Quantum entanglement
and the phases of matter

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March 30, 2012

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Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
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**Band insulators**

An even number of electrons per unit cell.
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.

Metals

An odd number of electrons per unit cell.
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
Modern phases of quantum matter

Not adiabatically connected to independent electron states:

*many-particle quantum entanglement*
Quantum superposition and entanglement
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom: \( |\uparrow\rangle \)

Hydrogen molecule:

\[
\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]

Superposition of two electron states leads to non-local correlations between spins
Quantum Entanglement: quantum superposition with more than one particle
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Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
Quantum superposition and entanglement
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes
Quantum superposition and entanglement

String theory

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Black holes
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)

S=1/2 spins

Examine ground state as a function of \( \lambda \)
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

At large \( \lambda \) ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ = \frac{1}{\sqrt{2}} \left( | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \right) \]
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighbor spins are “entangled” with each other. Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.
Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

No EPR pairs
\[
\lambda = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle - |\downarrow\uparrow\rangle)
\]
\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
TlCuCl$_3$

Quantum paramagnet at ambient pressure
TlCuCl$_3$

Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
\[ \lambda_c = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

"triplon"
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitations of $\text{TlCuCl}_3$ with varying pressure

Excitations of TlCuCl$_3$ with varying pressure

Broken valence bond ("triplon") excitations of the quantum paramagnet

Excitations of TlCuCl$_3$ with varying pressure

Spin waves above the Néel state

Excitations of TlCuCl$_3$ with varying pressure

S. Sachdev, arXiv:0901.4103

Higgs boson
First observation of the Higgs boson at the theoretically predicted energy!

Friday, March 30, 2012
\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point

• Long-range entanglement
• Long-range entanglement

• The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
• Long-range entanglement

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• The theory of the critical point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a CFT3
• Long-range entanglement

• The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

• The theory of the critical point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a CFT3.
Quantum superposition and entanglement

String theory

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String theory

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Black holes
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...
• A $D$-brane is a $d$-dimensional surface on which strings can end.
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The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.
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- In $d = 2$, we obtain strongly-interacting CFT3s. These are “dual” to string theory on anti-de Sitter space: AdS4.
A $D$-brane is a $d$-dimensional surface on which strings can end.

The low-energy theory on a $D$-brane has no gravity, similar to theories of entangled electrons of interest to us.

In $d = 2$, we obtain strongly-interacting CFT$_3$s. These are “dual” to string theory on anti-de Sitter space: AdS$_4$. 
Tensor network representation of entanglement at quantum critical point.
String theory near a D-brane

Emergent direction of AdS4

Friday, March 30, 2012
Tensor network representation of entanglement at quantum critical point

Emergent direction of AdS4

$d$-dimensional space

Brian Swingle, arXiv:0905.1317
Measure strength of quantum entanglement of region A with region B.

\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

Entanglement entropy \( S_{EE} = -\text{Tr} (\rho_A \ln \rho_A) \)
Entanglement entropy

$d$-dimensional space

depth of entanglement

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Most links describe entanglement within A.
Entanglement entropy

Links overestimate entanglement between A and B
Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.
The entanglement entropy of a region $A$ on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of $A$.

This can be seen both the string and tensor-network pictures

Brian Swingle, arXiv:0905.1317
AdS_{d+2} \quad R^{d,1} \\
Minkowski

Emergent holographic direction

Quantum matter with long-range entanglement

J. McGreevy, arXiv0909.0518
AdS_{d+2} \quad \mathbb{R}^{d,1} \quad \text{Minkowski}

CFT_{d+1}

Quantum matter with long-range entanglement

Emergent holographic direction
AdS\(_{d+2}\) → Minkowski

Area measures entanglement entropy

Emergent holographic direction

CFT\(_{d+1}\)

Quantum matter with long-range entanglement

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CFT3
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Quantum critical point with non-local entanglement in spin wavefunction

\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Classical spin waves

Dilute triplon gas

Quantum critical

Neel order

$T$

$\lambda$

$\lambda_c$

Thermally excited spin waves

Thermally excited triplon particles

Neel order

Quantum critical

\( \lambda_c \)
Classical spin waves

Dilute triplon gas

Quantum critical

Thermally excited spin waves

Thermally excited triplon particles

Neel order

Short-range entanglement

Thermally excited spin waves

Thermally excited triplon particles

Neel order

Quantum critical

Classical spin waves

Dilute triplon gas

Quantum critical

Excitations of a ground state with long-range entanglement

Quantum critical

Thermally excited spin waves

Thermally excited triplon particles

Neel order


Excitations of a ground state with long-range entanglement

Quantum critical

Needed: Accurate theory of quantum critical spin dynamics

Thermally excited spin waves

Thermally excited triplon particles

Neel order

String theory at non-zero temperatures

A 2+1 dimensional system at its quantum critical point
A 2+1 dimensional system at its quantum critical point.

A “horizon”, similar to the surface of a black hole!
Objects so massive that light is gravitationally bound to them.
Objects so massive that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius $R = \frac{2GM}{c^2}$
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
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Black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy).
A "horizon", whose temperature and entropy equal those of the quantum critical point.
String theory at non-zero temperatures

A “horizon”, whose temperature and entropy equal those of the quantum critical point.

Friction of quantum criticality = waves falling into black brane.

A 2+1 dimensional system at its quantum critical point.
A 2+1 dimensional system at its quantum critical point provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply).

A “horizon”, whose temperature and entropy equal those of the quantum critical point.

String theory at non-zero temperatures

Friday, March 30, 2012
Metals, “strange metals”, and high temperature superconductors

Insights from gravitational “duals”
High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Iron pnictides:
a new class of high temperature superconductors

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Superconductivity

Resistivity 
\[ \sim \rho_0 + AT^\alpha \]

BaFe$_2$(As$_{1-x}$P$_x$)$_2$


Short-range entanglement in state with Neel (AF) order

Resistivity 
\[ \sim \rho_0 + AT^\alpha \]


Superconductivity

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Resistivity

$\sim \rho_0 + AT^\alpha$


Superconductor

Bose condensate of pairs of electrons

Short-range entanglement
Superconductivity

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]

Resistivity
\[ \sim \rho_0 + AT^\alpha \]

Ordinary metal (Fermi liquid)


*Physical Review B* 81, 184519 (2010)
Sommerfeld-Bloch theory of ordinary metals

- Momenta with electron states empty
- Momenta with electron states occupied
Sommerfeld-Bloch theory of ordinary metals

Key feature of the theory: the Fermi surface

- Area enclosed by the Fermi surface $A = Q$, the electron density

- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$. 
Superconductivity

Resistivity $\sim \rho_0 + AT^\alpha$

Ordinary metal (Fermi liquid)


Resistivity $\sim \rho_0 + AT^\alpha$

$BaFe_2(As_{1-x}P_x)_2$


Strange Metal

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Resistivity

$\sim \rho_0 + A T^\alpha$


Classical spin waves
Dilute triplon gas
Quantum critical
Neel order
Ordinary Metal

$T$
$\lambda$

$\lambda_c$
Strange Metal

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Resistivity $\sim \rho_0 + AT^\alpha$


Strange Metal

\( \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \)

Resistivity
\( \sim \rho_0 + AT^\alpha \)


Strange Metal

Excitations of a ground state with long-range entanglement

Resistivity $\sim \rho_0 + AT^\alpha$

BaFe$_2$(As$_{1-x}$P$_x$)$_2$


Physical Review B 81, 184519 (2010)
Key (difficult) problem:

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement.
Challenge to string theory:

Describe quantum critical points and phases of metals
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Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals?
Challenge to string theory:

Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals?

Yes

S. Sachdev, Physical Review D 84, 066009 (2011)
Challenge to string theory:

Describe quantum critical points and phases of metals

Do the “holographic” gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals?
Challenge to string theory:

Describe quantum critical points and phases of metals

Do the “holographic” gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals?

Yes, lots of them, with many “strange” properties!
Challenge to string theory:

Describe quantum critical points and phases of metals

How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics?
How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics?

Challenge to string theory:

Describe quantum critical points and phases of metals

The simplest example of a “strange metal” is realized by fermions with a Fermi surface coupled to an Abelian or non-Abelian gauge field.
Challenge to string theory:

Describe quantum critical points and phases of metals

How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics?

Choose the theories with the proper entropy density

Checks: these theories also have the proper entanglement entropy and Fermi surface size!

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.
Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with long-range quantum entanglement.
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Much recent progress offers hope of a holographic description of “strange metals”