Quantum transitions of *d*-wave superconductors in a magnetic field

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Anatoli Polkovnikov
Subir Sachdev
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Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

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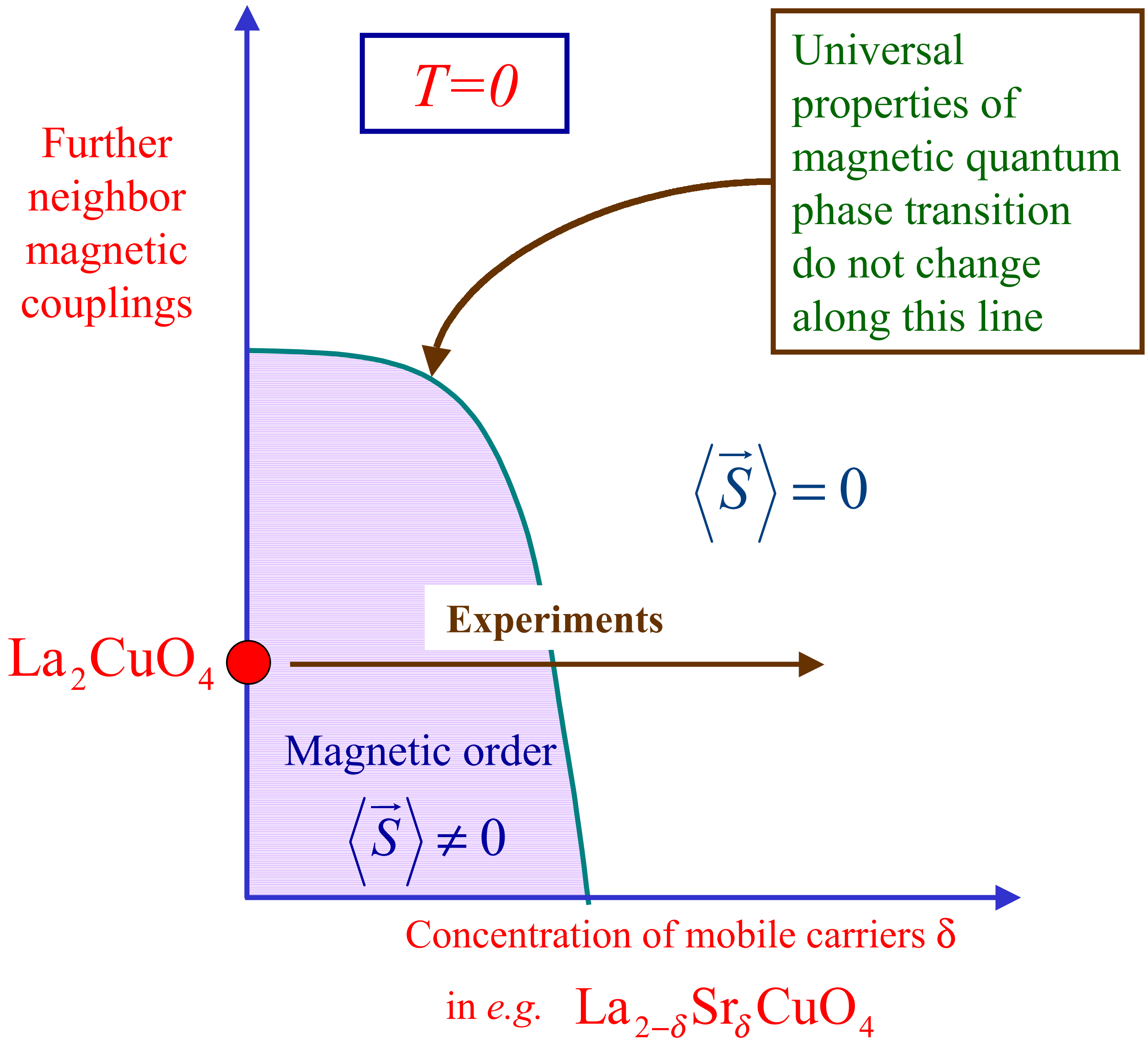

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

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<u>Outline</u>

I. Magnetic ordering transitions in two-dimensional quantum antiferromagnets: NMR measurements of Zn/Li impurities and neutron scattering measurements of phonon spectra.

II. Effect of magnetic field on antiferromagnetic order in superconductor.

    Comparison of predictions with neutron scattering experiments.

III. Conclusions

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I. Magnetic quantum transition in the insulator (δ=0)

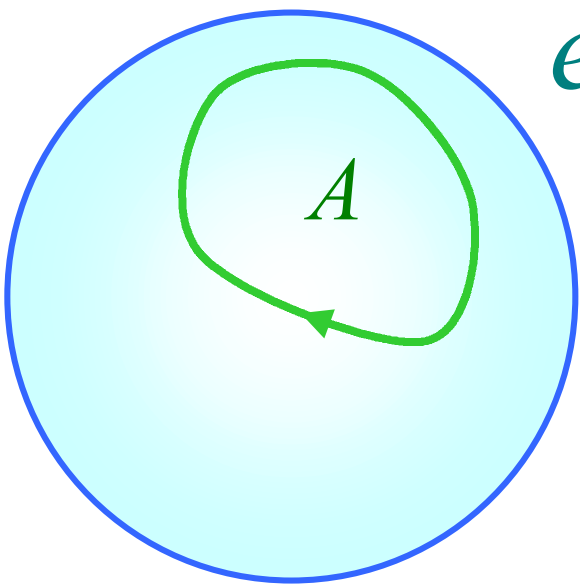
Neel order parameter ϕ_α $\alpha = 1,2,3$

Action:

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



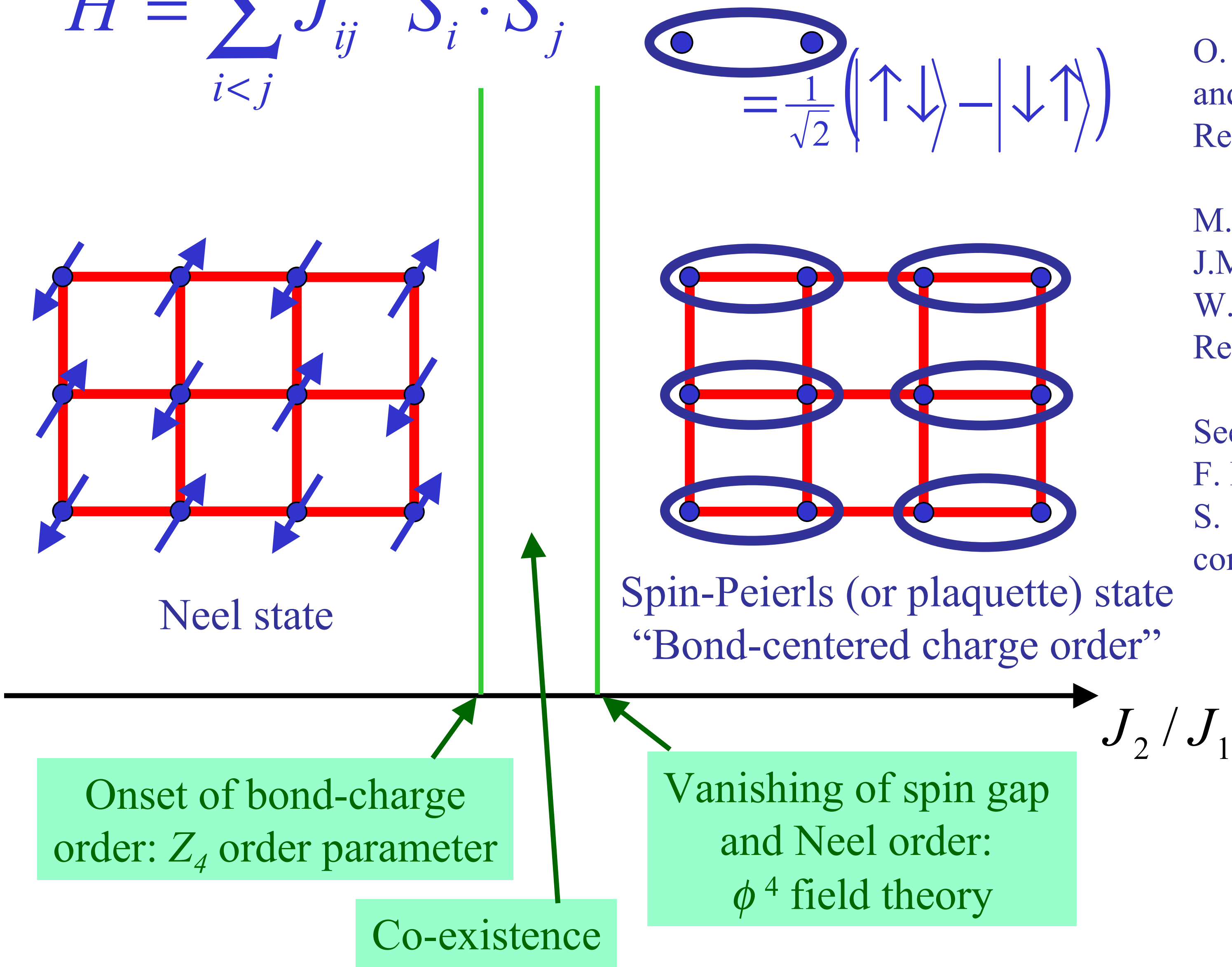
$$e^{iSA}$$

Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
e.g. spin-Peierls or plaquette
order (need not be quasi-1d)
"Dual order parameter"

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).

