Deconfined criticality in a doped random quantum Heisenberg magnet

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diagram for the IL structure encountered in their growth, a similar temperature-doping phase diagram for as-grown (solid circles) and reduced (open symbols) samples obtained by muon spin resonance as the pseudogap line will be described in Section 4.1. In the case of thin films, the growth phase diagram involves also oxygen pressure. However, these competing phases can lead to many erroneous conclusions on the physical properties and extra care has been taken separately the typical route used to grow high quality single crystals. We will then discuss the reduction conditions and epitaxial thin films. We should underline that CaCuO₄ do exists in the IL structure and favour the growth of small Ca as shown first by Siegrist et al. Unfortunately, these competing phases can lead to many erroneous conclusions on the physical properties and extra care has been taken separately the typical route used to grow high quality single crystals. We will then discuss the reduction conditions and epitaxial thin films. We should underline that Sr in SrCuO₄ can be substituted by iso-valent Ca as shown first by Siegrist et al. [32,33]. Moreover, hole-doped cuprates present a wealth of electronic phases like stripe order, pseudogap and antiferromagnetism and superconductivity on either side, suggesting that the mechanism modifying the electronic properties is most probably the same for both types of doping. Of course, there are many differences in the extent of the doping range for the antiferromagnetic phase on the electron-doped side and the very narrow range for cerium as the addition of Ca allows one to stabilize the IL crystal structure almost fully by two different methods: in-flux solidification and travelling solvent floating zone (TSFZ). The first group of techniques involves removing a tiny amount of oxygen in conditions approaching the decomposition line of the phase. In here, we will simply enumerate the contents with superconductivity. One can also observe the pseudogap line which will be described in Section 4.1. In the light blue region above the superconducting dome, strong antiferromagnetic order for the reduced samples in the doping range from 0.10 to 0.20 of the electron-doped samples.

**Fig. 3**

(a) shows single crystals. The solid grey circles result from a shift assuming that the antiferromagnetic order for the reduced samples in the doping range from 0.10 to 0.20 of the electron-doped samples.

**Fig. 4.**

(b) Temperature-transport properties of electron-doped cuprates (in red), antiferromagnetism (in blue) and superconductivity (in red), antiferromagnetism (in blue) and superconductivity (in red). Adapted from Ref.[6]. In the light blue region above the superconducting dome, strong antiferromagnetic order for the reduced samples in the doping range.

**Fig. 5.**

In the case of thin films, the growth phase diagram involves also oxygen pressure. However, these competing phases can lead to many erroneous conclusions on the physical properties and extra care has been taken separately the typical route used to grow high quality single crystals. We will then discuss the reduction conditions and epitaxial thin films. We should underline that CaCuO₄ do exists in the IL structure and favour the growth of small Ca as shown first by Siegrist et al. Unfortunately, these competing phases can lead to many erroneous conclusions on the physical properties and extra care has been taken separately the typical route used to grow high quality single crystals. We will then discuss the reduction conditions and epitaxial thin films. We should underline that Sr in SrCuO₄ can be substituted by iso-valent Ca as shown first by Siegrist et al. [32,33]. Moreover, hole-doped cuprates present a wealth of electronic phases like stripe order, pseudogap and antiferromagnetism and superconductivity on either side, suggesting that the mechanism modifying the electronic properties is most probably the same for both types of doping. Of course, there are many differences in the extent of the doping range for the antiferromagnetic phase on the electron-doped side and the very narrow range for cerium as the addition of Ca allows one to stabilize the IL crystal structure almost fully by two different methods: in-flux solidification and travelling solvent floating zone (TSFZ). The first group of techniques involves removing a tiny amount of oxygen in conditions approaching the decomposition line of the phase. In here, we will simply enumerate the contents with superconductivity. One can also observe the pseudogap line which will be described in Section 4.1. In the light blue region above the superconducting dome, strong antiferromagnetic order for the reduced samples in the doping range from 0.10 to 0.20 of the electron-doped samples.
La$_{2-x}$Sr$_x$CuO$_4$ and Ln$_{2-x}$Ce$_x$CuO$_4$

- **La$_{2-x}$Sr$_x$CuO$_4$**
  - Temperature-doping phase diagram for as-grown (solid circles) and reduced (open symbols) single crystals.
  - 

- **Ln$_{2-x}$Ce$_x$CuO$_4$**
  - Temperature-doping phase diagram for reduced samples.
  - The diagram shows the doping range for superconductivity (in red), antiferromagnetism (in blue) and $T_c$.

