Spooky action at a distance: in black holes and in the lab

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In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.
Quantum entanglement
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Interference of water waves
The double slit experiment

Bullets
The double slit experiment

Send electrons through the slits
The double slit experiment

Interference of electrons
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Is the electron a wave?

Interference of electrons
Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Unlike water waves, electrons arrive one-by-one (so is it like a particle?)

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons

No interference when you watch the electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Each electron passes through both slits!

Interference of electrons
Let $|L\rangle$ represent the state with the electron in the left slit.
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Principles of Quantum Mechanics: 1. Quantum Superposition
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.

Actual state of each electron is $|L\rangle + |R\rangle$. 
Quantum Entanglement: quantum superposition with more than one particle
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Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom: \( |\uparrow\rangle \)

Hydrogen molecule:

\[
\frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)
\]
Quantum Entanglement: quantum superposition with more than one particle
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Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away.
Quantum entanglement
Quantum entanglement

Black holes
Quantum entanglement

Black holes
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius $R = \frac{2GM}{c^2}$

$G$ Newton’s constant, $c$ velocity of light, $M$ mass of black hole
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

Black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away 0.1 seconds later!
The ring-down time \( \frac{8\pi GM}{c^3} \sim 8 \text{ milliseconds} \). Curiously, for essentially all types of black holes, the ring-down time equals

\[ \frac{\hbar}{k_B T_H}, \]

\( \hbar \) Planck’s constant, \( k_B \) Boltzmann’s constant.
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.
Quantum entanglement

Superconductors

Black holes
High temperature superconductors

$YBa_{2}Cu_{3}O_{6+x}$
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
The table shows the comparison of the measured slope of the experimental value, as discussed in the text and Supplementary Information, to the Planckian limit (Eq. 1; penultimate column), with the value predicted by the Planckian limit (Eq. 1; last column), although the ratio varies by a factor 5, the ratio of the experimental value, as discussed in the text and Supplementary Information, to the Planckian limit (Eq. 1; penultimate column), is 1.0 in all cases, so that the Planckian limit (Eq. 1; last column) varies by the same amount.

**Table 1 | Slope of T-linear resistivity vs Planckian limit in seven materials.**

<table>
<thead>
<tr>
<th>Material</th>
<th>$n$ ($10^{27}$ m$^{-3}$)</th>
<th>$m^*$ ($m_0$)</th>
<th>$A_1/d$ ($\Omega / K$)</th>
<th>$\hbar/(2e^2 T_F)$ ($\Omega / K$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi2212</td>
<td>$p = 0.23$</td>
<td>6.8</td>
<td>8.4 ± 1.6</td>
<td>8.0 ± 0.9</td>
<td>7.4 ± 1.4</td>
</tr>
<tr>
<td>Bi2201</td>
<td>$p \sim 0.4$</td>
<td>3.5</td>
<td>7 ± 1.5</td>
<td>8 ± 2</td>
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<tr>
<td>LSCO</td>
<td>$p = 0.26$</td>
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<td>9.8 ± 1.7</td>
<td>8.2 ± 1.0</td>
<td>8.9 ± 1.8</td>
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<td>Nd-LSCO</td>
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<td>12 ± 4</td>
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Slope of T-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

Current flow without quasiparticles.
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Slope of $T$-linear resistivity vs Planckian limit in seven materials.

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\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}
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Remarkably similar universality in strange metals and black holes!
Quantum entanglement

Black holes

Superconductors
Quantum entanglement

Black holes

Superconductors

A “toy model” which describes both a superconductor and a black hole!
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites
Entangle electrons pairwise randomly

The SYK model
The SYK model

Entangle electrons pairwise randomly
Entangle electrons pairwise randomly
Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly
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Entangle electrons pairwise randomly
The SYK model

This describes both a superconductor and a black hole!
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space ($\zeta$) and one time dimension.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. The quantum versions of Maxwell’s and Einstein’s equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. This has led to a deeper understanding of entanglement in superconductors and of Hawking’s black hole information “paradox.”
Quantum entanglement

Black holes

Superconductors

A “toy model” which describes both a superconductor and a black hole!