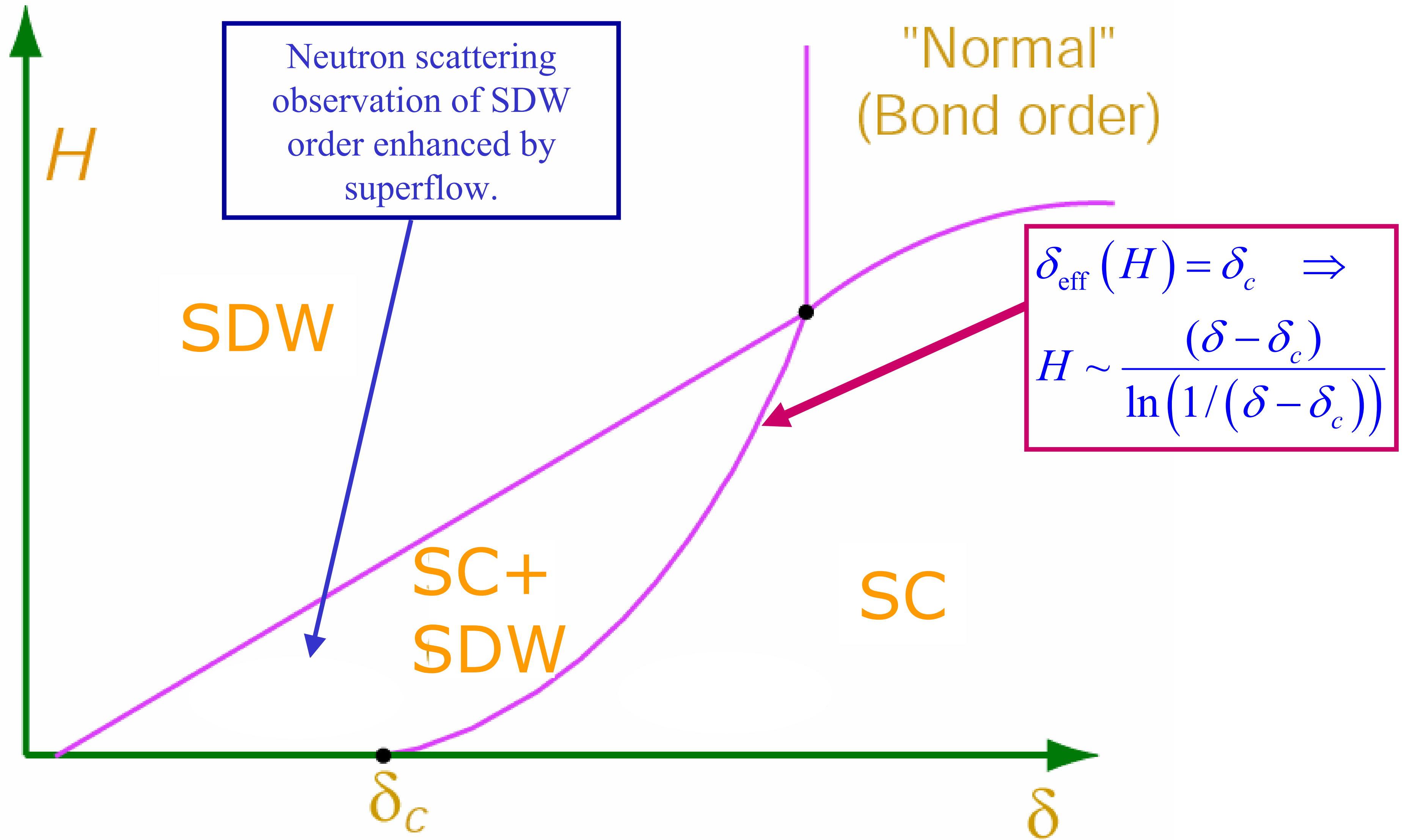
$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as an effective "doping" δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C \frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

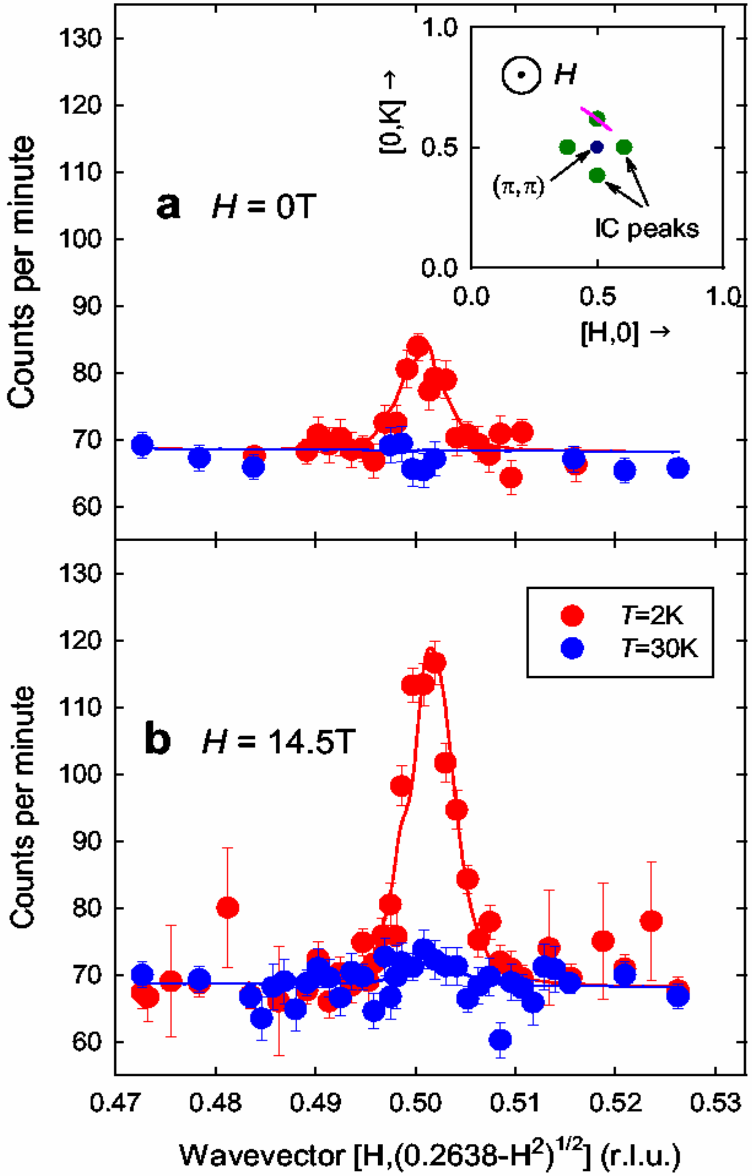
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phase diagram of a superconductor in a magnetic field