See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .

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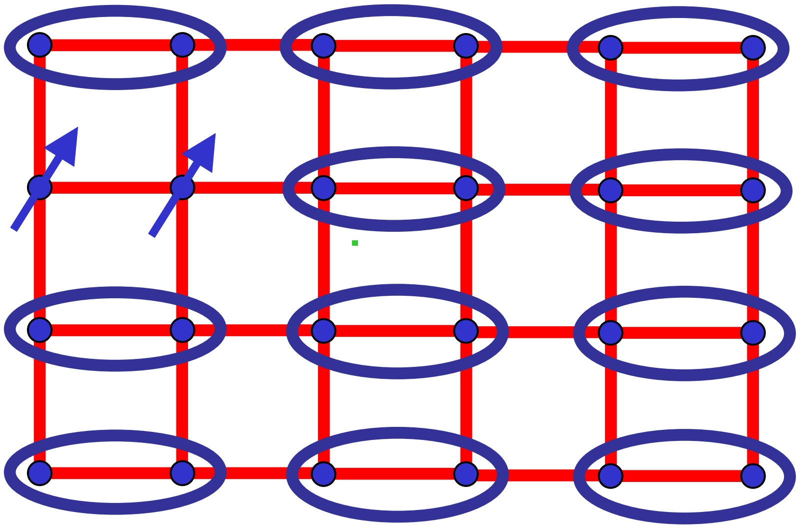
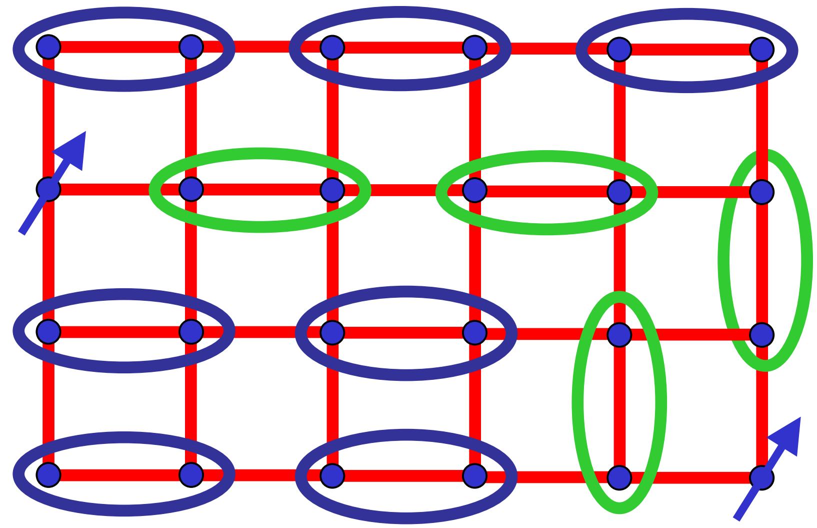
Properties of paramagnet with bond-charge-order

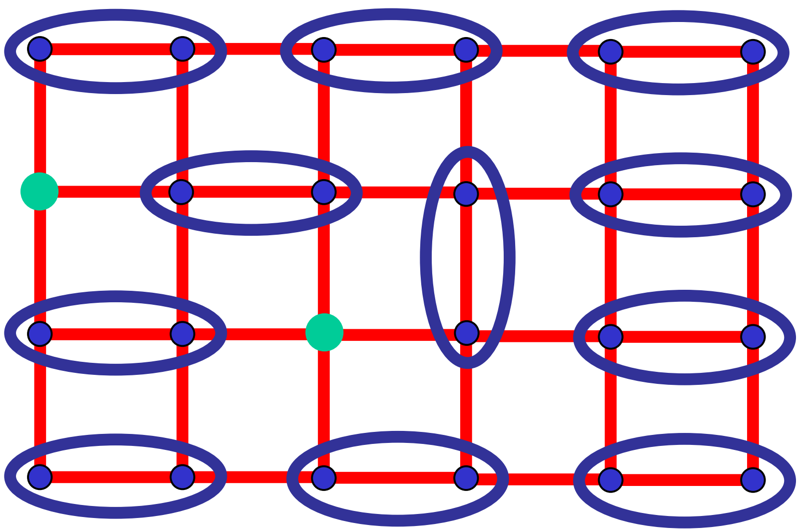
Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta} \qquad \Delta \rightarrow \text{Spin gap}$$





S=1/2 spinons are ***confined*** by a linear potential.



S=0 holes can similarly be ***confined in pairs***.

E. Fradkin and S. Kivelson, Mod. Phys. Lett B **4**, 225 (1990).

S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

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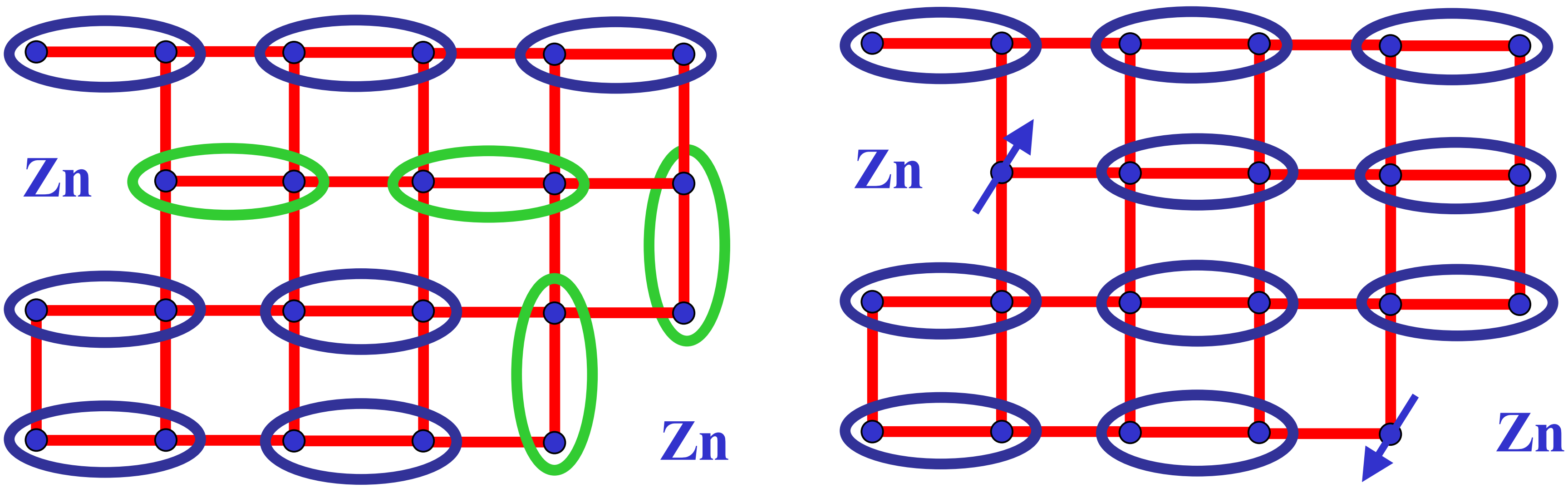
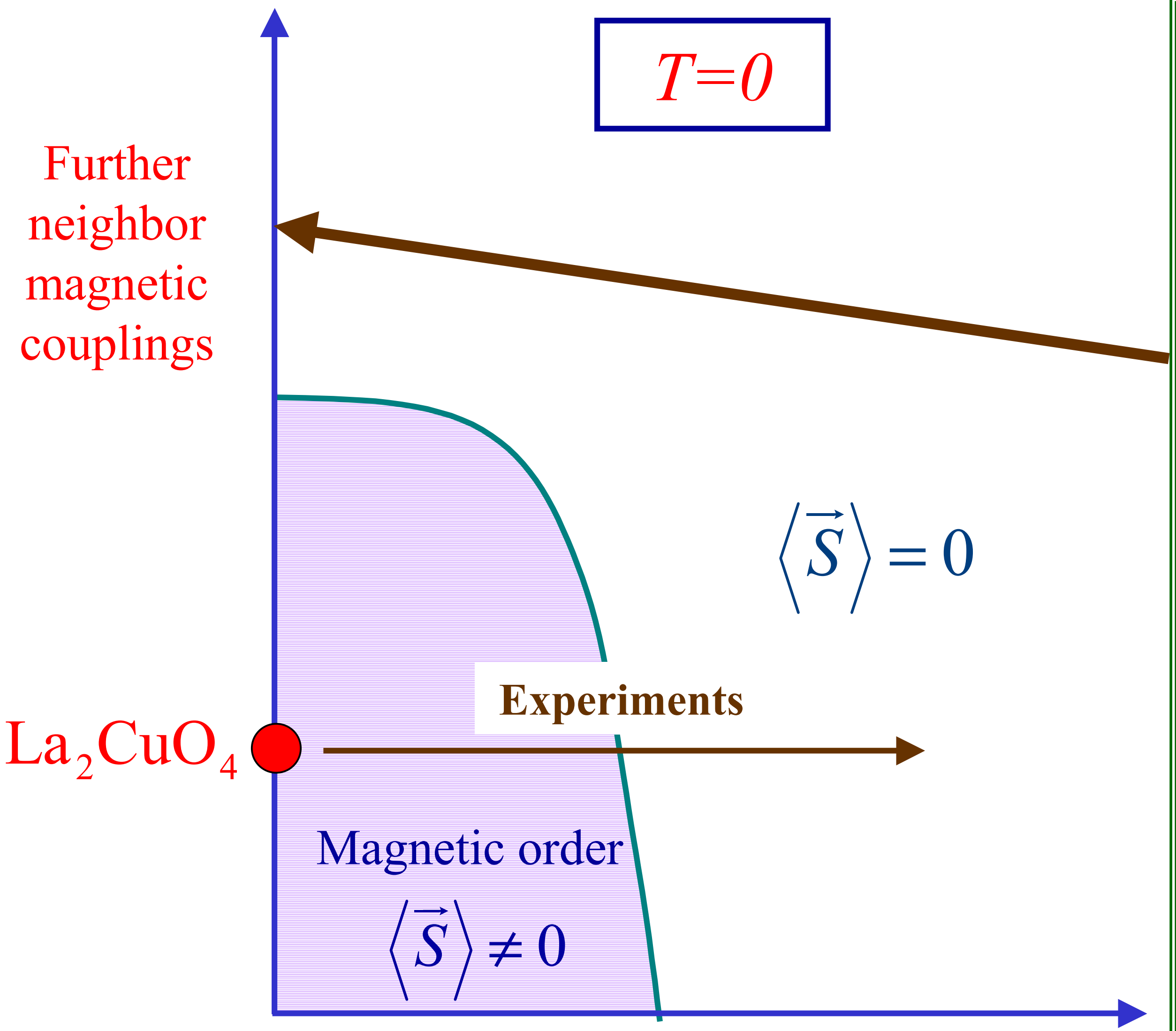
Effect of static non-magnetic impurities (Zn or Li)



Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).
J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001)
D. L. Sisson, S. G. Doettinger, A. Kapitulnik, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **61**, 3604 (2000).



Concentration of mobile carriers δ

in *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

Summary:

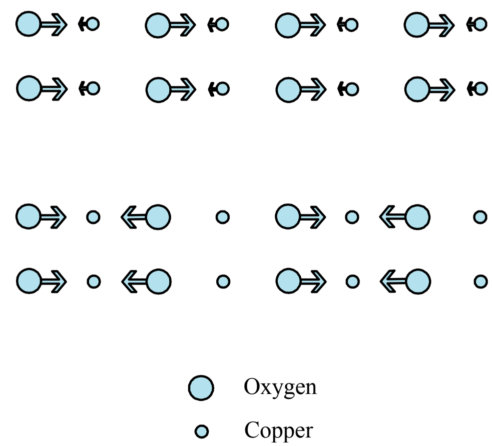
Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
   (resonance mode)
2. Broken translational symmetry:- bond charge order.
   (phonon spectra)
3. Pairing of holes.
4. S=1/2 moments near non-magnetic impurities.
   (NMR experiments)

Properties of paramagnetic insulator survive for a finite range of δ

Neutron scattering measurements of phonon spectra

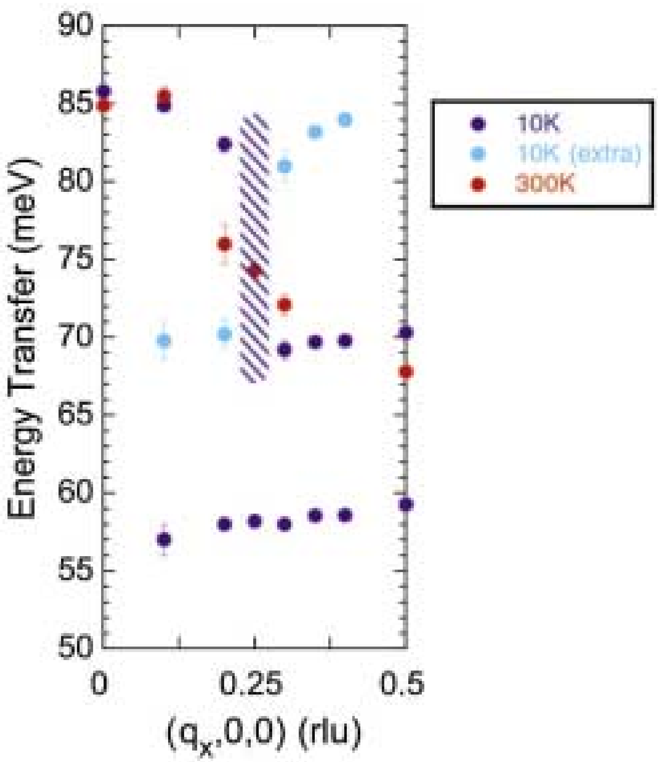
Discontinuity in the dispersion of a LO phonon of the O ions at wavevector $k = \pi/2$: evidence for bond-charge order with period $2a$



