Quantum criticality in insulators, metals and superconductors

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Talk online: Google Sachdev
\[ \langle S_j \rangle = N_1 \cos(K \cdot r_j) + N_2 \sin(K \cdot r_j) \]

Collinear spins: \( N_1 \times N_2 = 0 \)

Non-collinear spins: \( N_1 \times N_2 \neq 0 \)

States on both sides of critical point could be either
(A) Insulators
(B) Metals
(C) Superconductors

Quantum critical point
(A) Insulators
Coupled ladder antiferromagnet
\( S = \frac{1}{2} \) spins on coupled 2-leg ladders

\[
H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

\( 0 \leq \lambda \leq 1 \)


Square lattice antiferromagnet

Experimental realization: $La_2CuO_4$

Ground state has long-range collinear magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$
\begin{align*}
\lambda \text{ close to 0} & \quad \text{Weakly coupled ladders} \\
\text{Paramagnetic ground state} & \quad \langle \vec{S}_i \rangle = 0
\end{align*}
Excitation: $S=1$ exciton (spin collective mode, “triplon”)

Energy dispersion away from wavevector $\vec{K}$

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$S=1/2$ spinons are confined by a linear potential.
Quantum paramagnet
$|\langle \tilde{S} \rangle| = 0$

Neel state
$|\langle \tilde{S} \rangle| = N_0$

Neel order $N_0$

Spin gap $\Delta$

$T=0$
Field theory for quantum criticality

$\lambda$ close to $\lambda_c$ : use “soft spin” field

$$S_b = \int d^2x d\tau \left[ \frac{1}{2} \left( \left( \nabla_x \phi_\alpha \right)^2 + c^2 \left( \partial_\tau \phi_\alpha \right)^2 + (\lambda_c - \lambda) \phi_\alpha^2 \right) + \frac{u}{4!} \left( \phi_\alpha^2 \right)^2 \right]$$

$\phi_\alpha \rightarrow$ 3-component antiferromagnetic order parameter

Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar \omega/k_B T$

$$\chi(\omega,T) = T^n g\left( \hbar \omega/k_B T \right)$$

Exciton scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling $u$

(B) Metals
Spin density wave order in the presence of a Fermi surface
Low energy "paramagnon" excitations near the Fermi surface

Damping by fermionic quasiparticles leads to

\[ S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} |\vec{\phi}(q,\omega)|^2 \left( q^2 + |\omega| + \Gamma(\delta,T) \right) \]


Characteristic paramagnon energy at finite temperature \( \Gamma(0,T) \sim T^p \) with \( p > 1 \).

Arises from non-universal *corrections* to scaling, generated by \( \vec{\phi}^4 \) term.

(C) Superconductors
Co-existence of superconductivity and spin-density wave order
If \( \vec{K} \) does not exactly connect two nodal points, critical theory is as in an insulator.

Otherwise, new theory of coupled excitons and nodal quasiparticles.

Effect of an applied magnetic field
(A) Insulators

Zeeman term: only effect in coupled ladder system

Characteristic field $g\mu_B H = \Delta$, the spin gap

$1 \text{ Tesla} = 0.116 \text{ meV}$

Effect is negligible over experimental field scales
(B) Metals

Weak effects (shifts in phase boundary) of order $H^2$ at small $H$.

(First order) transitions involving changes in Fermi surface topology at large $H$
The suppression of SC order appears to the SDW order as an effective "doping" $\delta$:

$$\delta_{\text{eff}}(H) = \delta - C \frac{H}{H_{c_2}} \ln \left( \frac{3H_{c_2}}{H} \right)$$

Neutron scattering observation of SDW order enhanced by superflow.

\[
\delta_{\text{eff}}(H) = \delta_c \implies H \approx \frac{(\delta - \delta_c)}{\ln\left(\frac{1}{\delta - \delta_c}\right)}
\]

Phase diagram of a superconductor in a magnetic field

Neutron scattering of La$_{2-x}$Sr$_x$CuO$_4$ at $x=0.1$


**Solid line - fit to:**

\[
I(H) = a \frac{H}{H_{c2}} \ln \left( \frac{H_{c2}}{H} \right)
\]

Neutron scattering observation of SDW order enhanced by superflow.