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Fig. 3 (a) shows the temperature-doping phase diagrams for hole-doped La$_{2-x}$Sr$_x$CuO$_4$, electron-doped Ce$_x$CuO$_4$, and electron-doped Ce$_x$CuO$_4$ as a function of doping. (b) Temperature-decomposition line of the phase. In here, we will simply enumerate the directional growth of millimeter size high-quality single crystals.

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Fig. 4 (a) shows the temperature-decomposition line of the phase. In here, we will simply enumerate the directional growth of millimeter size high-quality single crystals. The first group of techniques involves the directional growth of millimeter size high-quality single crystals.

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Fig. 5 shows the doping range of antiferromagnetic order for the reduced samples in the doping phase diagram for La$_{2-x}$Sr$_x$CuO$_4$. The solid circles result from a shift assuming that the large magnetic fluctuations and the resulting structural impacts.

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Unfortunately, these competing phases can lead to many erroneous instances, they may appear during the post-annealing process. In fact, in some oxide phases can even be intercalated in the bulk as an epitaxial layer and are difficult to eliminate completely. In fact, in some cases, superconductivity may even be suppressed.

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And the superconducting transition $T_c$.

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We should underline that Sr in SrCuO$_2$ has been inserted successfully in (CaCuO$_2$)$_2$.$\text{BaCuO}_2$. In the case of thin films, the growth phase diagram involves also oxygen pressure. In the light blue region above the superconducting dome, strong antiferromagnetic order for the reduced samples in the doping phase diagram for as-grown (solid circles) and reduced (open symbols) single crystals. The solid grey circles result from a shift assuming that the large magnetic fluctuations and the resulting structural impacts.

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We will then discuss the reduction conditions, the physical properties are quite different in electron- and hole-doped samples. Electron-doped samples.

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In electron-doped cuprates and may be absent (and perhaps...
Hole doped cuprates


The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

Figure 6

Across the quantum critical point. a) Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p_0=0.08$ and $p'=0.19$. b) Normal-state Hall number $n_H$ in YBCO (blue circles (21), $p^*=0.19$) and Nd-LSCO (red squares (4), $p^*=0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

5. PSEUDOGAP PHASE

DOS: Density of states ($N_F$): Condensation energy $E_c$: Lower critical field $H_{c1}$: Residual linear term in the specific heat, $C(T)$ at $T=0$: purely electronic

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p^*$; 2) the opening of a partial spectral gap below $T^*$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p^*$. The new data show that the pseudogap does not simply cause a loss of DOS below $p^*$; instead, there is huge peak in the DOS at $p^*$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p^*$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).

5.1. Density of states

5.1.1. Condensation energy. One way to access the DOS, $N_F$, is via the superconducting condensation energy $E_c$, since $E_c=\frac{N_F}{4}$, where $0$ is the $d$-wave gap maximum. Experimentally, and in the framework of BCS theory, $E_c$ can be measured using the upper and lower critical fields, $H_{c2}$ and $H_{c1}$, to get the thermodynamic field $H_c$ via $H_{c2}^2=H_{c1}H_{c2}/(\ln(\frac{\pi}{\lambda})+0.5)$, given that $E_c=H_{c2}/2\mu_0$. In Fig. 2b, we plot $E_c/T_{c2}$ vs $p$ thus obtained for YBCO (17). We see that $E_c/T_{c2}/N_F$ drops by a factor 8-9 between $p=0.18$ and $p=0.1$, in agreement with the drop reported earlier from an analysis of specific heat data measured in low fields up to $T>T_c$ in YBCO (71) and Bi2212 (72). Note
Precision Measurement of the Node

I. M. Vishik et. al., PNAS (2012)
Two “gaps” for $p < 0.19$ ($T_c \approx 86$ K)

S.D. Chen et al., Science 2019
One gap for $p > 0.19$ ($T_c \approx 81$ K)
1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model
1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model
Insulating $J$ model

\[ H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 \]

$J_{ij}$ random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$
**Insulating $J$ model**

\[ Z = \int \mathcal{D} \vec{S}(\tau) \delta(\vec{S}^2 - 1) e^{-S} \]

\[ S = \int d\tau i \vec{A}(\vec{S}) \cdot \frac{d\vec{S}}{d\tau} - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \]

\[ \nabla_{\vec{S}} \times \vec{A}(\vec{S}) = \frac{1}{2} \vec{S}. \]
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\[ \vec{\nabla}_{\vec{S}} \times \vec{A}(\vec{S}) = \frac{1}{2} \vec{S}. \]

From this action we compute

\[ \overline{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_Z \]

and then impose the self-consistency condition

\[ Q(\tau) = \overline{Q}(\tau). \]

S. Sachdev and J. Ye, PRL \textbf{70}, 3339 (1993)
**Insulating J model: large M limit**

Express the spin operator in terms of fermions $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1\ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large $M$ limit we obtain for the fermion Green’s function $G$ and self energy $\Sigma$ (same as the SYK equations)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Insulating $J$ model

\[ H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Numerical studies for SU(2) spin-1/2 show spin-glass order!

