Quantum phase transitions in Mott insulators and *d*-wave superconductors

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Ying Zhang

Science **286**, 2479 (1999).

Transparencies on-line at
http://pantheon.yale.edu/~subir

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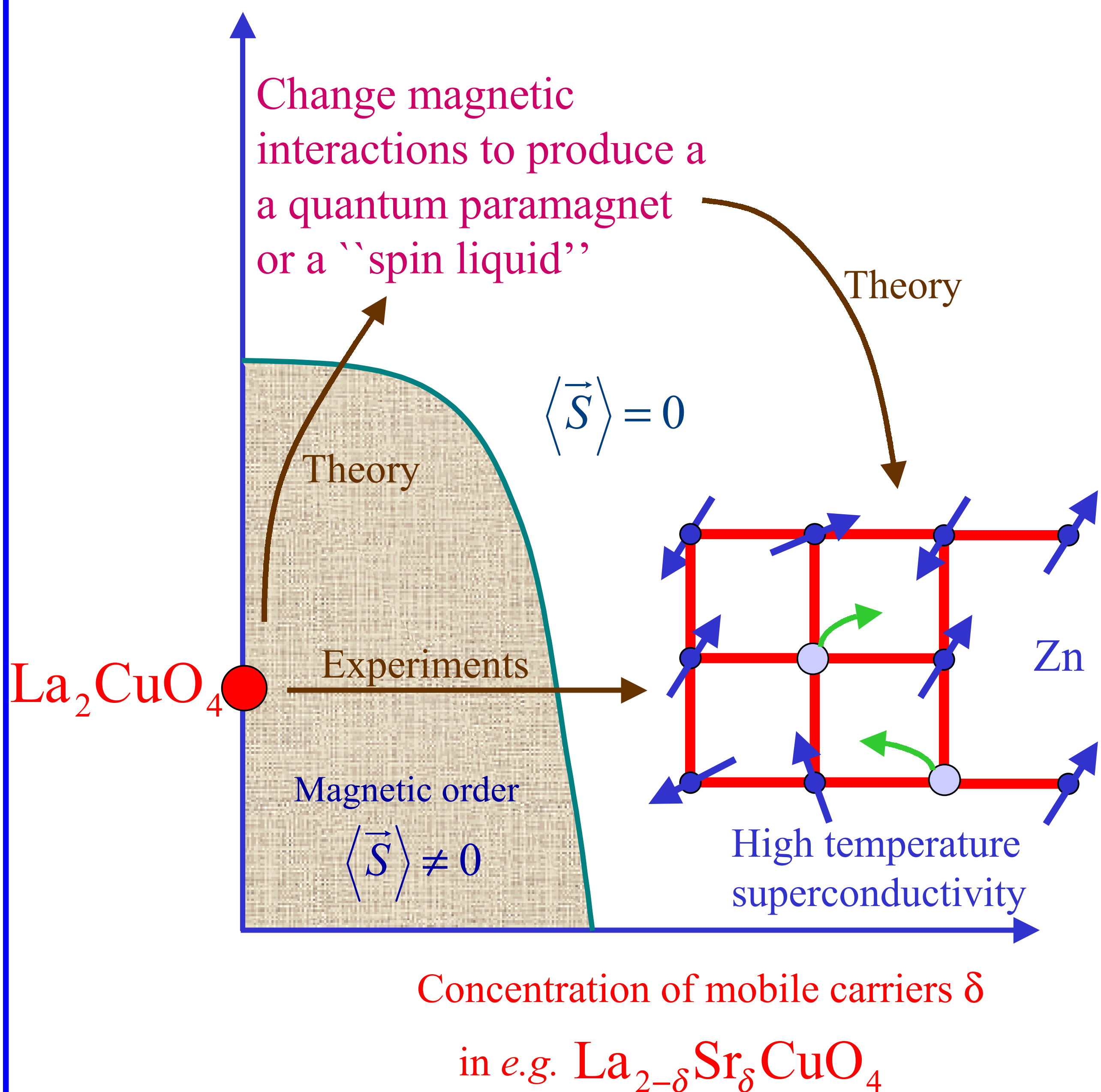


Change magnetic
interactions to produce a
a quantum paramagnet
or a ``spin liquid''
Theory
Theory
$\langle \vec{S} \rangle = 0$
La_2CuO_4
Experiments
Zn
Magnetic order
$\langle \vec{S} \rangle \neq 0$
High temperature
superconductivity
Concentration of mobile carriers δ
in *e.g.* $La_{2-\delta}Sr_\delta CuO_4$

Outline

1. Neel and paramagnetic ground states of two-dimensional antiferromagnets

2. Magnetic phase transitions in *d*-wave superconductors

3. Experimental evidence for theoretical framework

   (a) Inelastic neutron scattering measurements of phonon and spin spectra

   (b) NMR relaxation rates

   (c) Experiments on Zn impurities: NMR, neutron scattering, and STM

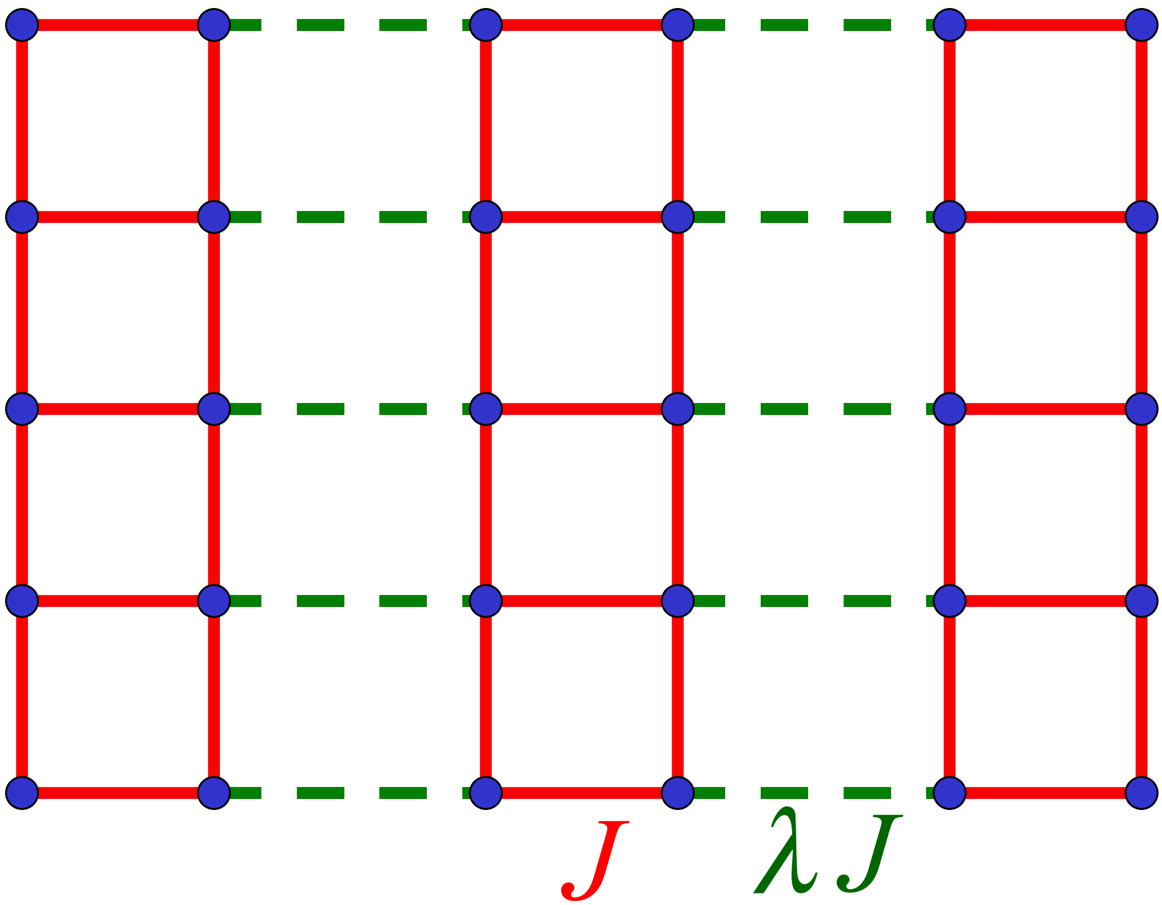
4. Damping of nodal quasi-particles: quantum phase transitions in *d*-wave superconductors with a spin-singlet order parameter at zero momentum.

5. Conclusions

1. Paramagnetic and Neel states of Mott insulators

(Katoh and Imada;
Tworzydlo, Osman, van Duin and Zaanen)

S=1/2 spins on coupled 2-leg ladders



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

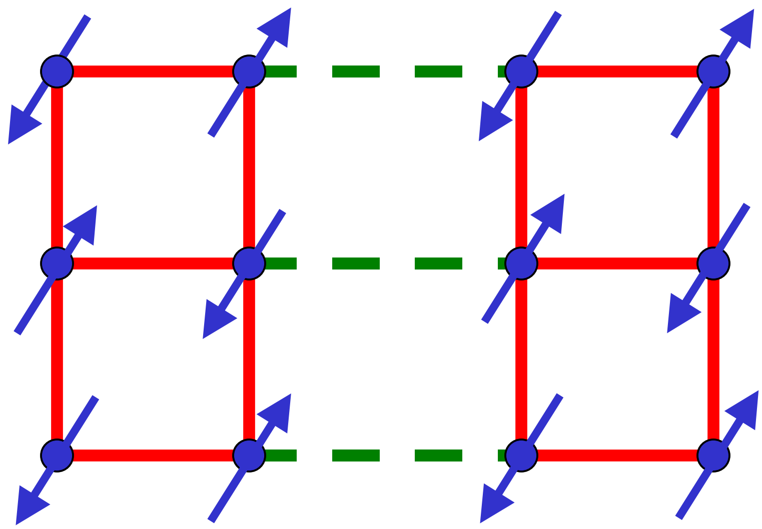
Follow ground state as a function of λ

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



Ground state has long-range magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