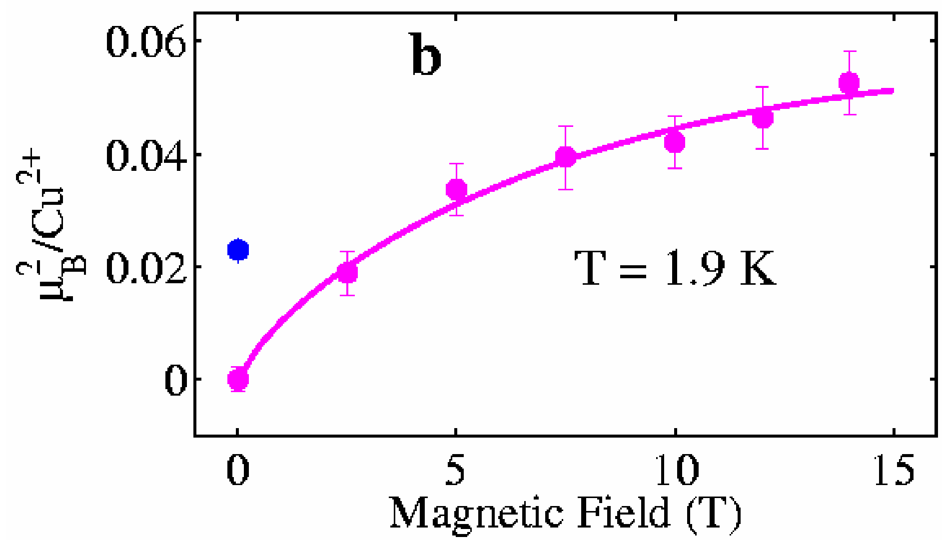


E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



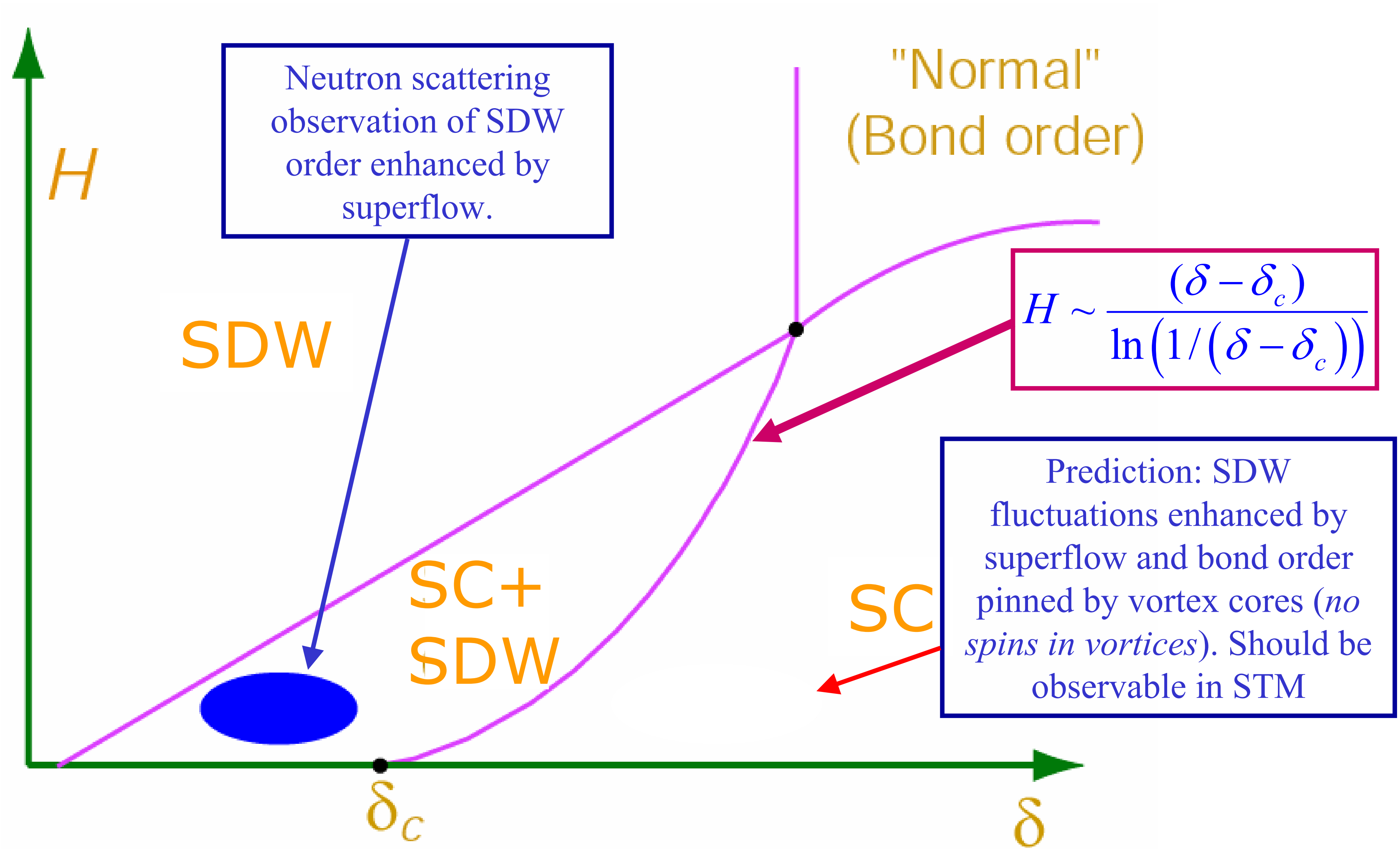
B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