Prediction: SDW fluctuations enhanced by superflow and bond order pinned by vortex cores (no spins in vortices). Should be observable in STM.

\[ H \sim \frac{(\delta - \delta_c)}{\ln(1/(\delta - \delta_c))} \]

Phase diagram of a superconductor in a magnetic field

- SDW
- SC
- SC+SDW

Vortex-induced LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ integrated from 1meV to 12meV

Our interpretation: LDOS modulations are signals of bond order of period 4 revealed in vortex halo


Similar results apply to quantum critical points with other “conventional” (fermion bilinear) order parameters 
*e.g.* charge density wave, orbital currents...
Compact U(1) gauge theory: bond order and confined spinons in $d=2$
Paramagnetic ground state of coupled ladder model
Can such a state with *bond order* be the ground state of a system with full square lattice symmetry?
Write down path integral for quantum spin fluctuations

**Key ingredient: Spin Berry Phases**
Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases

\[ e^{iSA} \]
\( A_{a\mu} \rightarrow \) oriented area of spherical triangle

formed by \( N_a, N_{a+\mu} \), and an arbitrary reference point \( N_0 \)
Change in choice of $n_0$ is like a “gauge transformation”

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

($\gamma_a$ is the oriented area of the spherical triangle formed by $N_a$ and the two choices for $N_0$).

The area of the triangle is uncertain modulo $4\pi$, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the paramagnetic phase.
Simplest effective action for $A_{a\mu}$ fluctuations in the paramagnet

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left( \frac{1}{2e^2} \sum_{\Box} \cos \left( \frac{1}{2} (\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_a \rightarrow \pm 1$ on two square sublattices.

This is compact QED in $d+1$ dimensions with static charges $\pm 1$ on two sublattices.

This theory can be reliably analyzed by a duality mapping.

$d=2$: The gauge theory is always in a confining phase and there is bond order in the ground state.

$d=3$: A deconfined phase with a gapless “photon” is possible.

Bond order in a frustrated $S=1/2$ XY magnet


First *large scale* numerical study of the destruction of Neel order in a $S=1/2$ antiferromagnet with full square lattice symmetry

\[
H = 2J \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijk \rangle \subset \square} \left( S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)
\]

See also C. H. Chung, Hae-Young Kee, and Yong Baek Kim, cond-mat/0211299.
Competing order parameters in the cuprate superconductors

1. Pairing order of BCS theory

   (Bose-Einstein) condensation of $d$-wave Cooper pairs

Orders associated with proximate Mott insulator

2. Collinear magnetic order

3. Bond/charge/stripe order

   (couples strongly to half-breathing phonons)

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* 83, 3916 (1999);
Compact U(1) gauge theory:
Deconfined spinons and quantum criticality in heavy fermion compounds in $d=3$
(talk by Matthias Vojta on Friday 10:00)
A new phase: Fractionalized Fermi Liquid (FL*)

\[ H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj} \]

\( f \)-electrons form a spin liquid with neutral spinon excitations. The state has “topological order”. The topological order can be detected by the violation of Luttinger’s theorem. It can only appear in dimensions \( d > 1 \)

\[ 2 \times \frac{v_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) = (n_T - 1)(\text{mod } 2) \]


Phase diagram \((U(1), d=3)\)

- Sharp transition at \(T=0\) in \(d=3\) compact \(U(1)\) gauge theory; compactness “irrelevant” at critical point

- No transition for \(T>0\) in \(d=3\) compact \(U(1)\) gauge theory; compactness essential for this feature
Phase diagram ($U(1), d=3$)

- Fermi surface volume does not include local moments
- Specific heat $\sim T \ln T$
- Violation of Wiedemann-Franz
- Resistivity $\sim 1/\ln(1/T)$
- Specific heat $\sim T \ln T$
- Violation of Wiedemann-Franz
Conclusions

I. Cuprate superconductors:
Magnetic/bond order co-exist and compete with superconductivity at low doping.
Theory of quantum phase transitions provides a description of “fluctuating order” in the superconductor.

II. “Hidden order” in heavy fermion systems
Fractionalized states (FL* and SDW*) lead to strongly interacting quantum criticality.