Tuning order in the cuprate superconductors by a magnetic field

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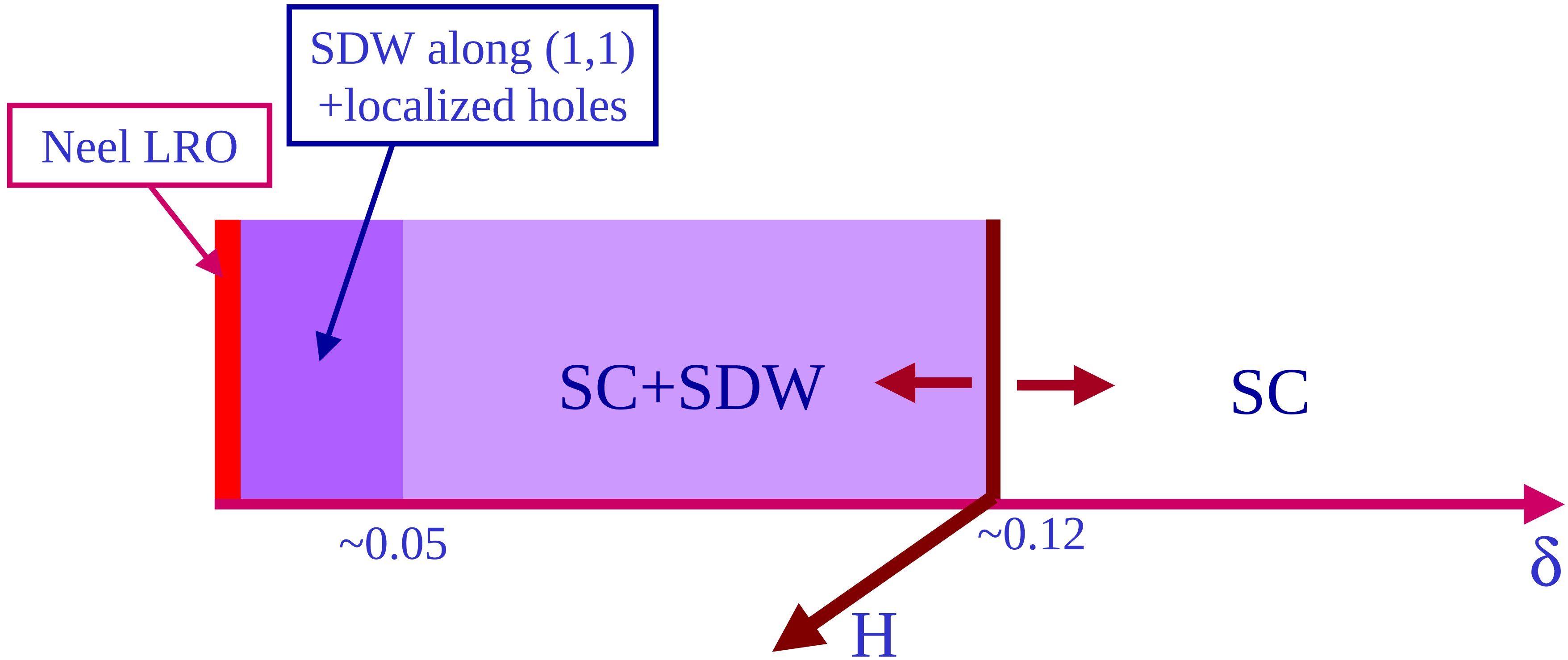
Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

Zero temperature phases of the cuprate superconductors as a function of hole density



Theory for a system with strong interactions:
describe SC and SC+SDW phases by expanding in the
deviation from the quantum critical point between them.

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Outline

I. Magnetic ordering transitions in the insulator (δ=0).

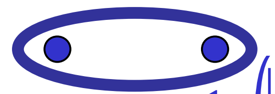
II. Theory of SC+SDW to SC quantum transition

III. Phase diagrams of above in an applied magnetic field
Comparison of predictions with experiments.

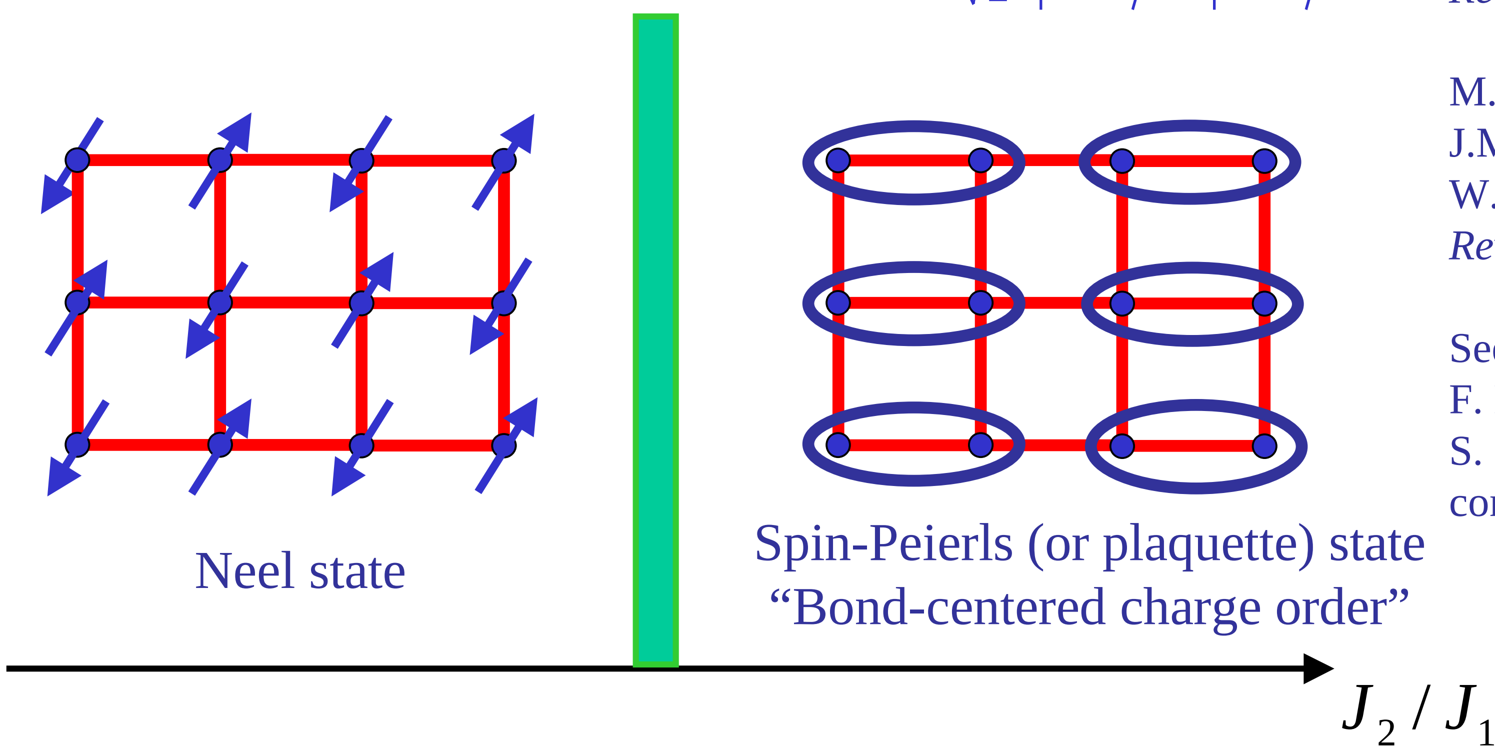
IV. Conclusions

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions (say)

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$



N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, *Phys. Rev.* B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, *Phys. Rev.* B **62**, 14844 (2000).

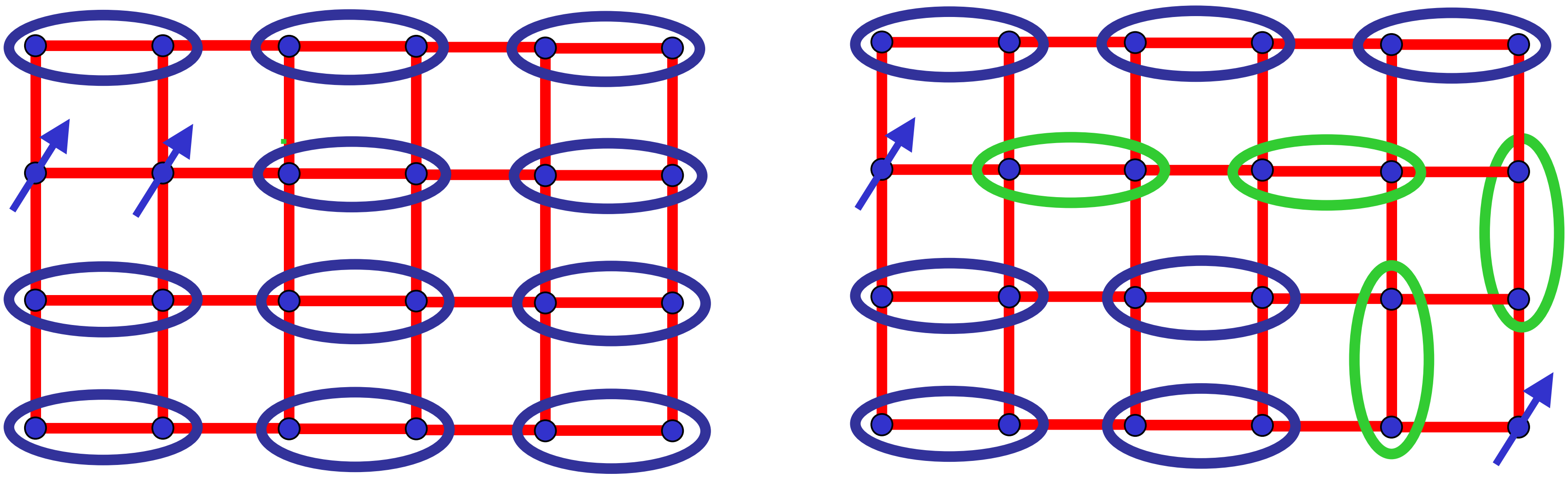
See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .

Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

$\Delta \rightarrow$ Spin gap



S=1/2 spinons are ***confined***
by a linear potential.

Transition to Neel state $\Rightarrow$ Bose condensation of ϕ_α

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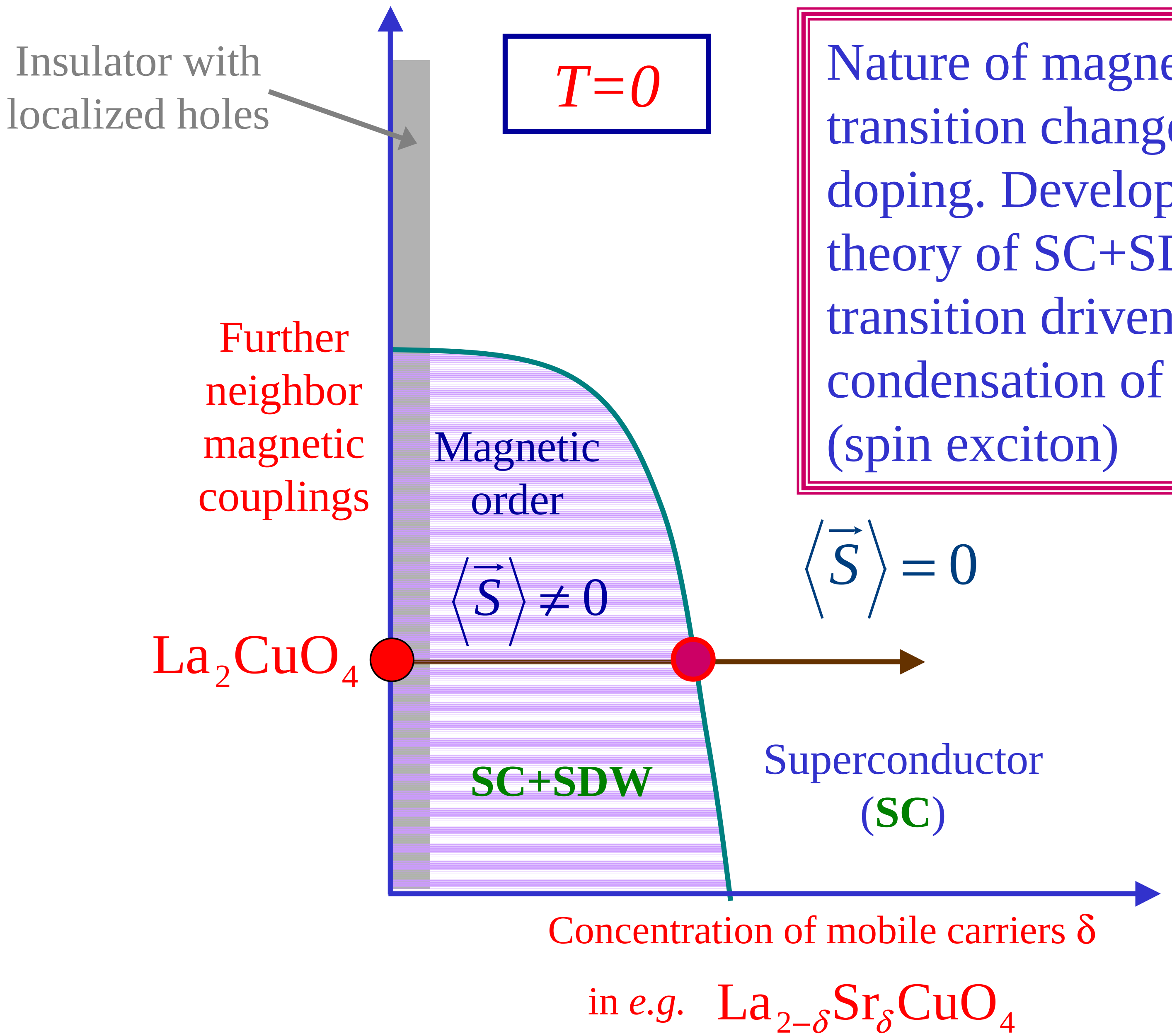


