Quantum entanglement and the phases of matter

Colloquium Ehrenfestii
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sachdev.physics.harvard.edu
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
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**Band insulators**

An even number of electrons per unit cell
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

Superconductors

\[ E \quad k \]
Modern phases of quantum matter

Not adiabatically connected
to independent electron states:
Modern phases of quantum matter

Not adiabatically connected to independent electron states:

many-particle, long-range quantum entanglement
Quantum superposition and entanglement
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom: \( \uparrow \)

Hydrogen molecule:

\[
\frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\]

Superposition of two electron states leads to non-local correlations between spins
Quantum Entanglement: quantum superposition with more than one particle
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Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
Mott insulator: Triangular lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighbor model has non-collinear Neel order
**Mott insulator: Triangular lattice antiferromagnet**

Quantum “disordered” state with exponentially decaying spin correlations.

non-collinear Néel state

\[ S_c \]

\[ S \]
Mott insulator: Triangular lattice antiferromagnet

Quantum “disordered” state with exponentially decaying spin correlations.

$Z_2$ spin liquid with long-range entanglement.

non-collinear Néel state

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

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Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell

Mott insulator: Triangular lattice antiferromagnet

$\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Topological order in the $\mathbb{Z}_2$ spin liquid ground state

$|\Psi\rangle \Rightarrow$ Ground state of entire system,

$\rho = |\Psi\rangle \langle \Psi|$ 

$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$

Entanglement entropy $S_{EE} = -\text{Tr} \left( \rho_A \ln \rho_A \right)$
Entanglement entropy of a band insulator:

\[ S_{EE} = aL - \exp(-bL) \]

where \( L \) is the perimeter of the boundary between A and B.
Entanglement entropy of a $Z_2$ spin liquid:

$$S_{EE} = aL - \ln(2)$$

where $L$ is the perimeter of the boundary between A and B. The $\ln(2)$ is a universal characteristic of the $Z_2$ spin liquid, and implies long-range quantum entanglement.

Promising candidate: the kagome antiferromagnet

Numerical evidence for a gapped spin liquid:

Young Lee,
APS meeting, March 2012

ZnCu$_3$(OH)$_6$Cl$_2$ (also called Herbertsmithite)
Quantum superposition and entanglement
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)

S=1/2 spins

Examine ground state as a function of \( \lambda \)
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

At large $\lambda$ ground state is a “quantum paramagnet” with spins locked in valence bond singlets

$$= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighor spins are “entangled” with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.
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No EPR pairs
\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
TlCuCl$_3$

Quantum paramagnet at ambient pressure
TlCuCl$_3$

Neel order under pressure

Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

"triplon"
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitations of TiCuCl$_3$ with varying pressure

Excitations of TlCuCl$_3$ with varying pressure

Broken valence bond ("triplon") excitations of the quantum paramagnet

Excitations of TlCuCl$_3$ with varying pressure

Spin waves above the Néel state

Excitations of TlCuCl$_3$ with varying pressure

Higgs boson
First observation of the Higgs boson at the theoretically predicted energy!

S. Sachdev, arXiv:0901.4103

\[ \lambda = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
Quantum critical point with non-local entanglement in spin wavefunction

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Tensor network representation of entanglement at quantum critical point

D-dimensional space

Depth of entanglement

Entanglement entropy

$d$-dimensional space

depth of entanglement
Entanglement entropy

$d$-dimensional space

Most links describe entanglement within $A$

depth of entanglement
Entanglement entropy

$d$-dimensional space

Links overestimate entanglement between A and B
Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.
Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where $\gamma$ is a universal number associated with the quantum critical point.

• Long-range entanglement
• Long-range entanglement

• The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
• Long-range entanglement

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• The theory of the critical point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a CFT3
• Long-range entanglement

• The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

• The theory of the critical point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a CFT3
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
String theory

- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...
A $D$-brane is a $d$-dimensional surface on which strings can end.
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• The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.
A $D$-brane is a $d$-dimensional surface on which strings can end.

The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.

In $d = 2$, we obtain strongly-interacting CFT$3$s. These are “dual” to string theory on anti-de Sitter space: $\text{AdS}_4$. 
- A $D$-brane is a $d$-dimensional surface on which strings can end.
- The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.
- In $d = 2$, we obtain strongly-interacting CFT3s. These are “dual” to string theory on anti-de Sitter space: AdS4.
Tensor network representation of entanglement at quantum critical point

d-dimensional space

depth of entanglement
String theory near a D-brane

Emergent direction of AdS4
Tensor network representation of entanglement at quantum critical point