$k = 0$

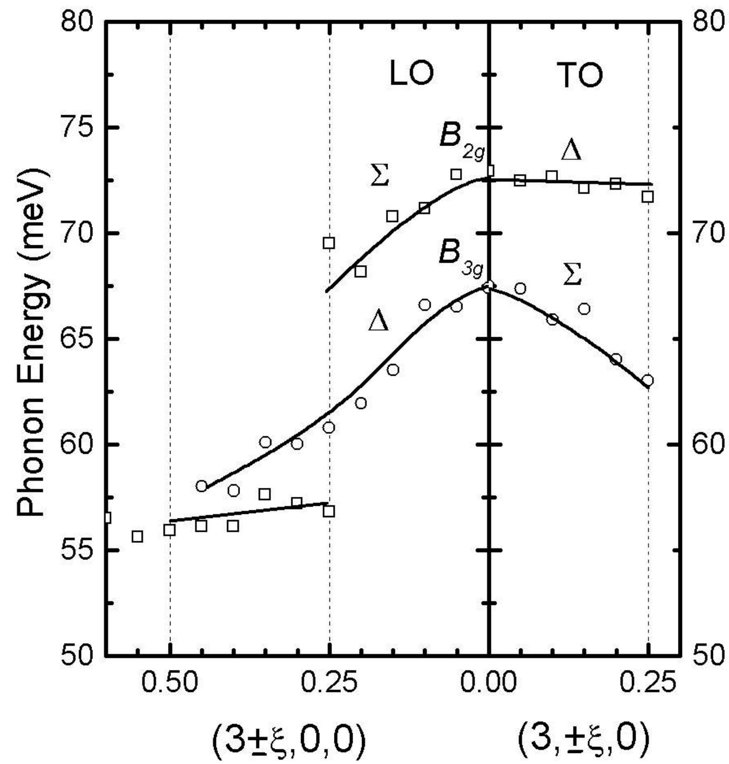
$k = \pi$

$La_{1.85}Sr_{0.15}CuO_4$



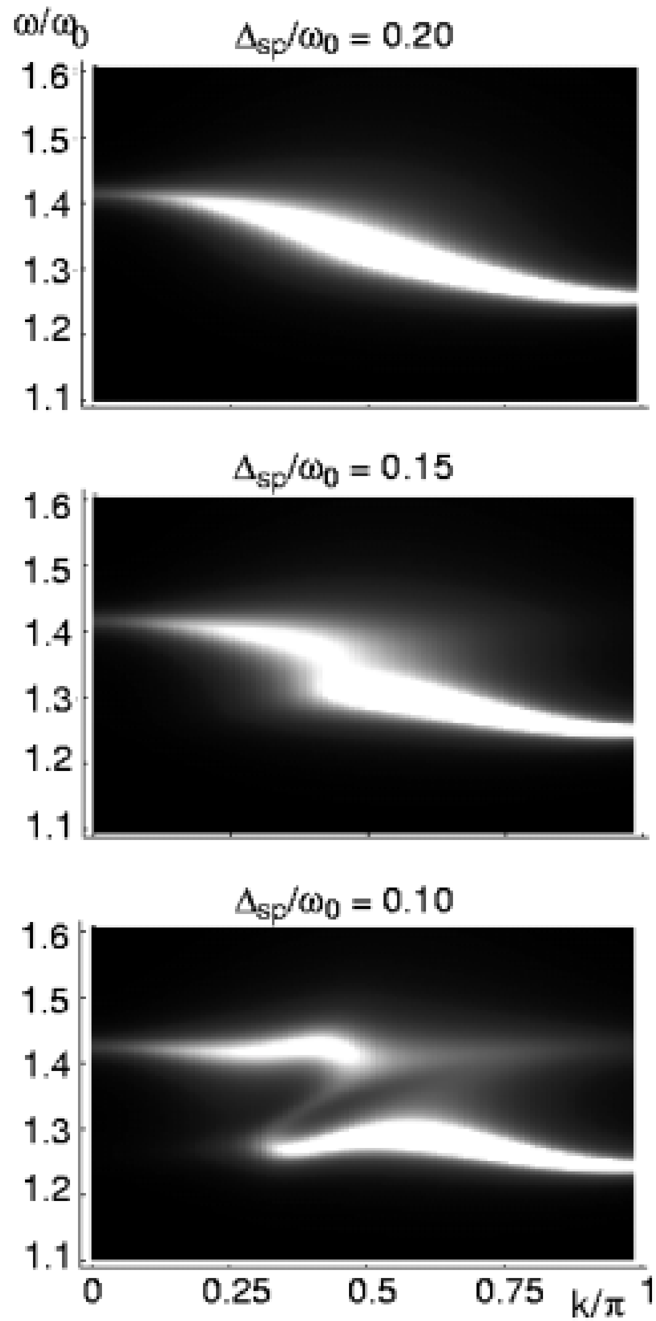
R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj. G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (
L. Pintschovius and M. Braden, Phys. Rev. B **60**, R15039 (1999).

$YBa_2Cu_3CuO_{6.95}$

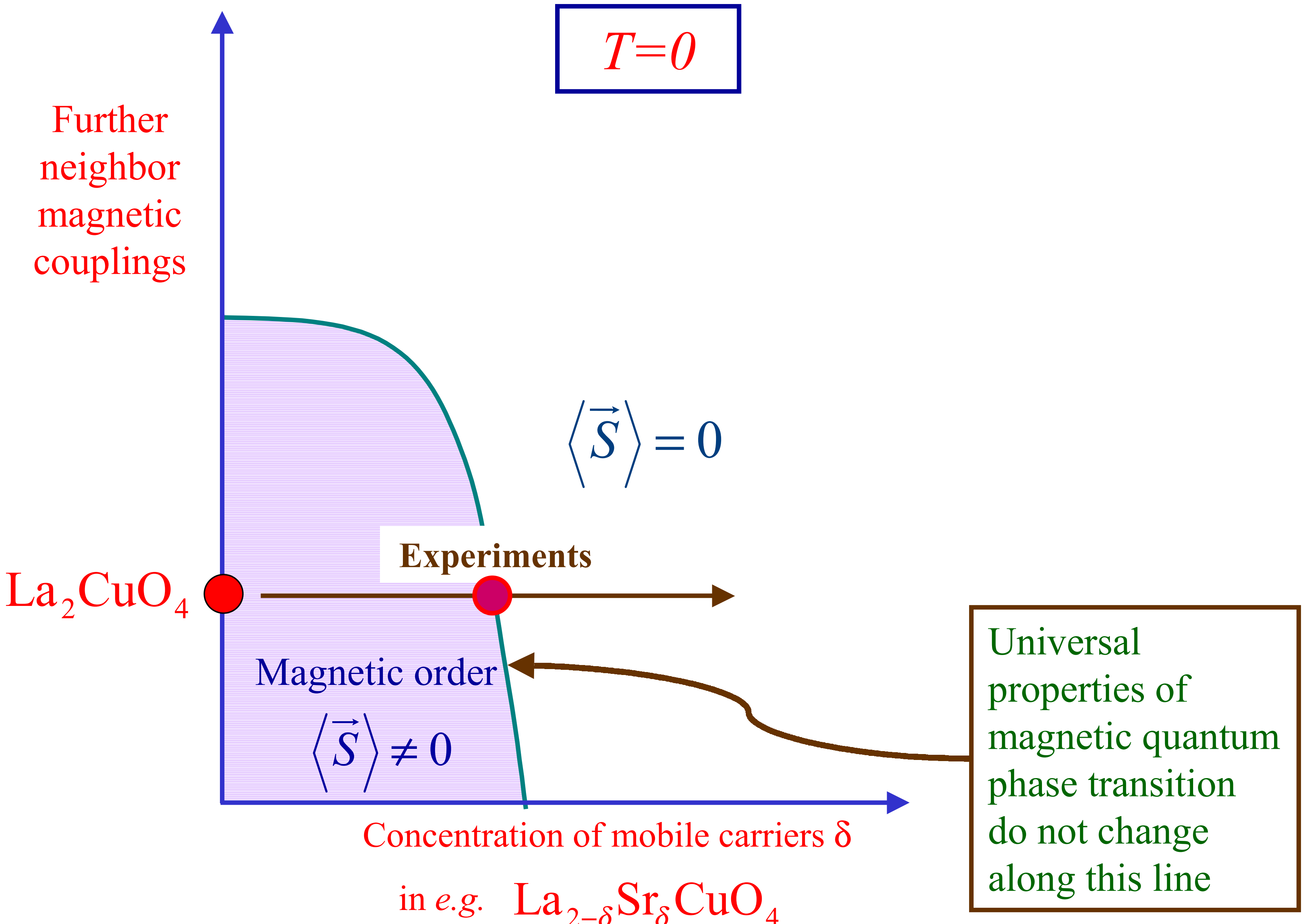


R. J. McQueeney, T. Egami, J.-H. Chung, Y. Petrov, M. Yethiraj, M. Arai, Y. Inamura, Y. Endoh, C. Frost and F. Dogan, cond-mat/0105593.

Computation of phonon damping by non-linear coupling to fluctuating spin-Peierls order with period *2a*