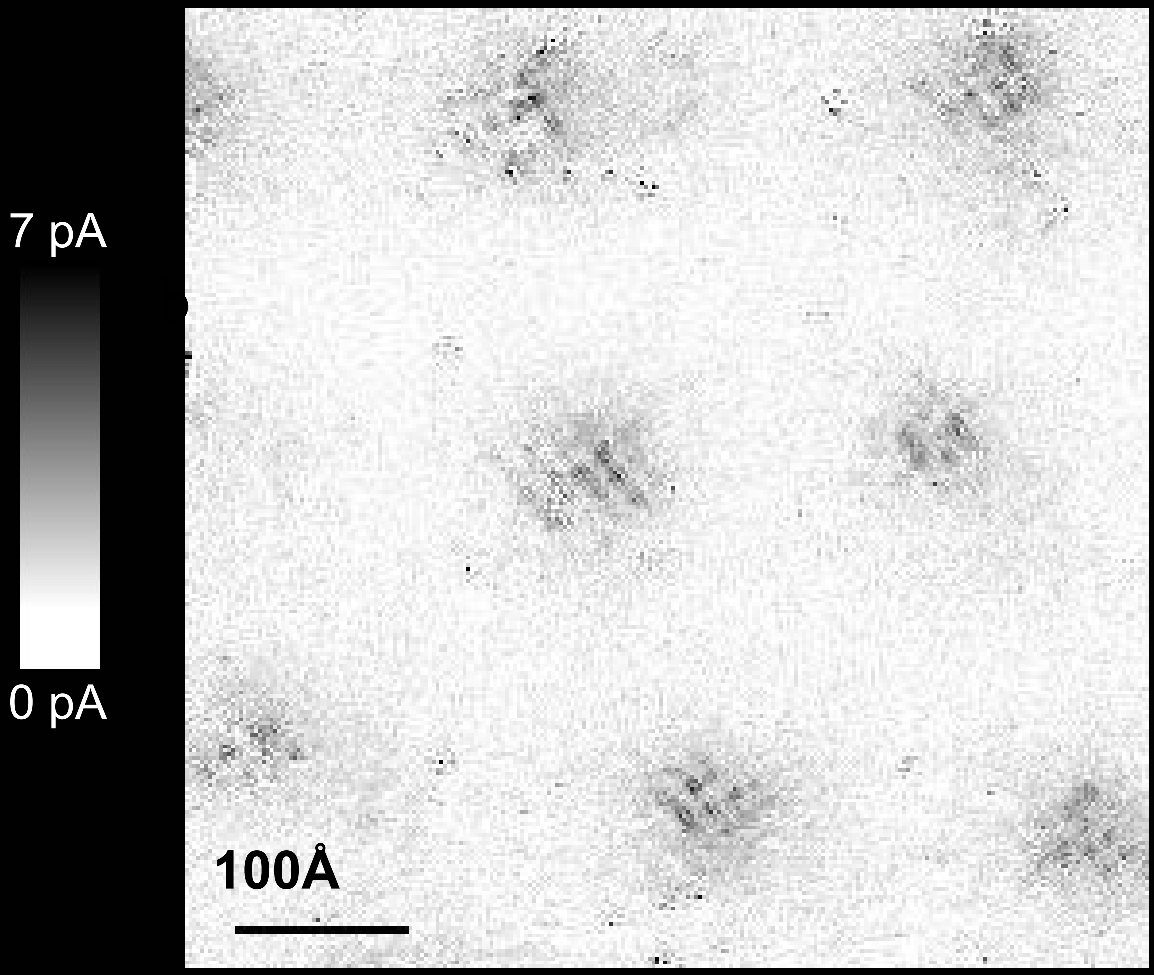
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev*. B **62**, R14677 (2000).

