

Quantum criticality in insulators, metals and superconductors

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Colloquium article in *Reviews of Modern Physics*, July 2003,
cond-mat/0211005.



Talk online:
[Google](#) [Sachdev](#)



SDW

$$T=0$$

$$\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$$

$$\langle \mathbf{S}_j \rangle = 0$$

Collinear spins: $N_1 \times N_2 = 0$

Non-collinear spins: $N_1 \times N_2 \neq 0$

Pressure,
carrier concentration,....

Quantum critical point

States on both sides of critical point
could be either (A) Insulators
(B) Metals
(C) Superconductors

(A) Insulators

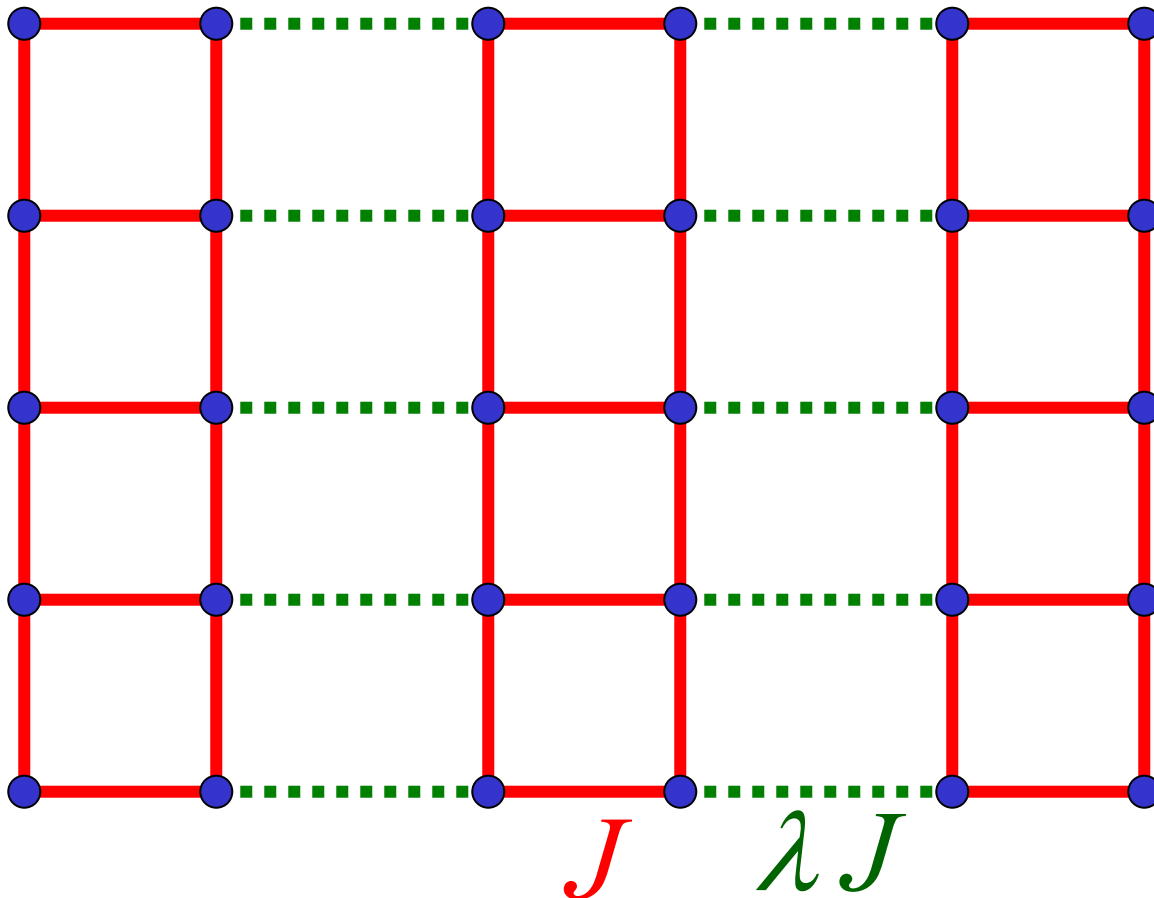
Coupled ladder antiferromagnet

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).

J. Tworzydło, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev. B* **59**, 115 (1999).

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev. B* **65**, 014407 (2002).

$S=1/2$ spins on coupled 2-leg ladders



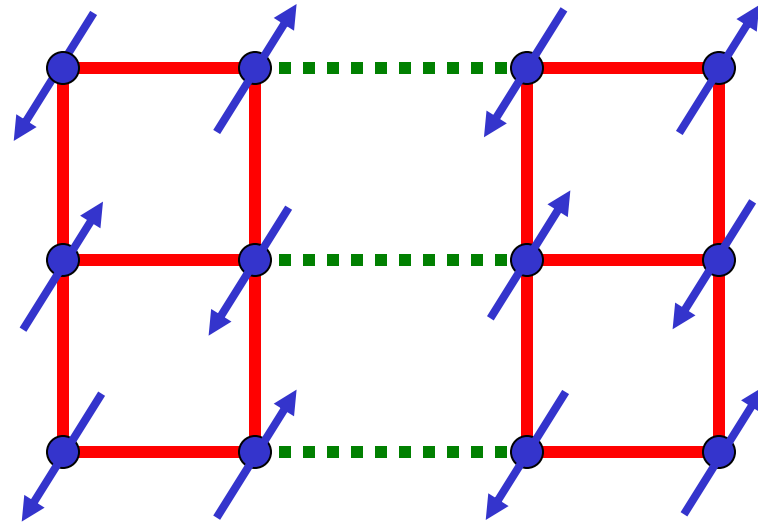
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



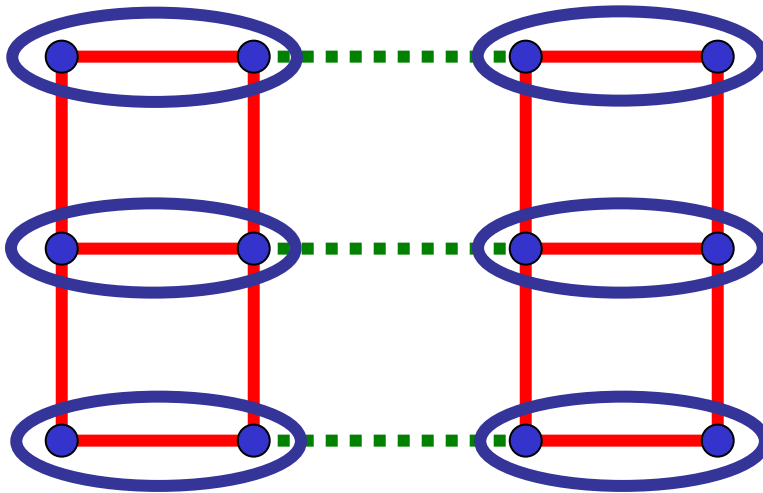
Ground state has long-range
collinear magnetic (Neel) order

$$\langle \vec{S}_i \rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled ladders



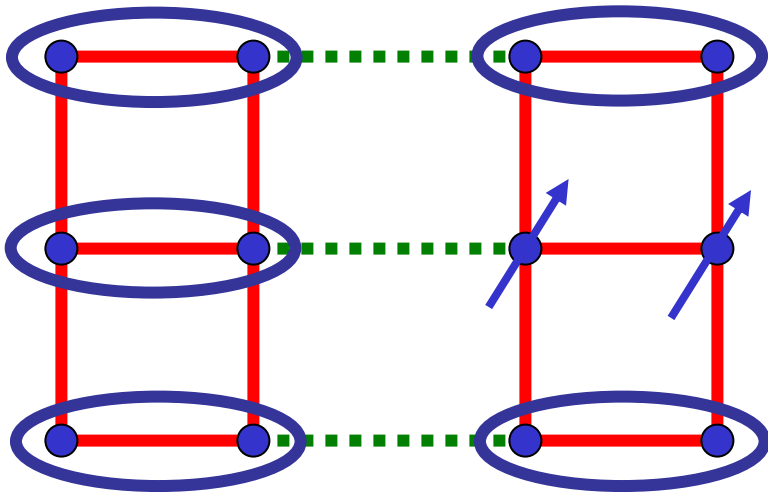
$$\text{blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Paramagnetic ground state

$$\langle \vec{S}_i \rangle = 0$$

λ close to 0

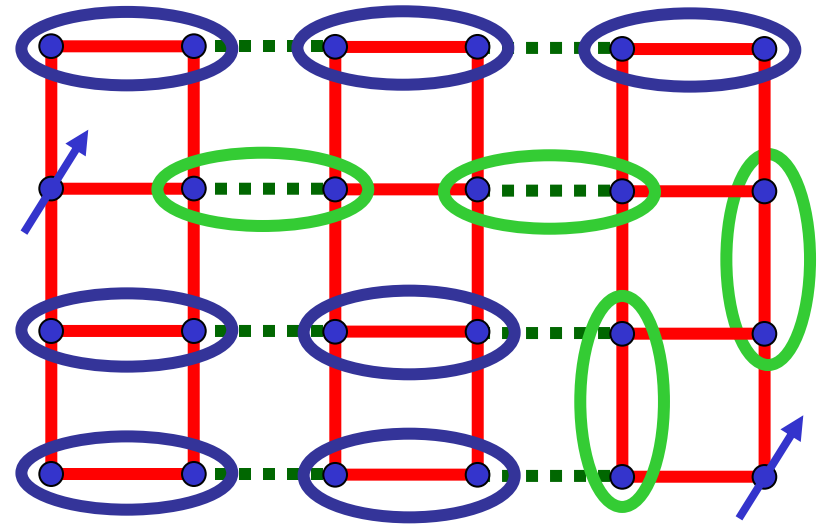
Excitations



Excitation: $S=1$ *exciton*
 (spin collective mode, “triplon”)