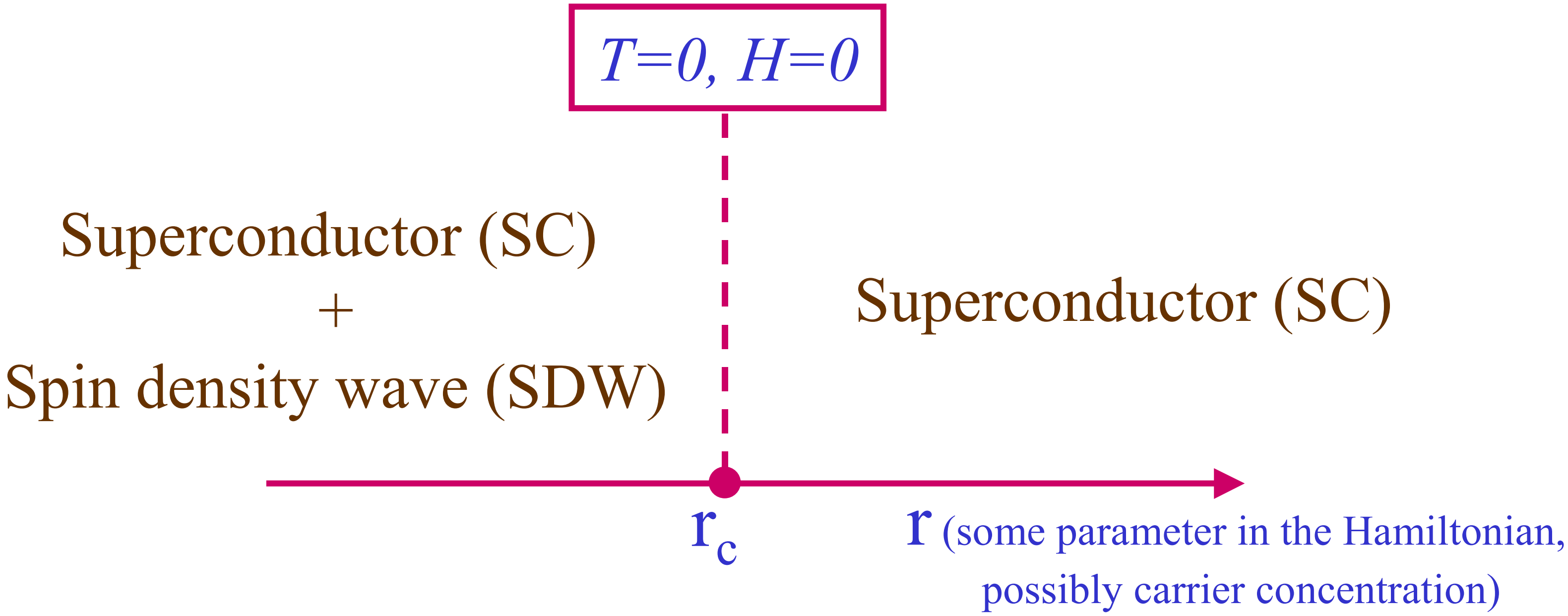


K. Park and S. Sachdev, cond-mat/0104519



S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

III. Effect of magnetic field on SDW order in SC phase



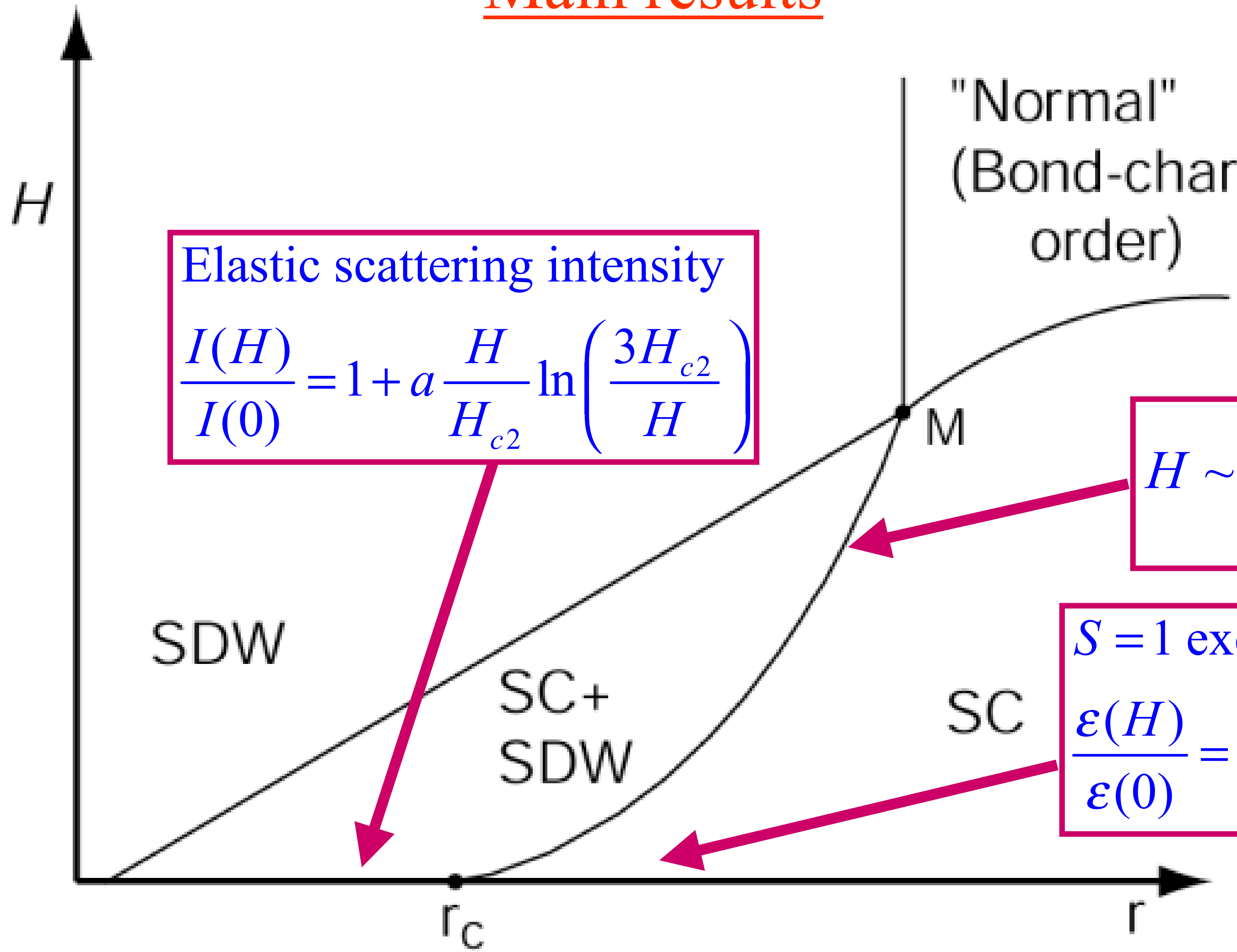
Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001).
Y. Sidis, C. Ulrich, P. Bourges, *et al.*, cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al*., preprint.

Main results

(extreme Type II superconductivity)

$T=0$



Elastic scattering intensity

$$\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(r-r_c)}{\ln\left(1/\left(r-r_c\right)\right)}$$

$S=1$ exciton energy

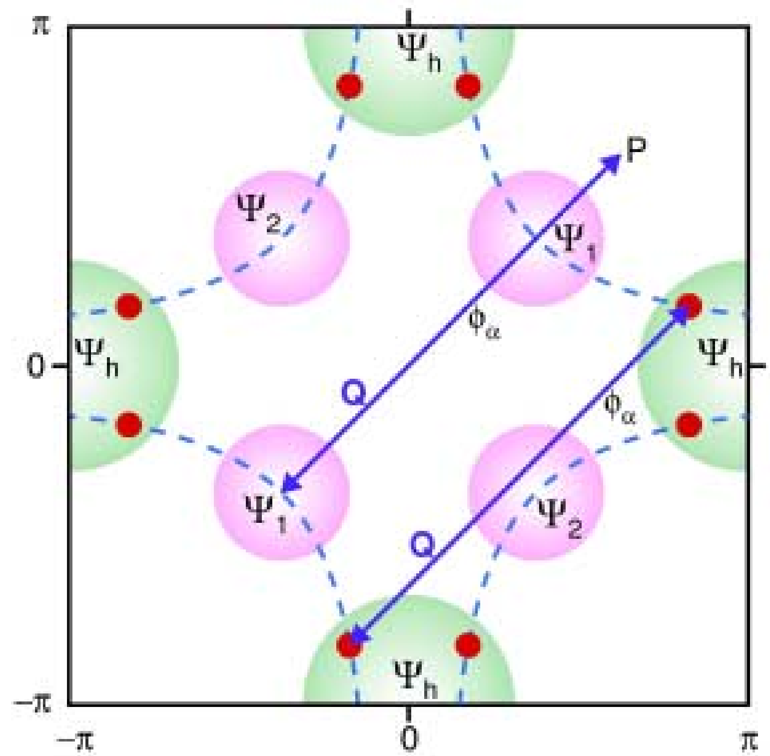
$$\frac{\varepsilon(H)}{\varepsilon(0)} = 1 - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

- All functional forms are **exact**.
- Similar results apply to other competing orders *e.g.* SC + staggered flux

E. Demler, S. Sachdev, and Y. Zhang, Phys. Rev. Lett. in press, cond-mat/0103192.

Structure of quantum theory

• Charge-order is not critical: can neglect Berry phases.



• Generically, momentum conservation prohibits decay of $S=1$ exciton ϕ_α into $S=1/2$ fermionic excitations at low energies. Virtual pairs of fermions only renormalize parameters in the effective action for ϕ_α .

• Zeeman coupling only leads to corrections at order H^2

• Simple Landau theory couplings between ϕ_α and superconducting order ψ are allowed (S.-C. Zhang, Science **275**, 1089 (1997)), *e.g.*:

$$V\left(\phi_\alpha^2\right) \rightarrow V\left(\phi_\alpha^2\right) + \lambda \phi_\alpha^2 \left|\psi\right|^2$$

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

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- Theory should account for quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a static Ginzburg-Landau theory

Action $F_{GL}/T + \mathcal{S}_b + \mathcal{S}_c$

$$F_{GL} = \int d^2x \left[-|\psi|^2 + \frac{|\psi|^4}{2} + \left|(\nabla_x - iA)\psi\right|^2 \right]$$

$$\mathcal{S}_c = \int d^2x d\tau \left[\frac{v}{2} \phi_\alpha^2 |\psi|^2 \right]$$

$$\mathcal{S}_b = \int d^2x \int_0^{1/T} d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

Self-consistent Hartree theory of quantum spin fluctuations (large N limit)

$$\chi(x,x',\omega_n)\delta_{\alpha\beta} = \left\langle \phi_\alpha(x,\omega_n)\phi_\beta(x',-\omega_n)\right\rangle$$