Phase diagram of a superconductor in a magnetic field



E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



Our interpretation:
LDOS modulations are signals of bond order of period 4 revealed in vortex halo

See also:
S. A. Kivelson, E. Fradkin, V. Oganesyan, I. P. Bindloss, J. M. Tranquada, A. Kapitulnik, and C. Howald, cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

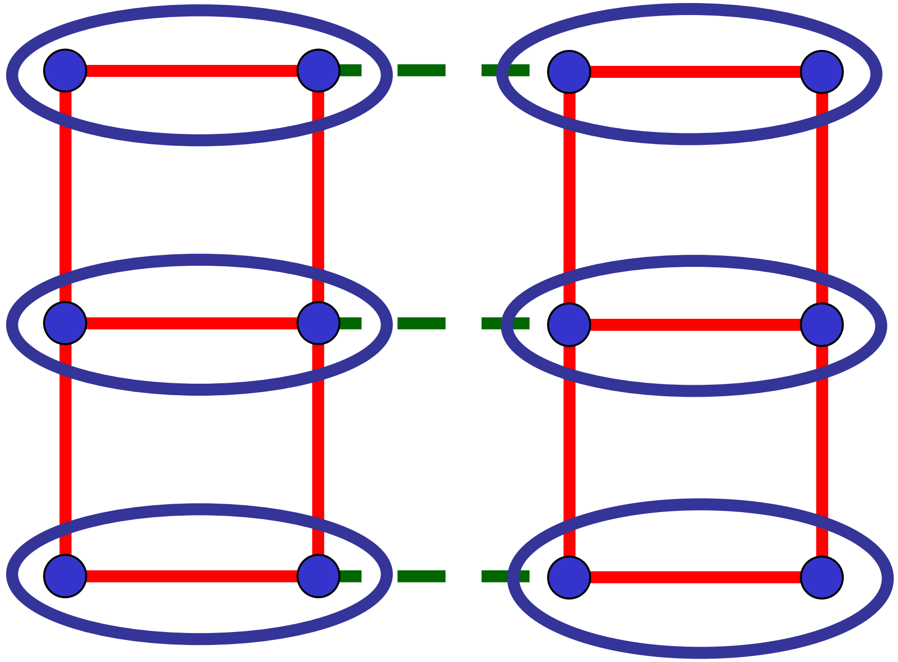
Similar results apply to quantum critical points with other "conventional" (fermion bilinear) order parameters

e.g. charge density wave, orbital currents…

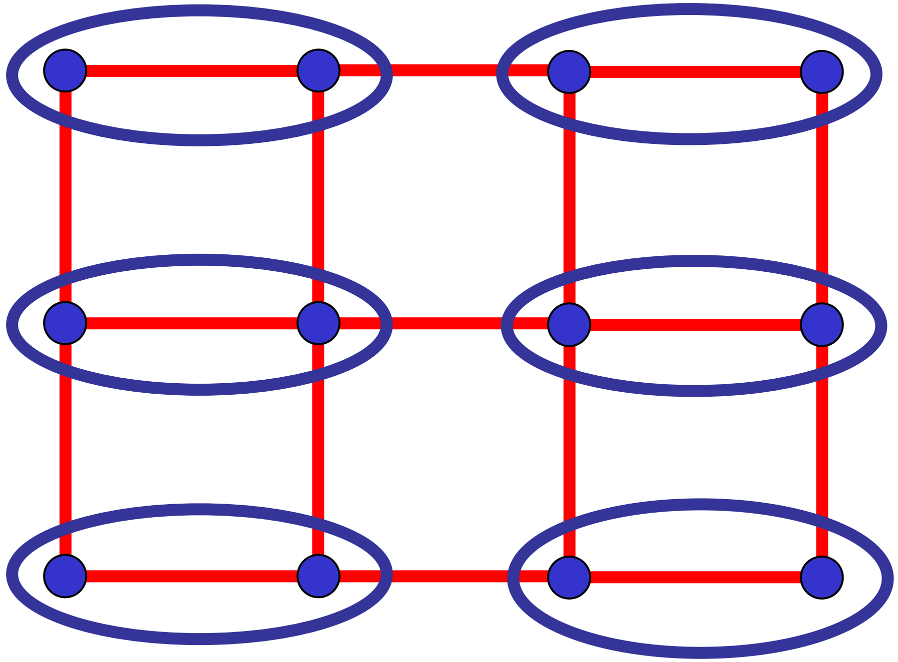
Compact U(1) gauge theory:

bond order and confined spinons in $d=2$

Paramagnetic ground state of coupled ladder model

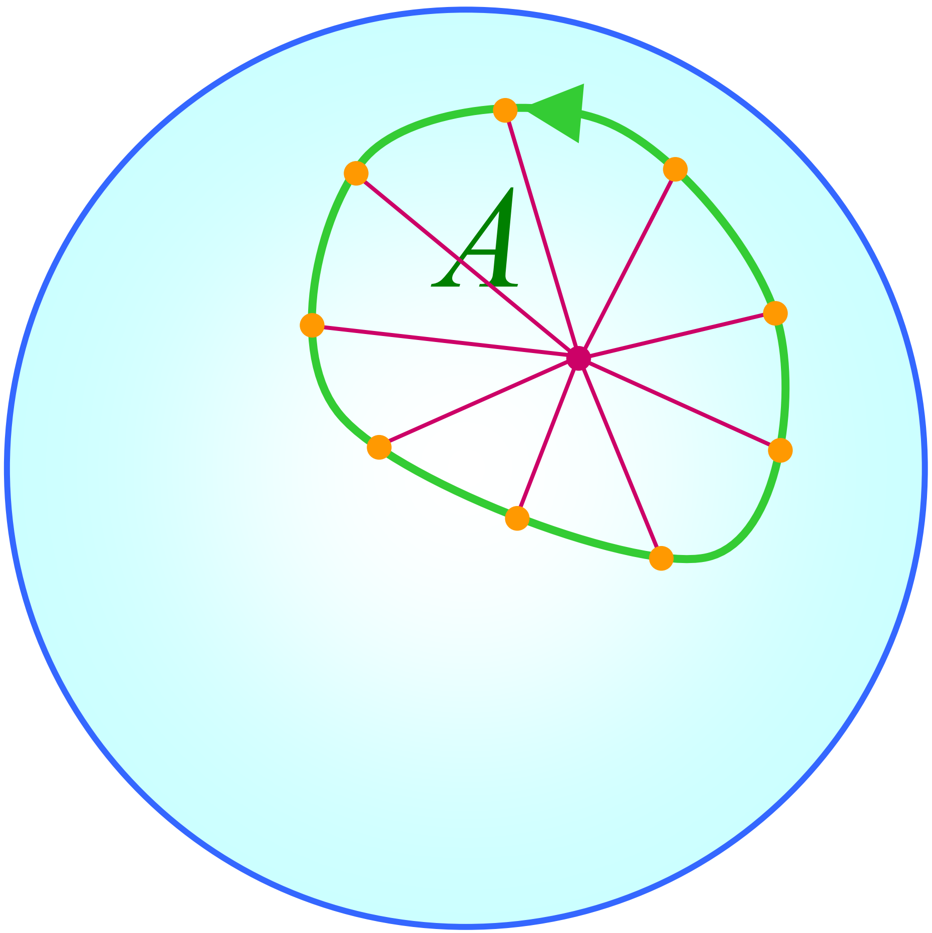


Can such a state with ***bond order*** be the ground state of a system with full square lattice symmetry ?