$d$-dimensional space

Emergent direction of AdS4

Brian Swingle, arXiv:0905.1317
Entanglement entropy

$A$

$d$-dimensional space

Emergent direction of AdS4

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.
AdS\(_{d+2}\) \[\rightarrow\] \(\mathbb{R}^{d,1}\) Minkowski

Emergent holographic direction

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Quantum matter with long-range entanglement

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J. McGreevy, arXiv0909.0518
Quantum matter with long-range entanglement

AdS_{d+2} \quad R^{d,1}

Minkowski

Emergent holographic direction

CFT_{d+1}
AdS_{d+2} \quad \rightarrow \quad \mathbb{R}^{d,1} \quad \text{Minkowski}

\text{Area measures entanglement entropy}

$\mathcal{CFT}_{d+1}$ Quantum matter with long-range entanglement

Emergent holographic direction

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CFT3
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Quantum critical point with non-local entanglement in spin wavefunction

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Classical spin waves

Dilute triplon gas

Quantum critical Neel order

\[ \lambda_c \]

Neel order
Classical spin waves

Dilute triplon gas

Quantum critical

Neel order

$T$

$\lambda$

$\lambda_c$
Thermally excited spin waves

Quantum critical

Thermally excited triplon particles

Neel order
Classical spin waves

Dilute triplon gas

Quantum critical


Thermally excited spin waves

Thermally excited triplon particles

Neel order

$\lambda_c$
Classical spin waves

Quantum critical


Thermally excited spin waves

Thermally excited triplon particles

Short-range entanglement

Neel order

Wednesday, May 9, 2012
Classical spin waves

Dilute triplon gas

Quantum critical


Thermally excited spin waves

Thermally excited triplon particles

Neel order

\( \lambda_c \)
Excitations of a ground state with long-range entanglement

Thermally excited spin waves

Thermally excited triplon particles

Neel order

Quantum critical
Excitations of a ground state with long-range entanglement

Quantum critical

Needed: Accurate theory of quantum critical spin dynamics

Thermally excited spin waves

Thermally excited triplon particles

Neel order

$\lambda_c$
String theory at non-zero temperatures

A 2+1 dimensional system at its quantum critical point
String theory at non-zero temperatures

A “horizon”, similar to the surface of a black hole!

A 2+1 dimensional system at its quantum critical point
Black Holes

Objects so massive that light is gravitationally bound to them.
Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

Black hole horizon

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Quantum Entanglement across a black hole horizon

Black hole horizon
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy).
String theory at non-zero temperatures

A “horizon”, whose temperature and entropy equal those of the quantum critical point

A 2+1 dimensional system at its quantum critical point
String theory at non-zero temperatures

A “horizon”, whose temperature and entropy equal those of the quantum critical point

Friction of quantum criticality = waves falling into black brane

A 2+1 dimensional system at its quantum critical point
String theory at non-zero temperatures

A 2+1 dimensional system at its quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

A “horizon”, whose temperature and entropy equal those of the quantum critical point

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Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
Metals, “strange metals”, and high temperature superconductors

Insights from gravitational “duals”
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Iron pnictides:
a new class of high temperature superconductors
Resistivity \( \sim \rho_0 + A T^\alpha \)


Short-range entanglement
in state with Neel (AF) order

Resistivity
$\sim \rho_0 + AT^\alpha$

Superconductivity

Resistivity $\sim \rho_0 + AT^\alpha$

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Bose condensate of pairs of electrons

Short-range entanglement


*Physical Review B* 81, 184519 (2010)
Superconductivity

Resistivity \sim \rho_0 + AT^\alpha

BaFe_{2}(As_{1-x}P_x)_2

Ordinary metal (Fermi liquid)


Sommerfeld-Bloch theory of ordinary metals

Moments with electron states empty

Moments with electron states occupied

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Sommerfeld-Bloch theory of ordinary metals

Key feature of the theory: the Fermi surface

- Area enclosed by the Fermi surface $A = Q$, the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$. 
Superconductivity

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Resistivity
$\sim \rho_0 + AT^\alpha$

Ordinary metal (Fermi liquid)

Physical Review B 81, 184519 (2010)
$\text{Resistivity } \sim \rho_0 + AT^\alpha$

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Classical spin waves

Quantum critical


Neel order

\( \lambda_c \)
Classical spin waves

Dilute triplon gas

Quantum critical

Ordinary Metal

Neel order
Classical spin waves

Dilute triplon gas

Quantum critical

Strange Metal

Neel order

Ordinary Metal

$\lambda_c$
Strange Metal

BaFe$_2$(As$_{1-x}$P$_x$)$_2$

Resistivity $\sim \rho_0 + A T^\alpha$


Resistivity $\sim \rho_0 + AT^\alpha$

Strange Metal

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$


Excitations of a ground state with long-range entanglement

Resistivity $\sim \rho_0 + AT^\alpha$

Strange Metal

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$


Key (difficult) problem:

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement.
Challenge to string theory:

Describe quantum critical points and phases of metals
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Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals?
Challenge to string theory:

Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals?