Insulating $J$ model: RG

We assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}}.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic ($\phi_a$, $a = 1 \ldots 3$) bath.
We assume a power-law decay
\[ Q(\tau) \sim \frac{1}{|\tau|^{d-1}}. \]

Ignore the self-consistency condition for now. We decouple the \( \vec{S}(\tau) \cdot \vec{S}(0) \) interaction by introducing a bosonic \( (\phi_a, a = 1 \ldots 3) \) bath. Then the problem reduces to the Hamiltonian
\[
H_{\text{imp}} = \gamma_0 f^\dagger_\alpha \frac{\sigma^a_{\alpha\beta}}{2} f_\beta \phi_a(0) + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]
\]
where \( \pi_a \) is canonically conjugate to the field \( \phi_a \), \( \phi_a(0) \equiv \phi_a(x = 0) \), and we have the constraint
\[ f^\dagger_\alpha f_\alpha = 1. \]
We assume a power-law decay

\[ Q(\tau) \sim \frac{1}{|\tau|^{d-1}}. \]

Ignore the self-consistency condition for now. We decouple the \( \tilde{S}(\tau) \cdot \tilde{S}(0) \) interaction by introducing a bosonic \((\phi_a, a = 1 \ldots 3)\) bath. Then the problem reduces to the Hamiltonian

\[
H_{\text{imp}} = \gamma_0 f^{\dagger}_\alpha \sigma^a_{\alpha\beta} f^\beta \phi_a(0) + \frac{1}{2} \int d^d x \left[ \pi^2_a + (\partial_x \phi_a)^2 \right]
\]

where \(\pi_a\) is canonically conjugate to the field \(\phi_a, \phi_a(0) \equiv \phi_a(x = 0)\), and we have the constraint

\[ f^{\dagger}_\alpha f^\alpha = 1. \]

We identify \(Q(\tau)\) with temporal correlator of \(\phi_a(0)\), and it can be verified that this correlator decays as above.
We assume a power-law decay

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where $\pi_a$ is canonically conjugate to the field $\phi_a$, $\phi_a(0) \equiv \phi_a(x = 0)$, and we have the constraint

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S. Sachdev, Physica C 357, 78 (2001)
We assume a power-law decay

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Ignore the self-consistency condition for now. We decouple the \( \mathbf{S}(\tau) \cdot \mathbf{S}(0) \) interaction by introducing a bosonic (\( \phi_a, a = 1, \ldots, 3 \)) bath. Then the problem reduces to the Hamiltonian

\[
H_{\text{imp}} = \gamma_0 b_\alpha^{\dagger} \sigma_{\alpha\beta}^a b_\beta \phi_a(0) + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]
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where \( \pi_a \) is canonically conjugate to the field \( \phi_a, \phi_a(0) \equiv \phi_a(x = 0) \), and we have the constraint

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S. Sachdev, Physica C 357, 78 (2001)
Insulating $J$ model: RG

We can perform a RG analysis in a $\epsilon = 3-d$ expansion, while imposing the fermion constraint exactly. The two-loop $\beta$ function is

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 - \gamma^5 + \ldots$$

This has a stable fixed point at $\gamma^* = \epsilon/2 + \epsilon^2/4 + \ldots$.
Insulating $J$ model: RG

We can perform a RG analysis in a $\epsilon = 3 - d$ expansion, while imposing the fermion constraint exactly. The two-loop $\beta$ function is

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 - \gamma^5 + \ldots$$

This has a stable fixed point at $\gamma^* = \epsilon/2 + \epsilon^2/4 + \ldots$.

The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to all orders in $\epsilon$. This implies the correlator

$$\overline{Q}(\tau) = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|^{3-d}}.$$
Insulating $J$ model: RG

We can perform a RG analysis in a $\epsilon = 3-d$ expansion, while imposing the fermion constraint exactly. The two-loop $\beta$ function is

$$\beta(\gamma) = -\frac{\epsilon}{2} \gamma + \gamma^3 - \gamma^5 + \ldots$$

This has a stable fixed point at $\gamma^* = \epsilon/2 + \epsilon^2/4 + \ldots$.

The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to all orders in $\epsilon$. This implies the correlator

$$\overline{Q}(\tau) = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|^{3-d}}.$$

Finally, we impose the self-consistency condition $Q(\tau) = \overline{Q}(\tau)$, and obtain the same self-consistent result as in the large $M$ expansion

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}.$$
1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model
t-J model

\[ H = - \frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

We consider the hole-doped case, with no double occupancy.

\[ \alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1 \]

\[ J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2 \]
\[ t_{ij} \text{ random, } \overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2 \]
We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

\[
|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle
\]

\[
c_\alpha = f_\alpha b^\dagger
\]

\[
\vec{S} = \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta
\]

\[
f_\alpha^\dagger f_\alpha + b^\dagger b = 1
\]

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

### t-J model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]
We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a fermion $f$ (the holon) and a boson $b_\alpha$ (the spinon):

$$|0\rangle \Rightarrow f^\dag |v\rangle \quad , \quad c_\alpha^\dag |0\rangle \Rightarrow b_\alpha^\dag |v\rangle$$

$$c_\alpha = b_\alpha f^\dag$$

$$\vec{S} = \frac{1}{2} b_\alpha^\dag \sigma_{\alpha\beta} b_\beta$$

$$b_\alpha^\dag b_\alpha + f^\dag f = 1$$

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.
We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a fermion \( f \) (the holon) and a boson \( b_\alpha \) (the spinon):

\[
|0\rangle \Rightarrow f^\dagger |v\rangle \quad , \quad c^\dagger_\alpha |0\rangle \Rightarrow b^\dagger_\alpha |v\rangle
\]

\[
c_\alpha = b_\alpha f^\dagger
\]

\[
\vec{S} = \frac{1}{2} b^\dagger_\alpha \sigma_{\alpha\beta} b_\beta
\]

\[
b^\dagger_\alpha b_\alpha + f^\dagger f = 1
\]

The physical electron (\( c_\alpha \)) and spin (\( \vec{S} \)) operators are rotations in this SU(2|1) superspin space.

\[
SU(1|2) \equiv SU(2|1)
\]
\[ Z = \int \mathcal{D} f_\alpha(\tau) \mathcal{D} b(\tau) \mathcal{D} \lambda(\tau) e^{-S} \]

\[ S = \int d\tau \left[ f_\alpha^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + s_0 + i\lambda \right) f_\alpha(\tau) + b^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + i\lambda \right) b(\tau) - i\lambda \right] \]

\[ - t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.} \]

\[ - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \]
**t-J model**

\[
Z = \int \mathcal{D} f_\alpha(\tau) \mathcal{D} b(\tau) \mathcal{D} \lambda(\tau) e^{-S}
\]

\[
S = \int d\tau \left[ f_\alpha^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + s_0 + i\lambda \right) f_\alpha(\tau) + b^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + i\lambda \right) b(\tau) - i\lambda \right]
\]

\[
- t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.}
\]

\[
- \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') .
\]

From this action we determined the correlators

\[
\overline{R}(\tau - \tau') = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(\tau') \rangle_Z
\]

\[
\overline{Q}(\tau - \tau') = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(\tau') \rangle_Z
\]

and finally impose the self-consistency conditions

\[
R(\tau) = \overline{R}(\tau) , \quad Q(\tau) = \overline{Q}(\tau).
\]
\[Z = \int \mathcal{D}b_\alpha(\tau) \mathcal{D}f(\tau) \mathcal{D}\lambda(\tau) e^{-S}\]

\[S = \int d\tau \left[ b_\alpha^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + s_0 + i\lambda \right) b_\alpha(\tau) + f^\dagger(\tau) \left( \frac{\partial}{\partial \tau} + i\lambda \right) f(\tau) - i\lambda \right] \]

\[- t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.}\]

\[- \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') .\]

From this action we determined the correlators

\[\overline{R}(\tau - \tau') = -\frac{1}{2} \left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau') \right\rangle_Z\]

\[\overline{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_Z\]

and finally impose the self-consistency conditions

\[R(\tau) = \overline{R}(\tau) , \quad Q(\tau) = \overline{Q}(\tau) .\]
We assume power-law decays

\[ Q(\tau) \sim \frac{1}{|\tau|^{d-1}}, \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}}. \]

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic \((\phi_a, a = 1\ldots3)\) and fermionic \((\psi_\alpha)\) baths.
We assume power-law decays

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We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic \((\phi_a, a = 1 \ldots 3)\) and fermionic \((\psi_\alpha)\) baths. Then the problem reduces to the Hamiltonian

\[
H = (s_0 + \lambda) f_{\alpha}^\dagger f_{\alpha} + \lambda b^\dagger b + g_0 \left( f_{\alpha}^\dagger b \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 f_{\alpha}^\dagger \frac{\sigma_{\alpha\beta}^a}{2} f_\beta \phi_a(0)
\]

\[
+ \int |k|^r dk \ k \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]
\]

where \(a = (x, y, z)\), \(\sigma^a\) are Pauli matrices, \(\pi_a\) is canonically conjugate to the field \(\phi_a\), and \(\phi_a(0) \equiv \phi_a(x = 0), \psi_\alpha(0) \equiv \int |k|^r dk \psi_{k\alpha}.\)

**t-J model RG**

We assume power-law decays

\[
Q(\tau) \sim \frac{1}{|\tau|^{d-1}}, \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}}.
\]

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic \((\phi_a, a = 1\ldots3)\) and fermionic \((\psi_\alpha)\) baths. Then the problem reduces to the Hamiltonian

\[
H = (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda b^\dagger b + g_0 \left( f_\alpha^\dagger b \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 f_\alpha^\dagger \frac{\sigma_\alpha^a}{2} f_\beta \phi_a(0) \\
+ \int |k|^r dk \kappa_\alpha \psi_\kappa^\dagger \psi_\kappa + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]
\]

where \(a = (x, y, z)\), \(\sigma^a\) are Pauli matrices, \(\pi_a\) is canonically conjugate to the field \(\phi_a\), and \(\phi_a(0) \equiv \phi_a(x = 0), \psi_\alpha(0) \equiv \int |k|^r dk \psi_\kappa\). We identify \(Q(\tau)\) with temporal correlator of \(\phi_a(0)\), and \(R(\tau)\) with the temporal correlator of \(\psi_\alpha(0)\), and it can be verified that these correlators decay as above.

We assume power-law decays

\[ Q(\tau) \sim \frac{1}{|\tau|^{d-1}}, \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}}. \]

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic \((\phi_a, a = 1 \ldots 3)\) and fermionic \((\psi_\alpha)\) baths. Then the problem reduces to the Hamiltonian

\[
H = (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda b^\dagger b + g_0 (f_\alpha^\dagger b \psi_\alpha(0) + \text{H.c.}) + \gamma_0 f_\alpha^\dagger \frac{\sigma_\alpha^a}{2} f_\beta \phi_a(0) \\
+ \int |k|^r dk k \psi_k^{\dagger} \psi_k + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]
\]

The impurity superspin is coupled to a fermionic bath by \(g_0\), and to a bosonic bath by \(\gamma_0\), and \(s_0\) acts as a local field on the superspin - a superKondo problem!
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\[
H = (s_0 + \lambda) b^\dagger_\alpha b_\alpha + \lambda f^\dagger f + g_0 \left( b^\dagger_\alpha \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 b^\dagger_\alpha \frac{\sigma^a_{\alpha\beta}}{2} b_\beta \phi_a(0) \\
+ \int |k|^r dk \ k \ \psi^\dagger_{k\alpha} \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[ \pi^2_a + (\partial_x \phi_a)^2 \right]
\]

The impurity superspin is coupled to a fermionic bath by \(g_0\), and to a bosonic bath by \(\gamma_0\), and \(s_0\) acts as a local field on the superspin - a superKondo problem!
We can perform a RG analysis for small $\epsilon = 3 - d$ and $\bar{r} = (1 - r)/2$, while imposing the local constraint exactly. The one-loop $\beta$ functions are

$$
\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2,
$$

$$
\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma.
$$

$$
\beta(s) = -s + 3g^2s - g^2 + \frac{3}{4}\gamma^2.
$$

These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of $s$ to $\pm\infty$. 
We can perform a RG analysis for small \( \epsilon = 3 - d \) and \( \bar{r} = (1 - r)/2 \), while imposing the local constraint \textit{exactly}. The one-loop \( \beta \) functions are

\[
\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2,
\]
\[
\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma.
\]
\[
\beta(s) = -s + 3g^2s - g^2 + \frac{3}{4}\gamma^2.
\]

These equations have a fixed point with \( s \approx 0 \) with only one relevant direction, corresponding to the flow of \( s \) to \( \pm\infty \). The 3 states of the superspin are nearly degenerate at the fixed point, and the flows away from the fixed point correspond to different orientations of the field on the superspin: one side (overdoped) favors the holon, and the other side (underdoped) favors the spinon.
The scaling dimensions of the electron and spin operators can be determined to all orders in $\epsilon$ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}} , \quad \overline{Q}(\tau) = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|^{3-d}} .$$
t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in $\epsilon$ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2} \langle c_\alpha(\tau)c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \overline{Q}(\tau) = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|^{3-d}}.$$  

Finally, we impose the self-consistency conditions $R(\tau) = \overline{R}(\tau)$, $Q(\tau) = \overline{Q}(\tau)$ and obtain $r = 0$ ($\bar{r} = 1/2$) and $d = 2$ ($\epsilon = 1$), so that at the critical point we have

$$\langle c_\alpha(\tau)c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|} \quad , \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}.$$
Deconfined quantum critical point

$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$
Deconfined quantum critical point

\[ \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} \]
Deconfined quantum critical point

\[ \langle \mathbf{S}(\tau) \cdot \mathbf{S}(0) \rangle \sim \frac{1}{|\tau|} \]

Disordered Fermi liquid.
Condense holon \( b \),
\( f_{\alpha} \) carrier density \( 1 + p \)

\[ b^\dagger |v\rangle \]
\[ f_{\uparrow}^\dagger |v\rangle \quad f_{\downarrow}^\dagger |v\rangle \]

\[ \langle \mathbf{S}(\tau) \cdot \mathbf{S}(0) \rangle \sim \frac{1}{\tau^2} \]
**$t$-$J$ model phase diagram**

**SU(2|1) theory**
- Metallic spin glass.
- Condense spinon $b_\alpha$, $f$ carrier density $p$

**SU(1|2) theory**
- Disordered Fermi liquid.
- Condense holon $b$, $f_\alpha$ carrier density $1 + p$

Deconfined quantum critical point

\[ \langle \mathbf{\bar{S}}(\tau) \cdot \mathbf{\bar{S}}(0) \rangle \sim \frac{1}{|\tau|} \]

\[ \langle \mathbf{\bar{S}}(\tau) \cdot \mathbf{\bar{S}}(0) \rangle \sim \text{constant} \]

\[ \langle \mathbf{\bar{S}}(\tau) \cdot \mathbf{\bar{S}}(0) \rangle \sim \frac{1}{\tau^2} \]

---

$f^\dagger |v\rangle$

$b_\uparrow^\dagger |v\rangle$, $b_\downarrow^\dagger |v\rangle$

$m$
Each site has 3 states which we map to the space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle \quad , \quad c_\alpha = f_\alpha b^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large $M$ limit, let $\alpha = 1 \ldots M$, endow the boson with an ‘orbital’ index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$
Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green’s functions:

\[
\begin{align*}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{align*}
\]

Here \(\mu_f\) and \(\mu_b\) are chemical potentials chosen to satisfy

\[
\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p .
\]
Each site has 3 states which we map to the space of a boson $b$ (the holon) and a fermion $f$ (the spinon):

$$\begin{align*}
|0_i\rangle & = b^\dagger |v_i\rangle c^\dagger,
|v_i\rangle & = f^\dagger |0_i\rangle f^\dagger + b^\dagger b = 1.
\end{align*}$$

To obtain a large $M$ limit, let $\lambda = \frac{1}{M}$...$M$, endow the boson with an 'orbital' index $a = 1...M$ and send $M \to 1$ at fixed $k = \frac{M_0}{M}$.

Then $c^\dagger = f^\dagger b^\dagger a, f^\dagger f^\dagger + b^\dagger b^\dagger = M^2$.

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$\begin{align*}
G_b(i!n) & = \frac{1}{i!n + \mu b} \lambda b(i!n),
\lambda b(i!n) \lambda b(i!n) & = \frac{1}{2} k p,
\end{align*}$$

$$\begin{align*}
\Delta_f + \Delta_b & = \frac{1}{2},
\frac{\theta_f}{\pi} + \left(\frac{1}{2} - \Delta_f\right) \frac{\sin(2\theta_f)}{\sin(2\pi \Delta_f)} & = k p,
\frac{\theta_b}{\pi} + \left(\frac{1}{2} - \Delta_b\right) \frac{\sin(2\theta_b)}{\sin(2\pi \Delta_b)} & = \frac{1}{2} + p.
\end{align*}$$

The last two are analogs of 'Luttinger' relations, which follow from an anomaly matching argument (Yingei Gu, A. Kitaev, S. Sachdev, G. Tarnopolsky arXiv:1910.14099).
The critical solution which is self-consistent in both the $t$ and $J$ terms has $\Delta_b = \Delta_f = 1/2$, implying

$$\langle c_\alpha(\tau)c_\alpha^+(0) \rangle \sim \begin{cases} \frac{A_+}{|\tau|}, & \tau > 0 \\ -\frac{A_-}{|\tau|}, & \tau < 0 \end{cases}, \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}. $$

The same exponents are obtained to all orders in the $\epsilon, \bar{r}$ expansion, but with $A_+ = A_-$. 
1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model
Random \( t-J-U \) model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}
\]

\( \alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}, \)

\( t_{ij}, J_{ij} \) random, \( U > 0 \)
Large $N$ limit

\[ Z = \int \mathcal{D}c_\alpha(\tau) e^{-S} \]

\[ S = \int d\tau \left[ c_\alpha^\dagger(\tau) \left( \frac{\partial}{\partial \tau} - \mu \right) c_\alpha(\tau) + Un_\uparrow(\tau)n_\downarrow(\tau) \right] \]

\[ - t^2 \int d\tau d\tau' R^*(\tau - \tau') c_\alpha^\dagger(\tau)c_\alpha(\tau') \]

\[ - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \tilde{S}(\tau) \cdot \tilde{S}(\tau') . \]

From this action we determine the correlators

\[ \overline{R}(\tau - \tau') = -\frac{1}{2} \left< c_\alpha^\dagger(\tau)c_\alpha(\tau') \right>_Z \]

\[ \overline{Q}(\tau - \tau') = \frac{1}{3} \left< \tilde{S}(\tau) \cdot \tilde{S}(\tau') \right>_Z \]

and finally impose the self-consistency conditions

\[ R(\tau) = \overline{R}(\tau) , \quad Q(\tau) = \overline{Q}(\tau). \]
Random $t$-$J$-$U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$\alpha = \uparrow, \downarrow$, \quad $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$, \quad $n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$,

$t_{ij}, J_{ij}$ random, \quad $U > 0$

$doping \ p = \langle n_{i\uparrow} + n_{i\downarrow} - 1 \rangle$
Random $t$-$J$-$U$ model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_i^{\uparrow} n_i^{\downarrow}
\]

\[
\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},
\]

\[
t_{ij}, J_{ij} \text{ random}, \quad U > 0
\]

J model insulator

doping $p = \langle n_i^{\uparrow} + n_i^{\downarrow} - 1 \rangle$
Random $t$-$J$-$U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$
Random $t$-$J$-$U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

Spin glass

Insulator

$t$-$J$ model

$\frac{1}{U}$

doping $p = \langle n_{i\uparrow} + n_{i\downarrow} - 1 \rangle$
Random $t$-$J$-$U$ model

\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow} \]

$\alpha = \uparrow, \downarrow$, \quad $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$, \quad $n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$,

Disordered Fermi liquid \( \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{\tau^2} \)

SYK-like criticality \( \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} \)

Spin glass

Doping \( p = \langle n_{i\uparrow} + n_{i\downarrow} - 1 \rangle \)
Quantum Critical Point between a FL Metal and a Spin Glass Mott Insulator

Quantum critical (QC)

\[ Q(t) \sim \frac{1}{t} \]
\[ \text{Im} \Sigma(i\omega) \sim \sqrt{\omega} \]

SG

\[ Q(t) \sim 1 \]
\[ \text{Im} \Sigma(i\omega) \to \infty \]

FL

\[ Q(t) \sim \frac{1}{t^2} \]
\[ \text{Im} \Sigma(i\omega) \sim \omega \]

\( Q(t) \) at long time limit
\( \text{Im} \Sigma(i\omega) \) at zero-frequency limit

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-ah Kim, APS March meeting 2019
Quantum Critical Point between a FL Metal and a Spin Glass Mott Insulator

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-ah Kim, APS March meeting 2019
Random $t$-$J$-$U$ model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}
\]

\[\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},\]

Disordered Fermi liquid $\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{\tau^2}$

SYK-like criticality $\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$

Spin glass

Doping $p = \left\langle n_{i\uparrow} + n_{i\downarrow} - 1 \right\rangle$
Random $t$-$J$-$U$ model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}
\]

\[
\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},
\]

Disordered Fermi liquid $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{\tau^2}$

SYK-like criticality $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$

Spin glass

Doping $p = \langle n_{i\uparrow} + n_{i\downarrow} - 1 \rangle$
Half-filled $t$-$J$-$U$ model RG

Let us focus on the critical point at half-filling, and assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}}, \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}}.$$ 

We ignore the self-consistency condition for now. We also decouple the last two terms by introducing bosonic ($\phi_a$, $a = 1 \ldots 3$) and fermionic ($\psi_\alpha$) baths. Then the problem reduces to the Hamiltonian

$$H_{\text{imp}} = \frac{U}{2} (c_\alpha^\dagger c_\alpha - 1)^2 + V_0 c_\alpha^\dagger \psi_\alpha(0) + \text{H.c.} + \zeta [\phi_\alpha(0)]^2$$

$$+ \gamma_0 c_\alpha^\dagger \sigma^\alpha_{\alpha\beta} c_\beta \phi_a(0) + \int |k|^r dk \, k \, \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[ \pi_a^2 + (\partial_x \phi_a)^2 \right]$$

where $a = (x, y, z)$, $\sigma^a$ are Pauli matrices, $\pi_a$ is canonically conjugate to the field $\phi_a$, and $\phi_a(0) \equiv \phi_a(x = 0)$, $\psi_\alpha(0) \equiv \int |k|^r dk \, \psi_{k\alpha}$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and $R(\tau)$ with the temporal correlator of $\psi_\alpha(0)$, and it can be verified that these correlators decay as above.

L. Fritz and M. Vojta, PRB 70, 214427 (2004)
J. H. Pixley, S. Kirchner, K. Ingersent, and Qimiao Si, PRB 88, 245111 (2013)
Half-filled $t$-$J$-$U$ model RG

Now a RG analysis is possible for $d \approx 2$ and $r \approx 1/2$, which is perturbative in $U$. The one-loop equations are

$$
\beta_\gamma = \frac{(1 - 2r)}{2} \gamma + \frac{(d - 2)}{2} \gamma + \frac{\gamma U}{\pi (i A_0)^2} + \frac{\zeta \gamma}{2\pi},
$$

$$
\beta_U = (1 - 2r) U - \frac{\gamma^2}{8\pi},
$$

$$
\beta_\zeta = (d - 2) \zeta + \frac{\zeta^2}{2\pi} + \frac{\gamma^2}{2\pi A_0^2}.
$$
Half-filled $t$-$J$-$U$ model RG

Now a RG analysis is possible for $d \approx 2$ and $r \approx 1/2$, which is perturbative in $U$. The one-loop equations are

$$\beta_\gamma = \frac{(1 - 2r)}{2} \gamma + \frac{(d - 2)}{2} \gamma + \frac{\gamma U}{\pi (iA_0)^2} + \frac{\zeta \gamma}{2\pi},$$

$$\beta_U = (1 - 2r)U - \frac{\gamma^2}{8\pi},$$

$$\beta_\zeta = (d - 2)\zeta + \frac{\zeta^2}{2\pi} + \frac{\gamma^2}{2\pi A_0^2}.$$

The electron operator $c_\alpha$ does not receive any wavefunction renormalization. This implies the correlator

$$\overline{R}(\tau) = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}}.$$

The self-consistent value of $r$ is therefore $r = 0$.

There is a non-trivial renormalization of the spin operator $\vec{S}$ at a fixed point with $\zeta \neq 0$. Determining the self-consistent value of $d$ requires computation ...
$t$-$f$ model phase diagram

**SU(2$|$1) theory**

Metalllic spin glass.
Condense spinon $b_\alpha$, $f$ carrier density $p$

**SU(1$|$2) theory**

Disordered Fermi liquid.
Condense holon $b$, $f_\alpha$ carrier density $1 + p$

Deconffined quantum critical point

$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$

$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{\tau^2}$

$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \text{constant}$

$\langle f^\dagger |v\rangle$

$\langle b_{\uparrow}^\dagger |v\rangle$, $\langle b_{\downarrow}^\dagger |v\rangle$

$|v\rangle$

$p_c$

$s$
The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

Figure 6

Across the quantum critical point. a) Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p=0.08$ and $p\approx0.19$.

b) Normal-state Hall number $n_H(=V/eR_H)$ in the $T=0$ limit as a function of doping, in YBCO (blue circles (21), $p^*=0.19$) and Nd-LSCO (red squares (4), $p^*=0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

5. PSEUDOGAP PHASE

DOS: Density of states ($N_F$): Condensation energy $H_{c1}$: Lower critical field $H_{c1}$: Residual linear term in the specific heat, $C(T)$ at $T=0$, purely electronic

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p^*$; 2) the opening of a partial spectral gap below $T^*$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p^*$. The new data show that the pseudogap does not simply cause a loss of DOS below $p^*$; instead, there is huge peak in the DOS at $p^*$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p^*$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).