Write down path integral for quantum spin fluctuations

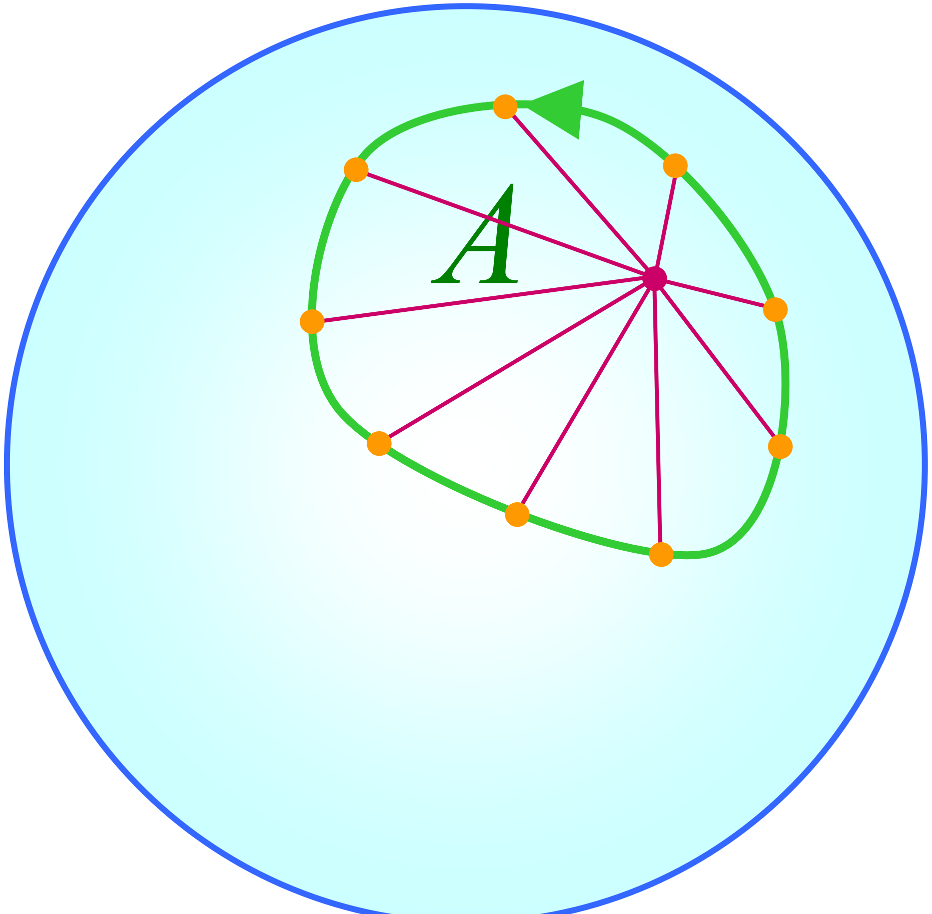
Key ingredient: Spin Berry Phases



$$e^{iSA}$$

Write down path integral for quantum spin fluctuations

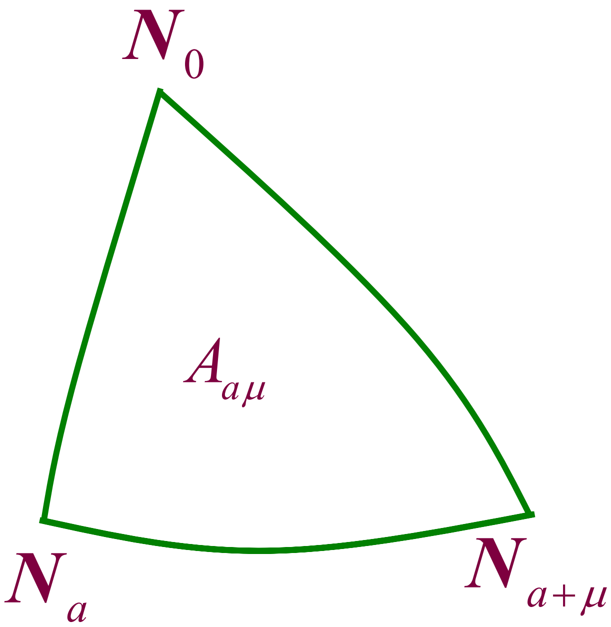
Key ingredient: Spin Berry Phases



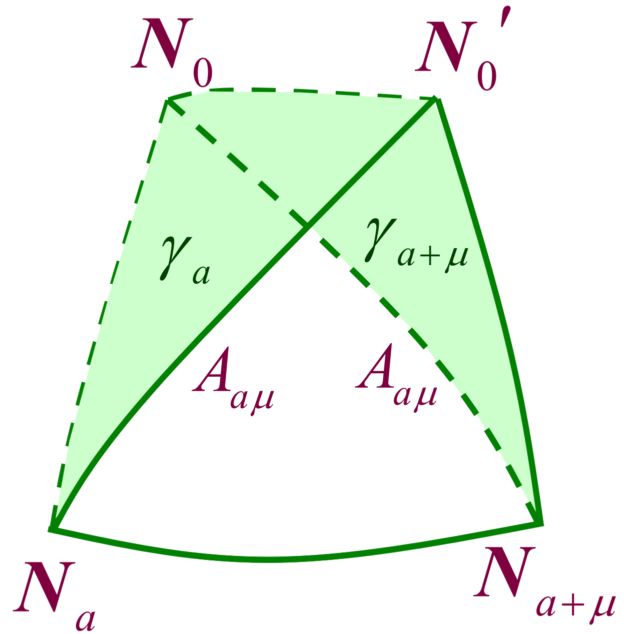
$$e^{iSA}$$

$A_{a\mu} \to$ oriented area of spherical triangle

formed by $\boldsymbol{N}_a$, $\boldsymbol{N}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{N}_0$



$A_{a\mu} \rightarrow$ oriented area of spherical triangle
formed by $\boldsymbol{N}_a$, $\boldsymbol{N}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{N}_0$



Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{N}_a$ and the two choices for $\boldsymbol{N}_0$).

The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the paramagnetic phase

Simplest effective action for $A_{a\mu}$ fluctuations in the paramagnet

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2}\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices.

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

***d*=2**: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

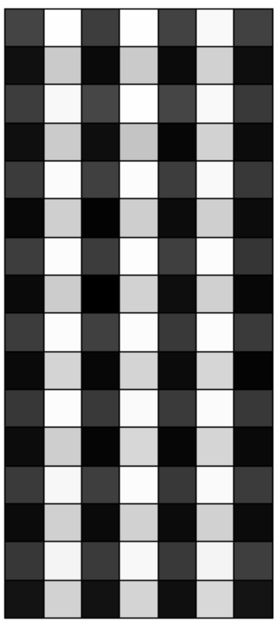
***d*=3**: A deconfined phase with a gapless "photon" is possible.

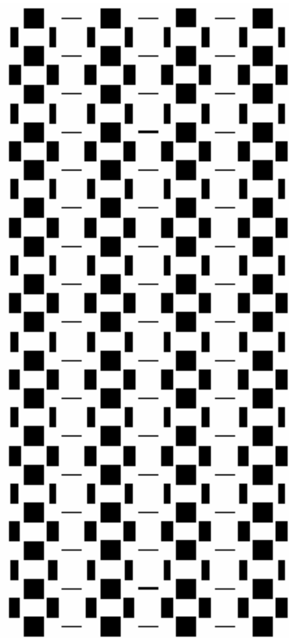
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev*. B **65**, 220405 (2002).

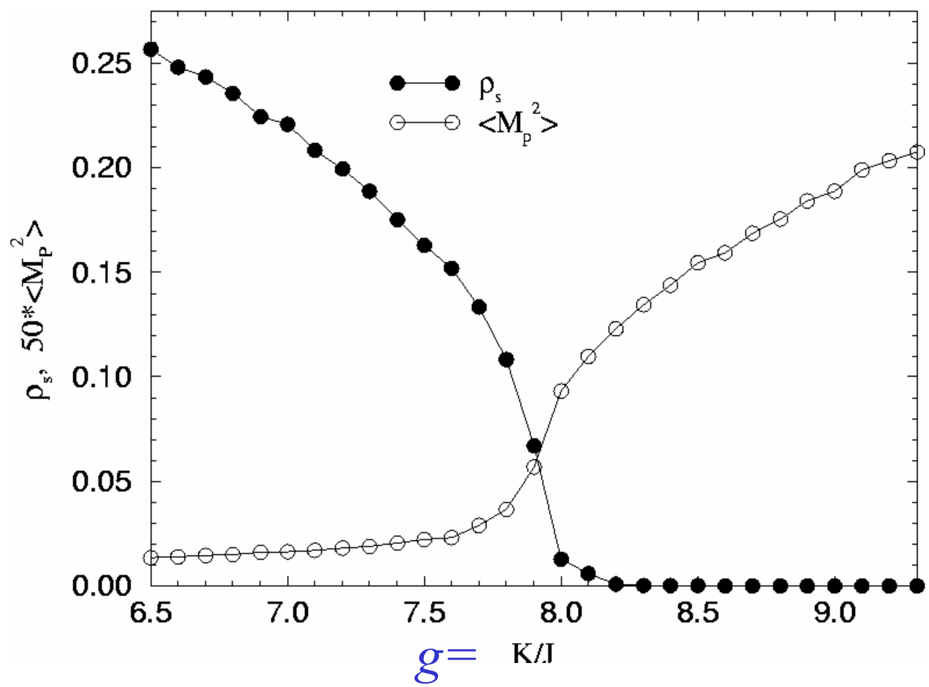
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First <u>*large scale*</u> numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry







$$H = 2J\sum_{\langle ij \rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl \rangle \subset \square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

See also C. H. Chung, Hae-Young Kee, and Yong Baek Kim, cond-mat/0211299.

Competing order parameters in the cuprate superconductors

1. Pairing order of BCS theory

(Bose-Einstein) condensation of *d*-wave Cooper pairs

Orders associated with proximate Mott insulator

2. Collinear magnetic order

3. Bond/charge/stripe order

(couples strongly to half-breathing phonons)

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000);
M. Vojta, Phys. Rev. B **66**, 104505 (2002).

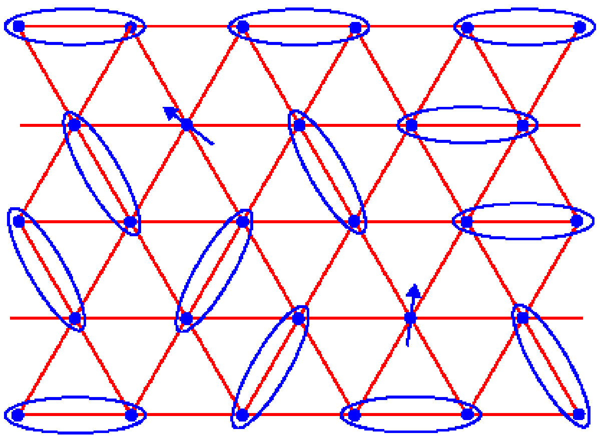
Compact U(1) gauge theory:

Deconfined spinons and quantum criticality in heavy fermion compounds in $d=3$

(talk by Matthias Vojta on Friday 10:00)

A new phase: Fractionalized Fermi Liquid (FL*)

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$



f-electrons form a spin liquid with neutral spinon excitations. The state has "*topological order*". The topological order can be detected by the violation of Luttinger's theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

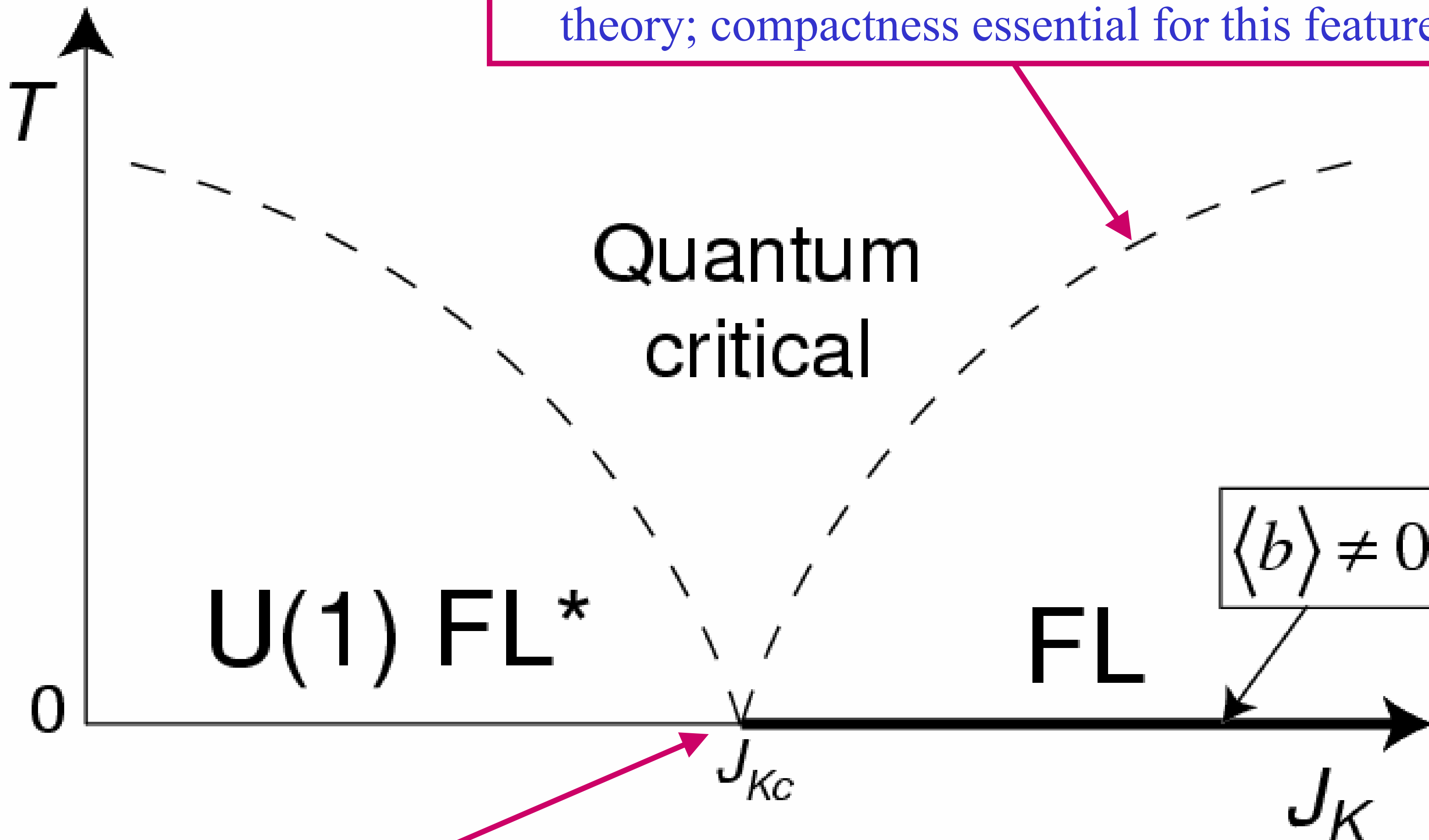
$$= (n_T - 1) (\text{mod } 2)$$

Precursor: S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev.* B 66, 045111 (2002).

T. Senthil, S. Sachdev and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).

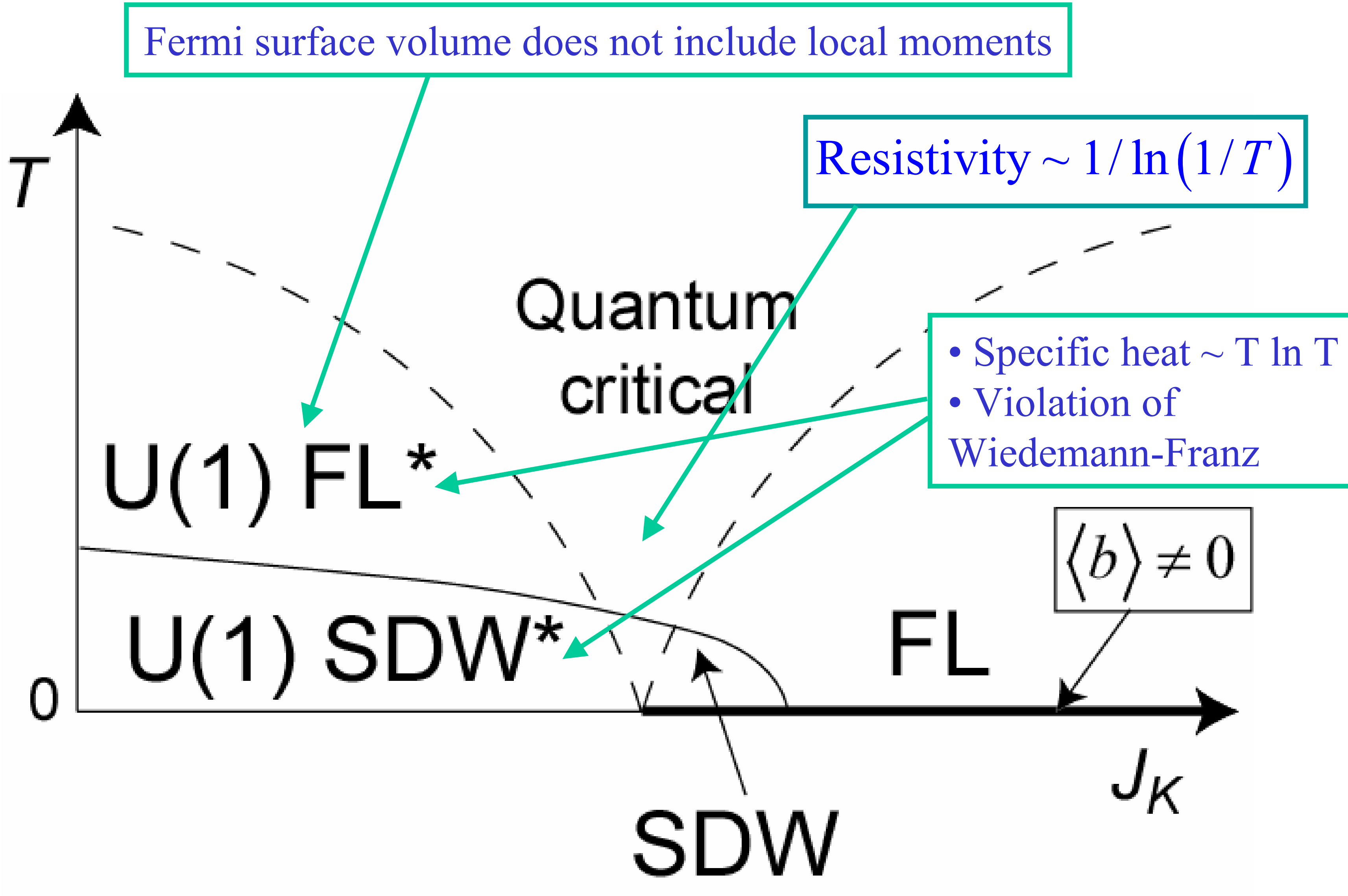
Phase diagram (U(1), *d*=3)

No transition for $T>0$ in $d=3$ compact U(1) gauge theory; compactness essential for this feature



Sharp transition at $T=0$ in $d=3$ compact U(1) gauge theory; compactness “irrelevant” at critical point

Phase diagram (U(1), *d*=3)



Conclusions

I. Cuprate superconductors:

Magnetic/bond order co-exist and compete with superconductivity at low doping.

Theory of quantum phase transitions provides a description of "fluctuating order" in the superconductor.

II. "Hidden order" in heavy fermion systems

Fractionalized states (FL* and SDW*) lead to strongly interacting quantum criticality.