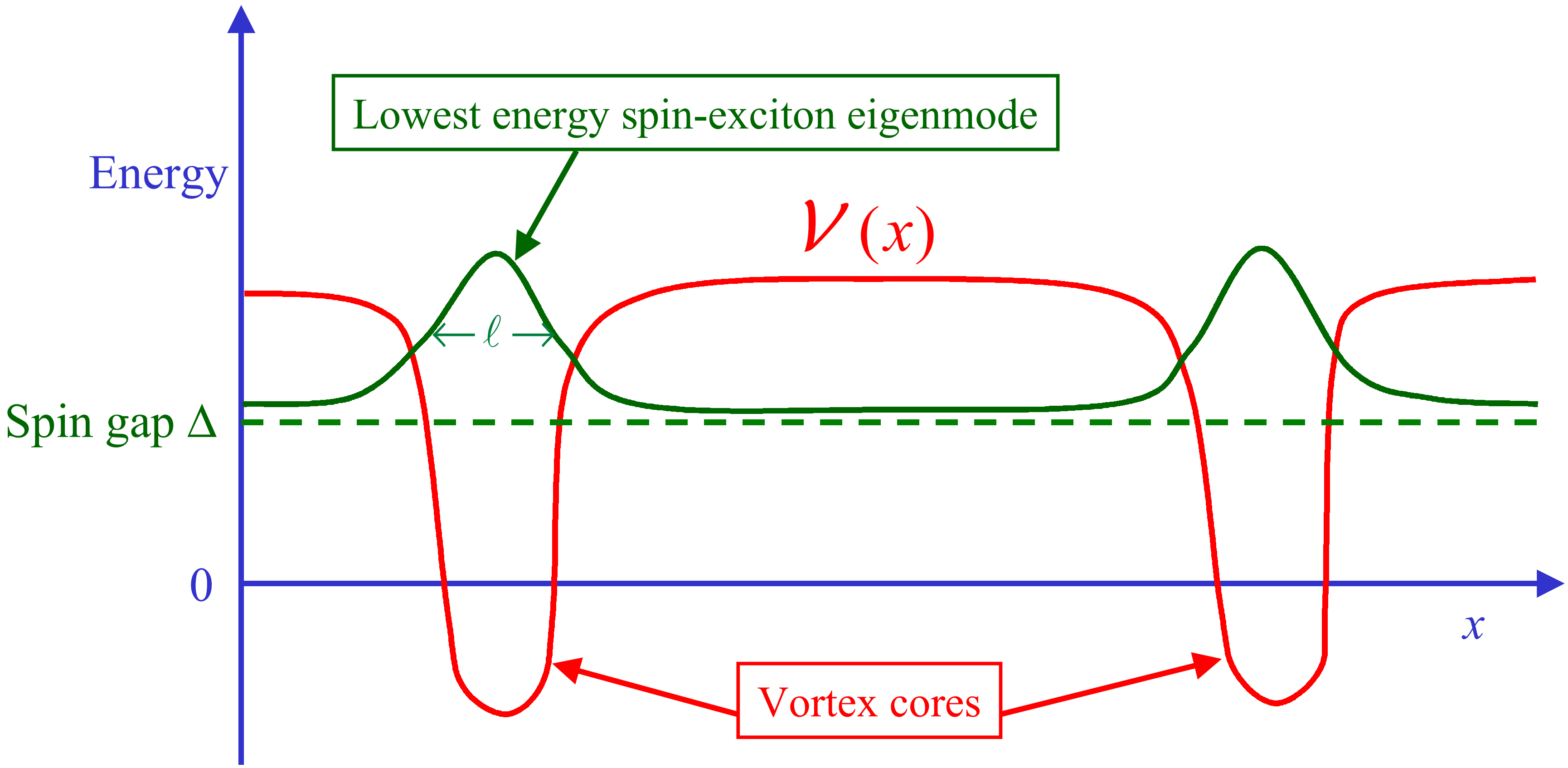
$$\left(\omega_n{}^2 - c^2\nabla_x^2 + \mathcal{V}(x)\right)\chi(x,x',\omega_n) = \delta(x-x')$$

$$\mathcal{V}(x) = r + v\left|\psi(x)\right|^2 + (NgT/6)\sum_{\omega_n}\chi(x,x,\omega_n)$$

$$\left[-1+\left|\psi(x)\right|^2 - \left(\nabla_x - i\vec{A}\right)^2\right]\psi(x) + (NvT/2)\sum_{\omega_n}\chi(x,x,\omega_n)\psi(x) = 0$$

$\mathcal{V}(x) \rightarrow$ local classical energy of spin fluctuations; can become negative in vortex cores for $v > 0$.

However, spin gap remains finite because of quantum fluctuations



As $\Delta \to 0$, $\ell \to \infty$, because of self interaction, g, of spin excitations.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L765 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

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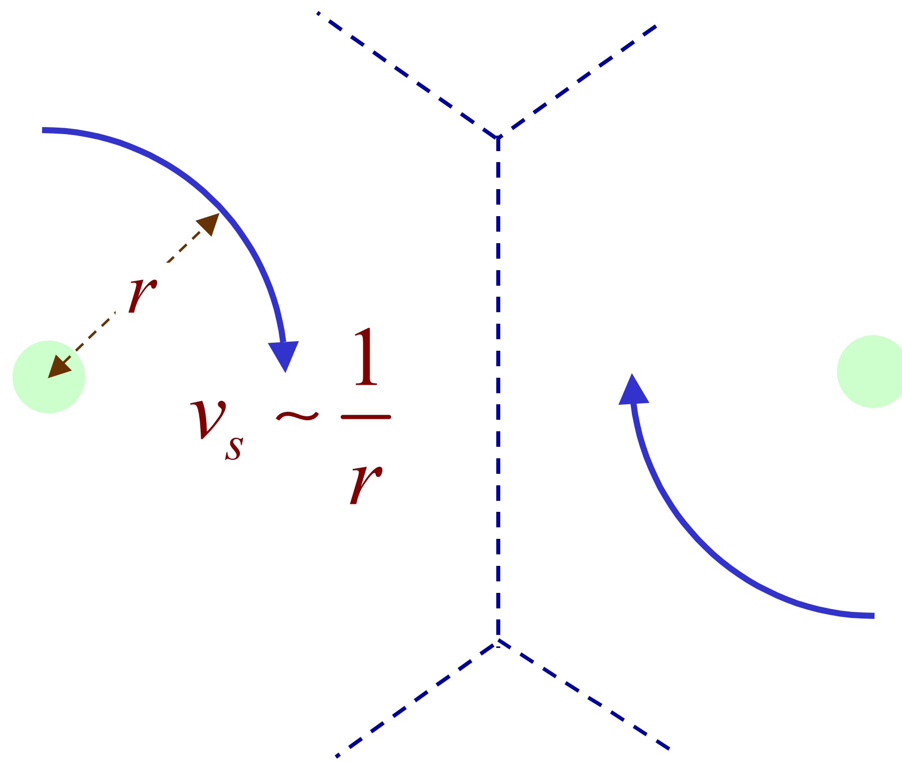
Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



$$v_s \sim \frac{1}{r}$$

Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

See D. P. Arovas *et al*., Phys. Rev. Lett. **79**, 2871 (1997)
for a different viewpoint.

Influence of $\psi(x)$ on extended spin eigenmodes:

$$|\psi(x)| = 1 - \frac{1}{2x^2}$$ outside each vortex core because of superflow kinetic energy

$$\left\langle |\psi(x)|^2 \right\rangle = 1 - \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

In SC phase, spin gap obeys:

$$\Delta(H) = \Delta(0) - \frac{24\pi c^2 v}{Ng\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

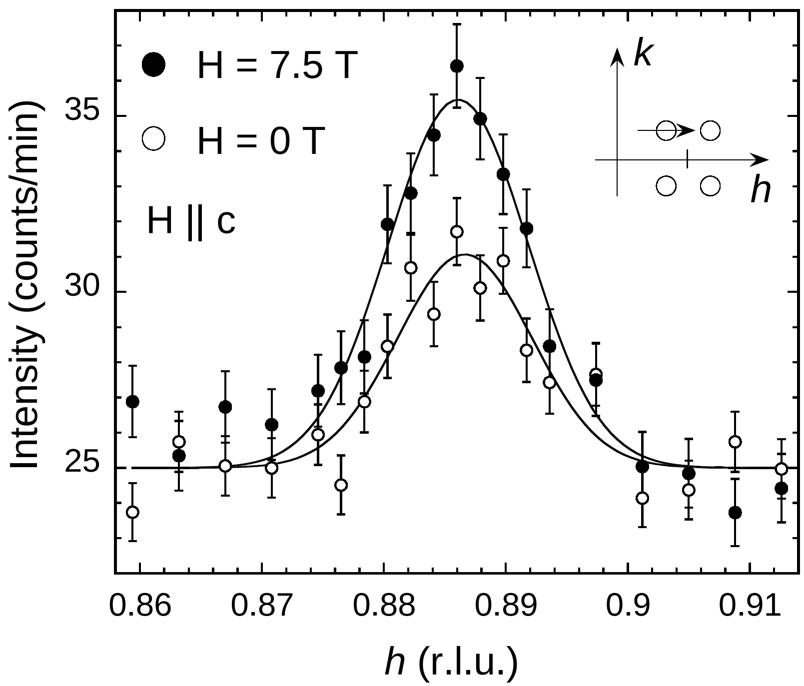
In SC+SDW phase, intensity of elastic scattering obeys:

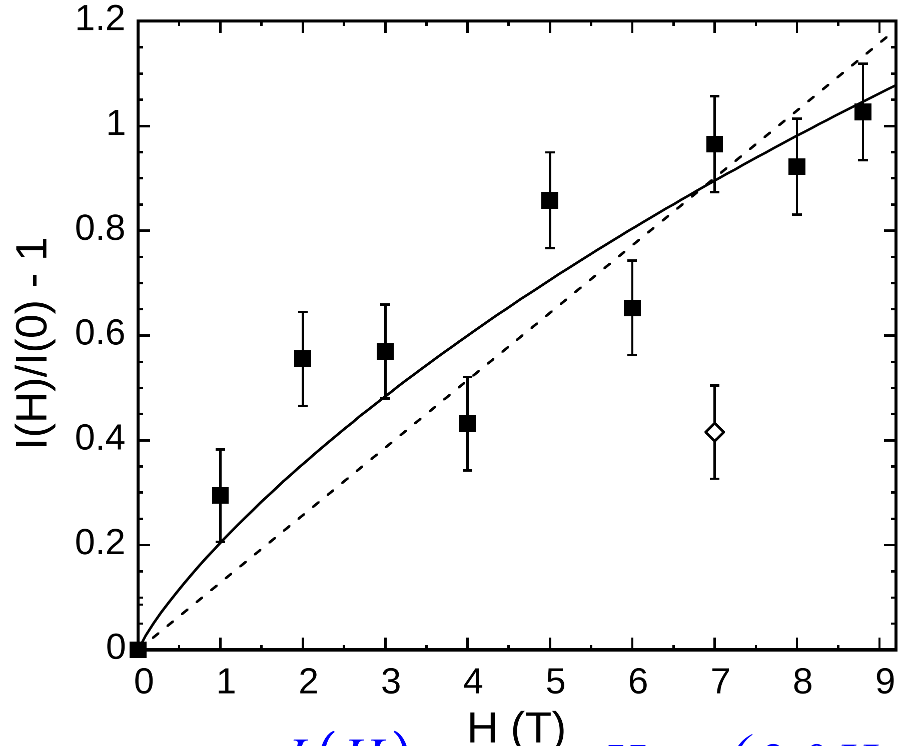
$$I(H) = I(0) + \frac{6v}{g\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas,

M. A. Kastner, and R.J. Birgeneau, preprint.





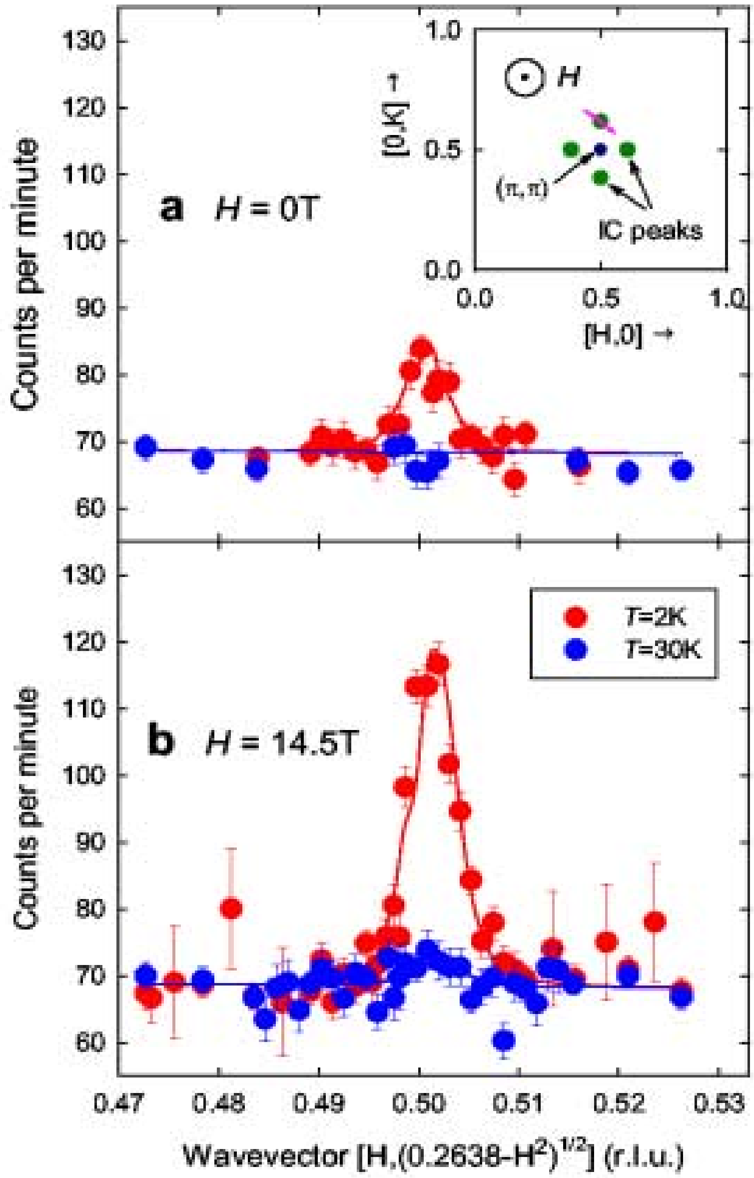
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

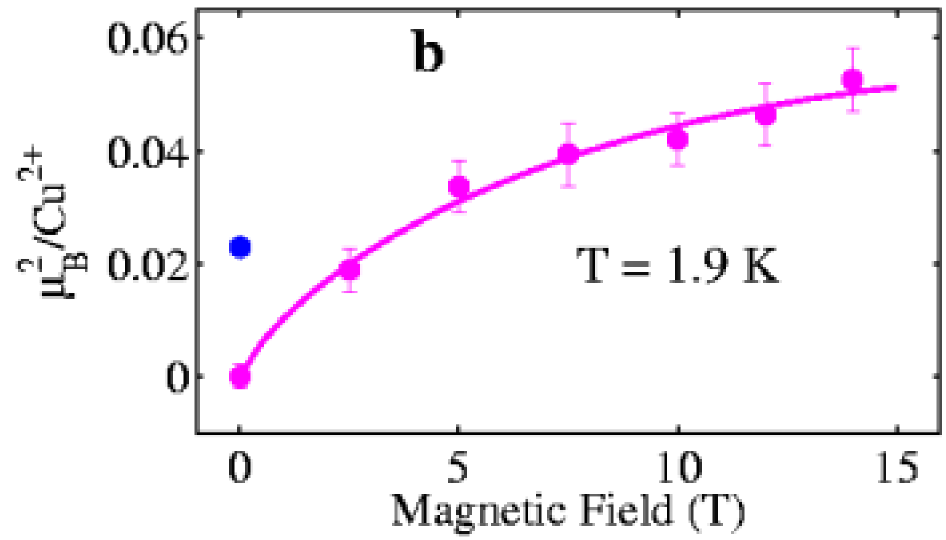
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*