Nature of magnetic ordering transition changes little upon doping. Develop quantum theory of SC+SDW to SC transition driven by condensation of a *S=1* boson (spin exciton)

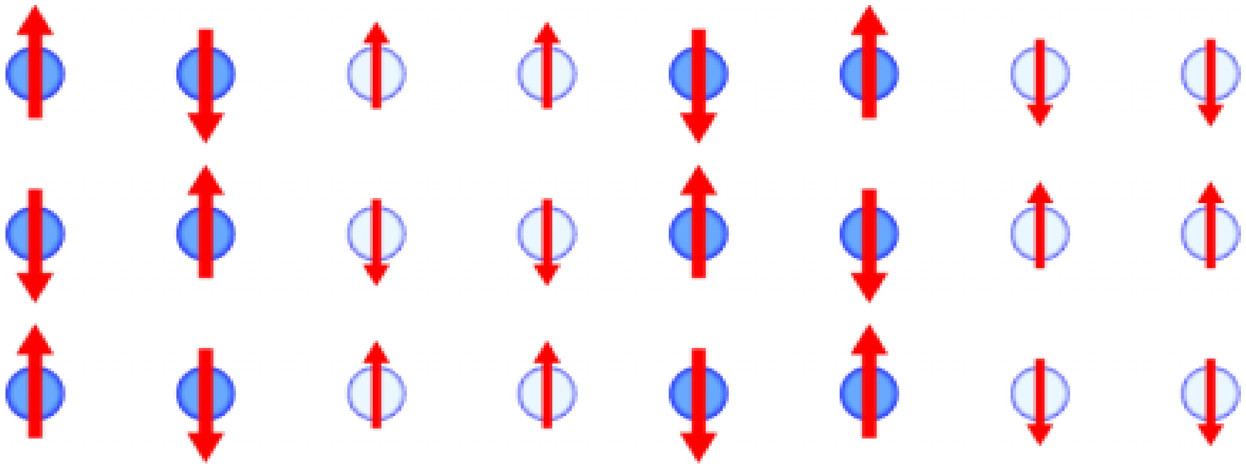
S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev*. B **49**, 11919 (1994)

II. Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a complex field $\left(\text{except for } \boldsymbol{K}=(\pi,\pi) \text{ when } e^{i\boldsymbol{K}\cdot\boldsymbol{r}} = (-1)^{r_x+r_y}\right)$



Wavevector $\boldsymbol{K}$=(3π/4,π)

Exciton wavefunction $\Phi_\alpha(\boldsymbol{r})$ describes *envelope* of this order. Phase of $\Phi_\alpha(\boldsymbol{r})$ represents sliding degree of freedom

Associated "charge" density wave order

$$\delta\rho(\boldsymbol{r}) \propto S_\alpha^2(\boldsymbol{r}) = \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, Phys. Rev. B **40**, 7391 (1989).

H. Schulz, J. de Physique **50**, 2833 (1989).

O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

Action for SDW ordering transition in the superconductor

$$S = \int d^2r d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + V\left(\Phi_\alpha\right) \right]$$

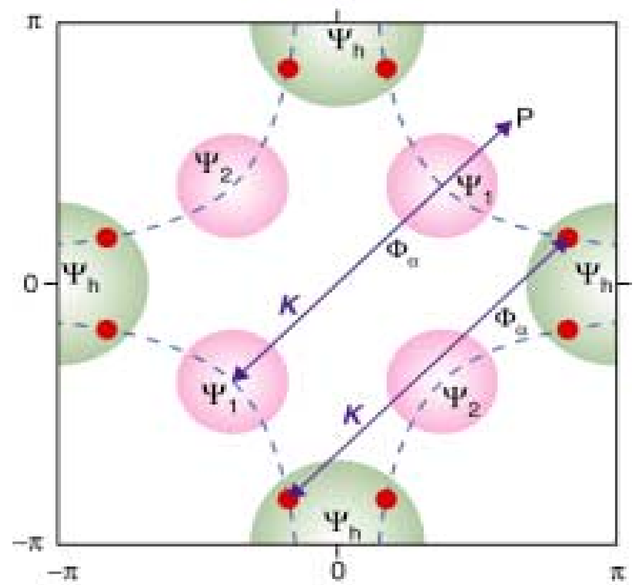
Similar terms present in action for SDW ordering in the insulator

Coupling to the *S*=1/2 Bogoliubov quasiparticles of the *d*-wave superconductor

Trilinear "Yukawa" coupling

$$\int d^2r d\tau \, \Phi_\alpha \Psi\Psi$$

is prohibited unless ordering wavevector is fine-tuned.

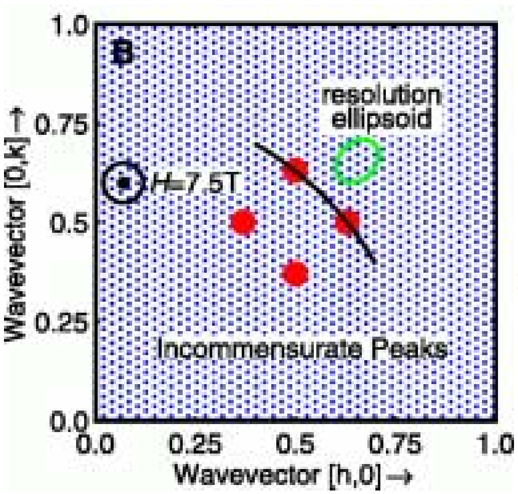


$$\kappa \sum_\alpha \int d^2r d\tau \left|\Phi_\alpha\right|^2 \Psi^\dagger \Psi \text{ is allowed}$$

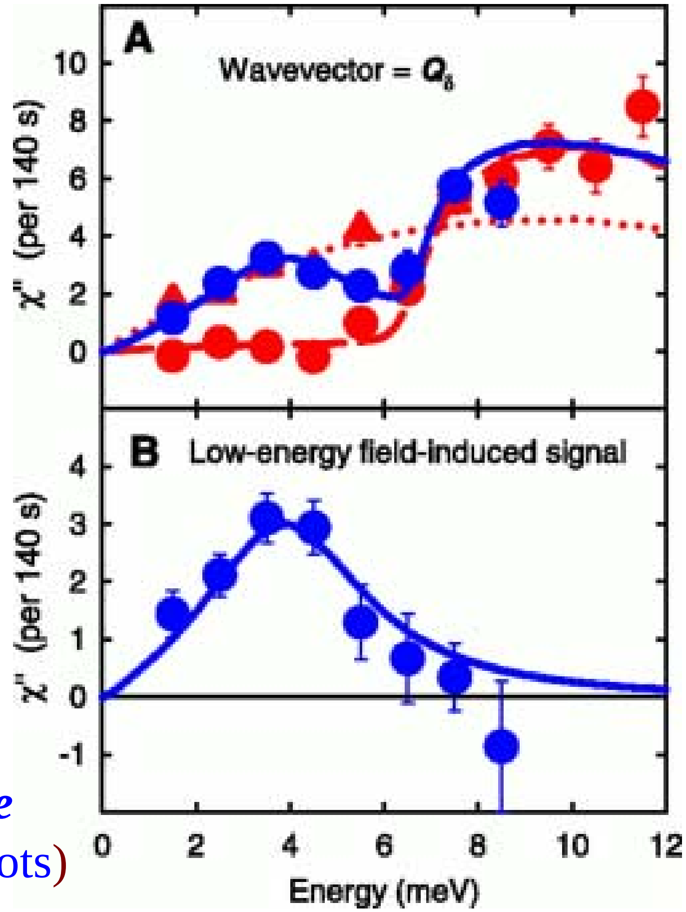
Scaling dimension of $\kappa = (1/\nu - 2) < 0 \;\Rightarrow\;$ irrelevant.

Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).



Peaks at $(0.5,0.5)\pm(0.125,0)$
and $(0.5,0.5)\pm(0,0.125)$
$\Rightarrow$ dynamic SDW of period 8



Neutron scattering off $La_{2-\delta}Sr_\delta CuO_4$ ($\delta = 0.163$, ***SC phase***
at low temperatures in H=0 (red dots) and H=7.5T (blue dots)

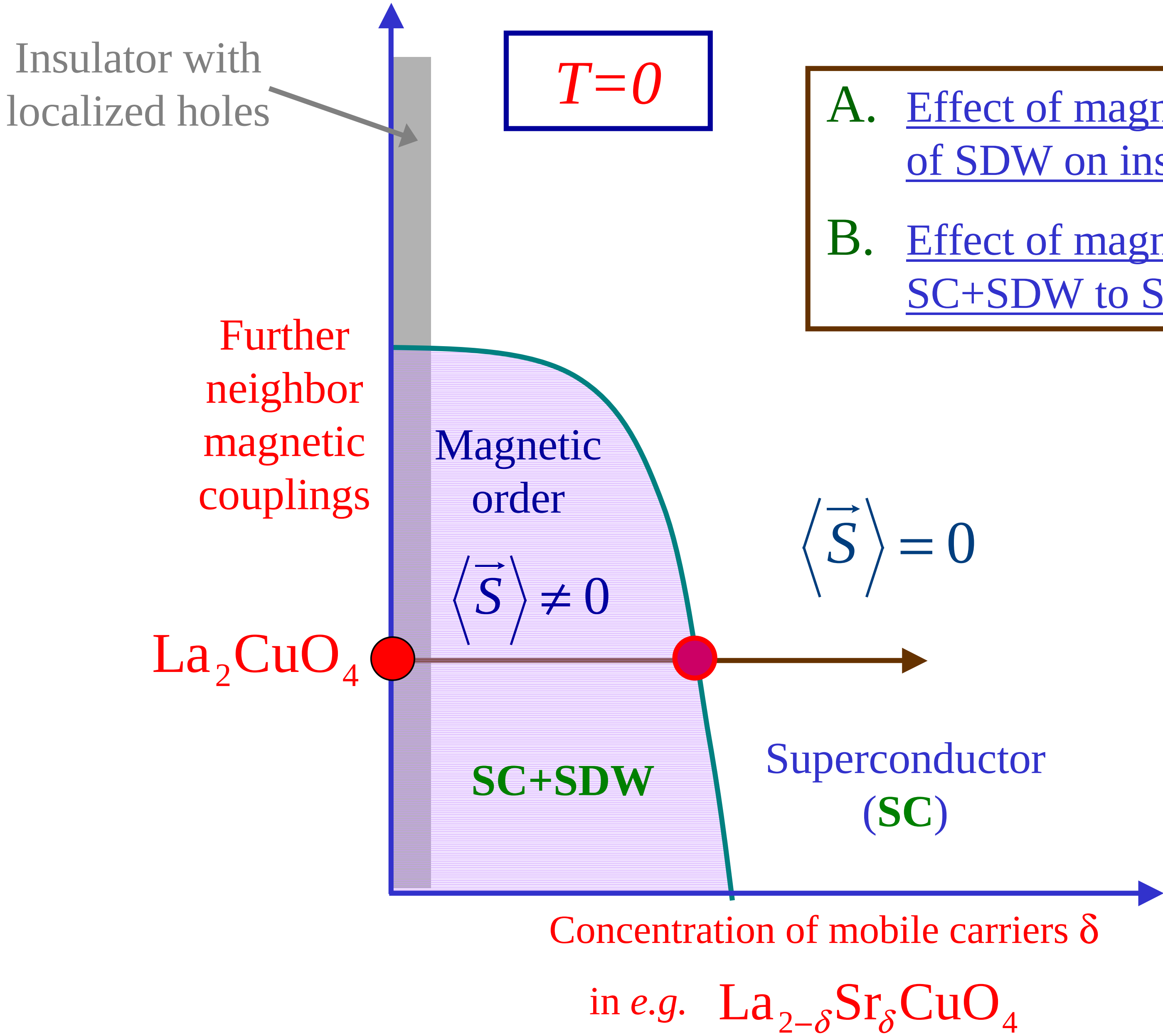
D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) suggested insulating Néel order in the cores of vortices. Using this picture of "spins in the vortices", the amplitude of the field-induced signal, and the volume-fraction of vortex cores (~10%), Lake *et al.* estimated that in such a model each spin in the vortex core would have a low-frequency moment equal to that in the insulating state at δ=0 (0.6 μ_B).