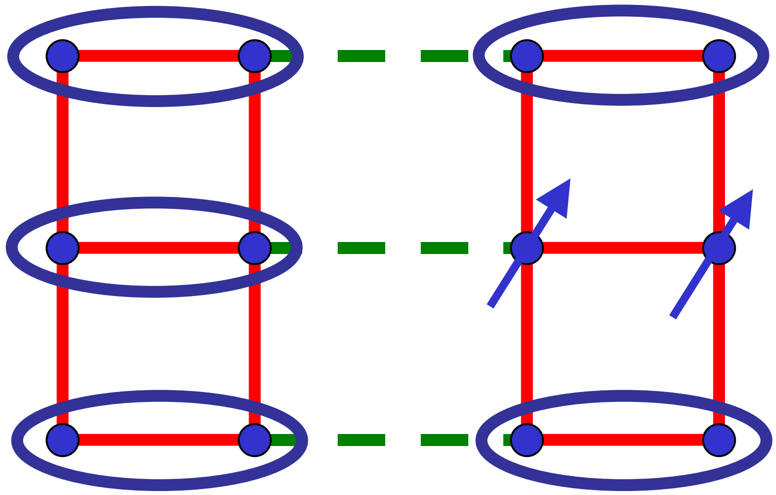
Excitations: 2 spin waves

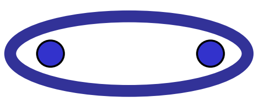
Quasiclassical wave dynamics at low T

(Chakravarty et al, 1989; Tyc et al, 1989)

λ close to 0

Weakly coupled ladders



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

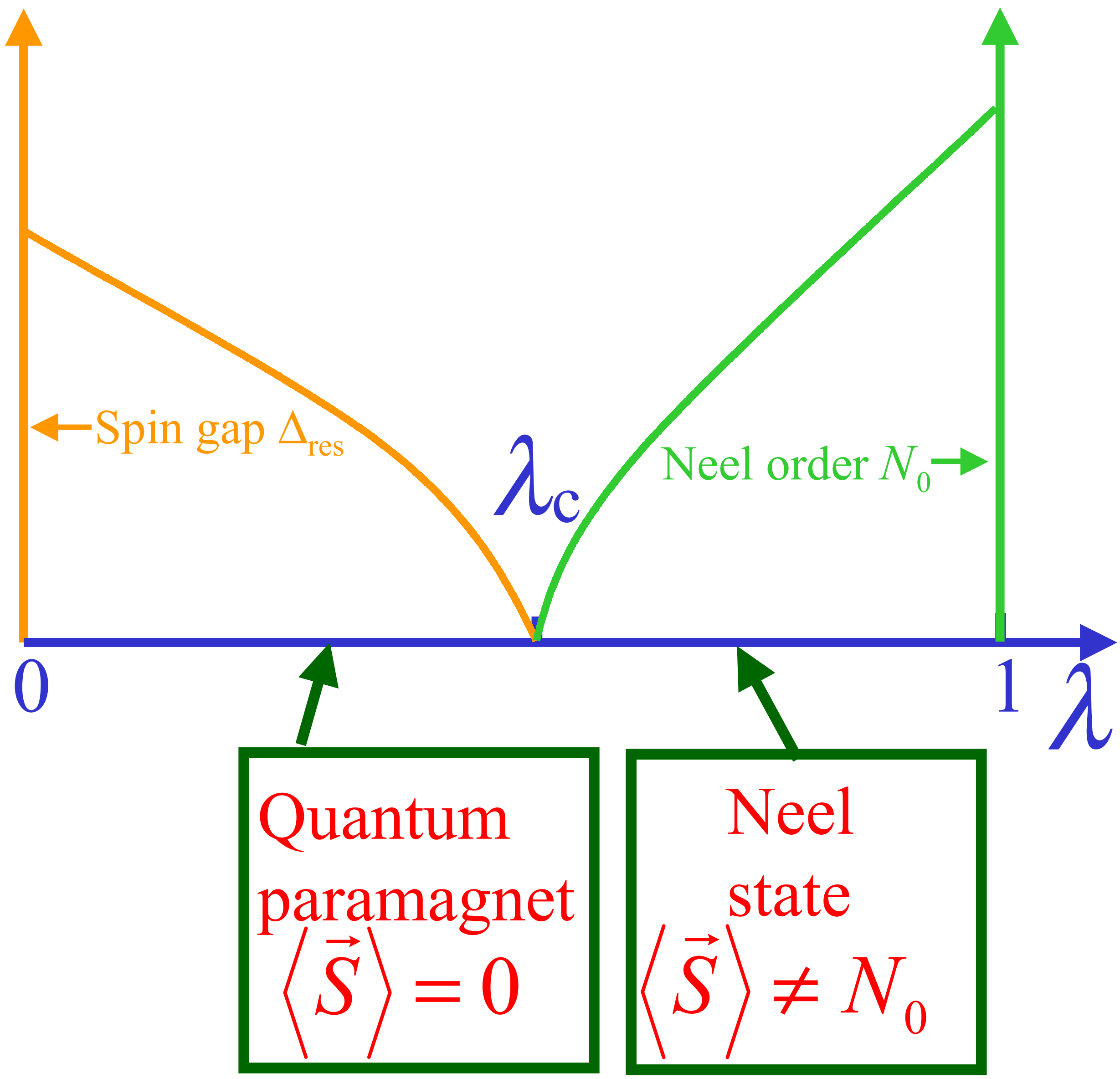
Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

Excitation: S=1, ϕ_α particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

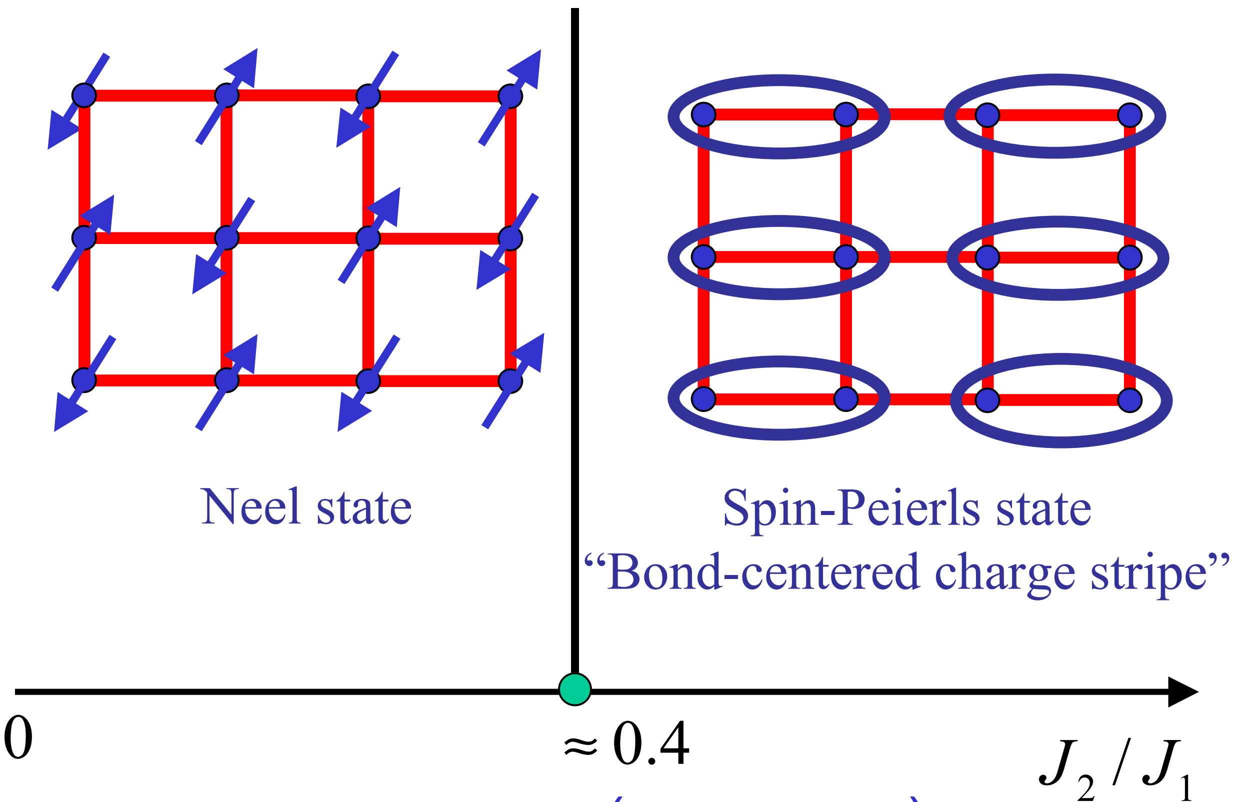
$$\varepsilon = \Delta_{res} + \frac{c^2 k^2}{2\Delta_{res}}$$

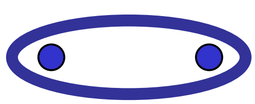
$\Delta_{res} \rightarrow$ Spin gap


Spin gap Δ_{res}
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Square lattice antiferromagnets

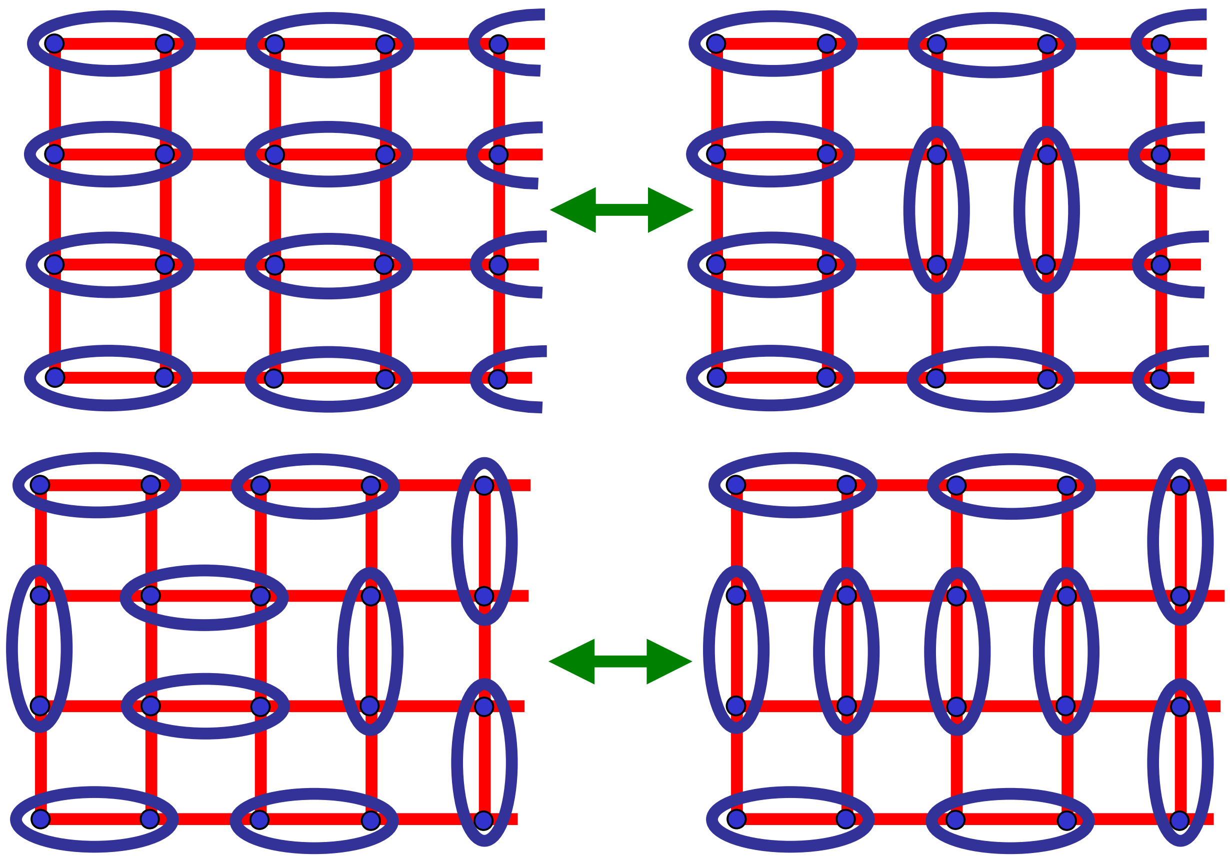
Square lattice with first (J_1) and second (J_2) neighbor exchange interactions



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
O. P. Sushkov, J. Oitmaa, and Z. Weihong, condmat/0007329.
M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, cond-mat/0002116.

Quantum dimer model –
D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left. These effects always lead to a broken square lattice symmetry near the transition to the Neel state.

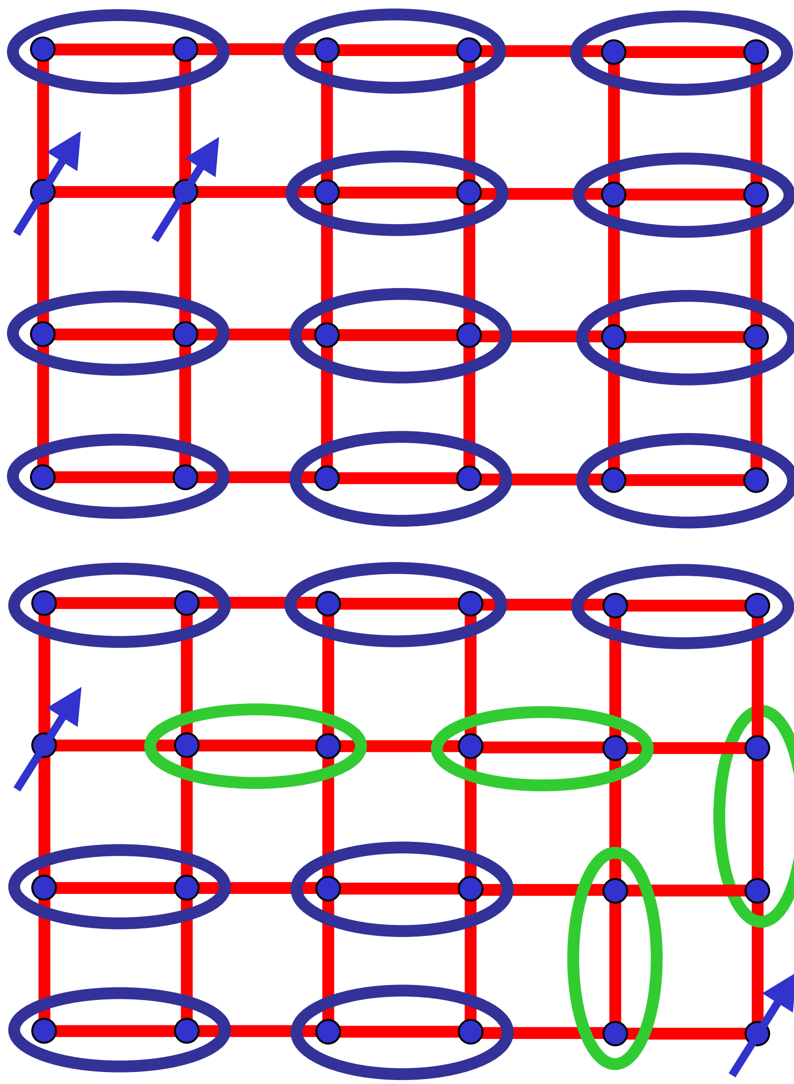
N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

Excitations

Stable S=1 particle

Energy dispersion $$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

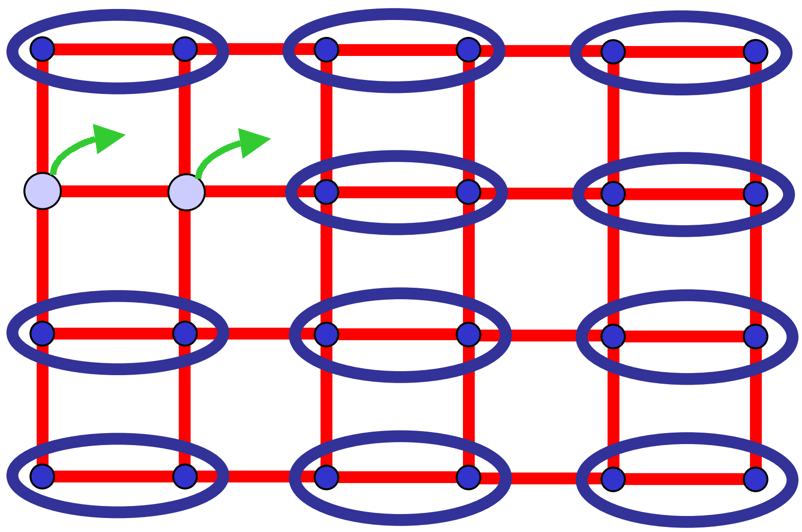
$\Delta \rightarrow$ Spin gap



S=1/2 spinons are linearly confined by the line of "defect" singlet pairs between them

2. Magnetic phase transitions in d-wave superconductors

A. Doping a paramagnet with confinement



Condensate of hole pairs

E. Fradkin and S. Kivelson, Mod. Phys. Lett B **4**, 225 (1990).
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

B. Doping a deconfined paramagnet

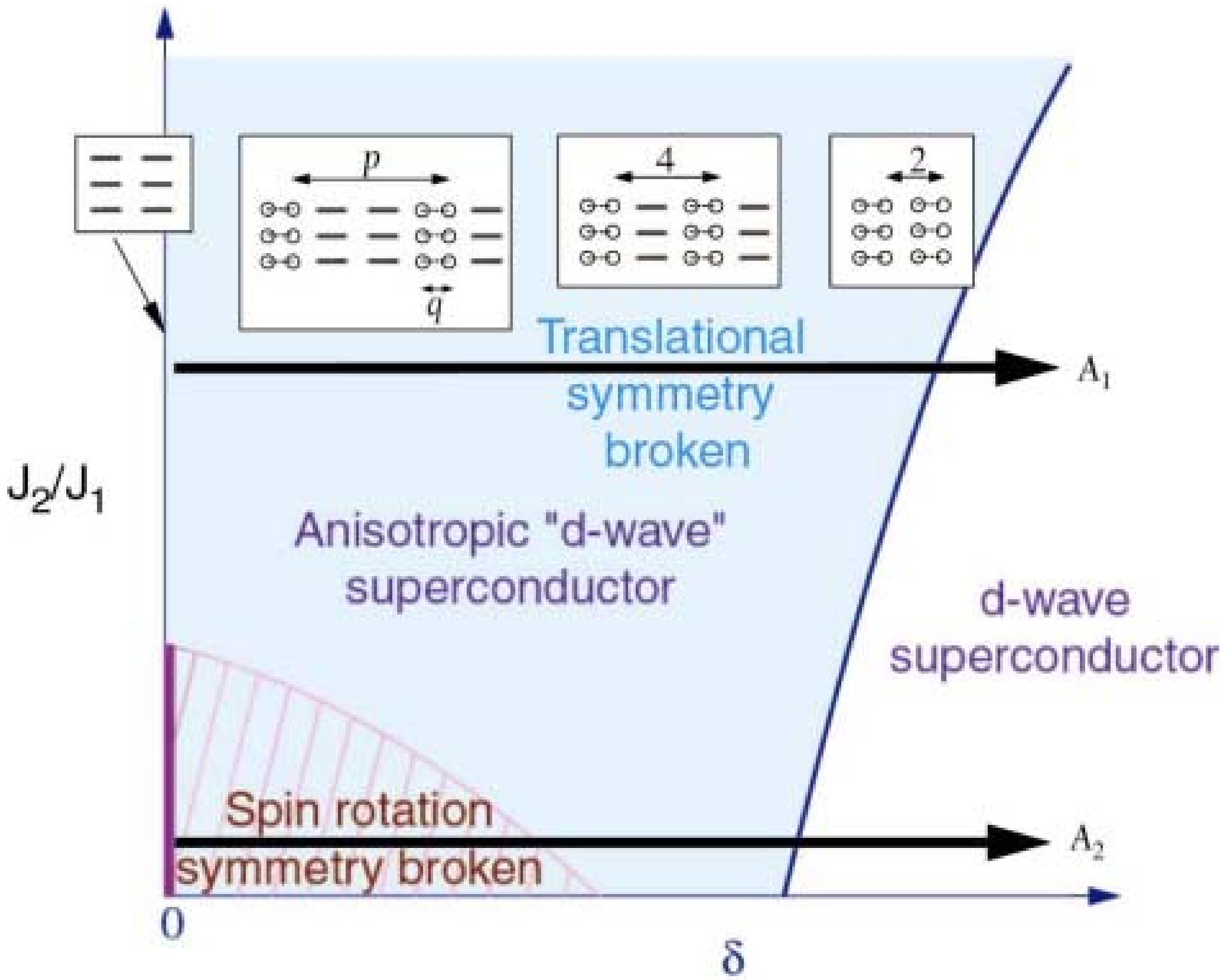
Single hole condensate

hc/(2e) flux quantum (S. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. 6, 353 (1988))

Stable hc/e vortices (S. Sachdev, Phys. Rev. B 45, 389 (1992))

+ other exotica (T. Senthil and M.P.A. Fisher, cond-mat/0006481)

Phase diagram for case A



Superconductivity coexists with stripe order

S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).
M. Vojta and S. Sachdev, Phys. Rev. Lett. **83**, 3916 (1999).
See also J. Zaanen, Physica C **217**, 317 (1999) and
S. Kivelson, E. Fradkin and V. Emery, Nature **393**, 550 (1998)

Outline

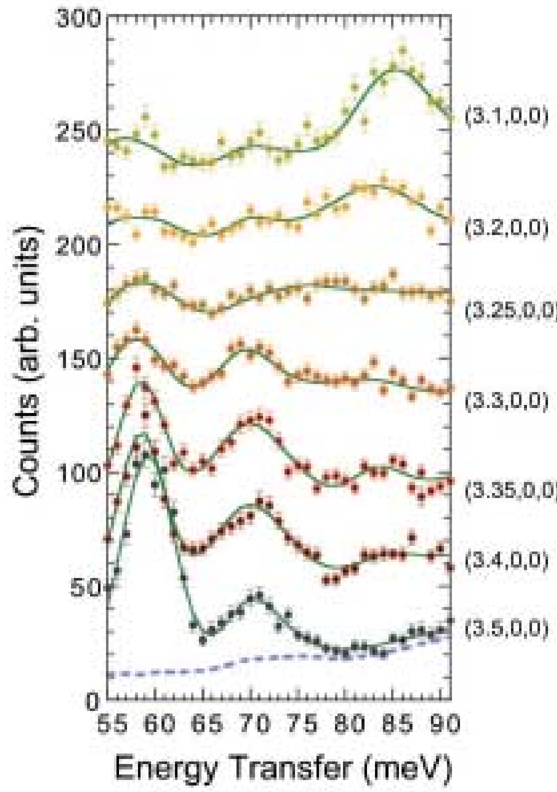
1. Neel and paramagnetic ground states of two-dimensional antiferromagnets
2. Magnetic phase transitions in d-wave superconductors
3. **Experimental evidence for theoretical framework**

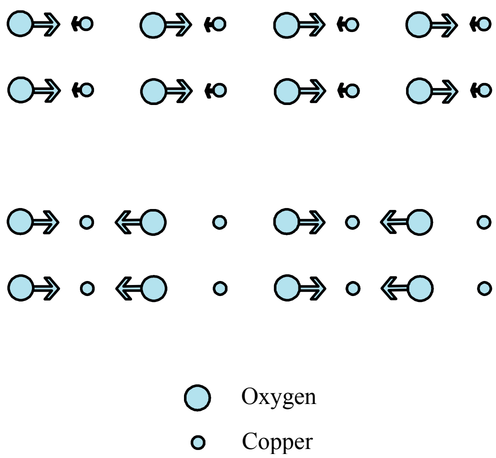
   (a) Inelastic neutron scattering measurements of phonon and spin spectra

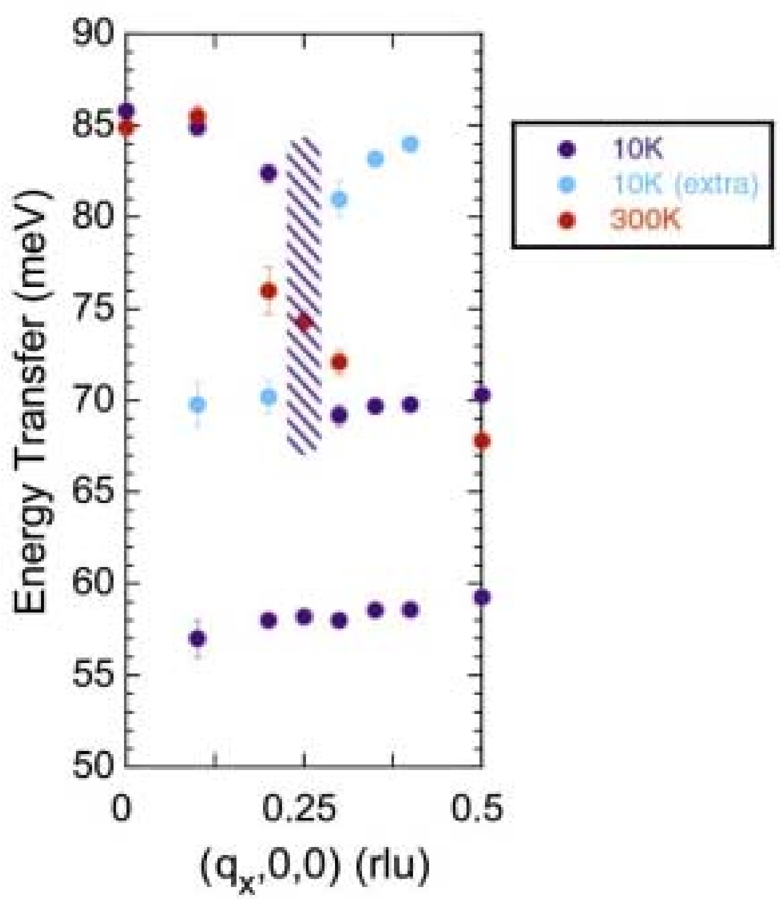
   (b) NMR relaxation rates

   (c) Experiments on Zn impurities: NMR, neutron scattering, and STM
4. Damping of nodal quasi-particles: quantum phase transitions in *d*-wave superconductors with a spin-singlet order parameter at zero momentum.
5. Conclusions

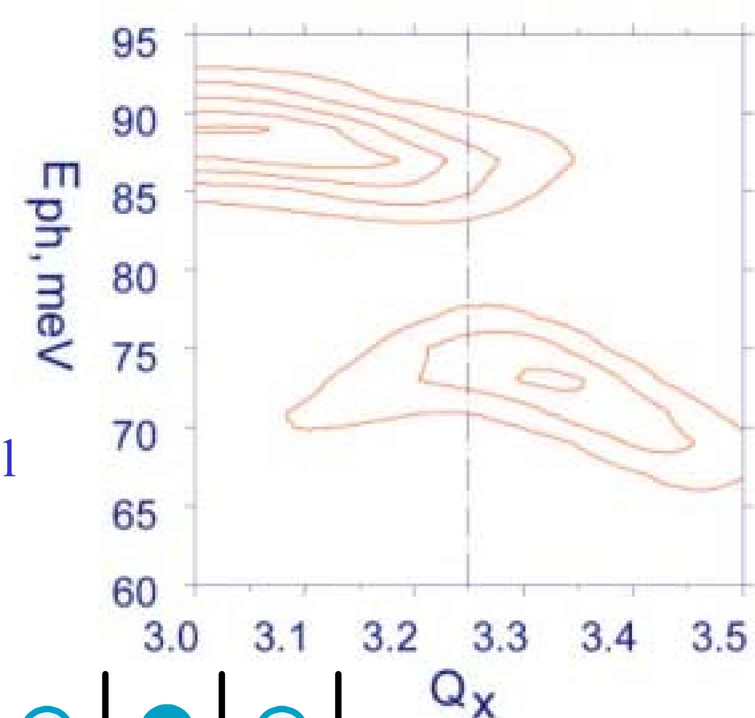
Neutron scattering measurements of phonon spectrum of superconducting $La_{1.85}Sr_{0.15}CuO_4$ by R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (1999)

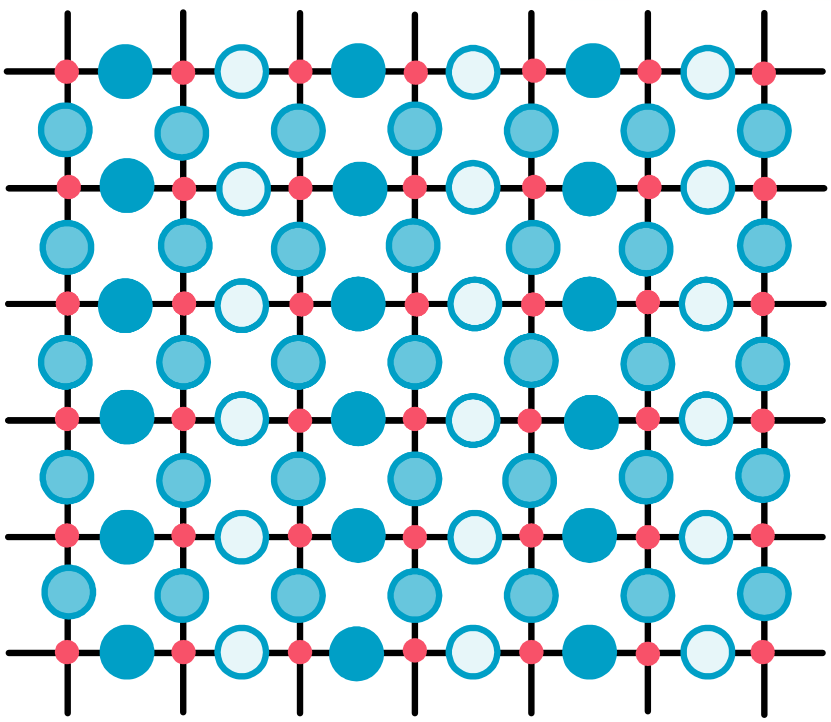






Computation of phonon spectrum by McQueeney *et al* using a simple model based on lattice modulation below

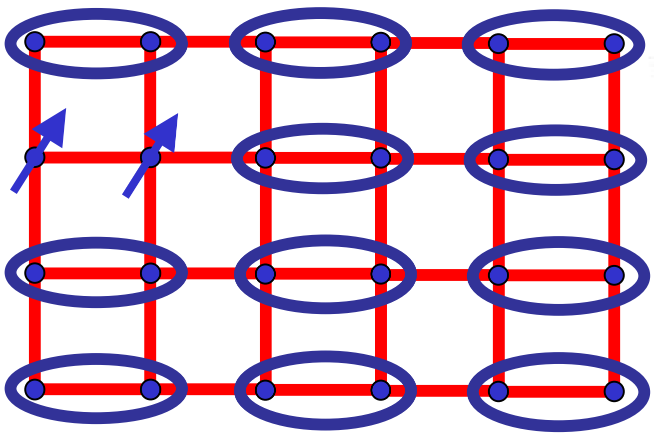


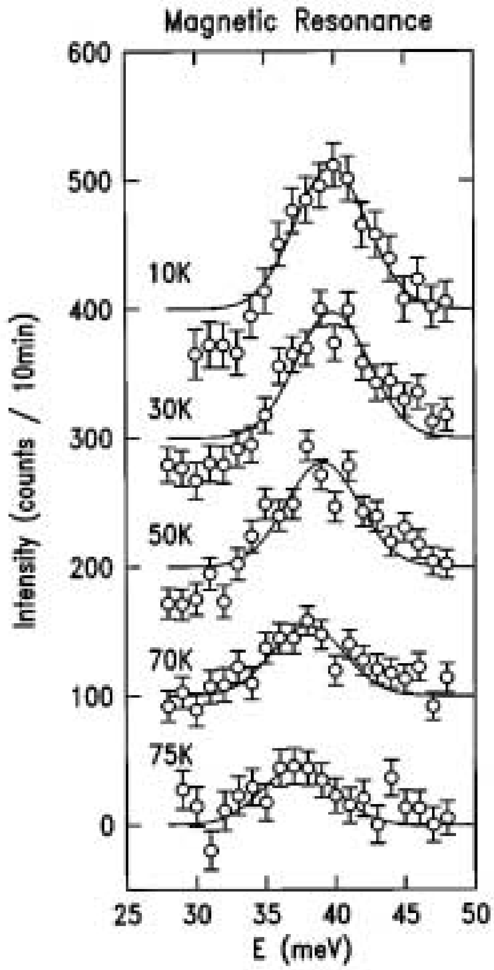


Evidence for predicted coexistence of spin-Peierls order and "d-wave" superconductivity.

S. Sachdev & N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

S=1 resonance mode in YBCO





H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

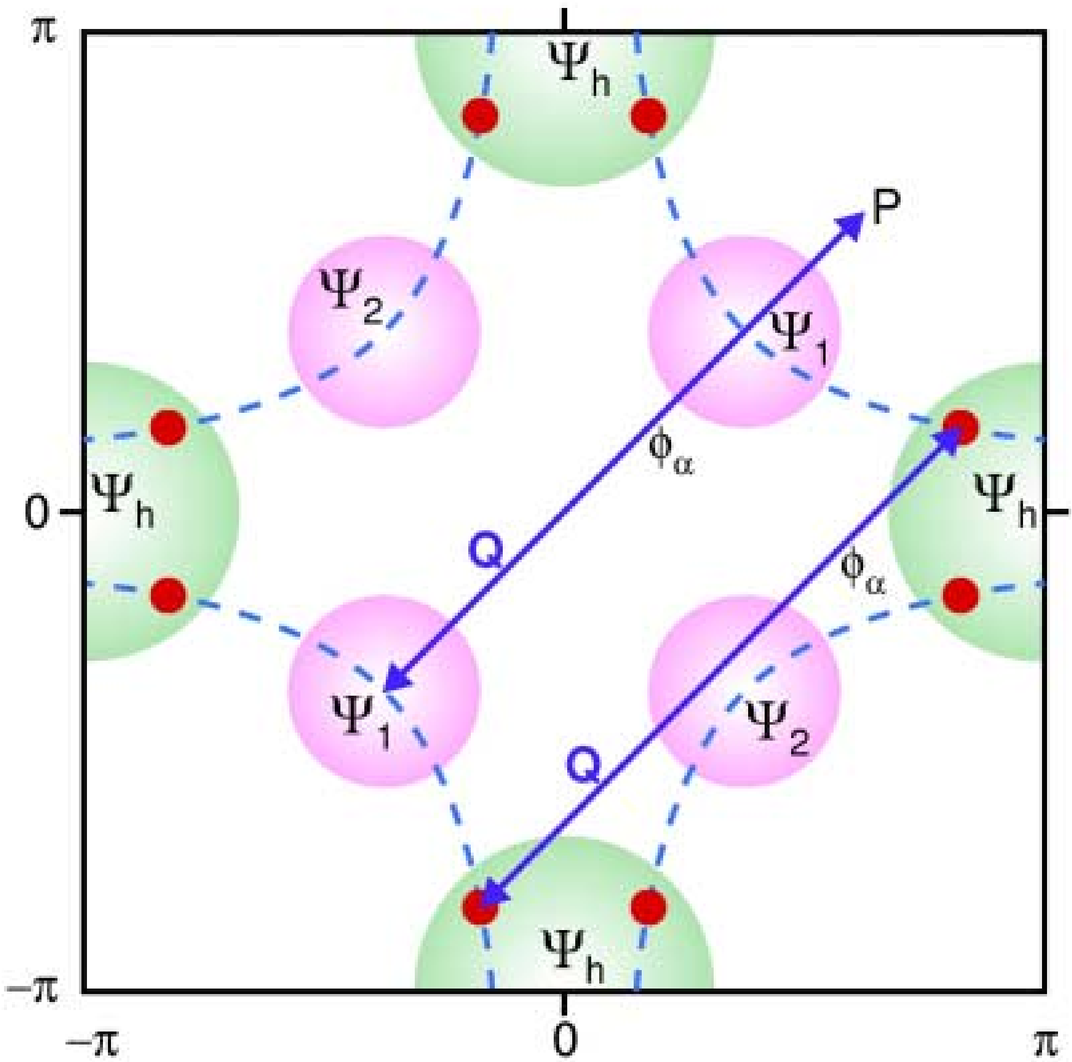
FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the $T=100$ K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$ - little observable damping at low T.

Coupling to superconducting quasiparticles unimportant.

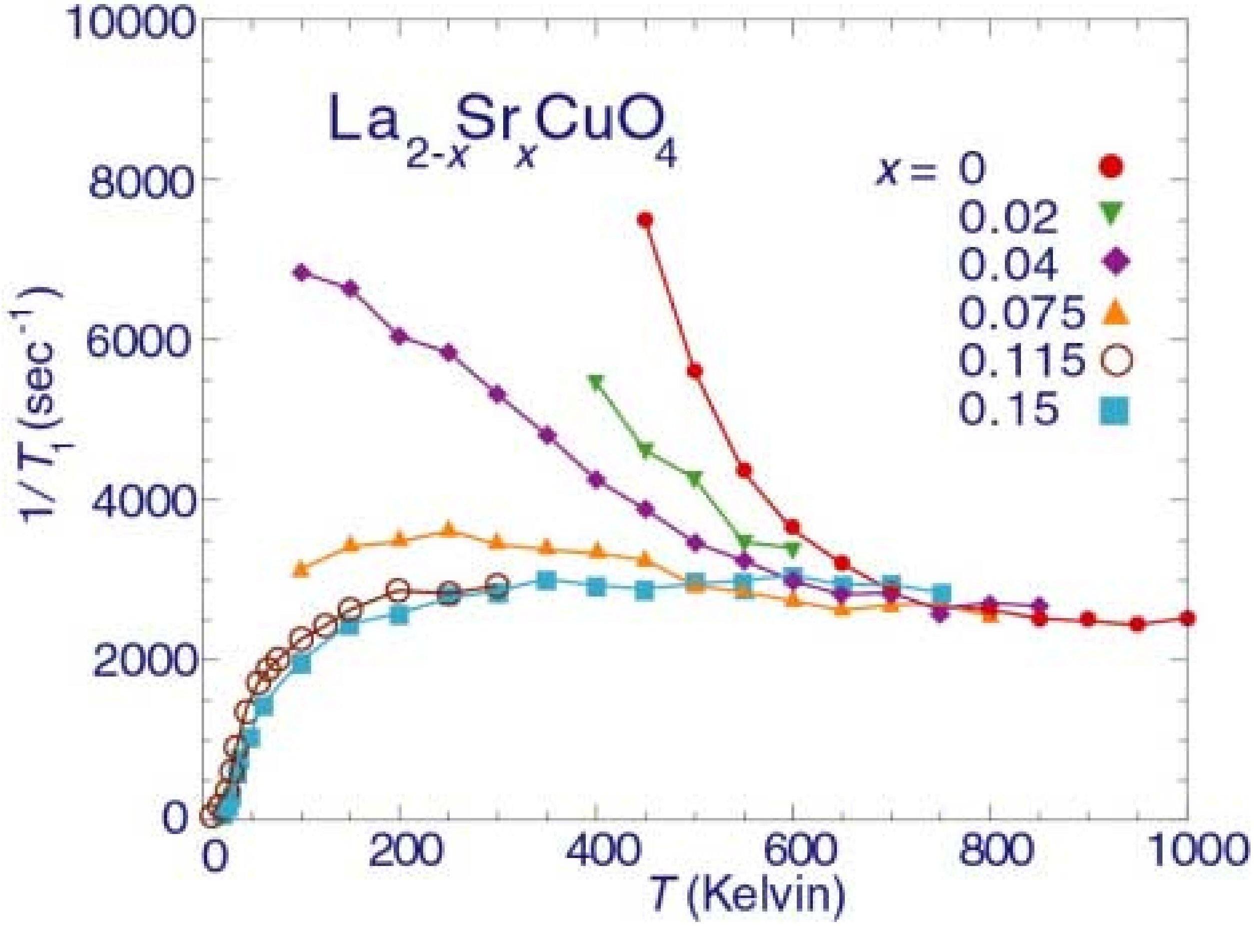
Continuously connected to S=1 particle in confined Mott insulator

Constraints from momentum conservation in d-wave superconductors



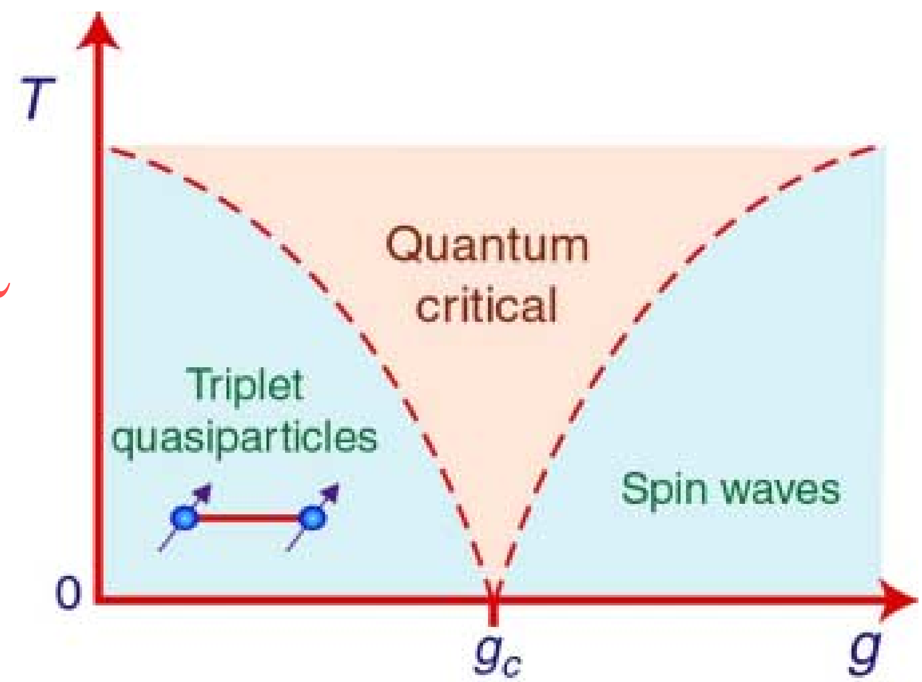
Collective magnetic excitations, ϕ_α , are not damped by fermionic Bogoliubov quasiparticles

(b) NMR



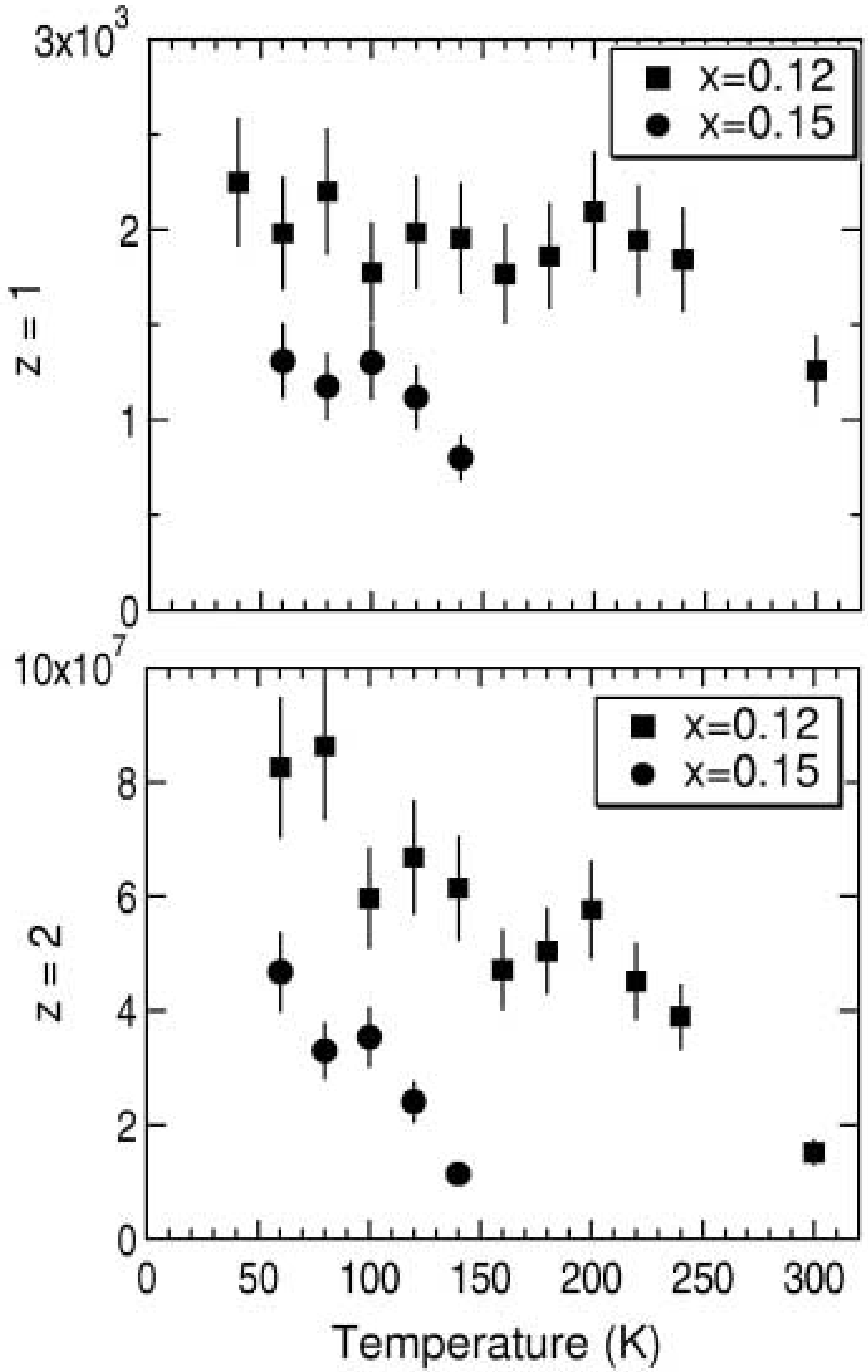
NMR measurements by Imai *et al* Phys. Rev. Lett. **70**, 1002 (1993)

$g \sim 1/\lambda$



T_1T/T_{2G}^{Z} for $La_{2-x}Sr_xCuO_4$

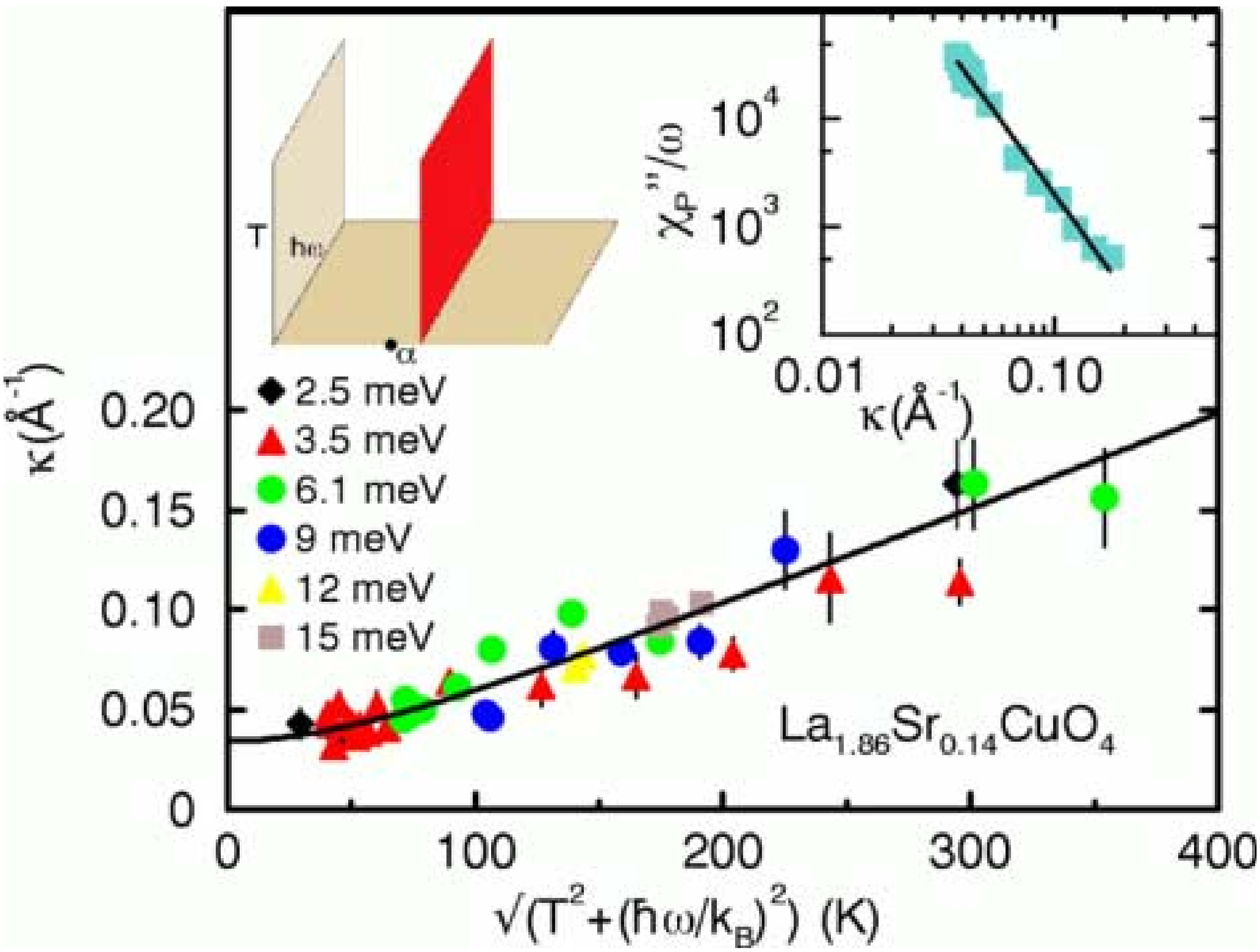
Fujiyama, Takigawa *et al* Phys. Rev. B **60**, 9801 (1999).

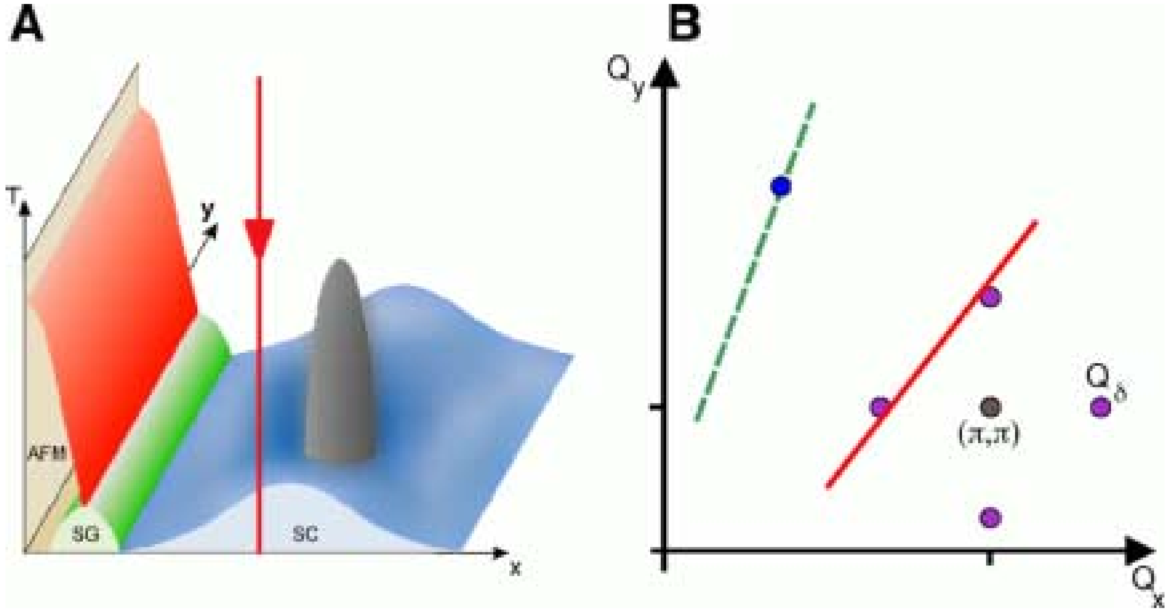


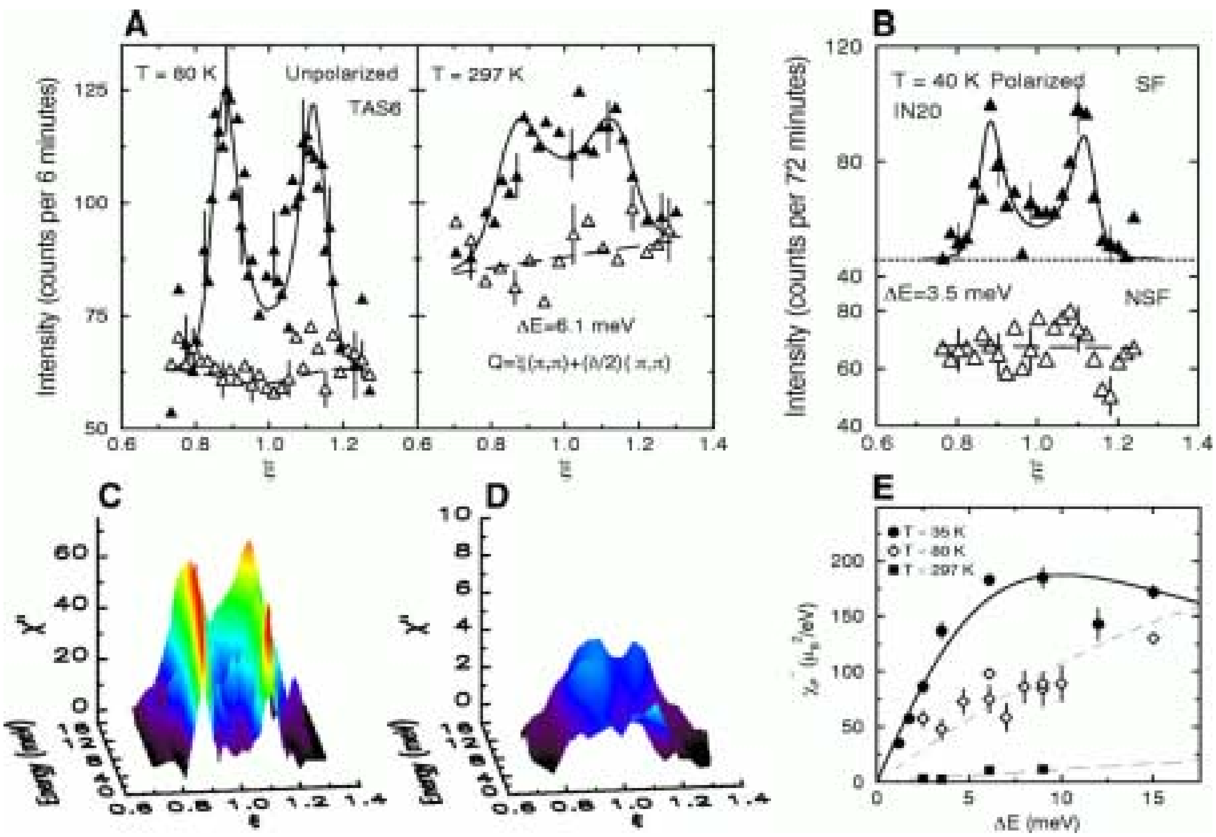
Inelastic neutron scattering near the magnetic ordering transitions

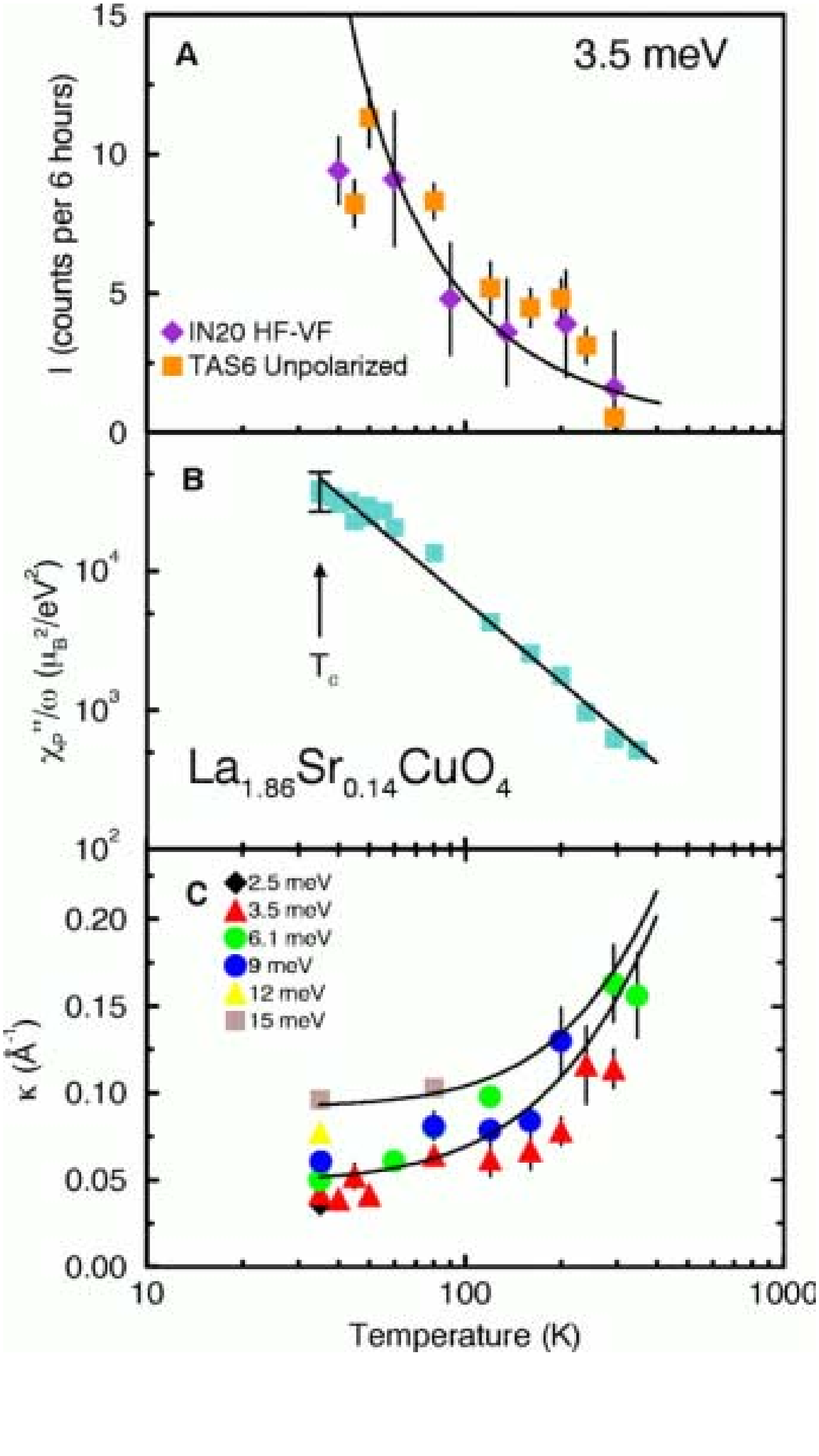
G. Aeppli, T. E. Mason, S. M. Hayden, H. A. Mook, and J. Kulda, Science **278**, 1432 (1997).

κ is the inverse correlation length




A
B
T
y
AFM
SG
SC
x
Q
y
Q
δ
(π,π)
Q
x


A
T = 80 K
Unpolarized
TAS6
T = 297 K
ΔE=6.1 meV
Intensity (counts per 6 minutes)
B
T = 40 K Polarized
SF
IN20
ΔE=3.5 meV
NSF
Intensity (counts per 72 minutes)
C
D
E
T = 35 K
T = 80 K
T = 297 K
ΔE (meV)



A
3.5 meV
I (counts per 6 hours)
IN20 HF-VF
TAS6 Unpolarized
B
χ_φ''/ω (μ_B^2/eV^2)
T_c
$La_{1.86}Sr_{0.14}CuO_4$
C
2.5 meV
3.5 meV
6.1 meV
9 meV
12 meV
15 meV
κ ($Å^{-1}$)
Temperature (K)

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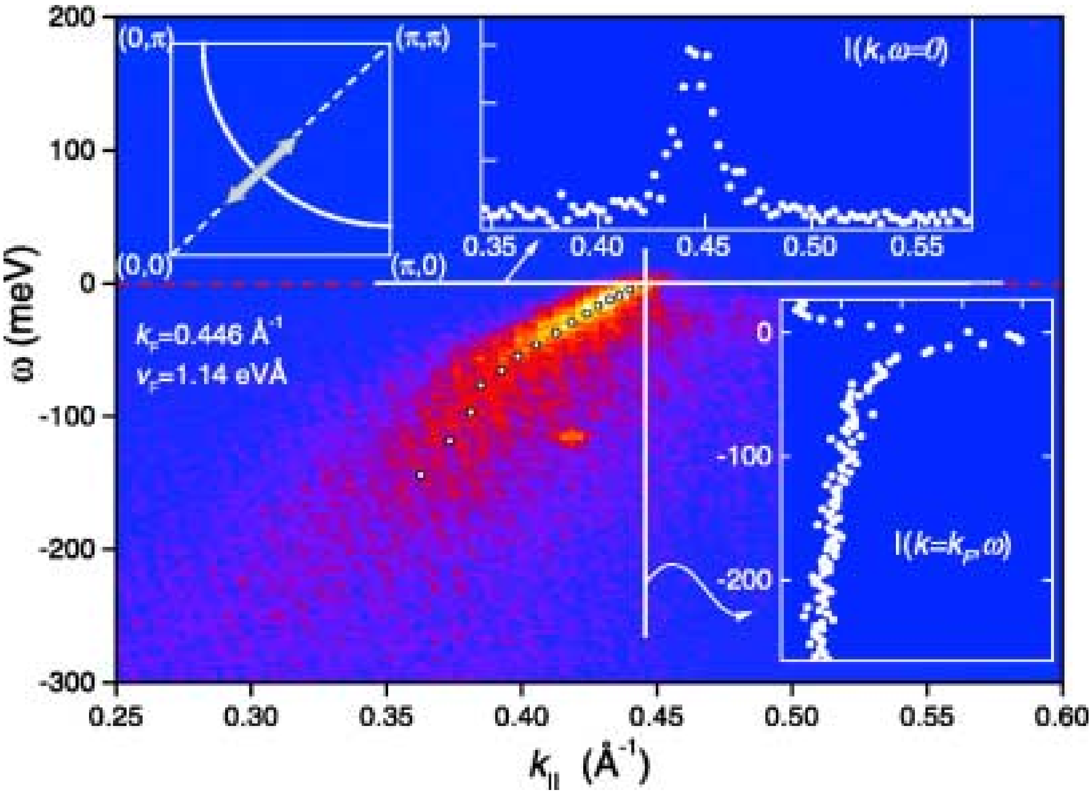
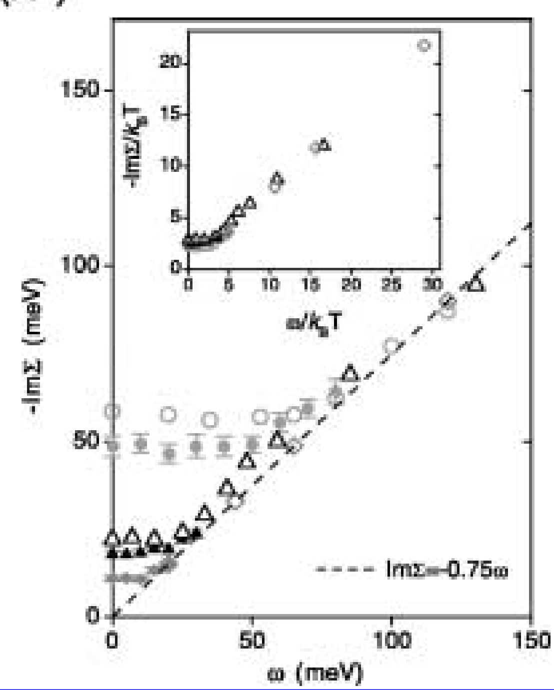
Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))



Quantum-critical damping of quasi-particles along (1,1)

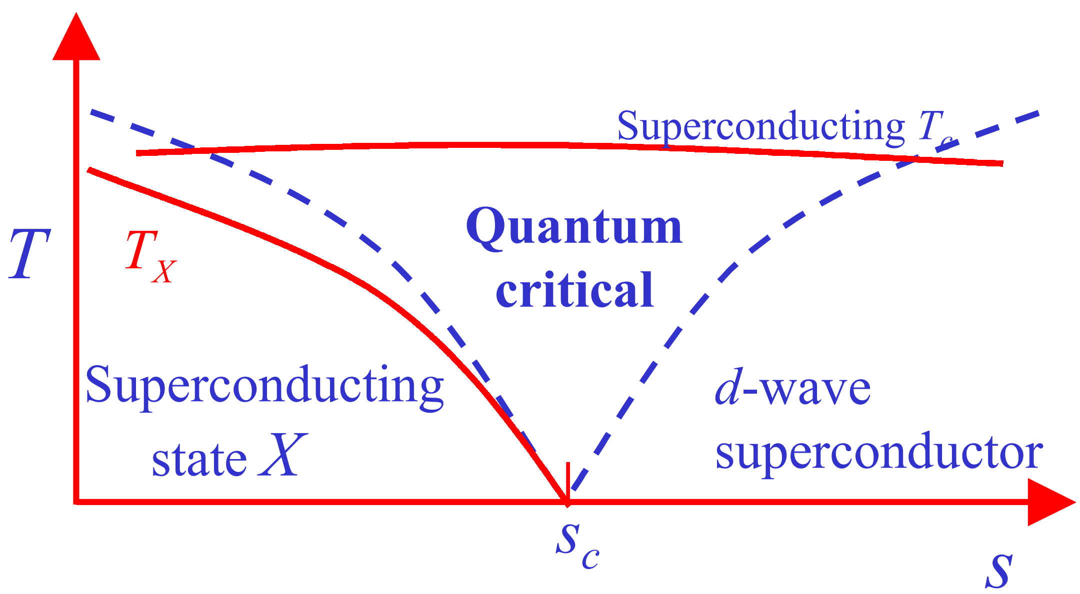


Goal: Classify theories in which, with minimal fine tuning, a *d*-wave superconductor has a fermionic quasiparticle momentum distribution curve (MDC), at the nodal points, with a width proportional to $k_B T$

In a Fermi liquid, MDC width $\sim T^2$

In a BCS d-wave superconductor, MDC width $\sim T^3$

Proximity to a quantum-critical point



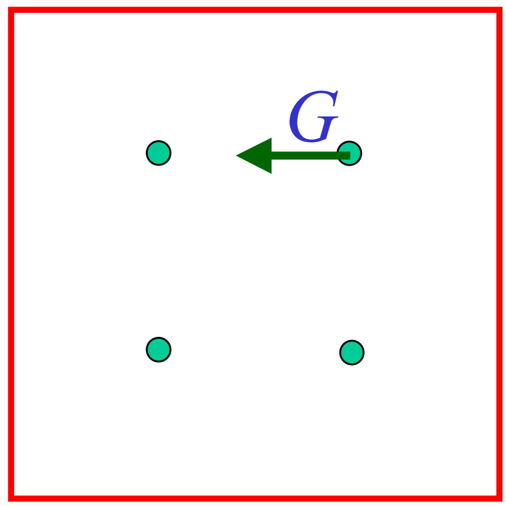
Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Nodal quasi-particles should be part of the critical-field theory.
3. Critical field theory should not be free – required to obtain damping in the scaling limit.

A spin-singlet, fermion bilinear, zero momentum order parameter for *X* is preferred.

e.g. An order parameter with momentum *G*:
Charge (or spin) density-wave order

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$



If *G* does not connect two nodal points, fermions are not part of the critical theory

Order parameter for X should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c^\dagger_{k\alpha} c_{k\alpha} \rangle \text{ (excitonic order)}$$

Complete group-theoretic classification

X has $d_{x^2-y^2}$ pairing plus

(A) is pairing

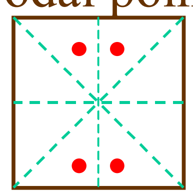
(B) id_{xy} pairing

→ fermion spectrum fully gapped

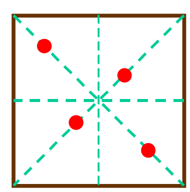
(C) ig pairing

Nodal points

(D) s pairing

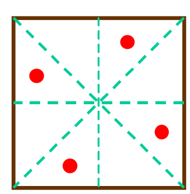


(E) d_{xy} excitons

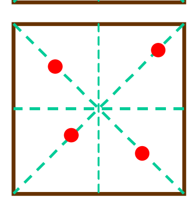


→ superconducting nematics

(F) d_{xy} pairing



(G) p excitons



Main results

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions.

Only cases (A) and (B) satisfy conditions 1,2,3

d_{xy} pairing vanishes along the (1,0),(0,1) directions, and so only case (B) does not strongly scatter the anti-nodal quasiparticles

Transition to d_{xy} pairing is expected with increasing J_2

Conclusions

1. Evidence from neutron scattering and NMR for a *z=1* quantum phase transition at which magnetic order vanishes.

2. Argued that many properties of the superconductor can be understood by adiabatic continuity from a reference paramagnetic Mott insulator with confinement – such a state *requires* *S=1* spin resonance, broken translational symmetry (stripe order), and moments near non-magnetic impurities.

3. Evidence for theoretically predicted bond-centered stripes in paramagnetic phase with *d*-wave superconductivity.

4. Damping of nodal quasiparticles may be associated with proximity to a quantum critical point to a $d_{x^2-y^2} + id_{xy}$ superconductor. Such a state is expected at larger second neighbor exchange.