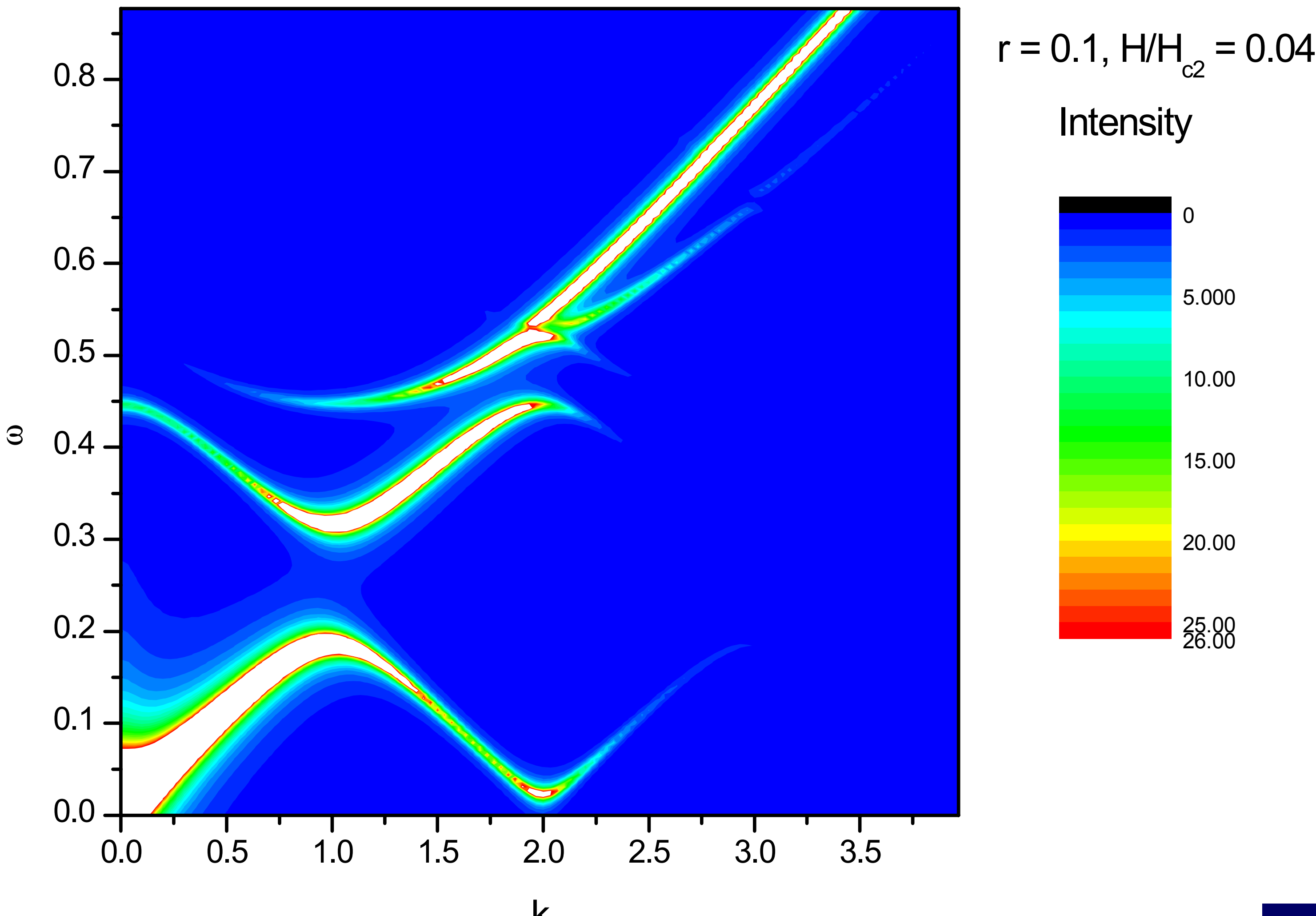




Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

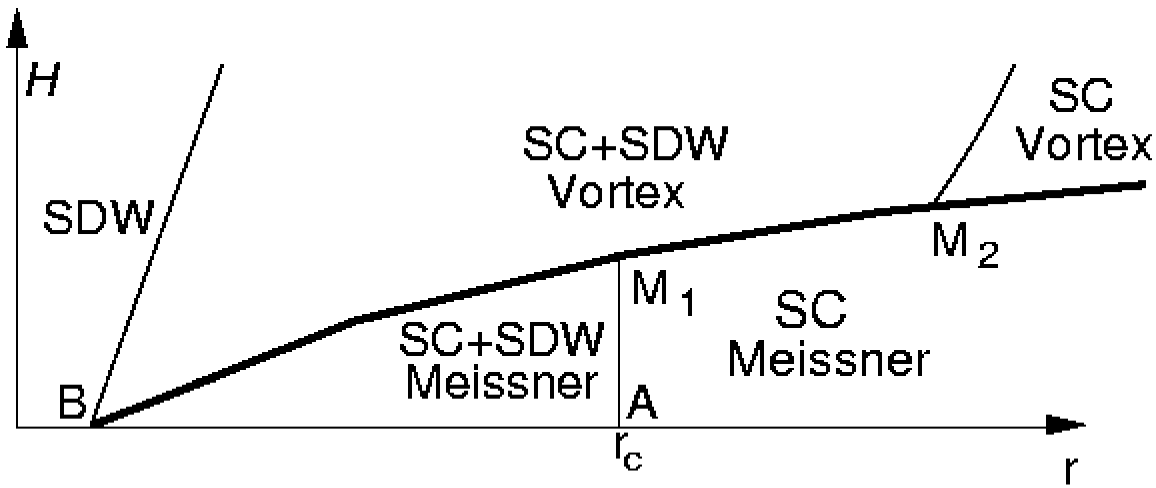
Presence of vortex lattice leads to supermodulation in the spin exciton spectrum

Computation of spin susceptibility $\chi''(k,\omega)$ in self-consistent large N theory of ϕ_α fluctuations in a vortex lattice

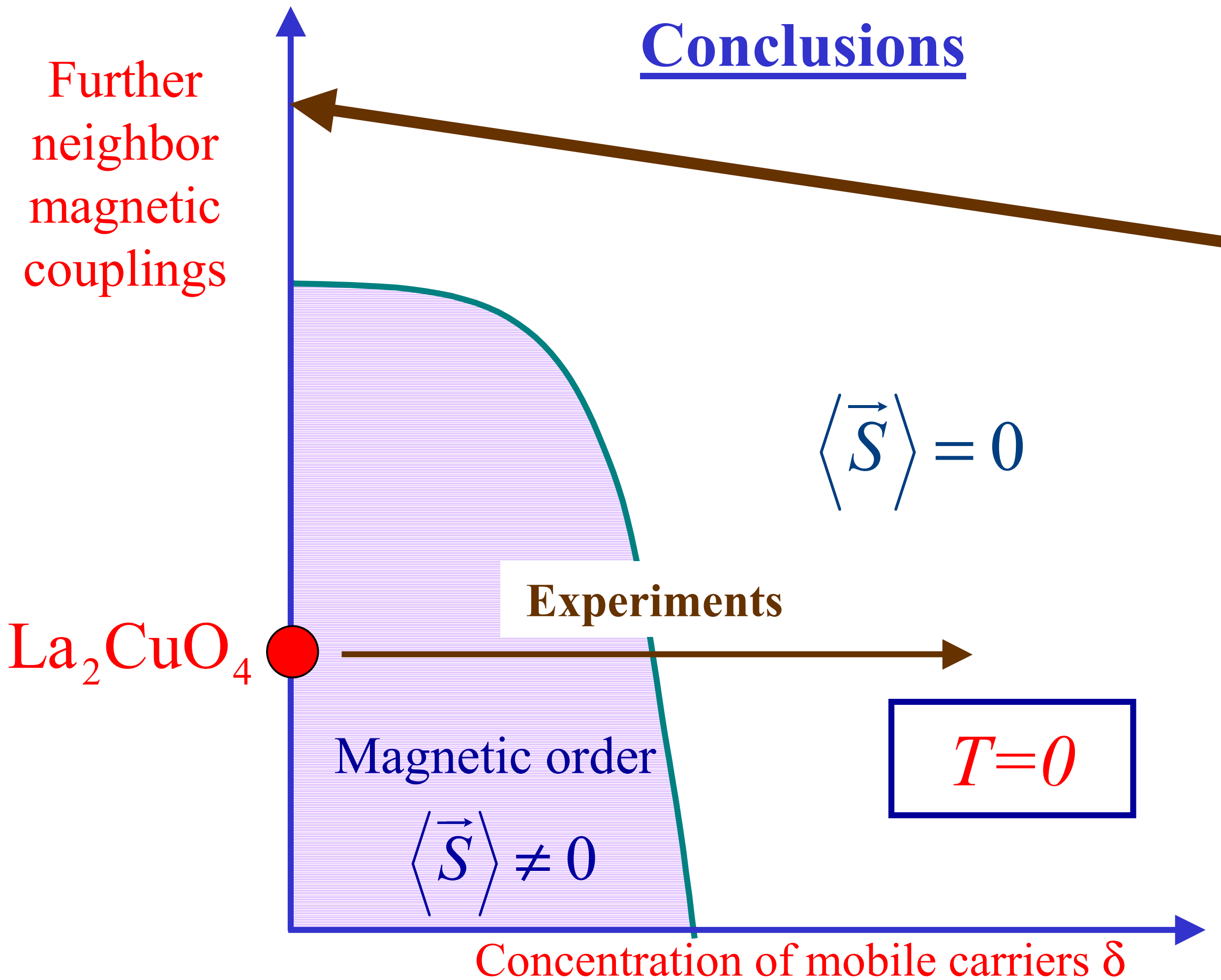


2π / (vortex lattice spacing)

Consequences of a finite London penetration depth (finite κ)



Conclusions



Confined, paramagnetic Mott insulator has

1. Stable *S*=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. Pairing of holes.
4. *S*=1/2 moments near non-magnetic impurities

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations