Metal-to-metal quantum phase transitions not described by symmetry breaking orders II

Strongly Correlated Quantum Materials and High Temperature Superconductors

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Talk online: sachdev.physics.harvard.edu
High temperature superconductors

\[ \text{CuO}_2 \text{ plane} \]

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor


We use angle-dependent magneto-resistance (ADMR) to measure the Fermi surface of the cuprate La$_{1.6-\delta}$Nd$_{0.4}$Sr$_x$CuO$_4$. Above the critical doping $p^*$—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below $p^*$, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a $Q = (\pi, \pi)$ wavevector. While static $Q = (\pi, \pi)$ antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

$p > p_c$ Large Fermi surface

$p < p_c$ Reconstructed Fermi surface
The remarkable underlying ground states of cuprate superconductors


Figure 6

Across the quantum critical point. a) Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p=0.08$ and $p'=0.19$.

b) Normal-state Hall number $n_H$ in the $T=0$ limit as a function of doping, in YBCO (blue circles (21), $p'=0.19$) and Nd-LSCO (red squares (4), $p'=0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

5. PSEUDOGAP PHASE

DOS: Density of states ($N_F$);

$E$: Condensation energy

$H_{c1}$: Lower critical field

$H_{c2}$: Residual linear term in the specific heat, $C(T)$ at $T=0$, purely electronic

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p'$; 2) the opening of a partial spectral gap below $T'$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p'$. The new data show that the pseudogap does not simply cause a loss of DOS below $p'$; instead, there is huge peak in the DOS at $p'$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p'$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).
Hidden magnetism at the pseudogap critical point of a high temperature superconductor
Nature Physics doi: 10.1038/s41567-020-0950-5

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Quasi-static magnetism in the pseudogap state of La_{2-x}Sr_{x}CuO_{4}.
Temperature – doping phase diagram representing \( T_{\text{min}} \), the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of \( T_{\text{min}} \) in zero-field, the dashed line (brown area) represents the extrapolated \( T_{\text{min}}(B=0) \). While not exactly equal to the freezing temperature \( T_{f} \) (see Fig. 2), \( T_{\text{min}} \) is closely tied to \( T_{f} \) and so is expected to have the same doping dependence, including a peak around \( p = 0.12 \) in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).
1. Metal-metal transition in the Kondo Lattice

2. Metal-metal transition in a one-band model
   A. FL* model of the pseudogap
   B. Ancilla qubits and ghost Fermi surfaces

3. Random $J$ model (insulator)
   \textit{RG analysis and exact exponent}

4. Random $t$-$J$ model (metals)
   \textit{Numerics, RG analysis and exact exponents}
Why random and all-to-all couplings?

Randomness is present in the real system. Randomness self-averages (except for certain correlators in the spin-glass phase) — Green's functions are the same on every site. The pseudogap-Fermi liquid transition is primarily a transition in many-body entanglement which survives presence of randomness. Introducing randomness removes the "distractions" of other parameters. The problem maps onto a model of a "quantum impurity" in a self-consistent environment. Closely related models are obtained in non-random models in the limit of large spatial dimension in extended dynamical mean-field theory. Analytic and numeric progress is possible, and we can compare their results!
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- Analytic and numeric progress is possible, and we can compare their results!
Random \( t-J \) model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j
\]

We consider the hole-doped case, with no double occupancy.

\[
\alpha = \uparrow, \downarrow, \quad \{ c_{i\alpha}, c_{j\beta}^\dagger \} = \delta_{ij} \delta_{\alpha \beta}, \quad \{ c_{i\alpha}, c_{j\beta} \} = 0
\]

\[
\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha \beta} c_{j\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i,\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p
\]

\[
J_{ij} \text{ random}, \quad \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2
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\[
t_{ij} \text{ random}, \quad \overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2
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Random \( t-J \) model

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Random $t\!-\!J$ model

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1. Random $J$ model (insulator)
   \textit{RG analysis and exact exponent}

2. SYK criticality
   \textit{Time reparameterizations and spectral functions}

3. Random $t$-$J$ model (metals)
   \textit{RG analysis and exact exponents}

4. Numerical results
   \textit{QMC and exact diagonalization}
Random $J$ model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j$$
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$J_{ij}$ random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

Fermionic spinons
Random $J$ model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} b_{i\beta}$, \quad \sum_{\alpha} b_{i\alpha}^\dagger b_{i\alpha} = 1$

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Bosonic spinons
Random $J$ model (insulator)

$$
Z = \int \mathcal{D} \vec{S}(\tau) \delta(\vec{S}^2 - 1) e^{-S_B - S_J}
$$

$$
S_B = \frac{i}{2} \int_0^1 du \int d\tau \vec{S} \cdot \left( \frac{\partial \vec{S}}{\partial T} \times \frac{\partial \vec{S}}{\partial u} \right)
$$

$$
S_J = -\frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau')
$$

---

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Random $J$ model (insulator)

\[ Z = \int \mathcal{D}\tilde{S}(\tau) \delta(\tilde{S}^2 - 1) e^{-S_B - S_J} \]

\[ S_B = \frac{i}{2} \int_0^1 du \int d\tau \tilde{S} \cdot \left( \frac{\partial \tilde{S}}{\partial \tau} \times \frac{\partial \tilde{S}}{\partial u} \right) \]

\[ S_J = -\frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \tilde{S}(\tau) \cdot \tilde{S}(\tau') . \]

From this action we compute

\[ \overline{Q}(\tau - \tau') = \frac{1}{3} \left\langle \tilde{S}(\tau) \cdot \tilde{S}(\tau') \right\rangle_Z \]

and then impose the self-consistency condition

\[ Q(\tau) = \overline{Q}(\tau). \]

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Random $J$ model (insulator): RG

We assume a power-law decay

\[ Q(\tau) \sim \frac{\gamma^2}{|\tau|^\alpha}. \]

Ignore the self-consistency condition for now. We decouple the $S(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic ($\phi_a, a = 1 \ldots 3$) bath. Then the problem reduces to the Hamiltonian

\[ H_{\text{imp}} = \gamma S_a \phi_a(0) + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right] \]

where $\pi_a$ is canonically conjugate to the field $\phi_a$, and $\phi_a(0) \equiv \phi_a(x = 0)$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and then we need $\alpha = d - 1$. 
Random $J$ model (insulator): RG

- The $\beta$-function of $\gamma$ can be computed order-by-order in $\epsilon = 2 - \alpha$

\[
\frac{d\gamma}{dl} = \frac{\epsilon \gamma}{2} - \gamma^3.
\]

- There is an attractive fixed point at $\gamma = \gamma^* = O(\sqrt{\epsilon})$.

- Because of the quantized Berry phase (Wess-Zumino-Witten) term, the renormalization of the coupling $\gamma$ is given only by the wavefunction renormalization. We can then prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in $\epsilon$. 

S. Sachdev, Physica C 357, 78 (2001)
Random $J$ model (insulator): RG

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- The self-consistency condition therefore yields

$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}.$$  

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- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.
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- The self-consistency condition therefore yields
  \[
  \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}.
  \]
  to all orders in $\epsilon$. 

This exponent can be understood from a SYK theory of spinons $f$, with $G_f \sim 1/\sqrt{\tau}$.
1. Random $J$ model (insulator)
   *RG analysis and exact exponent*

2. SYK criticality
   *Time reparameterizations and spectral functions*

3. Random $t$-$J$ model (metals)
   *RG analysis and exact exponents*

4. Numerical results
   *QMC and exact diagonalization*
The complex SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{a,b,c,d=1}^{N} U_{ab;cd} c_a^\dagger c_b^\dagger c_c c_d - \mu \sum_a c_a^\dagger c_a \]

\[ c_a c_b + c_b c_a = 0 \quad , \quad c_a c_b^\dagger + c_b c_a^\dagger = \delta_{ab} \]

\[ Q = \frac{1}{N} \sum_a c_a^\dagger c_a \]

\( U_{ab;cd} \) are independent random variables

with \( U_{ab;cd} = 0 \) and \( |U_{ab;cd}|^2 = U^2 \)

\[ G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau) \]

\[ \sum = \]

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Random $J$ model (insulator): SU($M$) symmetry

Express the spin operator in terms of fermionic spinons $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large $M$ limit we obtain the SYK equations for the spinon Green’s function $G$ and self energy $\Sigma$; similar results apply for bosonic spinons.

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
**Key properties**

1. The ground state is ‘critical’ and there are no quasiparticles.

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SYK criticality

A. Kitaev, KITP talk (2015)
J. Maldacena and D. Stanford, PRD 94, 106002 (2016)
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3. There is a non-zero extensive entropy as $T \to 0$

\[
\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = S_0(Q) \neq 0
\]

This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-NS_0}$ down to the ground state.
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4. Dynamic scaling of equilibrium and non-equilibrium properties in a universal time $\sim \hbar / (k_B T)$

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)
Syk criticality

Key properties

1. The ground state is 'critical' and there are no quasiparticles.
2. There is an emergent time reparameterization symmetry which is softly broken at high energies.
3. There is a non-zero extensive entropy as $\lim_{T \to 0} N \frac{S}{N} = S_0$, where $S_0$ is not due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N S_0}$ down to the ground state.
4. Dynamic scaling of equilibrium and non-equilibrium properties in a universal time $\sim T/k_B T$.
5. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent SL(2,R) symmetry. Specifically, the entropy is $S(T)/N = S_0(Q) + \gamma T$, where $\gamma$ propotional to the co-efficient of the Schwarzian.

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6. Spin correlations in the random $J$ model decay as

$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J.Ye, PRL 70, 3339 (1993)
The leading corrections to the critical spinon Green’s function arise from the time reparameterization soft mode, and these take the form

\[
\langle f_\alpha(\tau)f_\alpha^+(0) \rangle \sim \left[ \frac{\pi T}{\sin(\pi T \tau)} \right]^{1/2} \left( 1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)
\]

where \( \Phi_{\text{non-conformal}}(T\tau) \) is a computable (in the large \( M \) limit) scaling function, and \( \alpha_G \) is universally proportional to the co-efficient \( \alpha_S \) of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD 94, 106002 (2016)
Random $J$ model (insulator): SU($M$) symmetry

The local dynamic spin susceptibility, $\chi_L(i\omega_n) = \int_0^{1/T} d\tau \langle \vec{S}_i(\tau) \vec{S}_i(0) \rangle e^{i\omega_n \tau}$, obeys

$$\text{Im} \chi_L(\omega) \propto \tanh \left( \frac{\omega}{2T} \right)$$

A. Georges and O. Parcollet, PRB 59, 5341 (1999)
Random $J$ model (insulator): $SU(M)$ symmetry

The local dynamic spin susceptibility, $\chi_L(i\omega_n) = \int_0^{1/T} d\tau \langle \vec{S}_i(\tau) \vec{S}_i(0) \rangle e^{i\omega_n \tau}$, obeys

$$\mathrm{Im} \chi_L(\omega) \propto \tanh \left( \frac{\omega}{2T} \right) \left[ 1 - \alpha_S \omega \tanh \left( \frac{\omega}{2T} \right) + \ldots \right]$$

From the time reparameterization soft-mode

![Graph showing $\mathrm{Im} \chi_L(\omega)$ for $T = 0$.]
Random $J$ model (insulator): SU($M$) symmetry

The local dynamic spin susceptibility, $\chi_L(i\omega_n) = \int_0^{1/T} d\tau \langle \vec{S}_i(\tau)\vec{S}_i(0) \rangle e^{i\omega_n \tau}$, obeys

$$\text{Im}\chi_L(\omega) \propto \tanh\left(\frac{\omega}{2T}\right) \left[1 - \alpha_S \omega \tanh\left(\frac{\omega}{2T}\right) + \ldots\right]$$

The ratio $\alpha_S/\gamma$ is a universal number, which we have computed in the large $M$ limit

$$\frac{\alpha_S}{\gamma} = \frac{24}{\pi(2 + 3\pi)} , \quad M \to \infty .$$

(Recall the specific heat is $C = \gamma T$ per site and spin component.)

From the time reparameterization soft-mode
Exact diagonalization results of SU(2) random magnet
(recomputed by Henry Shackleton)

Large M theory (rescaled)

Incoherent contribution similar to SY spin liquid!
Exact diagonalization results of SU(2) random magnet
(recomputed by Henry Shackleton)

Incoherent contribution similar to SY spin liquid!

Spin glass order $q \approx 0.01$
Implies a local moment $2\sqrt{q} = 20\%$ of classical moment

Large M theory (rescaled)

Incoherent contribution similar to SY spin liquid!
1. Random $J$ model (insulator)
   *RG analysis and exact exponent*

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   *QMC and exact diagonalization*
Random $t$-$J$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.
Random t-J model (metal)

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Each site has 3 states which we map to the ‘superspin’ space of a boson \( b \) (the holon) and a fermion \( f_\alpha \) (the spinon):

\[
\begin{align*}
\begin{array}{c}
\uparrow \\
|v\rangle \\
\downarrow \\
\end{array}
\end{align*}
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\uparrow \\
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\downarrow \\
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\begin{align*}
\begin{array}{c}
\uparrow \\
|v\rangle \\
\downarrow \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
&c_\alpha = f_\alpha b^\dagger \\
&\vec{S} = \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\
&f_\alpha^\dagger f_\alpha + b^\dagger b = 1 \\
\text{U(1) gauge invariance, } & b \rightarrow be^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}
\end{align*}
\]

The physical electron \( (c_\alpha) \) and spin \( (\vec{S}) \) operators are rotations in this SU(1|2) superspin space.
Random t-J model (metal)

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Each site has 3 states which we map to the ‘superspin’ space of a fermion \( f \) (the holon) and a boson \( b_\alpha \) (the spinon):

\[
\begin{align*}
\hat{f} |v\rangle & \quad \hat{b}_{\uparrow} |v\rangle & \quad \hat{b}_{\downarrow} |v\rangle \\
\end{align*}
\]

\[
\begin{align*}
c_{\alpha} &= \hat{b}_\alpha \hat{f}^\dagger \\
\vec{S} &= \frac{1}{2} \hat{b}^\dagger_{\alpha} \sigma_{\alpha\beta} \hat{b}_{\beta} \\
\hat{b}^\dagger_{\alpha} \hat{b}_\alpha + \hat{f}^\dagger \hat{f} &= 1 \\
U(1) \text{ gauge invariance,} & \quad \hat{f} \to \hat{f} e^{i\phi}, \quad \hat{b}_{\alpha} \to \hat{b}_{\alpha} e^{i\phi}
\end{align*}
\]

The physical electron \( (c_{\alpha}) \) and spin \( (\vec{S}) \) operators are rotations in this SU(2|1) superspin space.
Random t-J model (metal)

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Each site has 3 states which we map to the ‘superspin’ space of a fermion \( f \) (the holon) and a boson \( b_\alpha \) (the spinon):

\[ f^\dagger |v\rangle \quad b^\dagger_\uparrow |v\rangle \quad b^\dagger_\downarrow |v\rangle \]

\[ c_\alpha = b_\alpha f^\dagger \]

\[ \vec{S} = \frac{1}{2} b^\dagger_\alpha \sigma_{\alpha\beta} b_\beta \]

\[ b^\dagger_\alpha b_\alpha + f^\dagger f = 1 \]

U(1) gauge invariance, \( f \rightarrow fe^{i\phi}, \quad b_\alpha \rightarrow b_\alpha e^{i\phi} \)

The physical electron (\( c_\alpha \)) and spin (\( \vec{S} \)) operators are rotations in this SU(2|1) superspin space.
Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $Q(\tau)$ and a ‘Zeeman superfield’ $s_0$.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX \textbf{10}, 021033 (2020)
The RG analysis is very similar to that for the $J$ model, except that the SU(2) spin is replaced by a SU(1|2) $\cong$ SU(2|1) superspin. One crucial difference is that there is now a 'Zeeman' field $s_0$ in superspin space which breaks the degeneracy between spinon and holon states. This becomes the single relevant perturbation at a critical fixed point where $s_0 = 0$ at leading order, i.e., the 3 states on each site are nearly degenerate at the critical point. The Wess-Zumino-Witten term in superspace now ensures the exact exponents at the fixed point $D_\sim S(\tau) \cdot \sim S(0) \sim 1 | \tau | \sim c \left( \tau \right) \sim \sim c \left( \tau \right) \sim 1 | \tau |$. These exponents do not have a quasiparticle interpretation. However, they can be understood (in a large $M$ limit of a SYK-like model with SU($M$) symmetry) by fractionalization of the electron into a spinon and holon, each of which decay as $1/\tau$. 

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Random $t$-$J$ model (metal): RG

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\begin{align*}
\text{Diagram:} \\
& \begin{array}{c}
\text{\begin{tabular}{c}
$\uparrow$ \\
$s_0 < 0$
\end{tabular}} \\
& \begin{array}{c}
\text{\begin{tabular}{c}
$\uparrow$ \\
$s_0 = 0$
\end{tabular}} \\
& \begin{array}{c}
\text{\begin{tabular}{c}
$\uparrow$ \\
$s_0 > 0$
\end{tabular}}
\end{array}
\end{array}
\end{array}
\end{align*}

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Random t-J model (metal): RG

- The RG analysis is very similar to that for the $J$ model, except that the SU(2) spin is replaced by a SU(1|2) $\cong$ SU(2|1) superspin.

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- The Wess-Zumino-Witten term in superspace now ensures the exact exponents at the fixed point

\[
\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}, \quad \langle c_\alpha(\tau)c_\alpha^\dagger(0) \rangle \sim \frac{1}{\tau}.
\]
Random t-J model (metal): RG

- The RG analysis is very similar to that for the $J$ model, except that the SU(2) spin is replaced by a SU(1|2) $\cong$ SU(2|1) superspin.

- One crucial difference is that there is now a ‘Zeeman’ field $s_0$ in superspin space which breaks the degeneracy between spinon and holon states. This becomes the single relevant perturbation at a critical fixed point where $s_0 = 0$ at leading order $i.e.$ the 3 states on each site are nearly degenerate at the critical point.

- The Wess-Zumino-Witten term in superspace now ensures the exact exponents at the fixed point

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\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} , \quad \langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{1}{\tau} .
\]

- These exponents do not have a quasiparticle interpretation. However, they can be understood (in a large $M$ limit of a SYK-like model with SU($M$) symmetry) by fractionalization of the electron into a spinon and holon, each of which decay as $1/\sqrt{\tau}$.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Deconfined quantum critical point/phase

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]
\[ \langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \]

\textit{t-J} phase diagram: RG using \textit{either} SU(2|1) \textit{or} SU(1|2)

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX \textbf{10}, 021033 (2020)
$t$-$J$ phase diagram: RG using *either* SU(2|1) or SU(1|2)

Deconfined quantum critical point/phase

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

$$\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Deconfined quantum critical point/phase

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]

\[ \langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \]

Zeroth order, \( p_c = 1/3 \)

RG flow
$t$-$J$ phase diagram: RG using either SU(2|1) or SU(1|2)

Deconfined quantum critical point/phase

\[
\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \\
\langle c_{i\alpha}(\tau)c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau}
\]

Zeroth order, $p_c = 1/3$

Disordered Fermi liquid.
Condense holon $b$

\[ f_\alpha \text{ carrier density } 1 + p \]

\[ b^{\dagger} |v\rangle \]

\[ f^{\dagger}_\uparrow |v\rangle \]

\[ f^{\dagger}_\downarrow |v\rangle \]

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2} \]

\[ \langle c_{i\alpha}(\tau)c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau} \]
$t$-$J$ phase diagram: RG using either SU(2|1) or SU(1|2)

SU(2|1) theory

Metallic spin glass. Condense spinon $b_{\alpha}$, $f$ carrier density $p$

Deconfined quantum critical point/phase

Disordered Fermi liquid. Condense holon $b$, $f_\alpha$ carrier density $1 + p$

RG flow

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
At the critical point/phase of the $t$-$J$ model, the Fermi liquid-like behavior of the electron Green’s function

$$\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

leads to a non-zero residual resistivity, $\rho(0) \neq 0$.

However, the critical state is not a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.
Annals of Physics,\n\textbf{418}, 168202 (2020)

Haoyu Guo

Yingfeu Gu
The leading corrections to the critical spinon Green’s function arise from the time reparameterization soft mode, and these take the form

$$\langle f_\alpha(\tau)f_\alpha^\dagger(0) \rangle \sim \left[ \frac{\pi T}{\sin(\pi T\tau)} \right]^{1/2} \left( 1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large $M$ limit) scaling function, and $\alpha_G$ is universally proportional to the co-efficient $\alpha_S$ of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD 94, 106002 (2016)
The leading corrections to the critical electron Green’s function arise from the time reparameterization soft mode, and these take the form

\[
\langle c_\alpha(\tau) c^{\dagger}_\alpha(0) \rangle \sim \left[ \frac{\pi T}{\sin(\pi T\tau)} \right] \left( 1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)
\]

where \( \Phi_{\text{non-conformal}}(T\tau) \) is a computable (in the large \( M \) limit) scaling function, and \( \alpha_G \) is universally proportional to the co-efficient \( \alpha_S \) of the Schwarzian action for the time reparameterization mode.
Time reparameterization soft mode

Computing the resistivity from this Green’s function via the Kubo formula, we find

\[ \rho(T) = \rho(0) \left( 1 + 8\alpha_G \frac{T}{J} + \ldots \right) \]
Time reparameterization soft mode

Computing the resistivity from this Green’s function via the Kubo formula, we find

\[ \rho(T) = \rho(0) \left( 1 + 8 \alpha_G \frac{T}{J} + \ldots \right) \]

In general, an operator with scaling dimension \( h \), yield a resistivity \( \rho(T) \sim T^{h-1} \). In the large \( M \) solution of the \( t-J \) model, we do find an operator with \( 1 < h < 2 \), but with a smaller prefactor.
Random $t$-$J$ model

- Proposed phase diagram of random $t$-$J$ model captures key characteristics of cuprates.

- Critical electron Green's function $G_c(\tau) \sim 1/\tau$ and local spin correlator $Q(\tau) \sim 1/|\tau|$.

- Can be interpreted in terms of fractionalization with spinon and holon correlators $\sim 1/\sqrt{\tau}$ (deconfined criticality).

- Linear-in-$T$ resistivity down to $T = 0$ at the critical point from the time reparameterization soft mode (but there could be other singular modes).

- Carrier density $p$ for $p < p_c$, and $1 + p$ for $p > p_c$.

- Extensive zero temperature entropy $\lim_{T \to 0} \lim_{N \to \infty} S/N > 0$. Related to entropy of extremal black holes.
1. Random $J$ model (insulator)
   \textit{RG analysis and exact exponent}

2. SYK criticality
   \textit{Time reparameterizations and spectral functions}

3. Random $t$-$J$ model (metals)
   \textit{RG analysis and exact exponents}

4. Numerical results
   \textit{QMC and exact diagonalization}
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_i^\uparrow n_i^\downarrow$$

\[ \alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_i^\alpha = c_{i\alpha}^\dagger c_{i\alpha}, \]

$J_{ij}$ random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

$t_{ij}$ random, $\overline{t_{ij}} = 0$, $\overline{t_{ij}^2} = t^2$

$U_H > 0$ non-random
Random $t$-$J$-$U_H$ model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}
\]
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\text{doping } p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$$

$n_{i\uparrow} + n_{i\downarrow} = 1$

$1/U_H$
Random $t$-$J$-$U_H$ model

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow} \]

Disordered Fermi liquid

\[
\left\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau} \\
\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}
\]

\[ n_{i\uparrow} + n_{i\downarrow} = 1 \]

doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$
Random $t$-$J$-$U_H$ model

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow} \]

1/\(U_H\)

\[ n_{i\uparrow} + n_{i\downarrow} = 1 \]

Spin glass

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant} \]

Disordered

\[ \langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \]

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2} \]

Doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$

L. Arrachea and M. J. Rozenberg,
PRB 65, 224430 (2002)
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

Disordered Fermi liquid

$$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

Metal-insulator transition

Spin glass Insulator

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$$

$1/U_H$

$n_{i\uparrow} + n_{i\downarrow} = 1$

0

doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

1. $1/U_H$

$1/U_H$

Disordered Fermi liquid

- $\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$
- $\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$

Metal-insulator transition with SYK criticality

- SYK criticality $\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$

- Spin glass Insulator $\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$

$n_{i\uparrow} + n_{i\downarrow} = 1$

doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$

Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS 117, 18341 (2020)
Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-1/2 fermions

Critical spin correlations: $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$

Resistivity $\rho \sim T$ to the lowest $T$ at the critical point

Onset of insulating gap and spin glass order co-incide.

Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS 117, 18341 (2020)
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

Disordered Fermi liquid

$$\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau}$$
$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

Metal-insulator transition with SYK criticality

Spin glass Insulator

Doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$
Random $t$-$J$-$U_H$ model

\[ H = - \frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow} \]

\[ \langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \]

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2} \]

\[ 1/U_H \]

Spin glass

Disordered Fermi liquid

Metal-insulator transition with SYK criticality

Insulator

\[ p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle \]
Random \( t-J-U_H \) model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}
\]

Metal-insulator transition with SYK criticality

Disordered Fermi liquid

\[
\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \quad \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}
\]

Metallic Spin glass

Spin glass Insulator
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

1/$U_H$

Metal-insulator transition with SYK criticality

Disordered Fermi liquid

Metallic Spin glass

Spin glass Insulator

$doping$ $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$
Random t-J mod

Exact diagonalization of clusters with $N$ finite

Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$
Random $t$-$J$ mod

Exact diagonalization of clusters with $N$ finite

Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$

Incoherent contribution similar to SY spin liquid!
Random $t$-$J$ mod

Exact diagonalization of clusters with $N$ finite

Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$

Remaining low frequency weight integrates to the spin glass order parameter $q$
Entropy at $T \gg 1$ determined by $\dim(\mathcal{H})$
Maximum entropy shifts at lower temperature
Maximum entropy shifts at lower temperature

![Graph showing the relationship between S/N and p for different values of N at T = 1.00]
Maximum entropy shifts at lower temperature
Large-N extrapolation of entropy density
Large-N extrapolation of entropy density

Entropy, $p = 1/6$

\[ S_0 = -0.010 \pm 0.011 \]
Large-N extrapolation of entropy density

$p = 1/4$

$S/N \approx 0.024 \pm 0.005$

$p = 1/4$
Large-N extrapolation of entropy density

Entropy, $p = 1/3$
Large-N extrapolation of entropy density

Entropy, $p = 1/2$
Non-zero \( s_0 \) at \( p = 1/4 \) is extrapolation-dependent.
Non-zero $s_0$ at $p = 1/4$ is extrapolation-dependent
Random $t$-$J$-$U_H$ model

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow} \]

Disordered Fermi liquid:
\[
\left\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau} \quad \left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}
\]

Metal-insulator transition with SYK criticality:
\[
\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}
\]

Metallic Spin glass

Spin glass Insulator

doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$

$\frac{1}{U_H}$

$p \approx 1/3$
Metal-metal quantum phase transitions

The ancilla qubit approach for non-random $t$-$J$ models, and the random $t$-$J$ model, have in common

- A metal-metal quantum phase transition with a change in carrier density from $p$ to $1 + p$.
- Fractionalization of the electron in the critical regime
- Unexpectedly large low $T$ entropy near the critical point (from ghost fermions, or the SYK black hole entropy).