Observed field-induced signal is much larger than anticipated.

III. Phase diagrams in a magnetic field.

A. Effect of magnetic field on onset of SDW on insulator

B. Effect of magnetic field on SC+SDW to SC transition

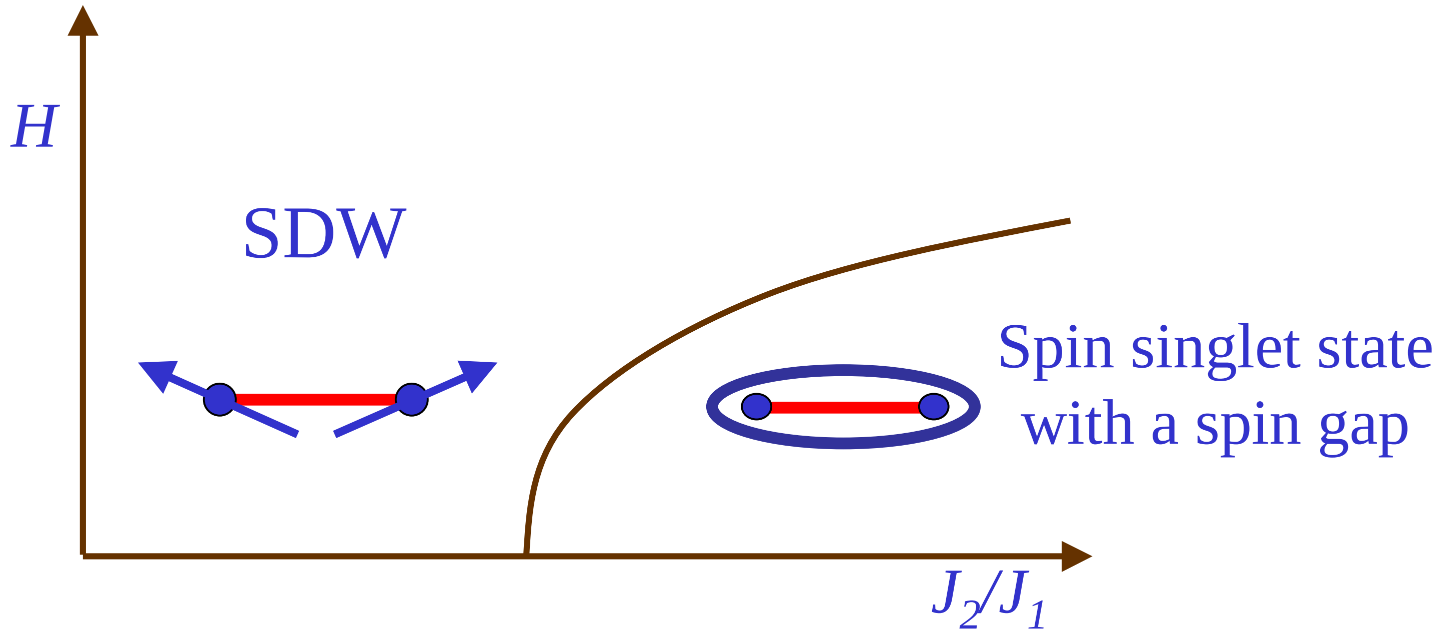


III. Phase diagrams in a magnetic field.

A. Effect of magnetic field on onset of SDW in the insulator

H couples via the Zeeman term

$$\left|\partial_\tau \Phi_\alpha\right|^2 \Rightarrow \left(\partial_\tau \Phi_\alpha^* - i\varepsilon_{\alpha\sigma\rho} H_\sigma \Phi_\rho\right)\left(\partial_\tau \Phi_\alpha - i\varepsilon_{\alpha\beta\gamma} H_\beta \Phi_\gamma\right)$$



Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Related theory applies to spin gap systems in a field and to double layer quantum Hall systems at $\nu=2$

III. Phase diagrams in a magnetic field.

(extreme Type II superconductivity)

B. Effect of magnetic field on SDW+SC to SC transition

Infinite diamagnetic susceptibility of *non-critical* superconductivity leads to a strong effect.

- Theory should account for *dynamic* quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a *static* Ginzburg-Landau theory

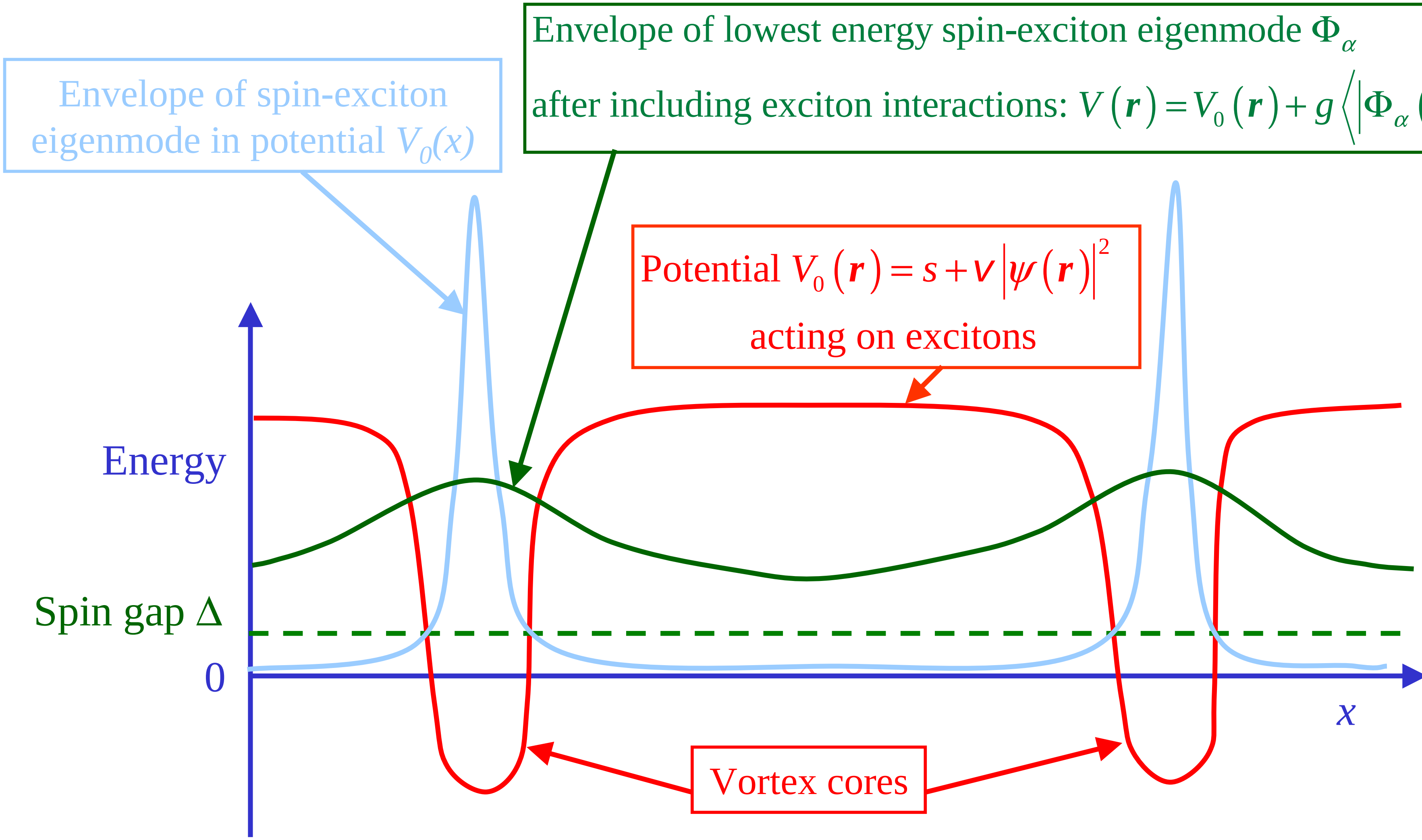
$$S_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

$$S_c = \int d^2 r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

$$F_{GL} = \int d^2 r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

$$Z\left[\psi(r)\right] = \int D\Phi(r,\tau)\, e^{-F_{GL} - S_b - S_c}$$

$$\frac{\delta \ln Z\left[\psi(r)\right]}{\delta \psi(r)} = 0$$

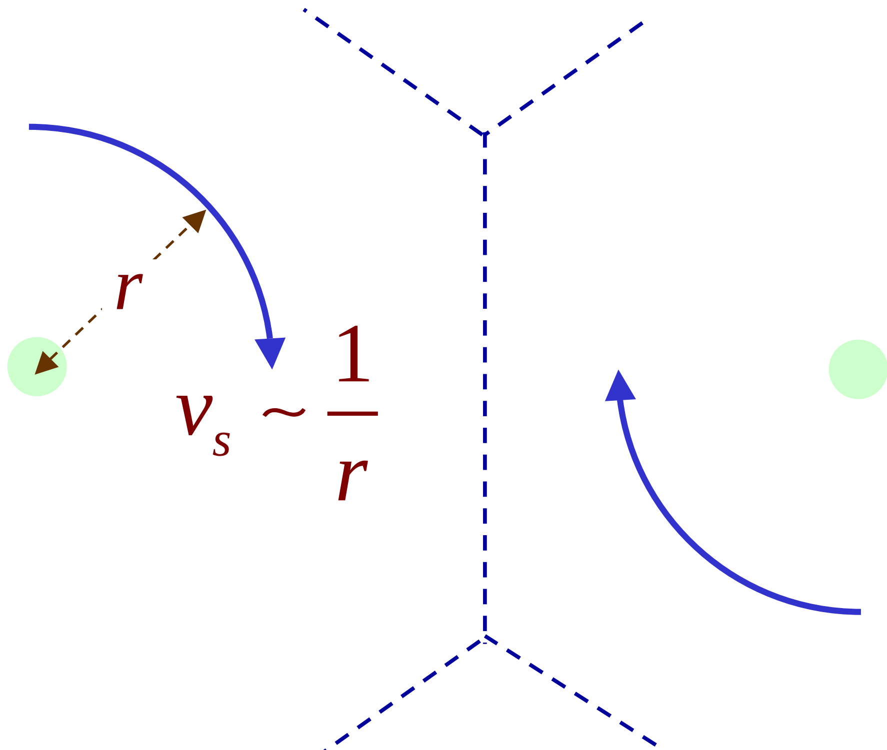


Strongly relevant repulsive interactions between excitons imply that low energy excitons must be extended.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

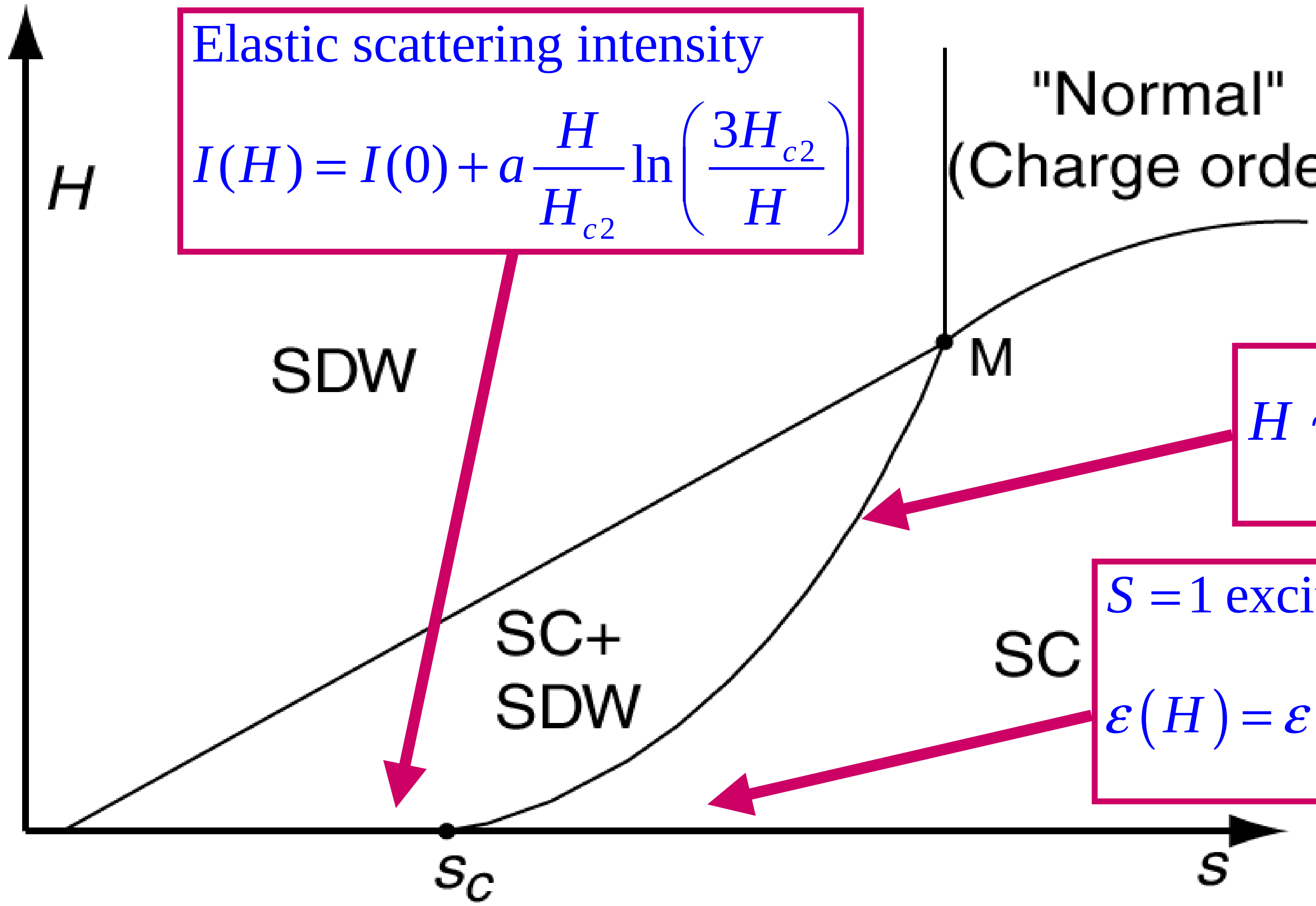
Tuning parameter s replaced by $s_{eff}\left(H\right) = s - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$

Main results

$T=0$

Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$



$$H \sim \frac{(s-s_c)}{\ln\left(1/(s-s_c)\right)}$$

$S=1$ exciton energy

$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

- All functional forms are **exact**.

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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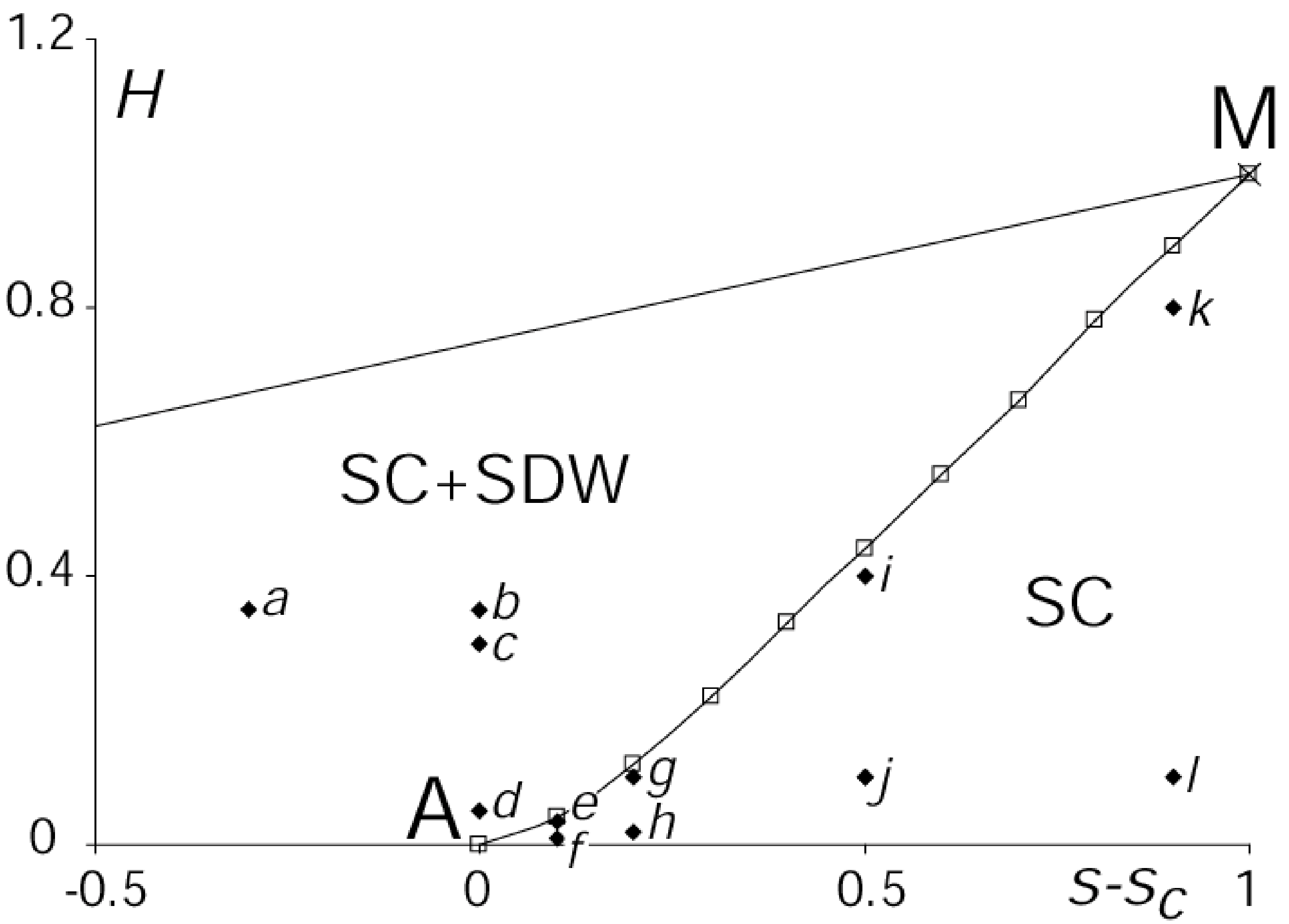
Full solution of self-consistent large *N* equations for phases and phase boundaries

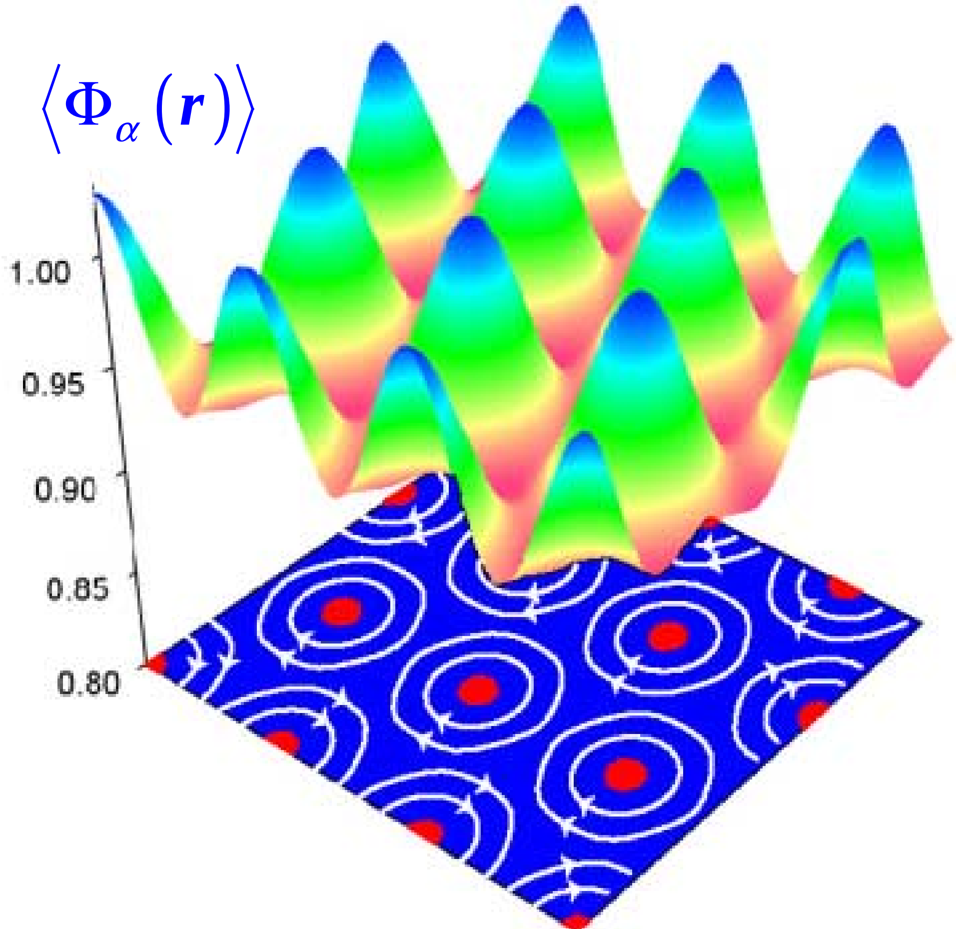
Y. Zhang, E. Demler, and S. Sachdev,
cond-mat/0112xxx, to appear shortly,
http://onsager.physics.yale.edu/sdw.pdf

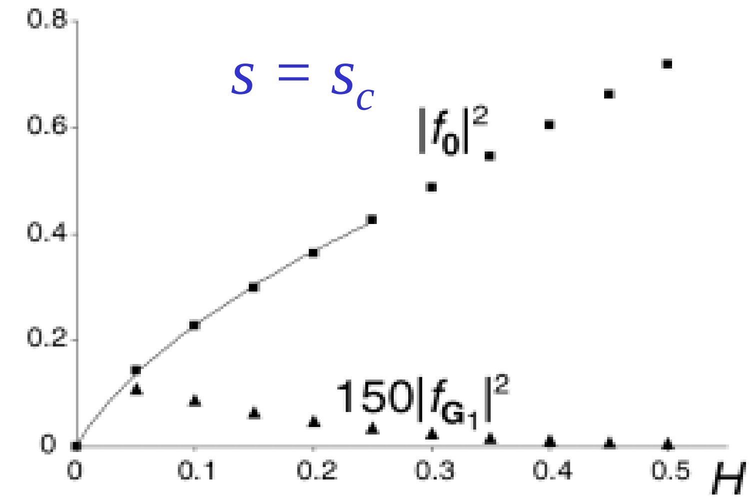


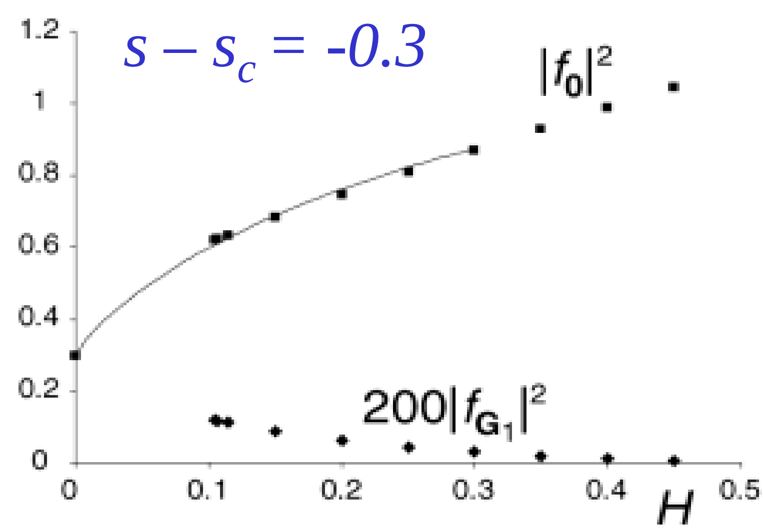
Structure of *long-range* SDW order in SC+SDW phase

Computation in a self-consistent "large *N*" theory

$\langle \Phi_\alpha(\boldsymbol{r}) \rangle$







Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} |f_{\boldsymbol{G}}|^2 \delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

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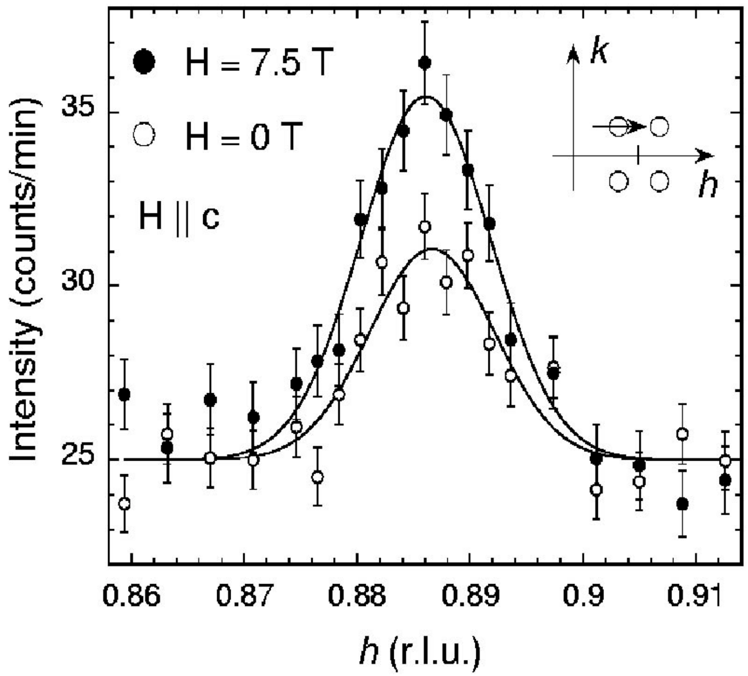
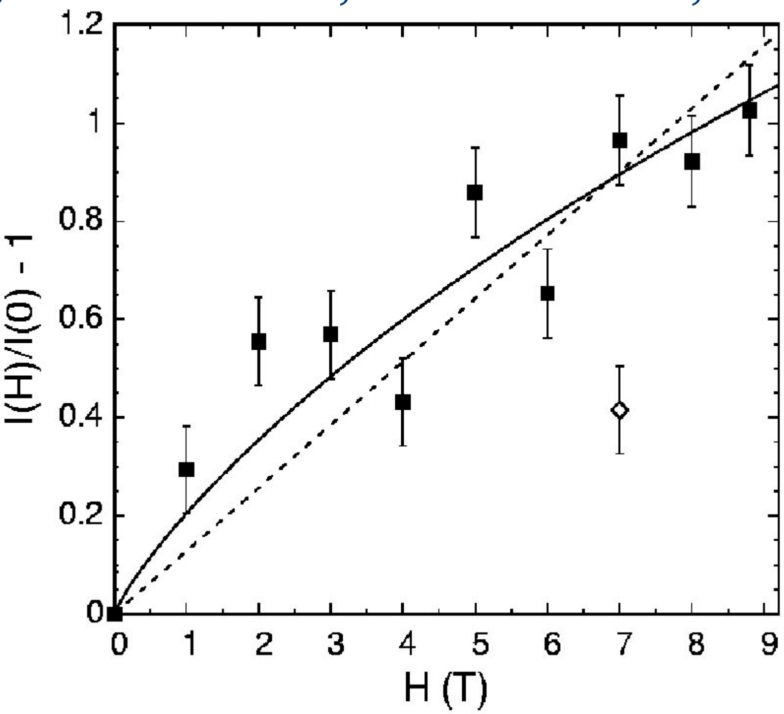
Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, preprint.





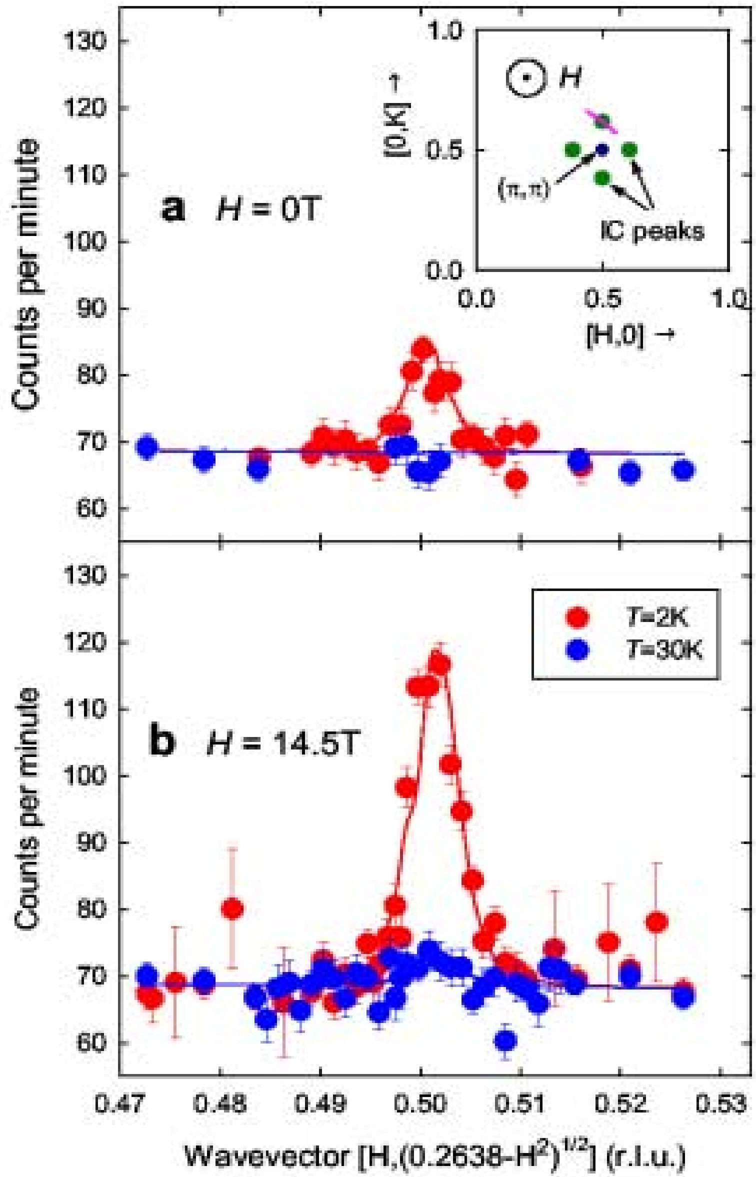
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

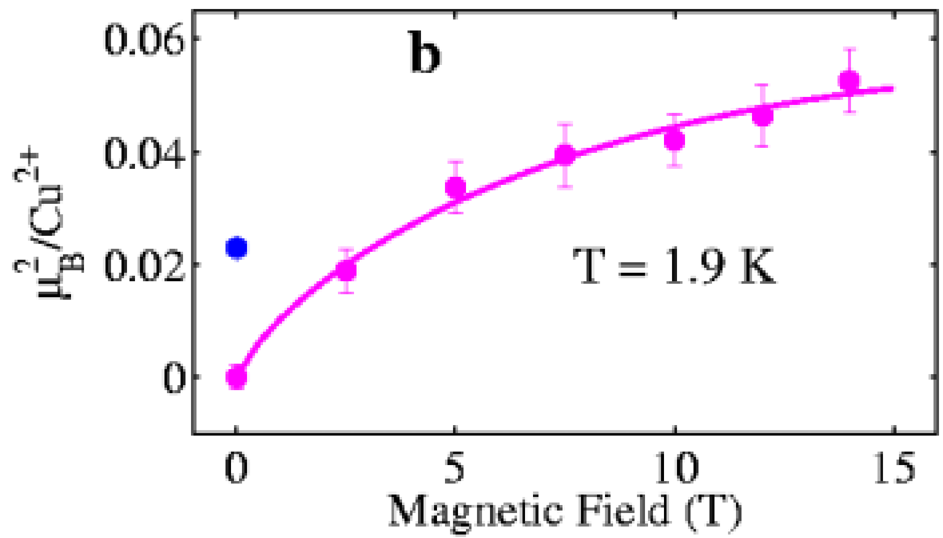
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*, preprint

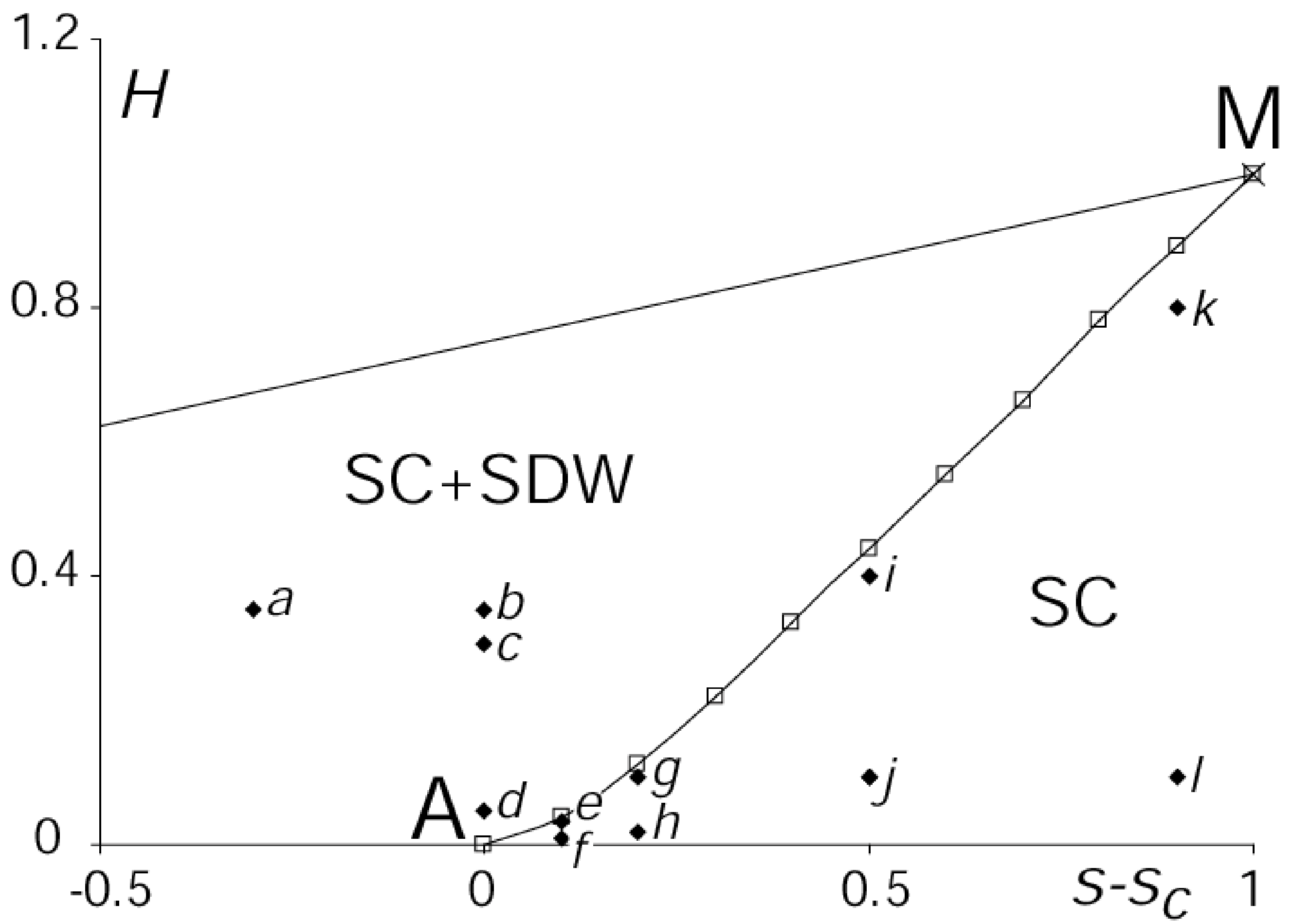




Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

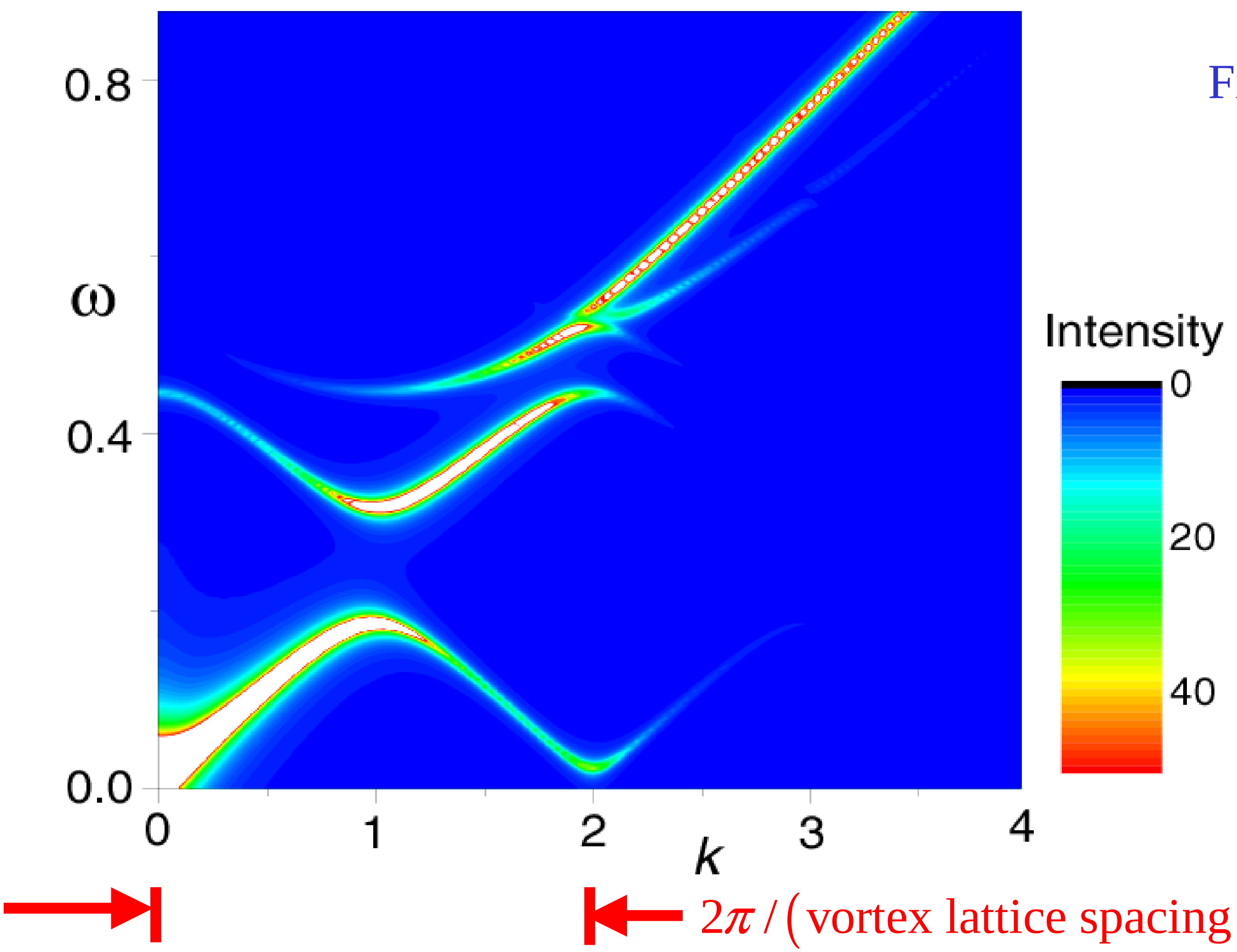
Full solution of self-consistent large *N* equations for phases and phase boundaries

Y. Zhang, E. Demler, and S. Sachdev,
cond-mat/0112xxx, to appear shortly,
http://onsager.physics.yale.edu/sdw.pdf



Dynamic SDW fluctuations in the SC phase

Computation of spin susceptibility $\chi''(k,\omega)$ in self-consistent large N theory of Φ_α fluctuations



Field H chosen to place the system close to boundary to SC+SDW phase

Point "e"

$2\pi/(\text{vortex lattice spacing})$

Spatially resolved NMR around vortices induced by a magnetic field in the superconducting state

Spatially resolved electronic structure inside and outside the vortex cores of a high-temperature superconductor

V. F. Mitrović[*], E. E. Sigmund[*], M. Eschrig[†], H. N. Bachman[*], W. P. Halperin[*], A. P. Reyes[‡], P. Kuhns[‡] & W. G. Moulton[‡]

* *Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
† *Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
‡ *National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA*

Puzzling aspects of high-transition-temperature (high-T_c) superconductors include the prevalence of magnetism in the normal state and the persistence of superconductivity in high magnetic fields. Superconductivity and magnetism generally are thought to be incompatible, based on what is known about conventional superconductors. Recent results[1], however, indicate that antiferromagnetism can appear in the superconducting state of a high-T_c superconductor in the presence of an applied magnetic field. Magnetic fields penetrate a superconductor in the form of quantized flux lines, each of which represents a vortex of supercurrents. Superconductivity is suppressed in the core of the vortex and it has been suggested that antiferromagnetism might develop there[2]. Here we report the results of a high-field nuclear-magnetic-resonance (NMR) imaging experiment[3–5] in which we spatially resolve the electronic structure of near-optimally doped $YBa_2Cu_3O_{7-\delta}$ inside and outside vortex cores. Outside the cores, we find strong antiferromagnetic fluctuations, whereas inside we detect electronic states that are rather different from those found in conventional superconductors.

Nature, **413**, 501 (2001).

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Pinning of CDW order by vortex cores in SC phase

Simplified effective action for extended SDW fluctuations

A. Polkovnikov, S. Sachdev,
M. Vojta, and E. Demler,
cond-mat/0110329

$$S_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s_{\text{eff}}\left(H\right)\left|\Phi_\alpha\right|^2 \right]$$

s has been reduced to s_{eff} by superflow.

Vortex cores have a preference for a particular location of local CDW order:

$$S_{\text{lat}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\Phi_\alpha^2\left(\boldsymbol{r}_v\right) e^{i\theta} + \text{c.c.} \right]$$

This induces static CDW order (without static SDW order) around vortex core:

$$\left\langle \Phi_\alpha^2\left(\boldsymbol{r}\right)\right\rangle = \sum_{x_v} \left(\frac{3}{8\pi^{3/2} c^{5/2} s_{\text{eff}}^{1/4}} \right) \zeta e^{-i\theta} \frac{e^{-2\left|\boldsymbol{r}-\boldsymbol{r}_v\right|\sqrt{s_{\text{eff}}}/c}}{\left|\boldsymbol{r}-\boldsymbol{r}_v\right|^{3/2}} \quad ; \quad \left\langle \Phi_\alpha\left(\boldsymbol{r}\right)\right\rangle = 0$$

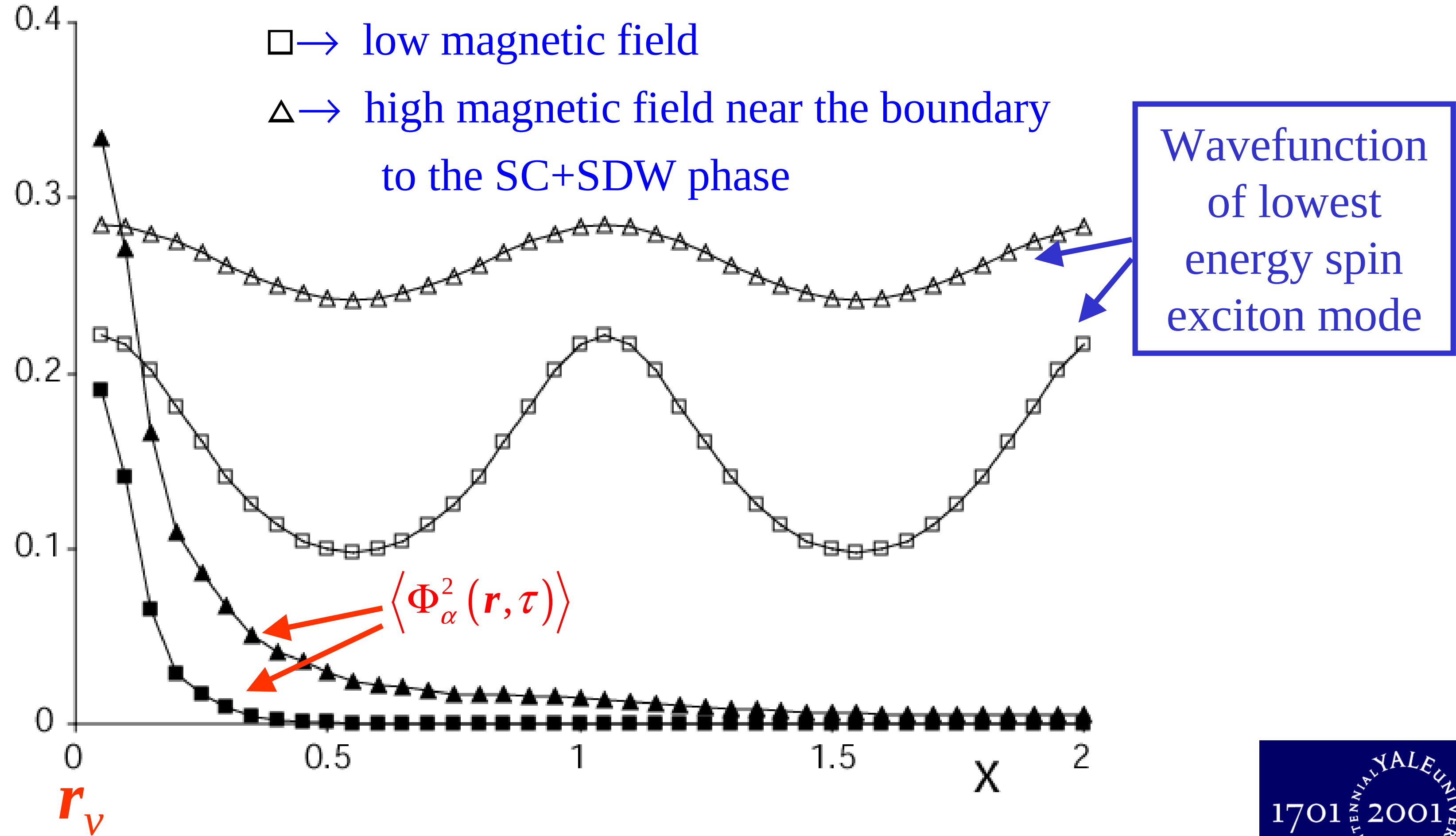
$$\delta\rho\left(\boldsymbol{r}\right) \propto \sum_\alpha \Phi_\alpha^2\left(\boldsymbol{r}\right) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.} \quad ; \quad S_\alpha\left(\boldsymbol{r}\right) = \Phi_\alpha\left(\boldsymbol{r}\right) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

Note **correlation length of CDW = (1/2) x (correlation of SDW)**

Pinning of CDW order by vortex cores in SC phase

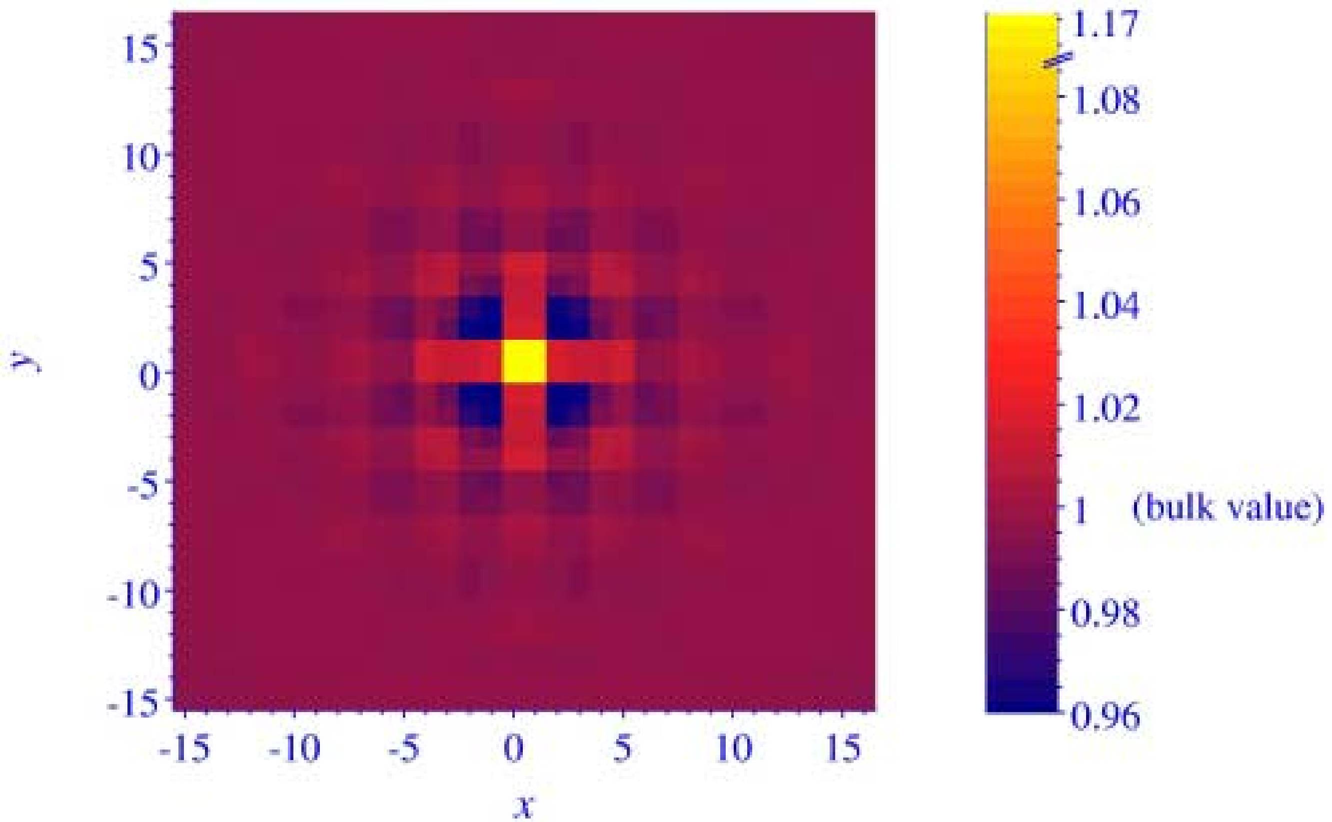
Computation in self-consistent large N theory

$$\left\langle \Phi_\alpha^2(\mathbf{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\mathbf{r},\tau)\Phi_\alpha^*(\mathbf{r}_v,\tau_1)\right\rangle^2$$



Simplified theoretical computation of modulation in local density of states at low energy due to CDW order induced by superflow and pinned by vortex core

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329



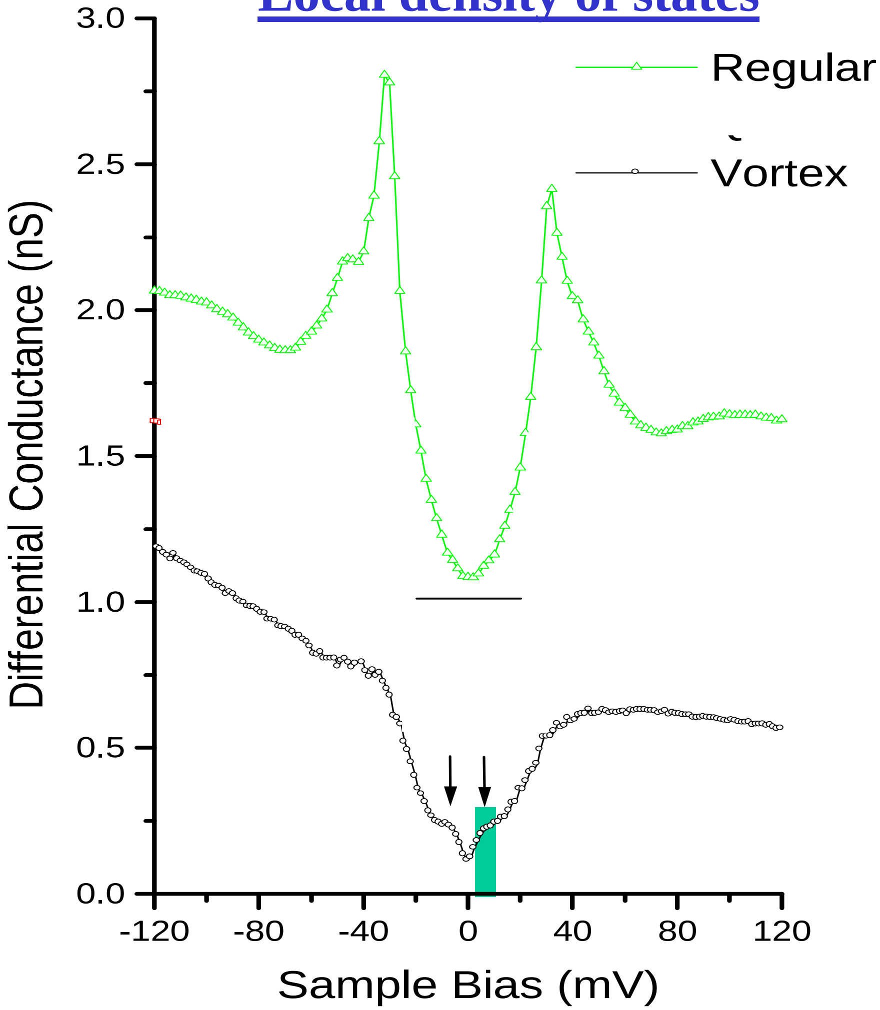
$$H=\sum_{ij}\left(-t_{ij}c_{i\sigma}^{\dagger}c_{j\sigma}+\Delta_{ij}c_{i\sigma}^{\dagger}c_{j,-\sigma}^{\dagger}+h.c.\right)+\sum_{i}\left[v\left(\boldsymbol{r}_i\right)-\mu\right]c_{i\sigma}^{\dagger}c_{i\sigma},$$

$$v\left(\boldsymbol{r}\right)=v_1\left\{\cos\left[\boldsymbol{K}_{cx}\cdot\left(\boldsymbol{r}-\boldsymbol{r}_0\right)\right]+\cos\left[\boldsymbol{K}_{cy}\cdot\left(\boldsymbol{r}-\boldsymbol{r}_0\right)\right]\right\}e^{-\left|\boldsymbol{r}-\boldsymbol{r}_0\right|/\xi_c}\left(\left|\boldsymbol{r}-\boldsymbol{r}_0\right|^2+1\right)^{-3/4}$$

(E) STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, preprint, *Science*, to appear.

Local density of states



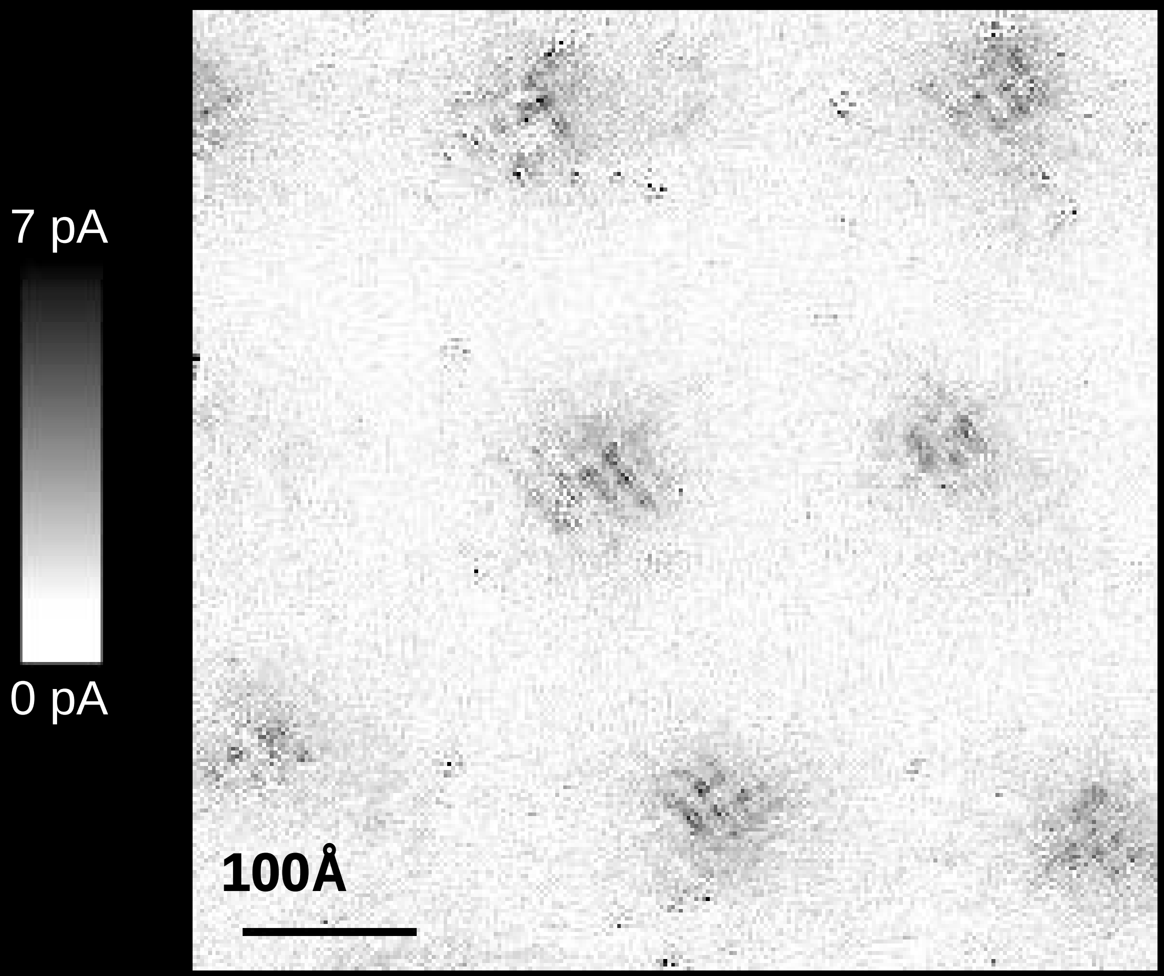
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