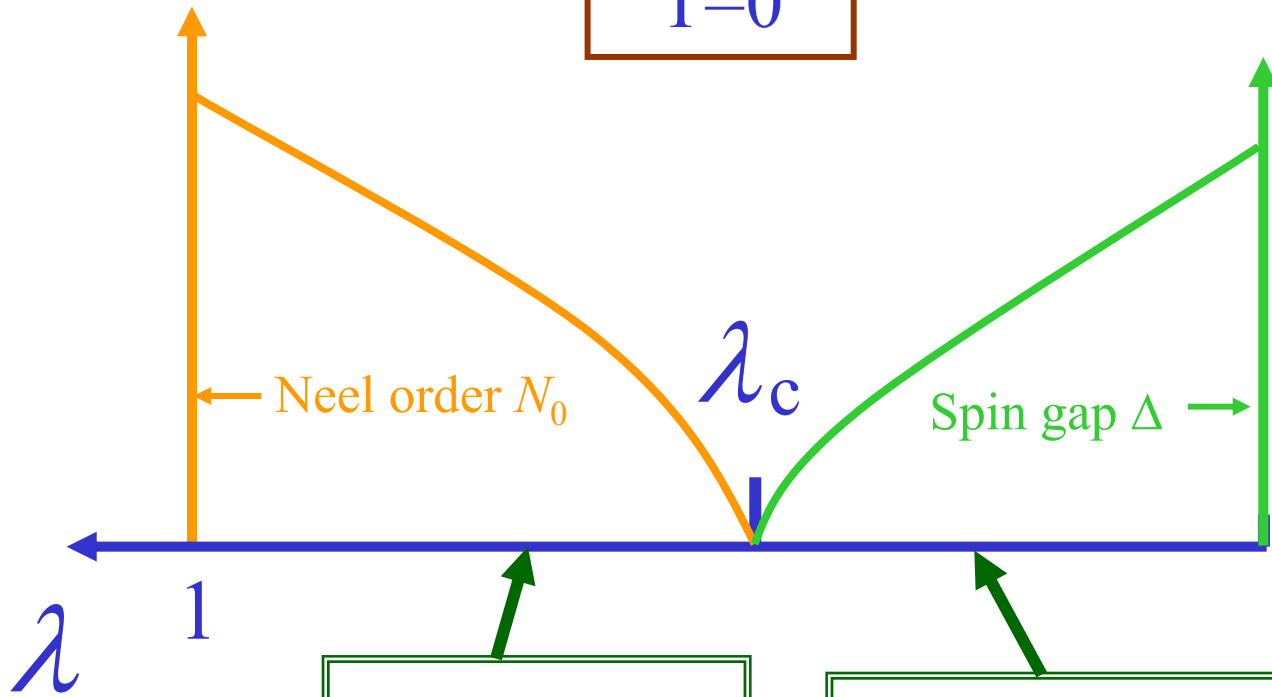
Energy dispersion away from
 wavevector \vec{K}

$$\epsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$



$S=1/2$ spinons are *confined*
 by a linear potential.

T=0



Neel state
 $\langle \vec{S} \rangle = N_0$

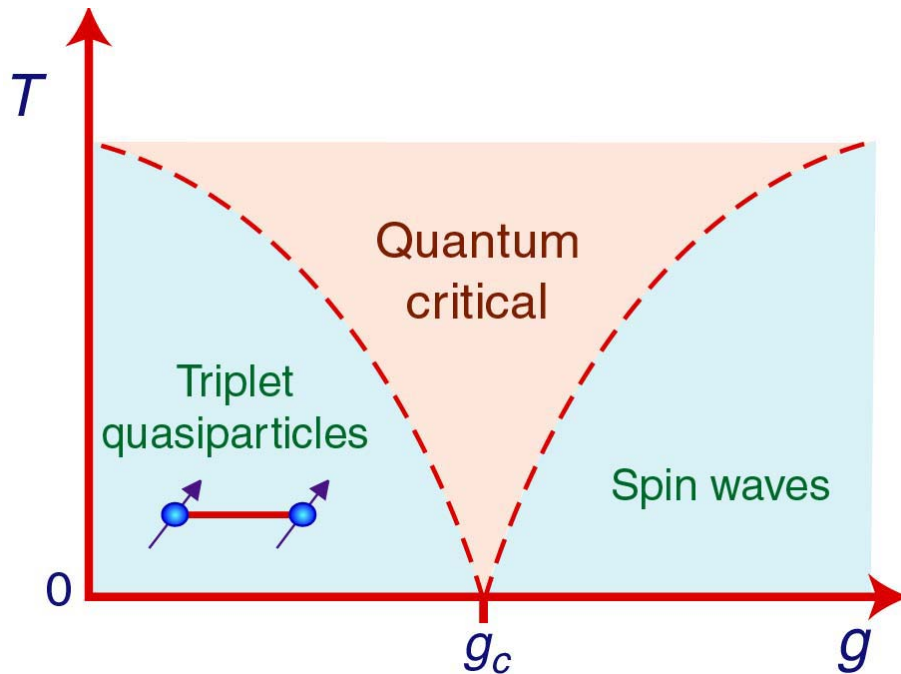
Quantum
paramagnet
 $\langle \vec{S} \rangle = 0$

Field theory for quantum criticality

λ close to λ_c : use “soft spin” field

$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + (\lambda_c - \lambda) \phi_\alpha^2 \right) + \frac{u}{4!} (\phi_\alpha^2)^2 \right]$$

$\phi_\alpha \longrightarrow$ 3-component antiferromagnetic order parameter



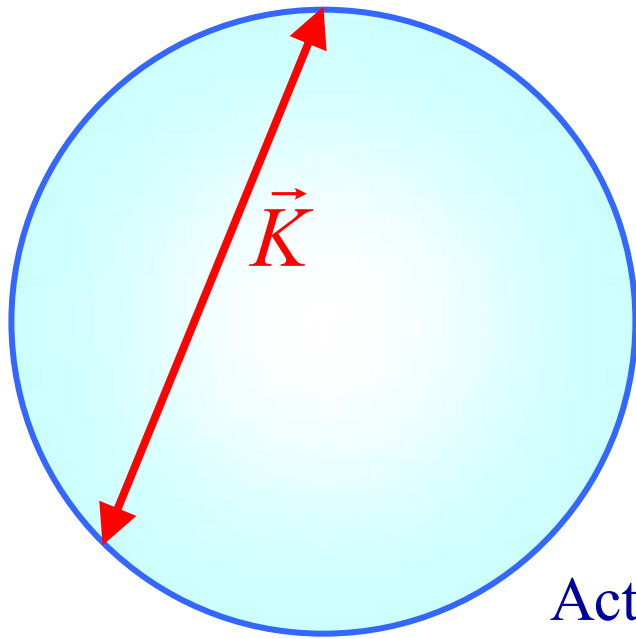
Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega/k_B T$

$$\chi(\omega, T) = T^n g(\hbar\omega/k_B T)$$

Exciton scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

(B) Metals

Spin density wave order in the presence of a
Fermi surface



Low energy “paramagnon” excitations
near the Fermi surface

Damping by fermionic quasiparticles leads to

Action:
$$S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} |\vec{\phi}(q, \omega)|^2 (q^2 + |\omega| + \Gamma(\delta, T))$$

M. T. Beal-Monod and K. Maki, *Phys. Rev. Lett.* **34**, 1461 (1975); J.A. Hertz, *Phys. Rev. B* **14**, 1165 (1976).

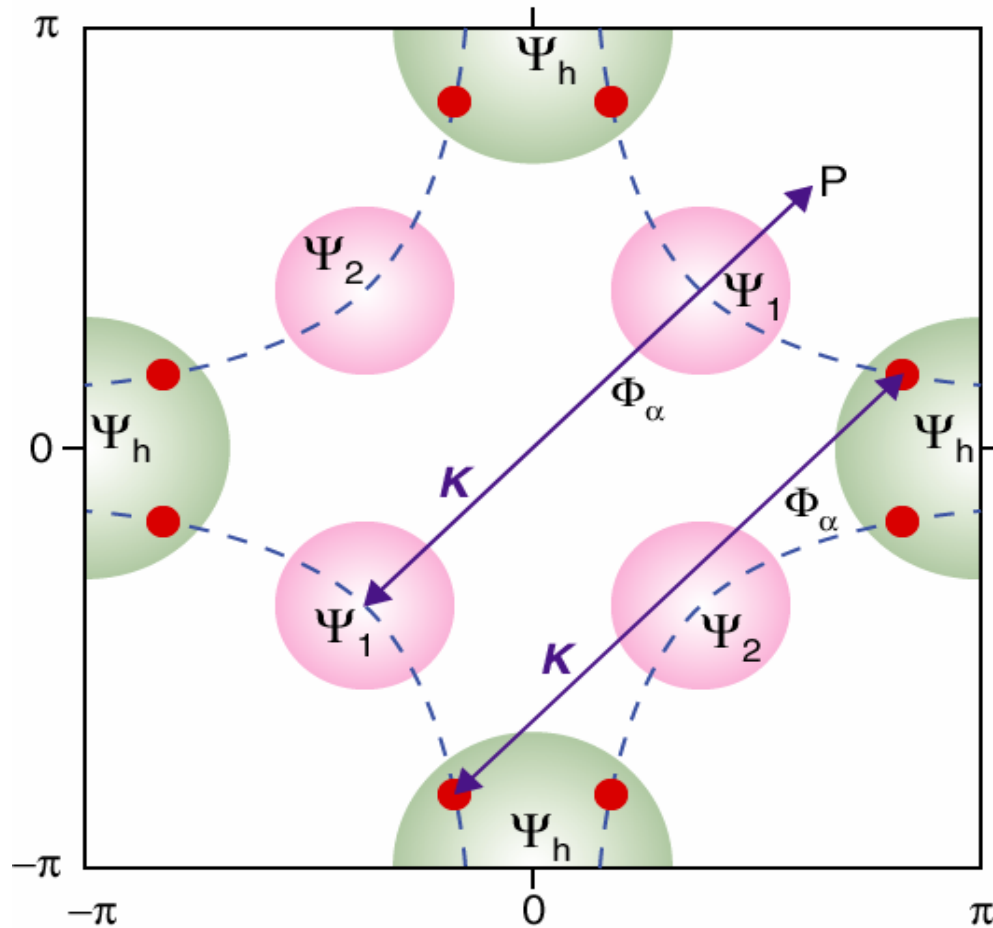
Characteristic paramagnon energy at finite temperature $\Gamma(0, T) \sim T^p$ with $p > 1$.

Arises from non-universal corrections to scaling, generated by $\vec{\phi}^4$ term.

J. Mathon, *Proc. R. Soc. London A*, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev. B* **10**, 4014 (1974);
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985);
G. G. Lonzarich and L. Taillefer, *J. Phys. C* **18**, 4339 (1985); A.J. Millis, *Phys. Rev. B* **48**, 7183 (1993).

(C) Superconductors

Co-existence of superconductivity and spin-density wave order



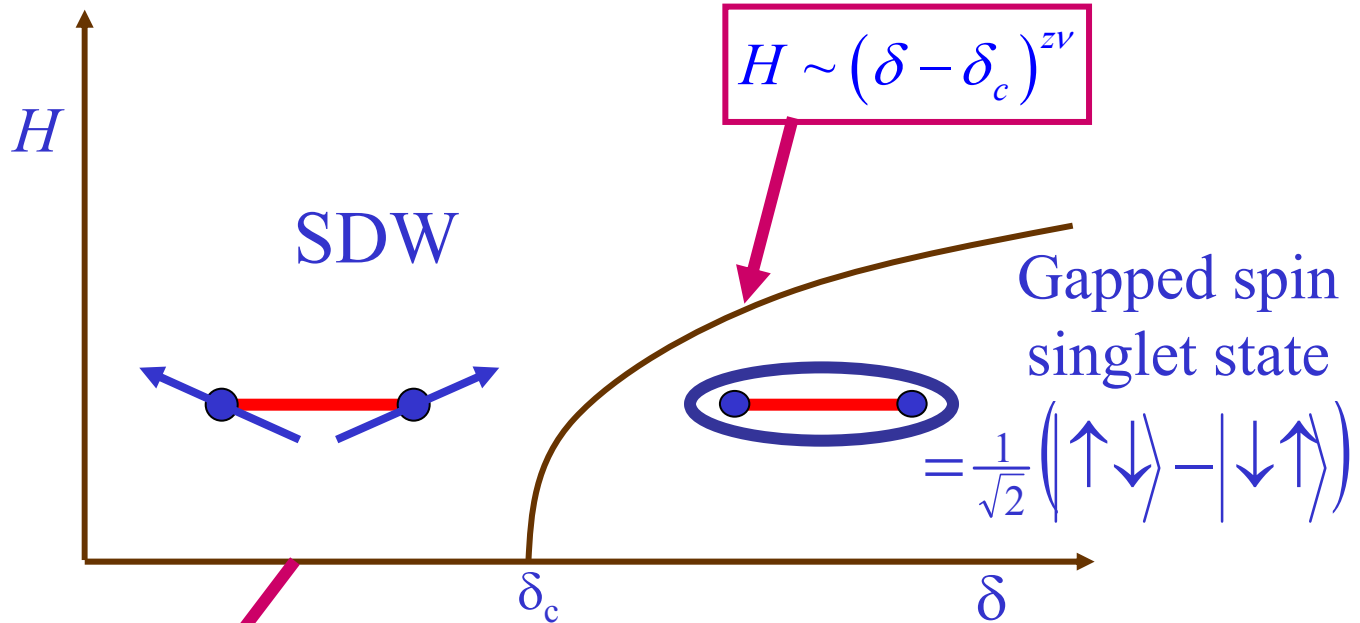
If \vec{K} does not exactly connect two nodal points,
critical theory is as in an insulator

Otherwise, new theory of coupled excitons and nodal quasiparticles

Effect of an applied magnetic field

(A) Insulators

Zeeman term: only effect in coupled ladder system



Characteristic field $g\mu_B H = \Delta$, the spin gap
1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Elastic scattering intensity

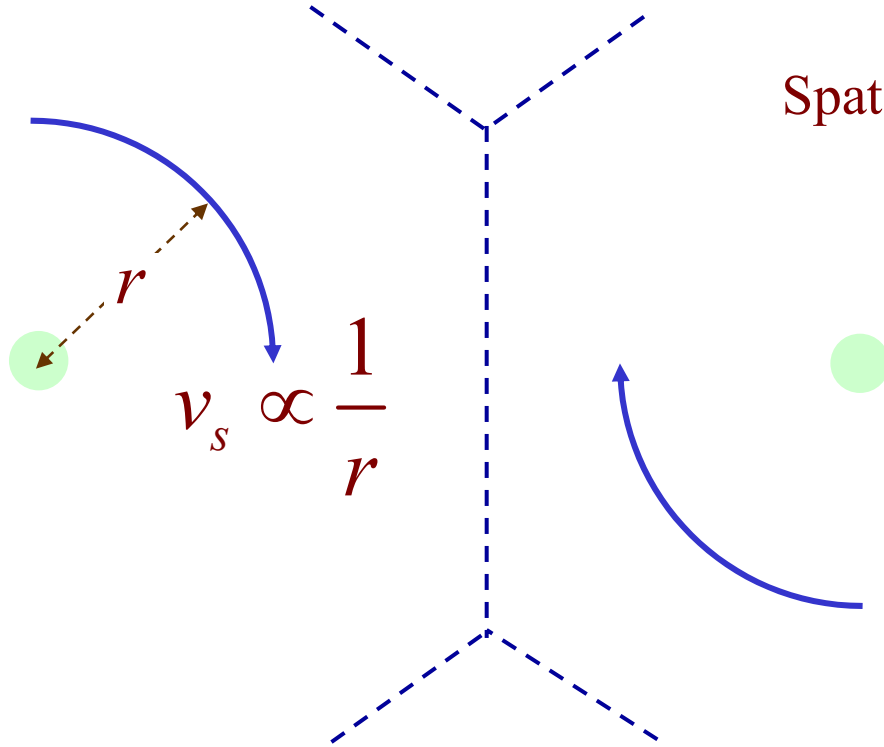
$$I(H) = I(0) + a \left(\frac{H}{J} \right)^2$$

(B) Metals

Weak effects (shifts in phase boundary) of order H^2 at small H .