Yes

S. Sachdev, Physical Review D 84, 066009 (2011)
Challenge to string theory:

Describe quantum critical points and phases of metals

Do the “holographic” gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals?
Challenge to string theory:

Describe quantum critical points and phases of metals

Do the “holographic” gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals?

Yes, lots of them, with many “strange” properties!

S.-S. Lee, Phys. Rev. D 79, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, Science 325, 439 (2009);
Challenge to string theory:

Describe quantum critical points and phases of metals

How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics?
Challenge to string theory:

Describe quantum critical points and phases of metals

How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics?

Choose the theories with the proper entropy density

Checks: these theories also have the proper entanglement entropy and Fermi surface size!

Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F L) \ln(k_F L)$

for a circular Fermi surface with Fermi momentum $k_F$, where $L$ is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the “1/12” prefactor modified.

AdS$_{d+2}$

Minkowski

CFT$_{d+1}$

Quantum matter with long-range entanglement

Emergent holographic direction
AdS \mathbb{R}^{d,1} + 2 \rightarrow \text{Emergent holographic direction}

\[
\tau \quad \text{Emergent holographic direction}
\]

Quantum matter with long-range entanglement
Abandon conformal invariance, and only require scale invariance at long lengths and times....
Consider the metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i \]
\[ t \rightarrow \zeta^z t \]
\[ ds \rightarrow \zeta^{\theta/d} ds. \]

This identifies \( z \) as the dynamic critical exponent (\( z = 1 \) for “relativistic” quantum critical points).

\( \theta \) is the violation of hyperscaling exponent.
Consider the metric which transforms under rescaling as

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\( \theta \) is the violation of hyperscaling exponent.

The most general choice of such a metric is

\[
d s^2 = \frac{1}{r^2} \left( -\frac{d t^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} d r^2 + d x_i^2 \right)
\]

AdS\( _{d+2} \) corresponds to \( \theta = 0, z = 1 \). We have used reparametrization invariance in \( r \) to choose so that it scales as \( r \rightarrow \zeta^{(d-\theta)/d} r \).
\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2 d(z-1)/(d-\theta)} + \frac{r^2 \theta / (d-\theta)}{} dr^2 + dx_i^2 \right) \]
\[
\begin{multline*}
\frac{1}{r^2} \left( - \frac{dt^2}{r^{2d(z-1)/(d-\theta)} + \frac{r^{2\theta}}{(d-\theta)} dr^2 + dx_i^2} \right)
\end{multline*}
\]

- The thermal entropy density scales as (Stefan-Boltzmann law or “hyperscaling”)

\[
S \sim T^{(d-\theta)/z}.
\]

where \(d-\theta\) is the effective dimension. A Fermi surface has excitations which disperse only in one direction, and so we require \(\theta = d - 1\).
\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2d(z-1)/(d-\theta)} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]

- The thermal entropy density scales as (Stefan-Boltzmann law or “hyperscaling”)
  \[ S \sim T^{(d-\theta)/z}. \]
  where \( d - \theta \) is the effective dimension. A Fermi surface has excitations which disperse only in one direction, and so we require \( \theta = d - 1 \).

- The entanglement entropy, \( S_E \), of an entangling region with boundary surface ‘area’ \( \Sigma \) scales as
  \[
  S_E \sim \begin{cases} 
  \Sigma, & \text{for } \theta < d - 1 \\
  \Sigma \ln \Sigma, & \text{for } \theta = d - 1 \\
  \Sigma^{\theta/(d-1)}, & \text{for } \theta > d - 1 
  \end{cases}
  \]

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2d(z-1)/(d-\theta)} + r^2\frac{\theta}{(d-\theta)} dr^2 + dx_i^2 \right) \]

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\Sigma^{\theta/(d-1)}, & \text{for } \theta > d - 1 
\end{cases}
\]

So we have log violation of area law for \( \theta = d - 1 \), just as expected for a Fermi surface!

- The null energy condition implies \( z \geq 1 + \frac{\theta}{d} \). This implies \( z \geq 3/2 \) in \( d = 2 \). Remarkably the value \( z = 3/2 \) is obtained from a field theory of a Fermi surface coupled to an emergent gauge field !!
Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.
Simplest examples of long-range entanglement are in insulating antiferromagnets: $\mathbb{Z}_2$ spin liquids and quantum critical points.
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with long-range quantum entanglement.
Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”
anti-de Sitter space

Emergent holographic direction

$AdS_{d+2}$

$R^{d,1}$ Minkowski
anti-de Sitter space