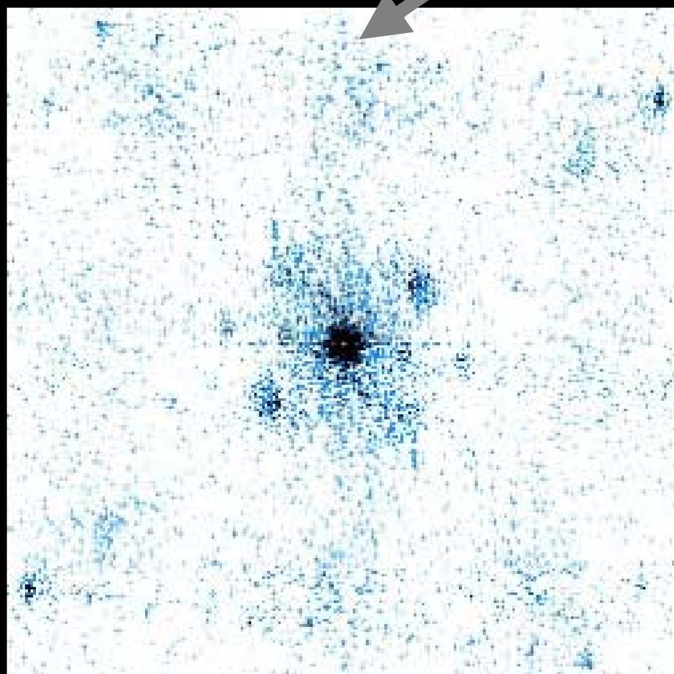
Berkeley University of California

Vortex-induced LDOS integrated from 1meV to 12meV

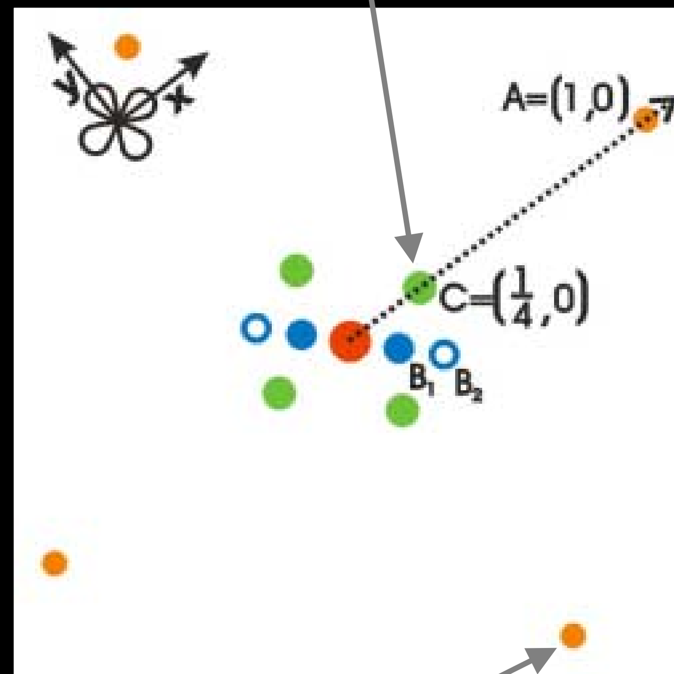


J. Hoffman *et al* Submitted.

Fourier Transform of Vortex-Induced LDOS map



K-space locations of vortex induced LDOS



K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

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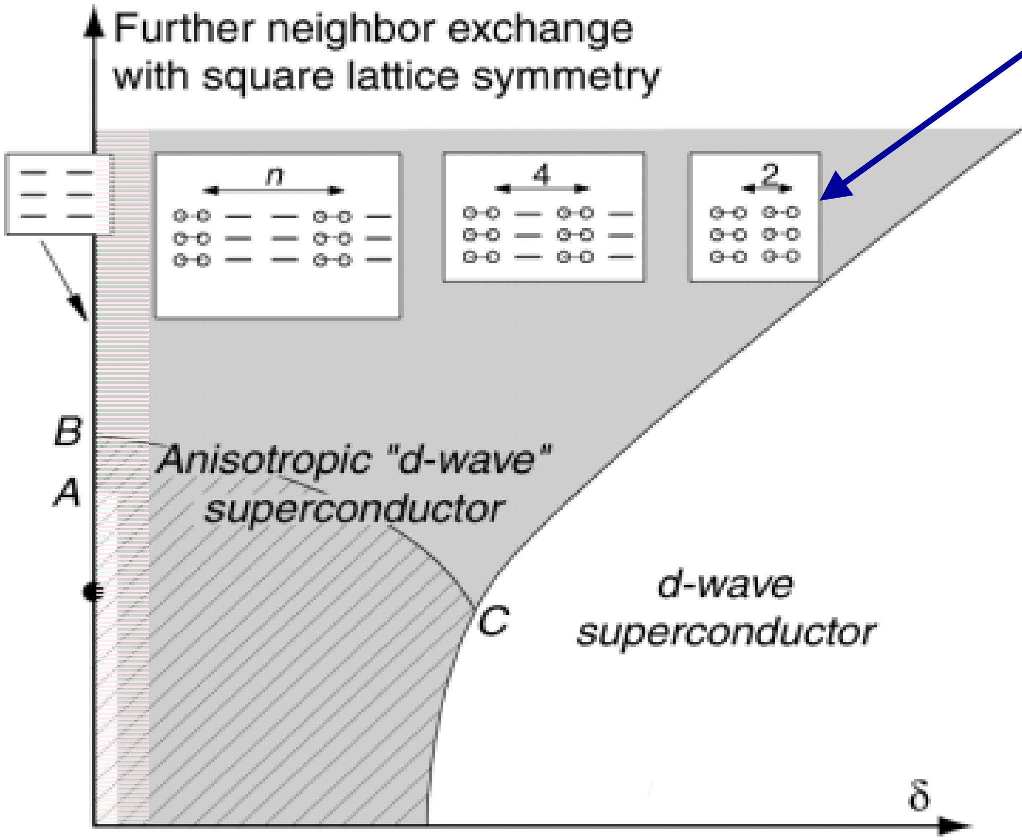
University of California

Why does CDW have period 4 ?



"Large N" theory in region with preserved spin rotation symmetry

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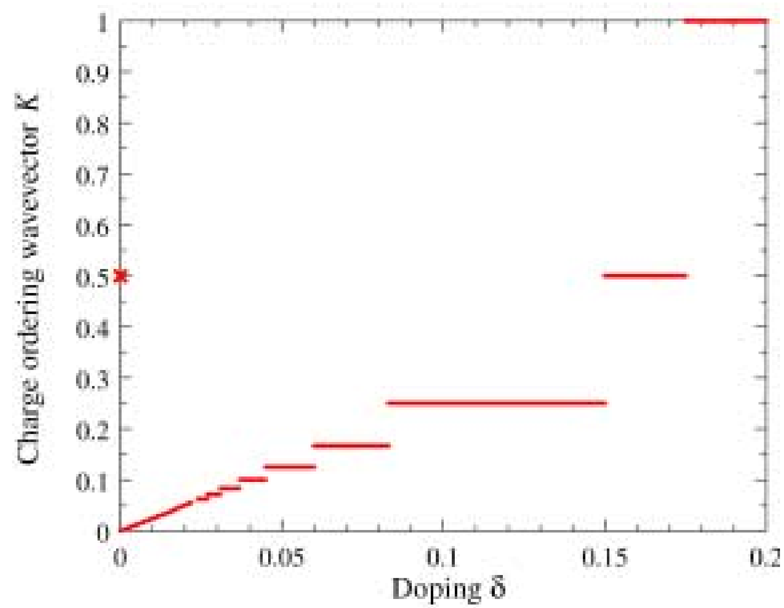
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).



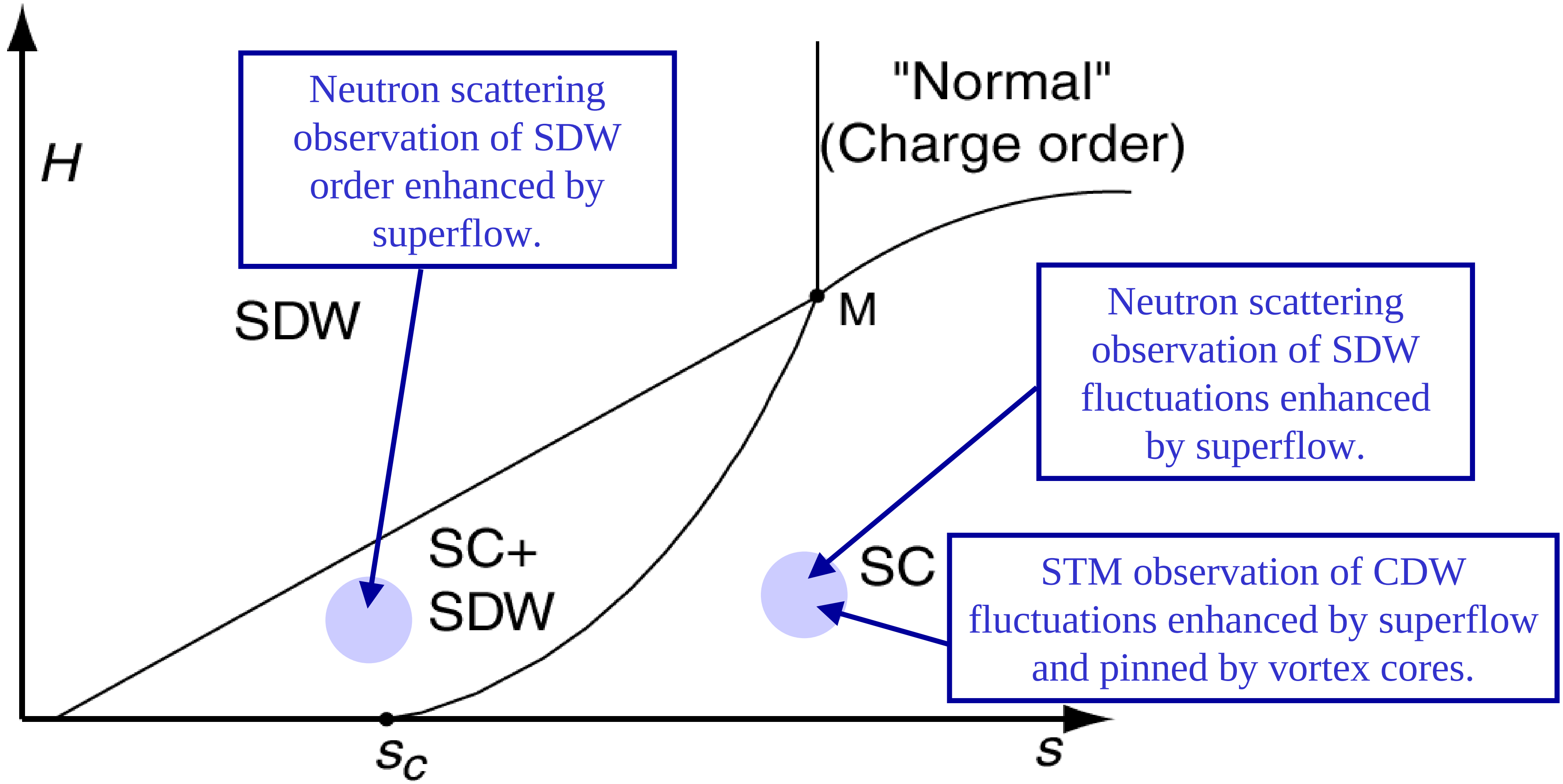
See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).

Effect of magnetic field on SDW+SC to SC transition

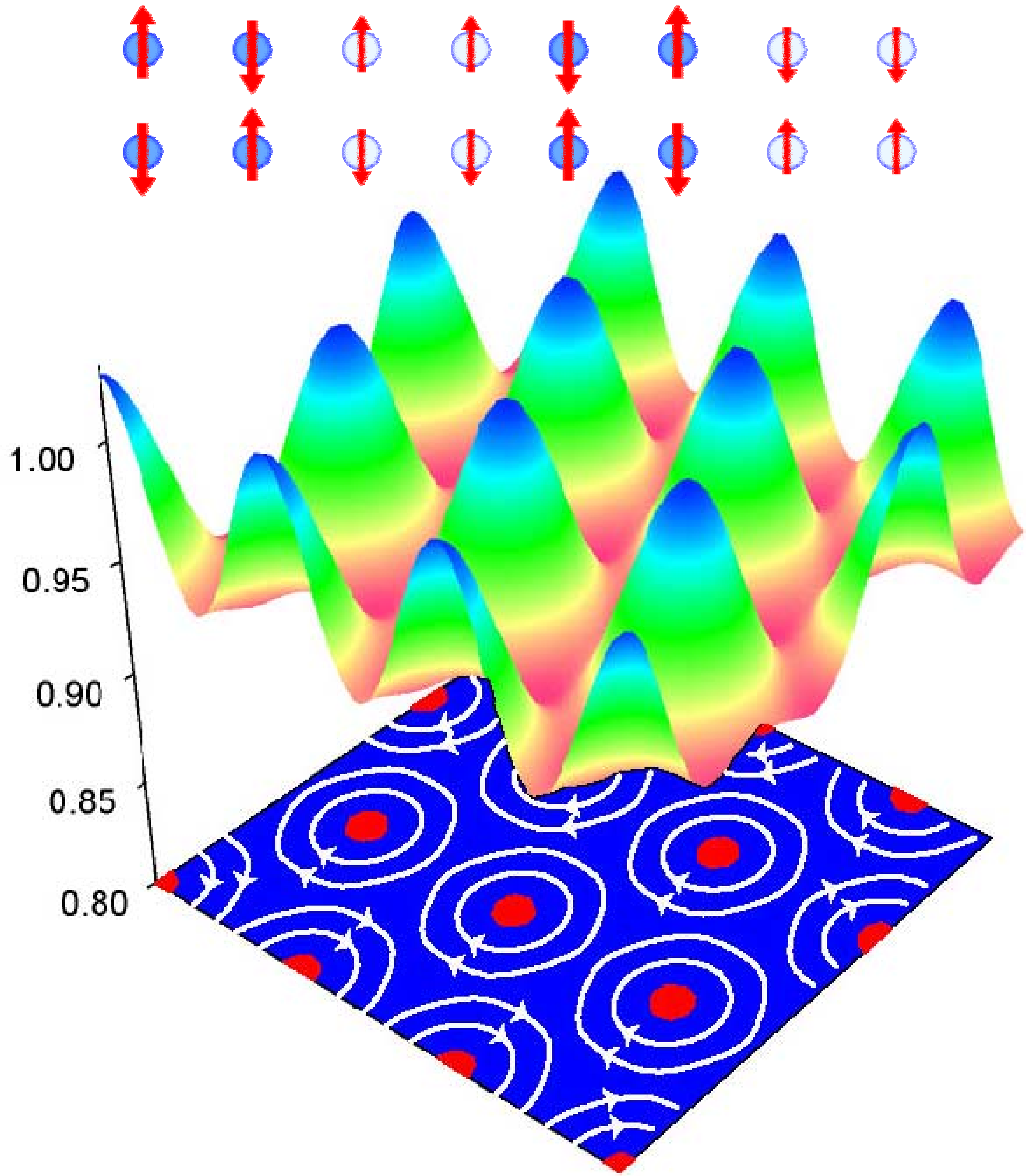
(extreme Type II superconductivity)

T=0

Main results



Prospects for studying quantum critical point between SC and SC+SDW phases by tuning H ?


1.00
0.95
0.90
0.85
0.80