(First order) transitions involving changes in Fermi surface topology at large H

(C) Superconductors



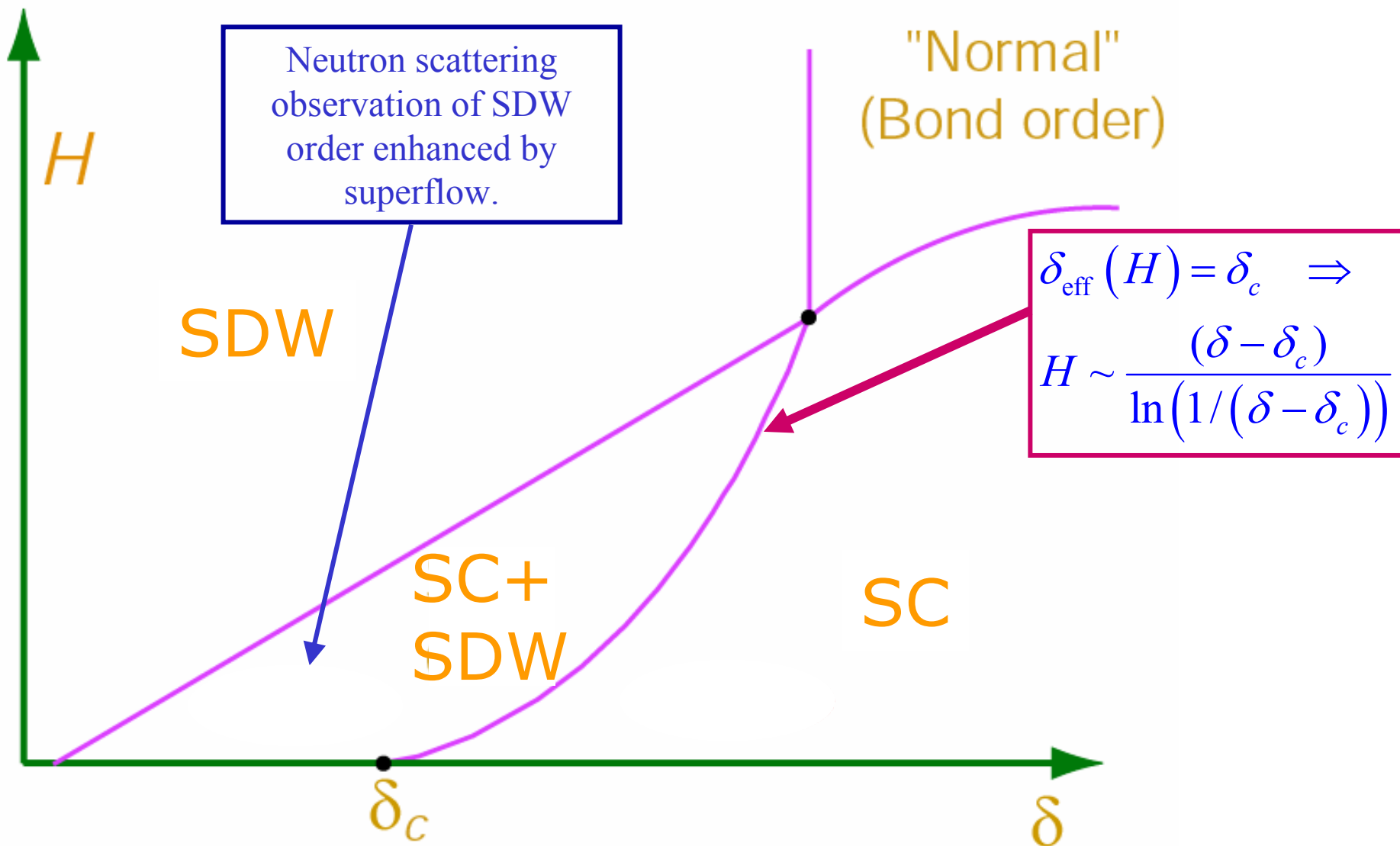
Spatially averaged superflow kinetic energy

$$\langle v_s^2 \rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as an effective "doping" δ :

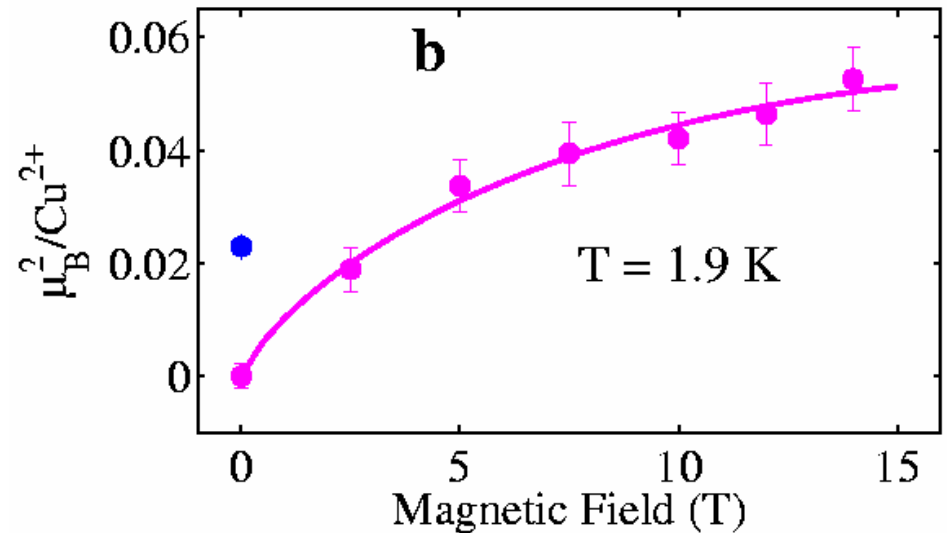
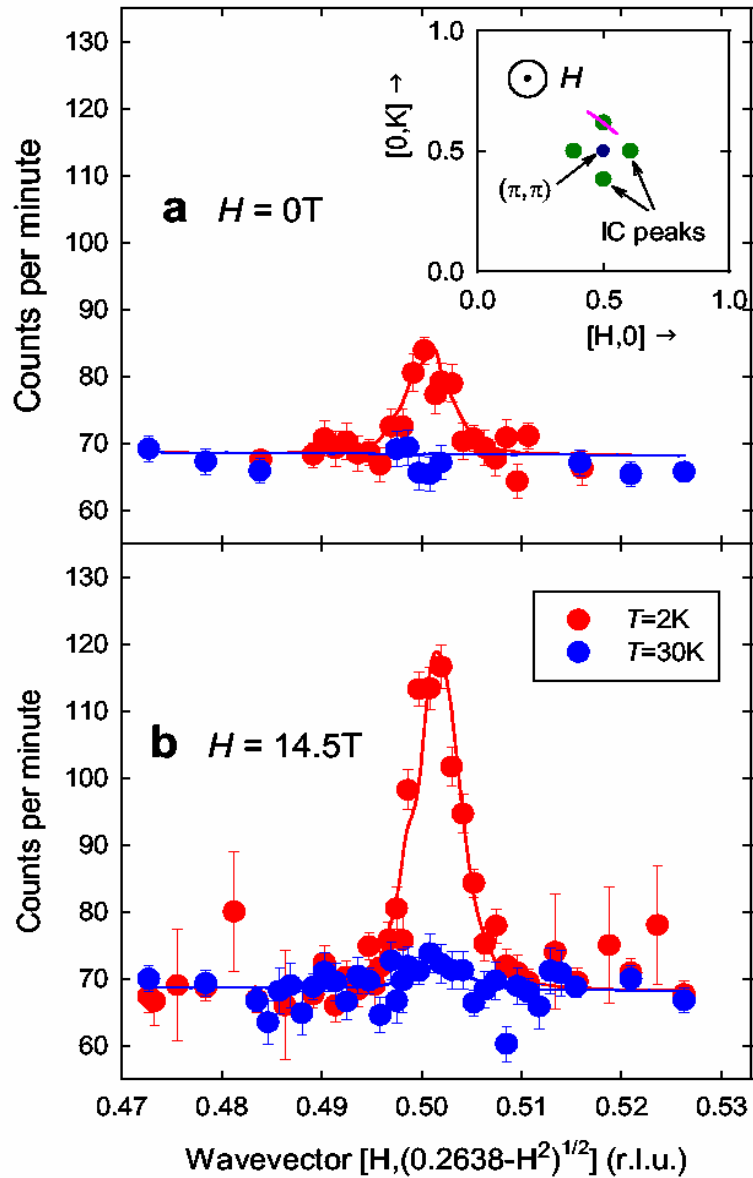
$$\delta_{\text{eff}}(H) = \delta - C \frac{H}{H_{c2}} \ln \left(\frac{3H_{c2}}{H} \right)$$

Phase diagram of a superconductor in a magnetic field



Neutron scattering of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ at $x=0.1$

B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



Solid line - fit to : $I(H) = a \frac{H}{H_{c2}} \ln \left(\frac{H_{c2}}{H} \right)$

See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev. B* **62**, R14677 (2000).

Phase diagram of a superconductor in a magnetic field

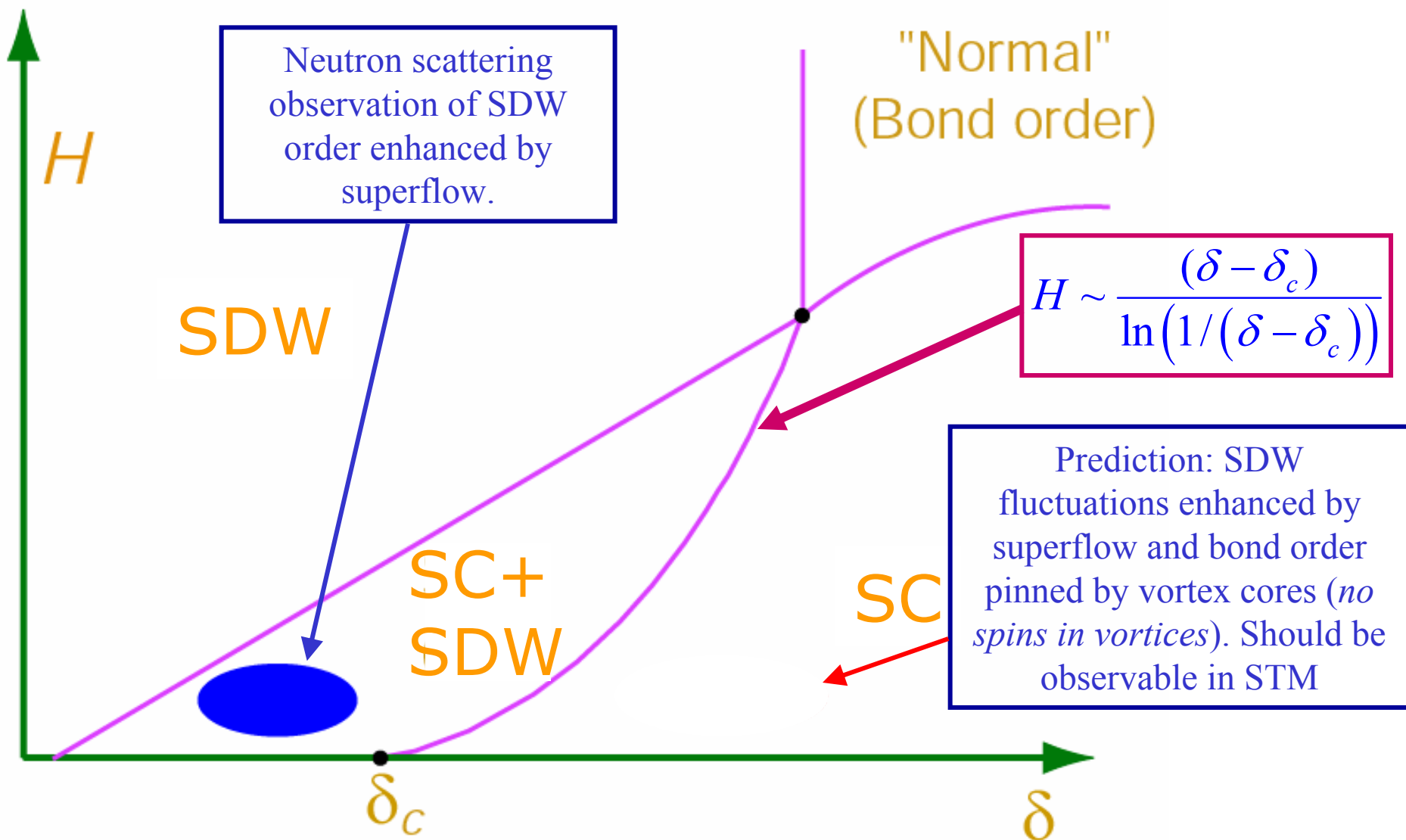
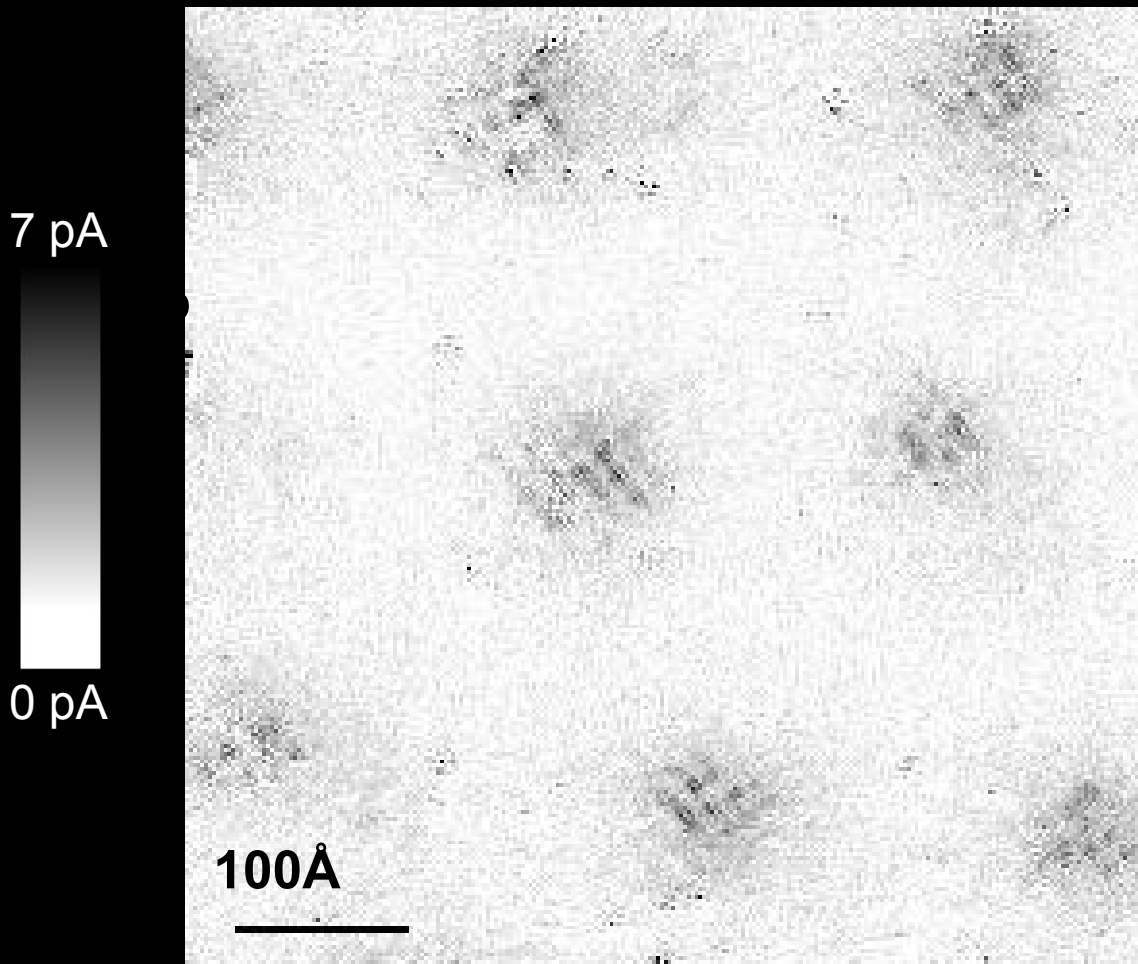


FIG. 1. FIG. 11. FIG. 12. FIG. 13. FIG. 14. FIG. 15. FIG. 16. FIG. 17. FIG. 18. FIG. 19. FIG. 20. FIG. 21. FIG. 22. FIG. 23. FIG. 24. FIG. 25. FIG. 26. FIG. 27. FIG. 28. FIG. 29. FIG. 30. FIG. 31. FIG. 32. FIG. 33. FIG. 34. FIG. 35. FIG. 36. FIG. 37. FIG. 38. FIG. 39. FIG. 40. FIG. 41. FIG. 42. FIG. 43. FIG. 44. FIG. 45. FIG. 46. FIG. 47. FIG. 48. FIG. 49. FIG. 50. FIG. 51. FIG. 52. FIG. 53. FIG. 54. FIG. 55. FIG. 56. FIG. 57. FIG. 58. FIG. 59. FIG. 60. FIG. 61. FIG. 62. FIG. 63. FIG. 64. FIG. 65. FIG. 66. FIG. 67. FIG. 68. FIG. 69. FIG. 70. FIG. 71. FIG. 72. FIG. 73. FIG. 74. FIG. 75. FIG. 76. FIG. 77. FIG. 78. FIG. 79. FIG. 80. FIG. 81. FIG. 82. FIG. 83. FIG. 84. FIG. 85. FIG. 86. FIG. 87. FIG. 88. FIG. 89. FIG. 90. FIG. 91. FIG. 92. FIG. 93. FIG. 94. FIG. 95. FIG. 96. FIG. 97. FIG. 98. FIG. 99. FIG. 100.

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev. B* **66**, 020401 (2002).

Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV



Our interpretation:
LDOS modulations are signals of bond order of period 4 revealed in vortex halo

See also:

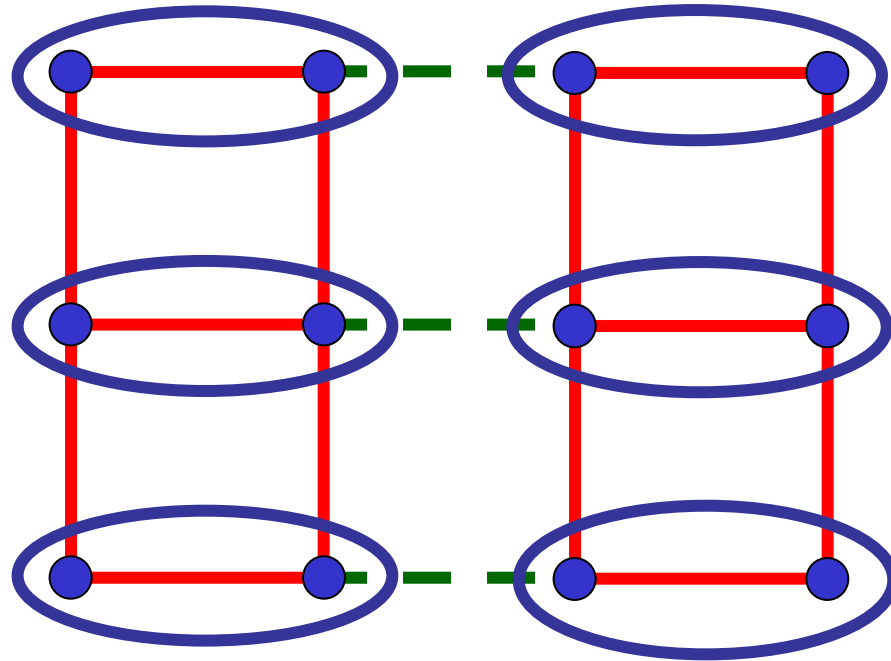
S. A. Kivelson, E. Fradkin,
V. Oganesyan, I. P. Bindloss,
J. M. Tranquada,
A. Kapitulnik, and
C. Howald,
[cond-mat/0210683](https://arxiv.org/abs/cond-mat/0210683).

J. Hoffman E. W. Hudson, K. M. Lang,
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,
and J. C. Davis, *Science* 295, 466 (2002).

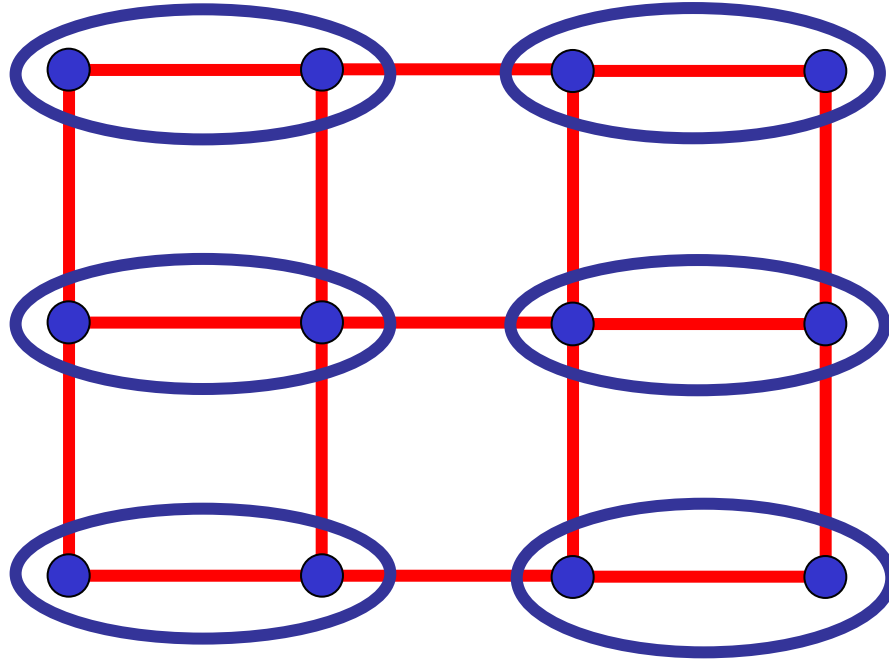
Similar results apply to quantum critical points
with other “conventional” (fermion bilinear)
order parameters
e.g. charge density wave, orbital currents...

Compact U(1) gauge theory:
bond order and confined spinons in $d=2$

Paramagnetic ground state of coupled ladder model

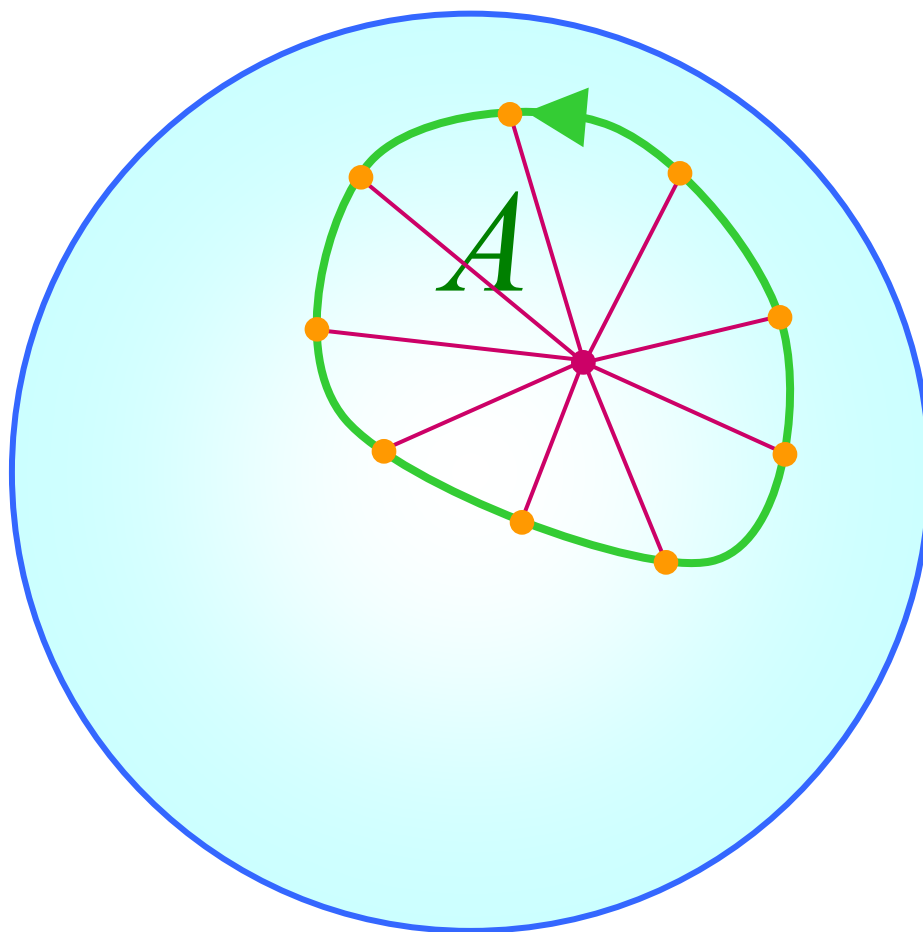


Can such a state with *bond order* be the ground state of a system with full square lattice symmetry ?



Write down path integral for quantum spin fluctuations

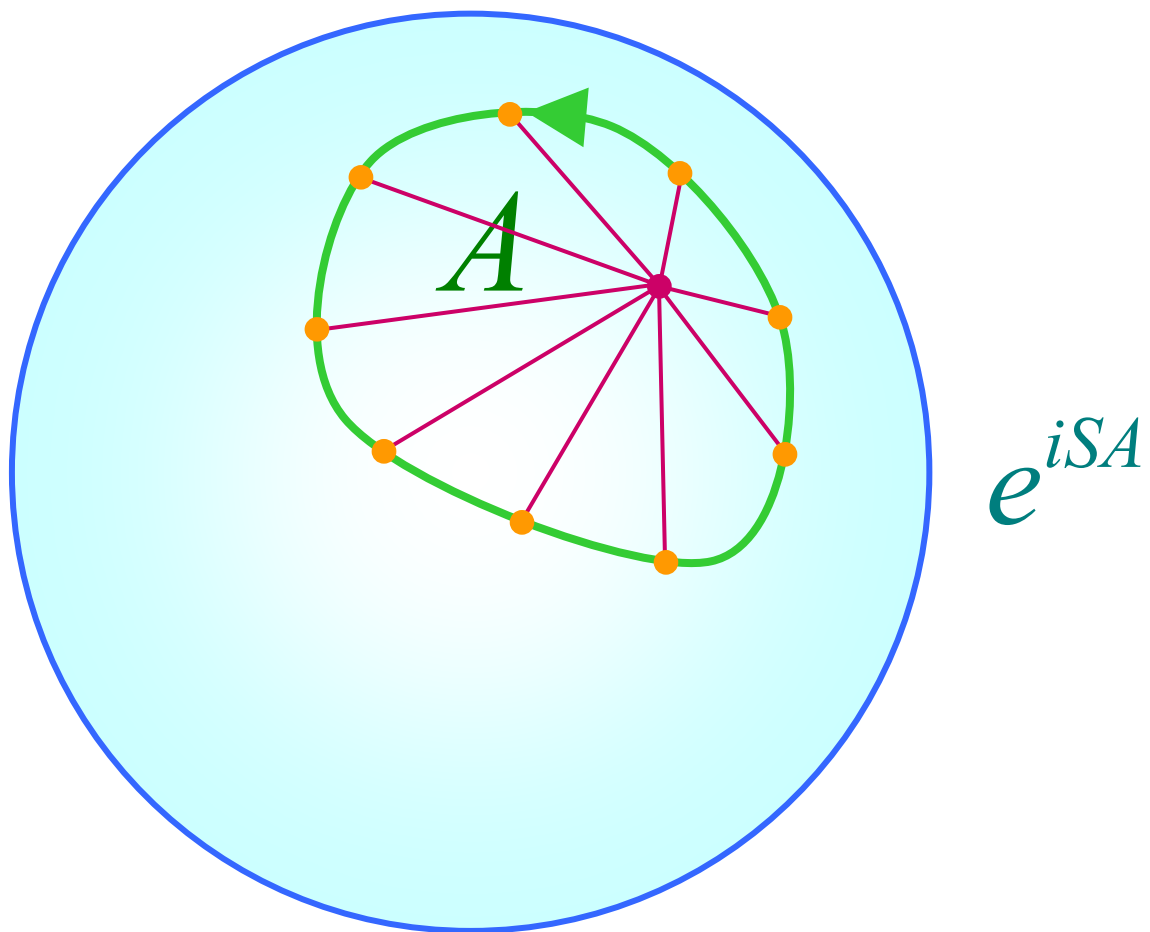
Key ingredient: Spin Berry Phases



$$e^{iSA}$$

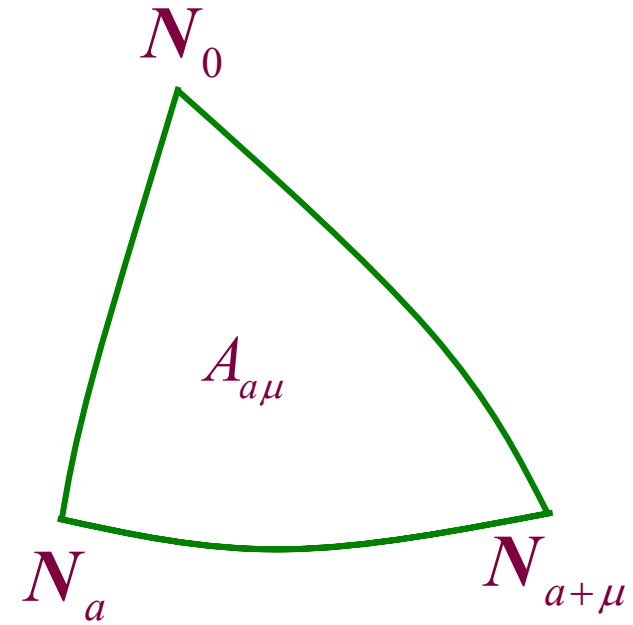
Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by N_a , $N_{a+\mu}$, and an arbitrary reference point N_0



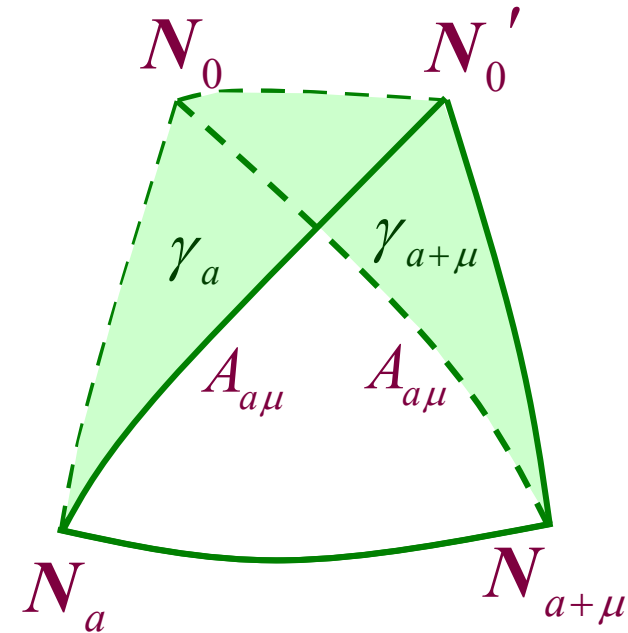
$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by N_a , $N_{a+\mu}$, and an arbitrary reference point N_0

Change in choice of n_0 is like a “gauge transformation”

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by N_a and the two choices for N_0).



The area of the triangle is uncertain modulo 4π , and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the paramagnetic phase

Simplest effective action for $A_{a\mu}$ fluctuations in the paramagnet

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left(\frac{1}{2e^2} \sum_{\square} \cos \left(\frac{1}{2} (\Delta_{\mu} A_{a\nu} - \Delta_{\nu} A_{a\mu}) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_a \rightarrow \pm 1$ on two square sublattices.

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

$d=2$: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

$d=3$: A deconfined phase with a gapless “photon” is possible.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

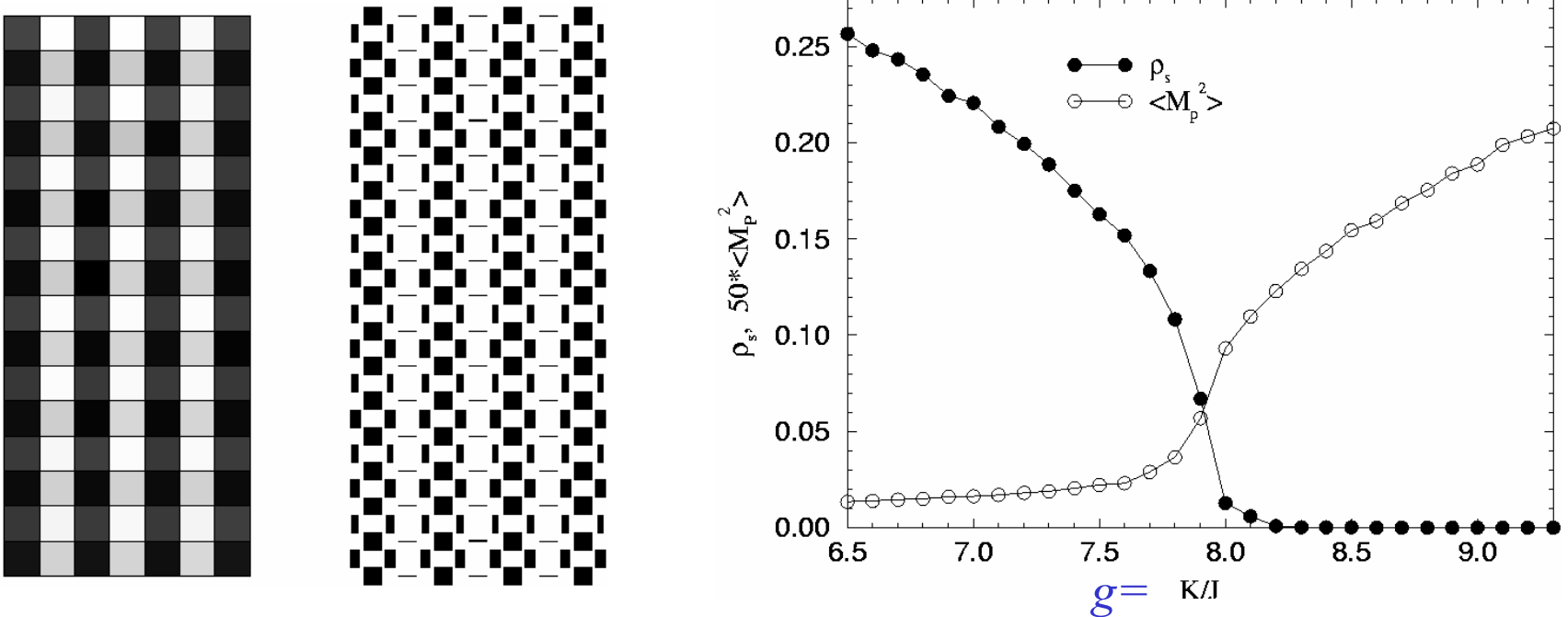
S. Sachdev and R. Jalabert, *Mod. Phys. Lett. B* **4**, 1043 (1990).

K. Park and S. Sachdev, *Phys. Rev. B* **65**, 220405 (2002).

Bond order in a frustrated $S=1/2$ XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a $S=1/2$ antiferromagnet with full square lattice symmetry



$$H = 2J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - K \sum_{\langle ijkl \rangle \square} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

See also C. H. Chung, Hae-Young Kee, and Yong Baek Kim, cond-mat/0211299.

Competing order parameters in the cuprate superconductors

1. Pairing order of BCS theory

(Bose-Einstein) condensation of d -wave Cooper pairs

Orders associated with proximate Mott insulator

2. Collinear magnetic order

3. Bond/charge/stripe order

(couples strongly to half-breathing phonons)

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991).

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);

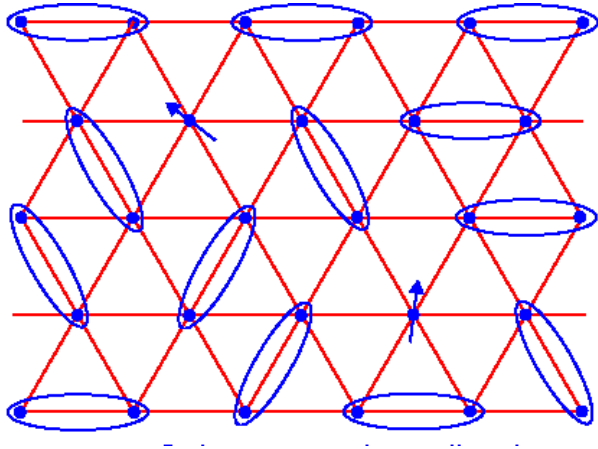
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev. B* **62**, 6721 (2000);

M. Vojta, *Phys. Rev. B* **66**, 104505 (2002).

Compact U(1) gauge theory:
Deconfined spinons and quantum criticality in
heavy fermion compounds in $d=3$
(talk by Matthias Vojta on Friday 10:00)

A new phase: Fractionalized Fermi Liquid (FL*)

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$



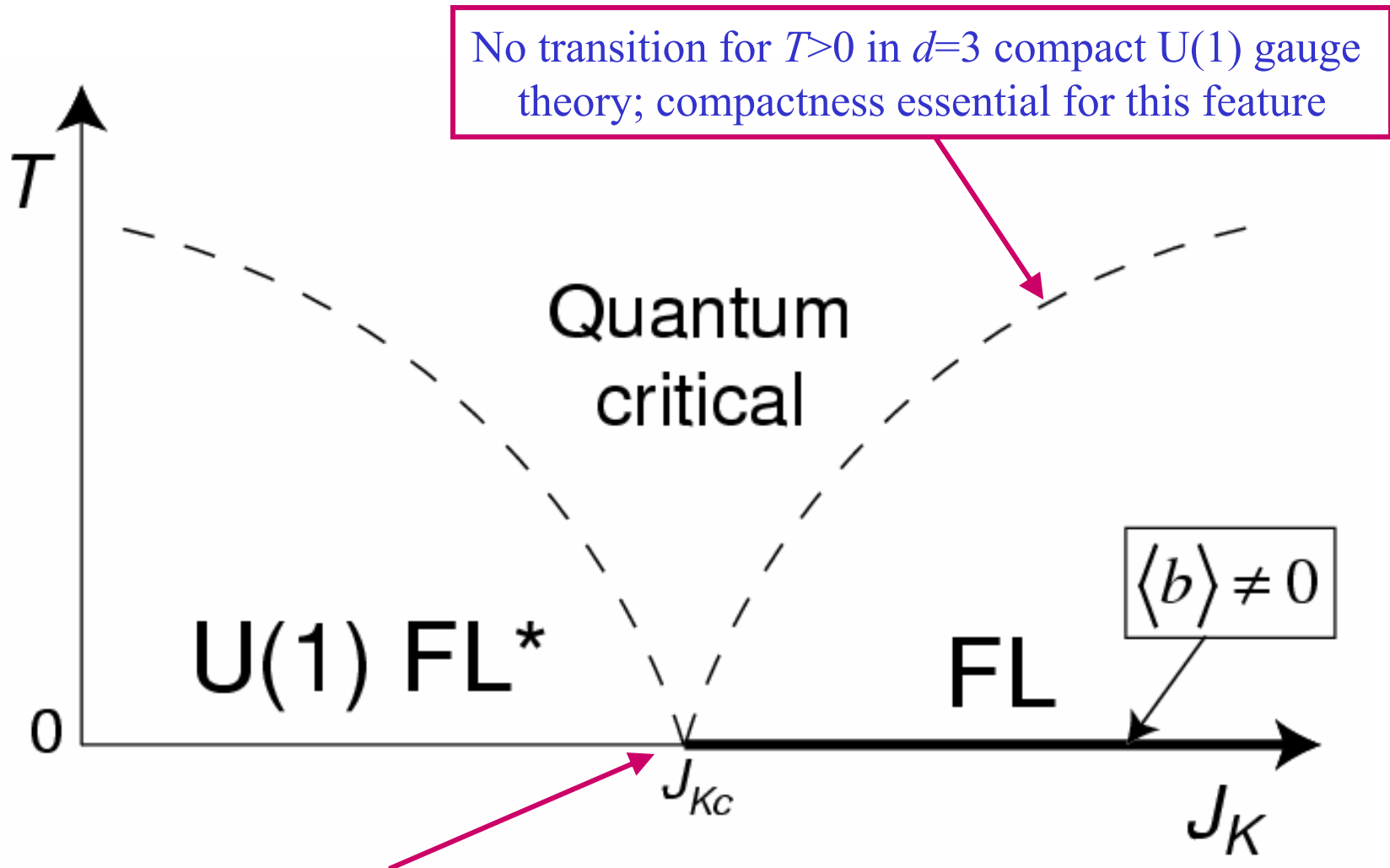
f-electrons form a spin liquid with neutral spinon excitations. The state has “*topological order*”. The topological order can be detected by the violation of Luttinger’s theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{V_0}{(2\pi)^d} (\text{Volume enclosed by Fermi surface}) \\ = (n_T - 1) (\text{mod } 2)$$

Precursor: S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev. B* 66, 045111 (2002).

T. Senthil, S. Sachdev and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).

Phase diagram (U(1), $d=3$)



No transition for $T > 0$ in $d=3$ compact U(1) gauge theory; compactness essential for this feature

Sharp transition at $T=0$ in $d=3$ compact U(1) gauge theory; compactness "irrelevant" at critical point

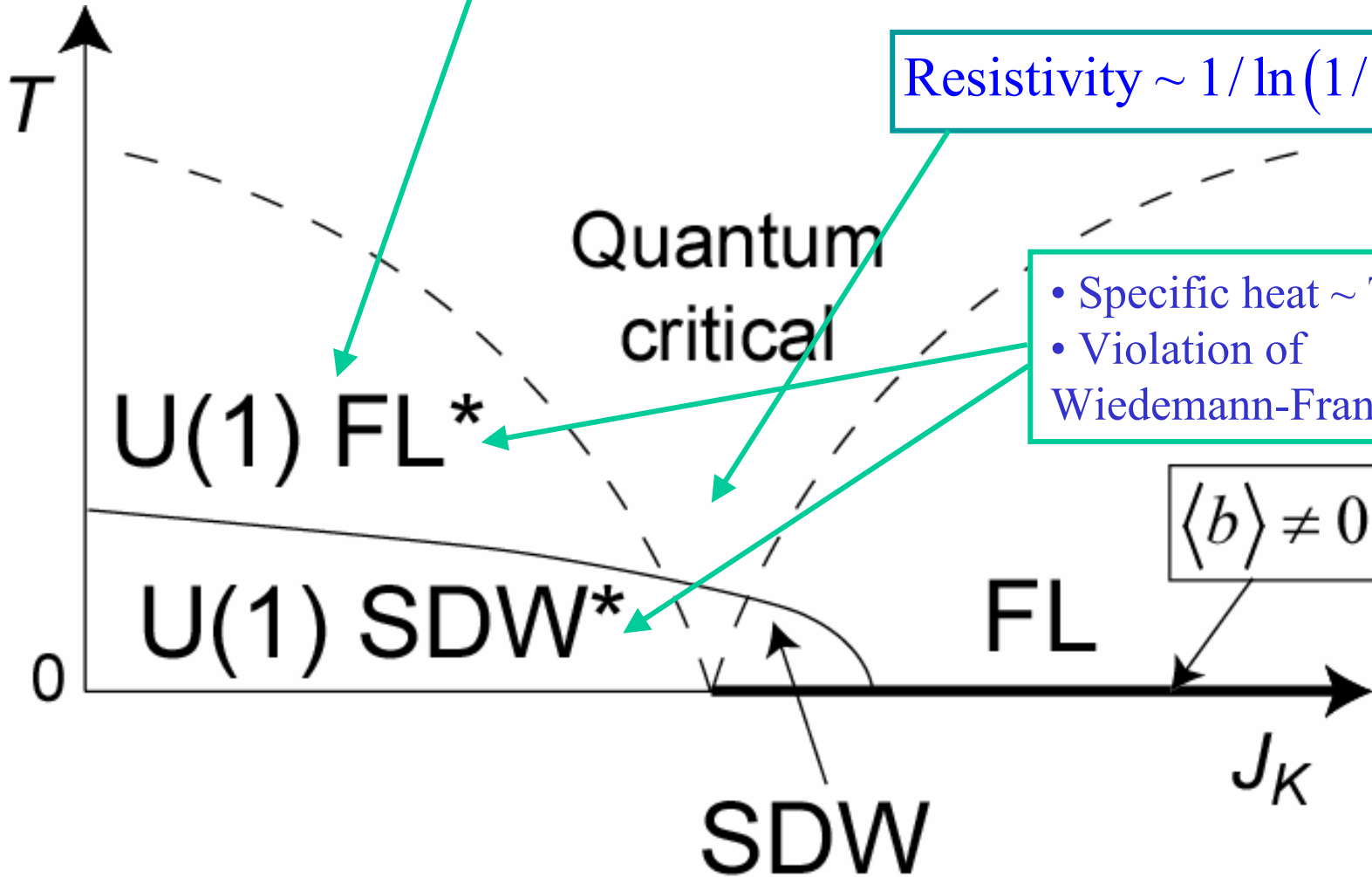
Phase diagram (U(1), $d=3$)

Fermi surface volume does not include local moments

Resistivity $\sim 1/\ln(1/T)$

- Specific heat $\sim T \ln T$
- Violation of Wiedemann-Franz

$\langle b \rangle \neq 0$



Conclusions

I. Cuprate superconductors:

Magnetic/bond order co-exist and compete with superconductivity at low doping.

Theory of quantum phase transitions provides a description of “fluctuating order” in the superconductor.

II. “Hidden order” in heavy fermion systems

Fractionalized states (FL* and SDW*) lead to strongly interacting quantum criticality.