Quantum phase transitions in Mott insulators and $d$-wave superconductors

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Transparencies on-line at http://pantheon.yale.edu/~subir
Concentration of mobile carriers $\delta$ in e.g. $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$.

Change magnetic interactions to produce a quantum paramagnet or a "spin liquid".

$\langle \vec{S} \rangle = 0$

Magnetic order $\langle \vec{S} \rangle \neq 0$
Outline

1. Neel and paramagnetic ground states of two-dimensional antiferromagnets

2. Magnetic phase transitions in $d$-wave superconductors

3. Experimental evidence for theoretical framework
   (a) Inelastic neutron scattering measurements of phonon and spin spectra
   (b) NMR relaxation rates
   (c) Experiments on Zn impurities: NMR, neutron scattering, and STM


5. Conclusions
1. **Paramagnetic and Neel states of Mott insulators**  

(Katoh and Imada; Tworzydlo, Osman, van Duin and Zaanen)

S=1/2 spins on coupled 2-leg ladders

\[ H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Follow ground state as a function of \( \lambda \)

\[ 0 \leq \lambda \leq 1 \]
Square lattice antiferromagnet

Experimental realization: $La_2CuO_4$

Ground state has long-range magnetic (Neel) order

$$\langle \tilde{S}_i \rangle = (-1)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves

Quasiclassical wave dynamics at low $T$

(Chakravarty et al, 1989; Tyc et al, 1989)
\( \lambda \) close to 0

Weakly coupled ladders

\[
\frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)
\]

Paramagnetic ground state \( \langle \hat{S}_i \rangle = 0 \)

Excitation: \( S=1 \), \( \phi_\alpha \) particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

\[
\varepsilon = \Delta_{\text{res}} + \frac{c^2 k^2}{2 \Delta_{\text{res}}}
\]

\( \Delta_{\text{res}} \rightarrow \text{Spin gap} \)
Quantum paramagnet $\langle \vec{S} \rangle = 0$

Neel order $N_0$

Spin gap $\Delta_{\text{res}}$

$\lambda_c$
Square lattice antiferromagnets

Square lattice with first ($J_1$) and second ($J_2$) neighbor exchange interactions

Neel state

Spin-Peierls state

“Bond-centered charge stripe”

\[
\approx 0.4
\]

\[
\frac{1}{\sqrt{2}} \left( |↑↓⟩ - |↓↑⟩ \right)
\]

Quantum dimer model –

Quantum “entropic” effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left. These effects always lead to a broken square lattice symmetry near the transition to the Neel state.

Excitations

Stable S=1 particle

Energy dispersion

\[ \varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2 \Delta} \]

\[ \Delta \rightarrow \text{Spin gap} \]

S=1/2 spinons are linearly confined by the line of “defect” singlet pairs between them
2. **Magnetic phase transitions in d-wave superconductors**

**A. Doping a paramagnet with confinement**

Condensate of hole pairs


**B. Doping a deconfined paramagnet**

Single hole condensate


+ other exotica (T. Senthil and M.P.A. Fisher, cond-mat/0006481)
Phase diagram for case A

Superconductivity coexists with stripe order

See also J. Zaanen, Physica C 217, 317 (1999) and
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5. Conclusions
Neutron scattering measurements of phonon spectrum of superconducting La$_{1.85}$Sr$_{0.15}$CuO$_4$ by R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. 82, 628 (1999)
Computation of phonon spectrum by McQueeney et al using a simple model based on lattice modulation below.

Evidence for predicted coexistence of spin-Peierls order and “d-wave” superconductivity.

Spin-1 collective mode in \textit{YBCO} - little observable damping at low T. 

Coupling to superconducting quasiparticles unimportant.

Continuously connected to S=1 particle in confined Mott insulator

Constraints from momentum conservation in d-wave superconductors

Collective magnetic excitations, \( \phi_\alpha \), are not damped by fermionic Bogoliubov quasiparticles

\[ g \sim \frac{1}{\lambda} \]
\( \frac{T_1 T}{T_{2G}^z} \) for La\(_{2-x}\)Sr\(_x\)CuO\(_4\)

Inelastic neutron scattering near the magnetic ordering transitions


\( \kappa \) is the inverse correlation length
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4. **Damping of nodal quasi-particles: quantum phase transitions in d-wave superconductors with a spin-singlet order parameter at zero momentum.**

5. Conclusions
Photoemission on BSSCO

(Valla et al Science 285, 2110 (1999))

Quantum-critical damping of quasi-particles along (1,1)
Goal: Classify theories in which, with minimal fine tuning, a $d$-wave superconductor has a fermionic quasiparticle momentum distribution curve (MDC), at the nodal points, with a width proportional to $k_B T$

In a Fermi liquid, MDC width $\sim T^2$

In a BCS $d$-wave superconductor, MDC width $\sim T^3$
Proximity to a quantum-critical point

Superconducting state $X$

Quantum critical

Superconducting $T_c$

$d$-wave superconductor

$T$

$T_X$

$s_c$
Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Nodal quasi-particles should be part of the critical-field theory.
3. Critical field theory should not be free – required to obtain damping in the scaling limit.
A spin-singlet, fermion bilinear, zero momentum order parameter for $X$ is preferred.

*e.g.* An order parameter with momentum $G$:
Charge (or spin) density-wave order

$$\delta \rho \sim \text{Re}[\Phi_x e^{iGx} + \Phi_y e^{iGy}]$$

If $G$ does not connect two nodal points, fermions are not part of the critical theory.
Order parameter for $X$ should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c_{k\alpha}^\dagger c_{k\alpha} \rangle \text{ (excitonic order)}$$

**Complete group-theoretic classification**

$X$ has $d_{x^2-y^2}$ pairing plus

- (A) $is$ pairing
- (B) $id_{xy}$ pairing
- (C) $ig$ pairing
- (D) $s$ pairing
- (E) $d_{xy}$ excitons
- (F) $d_{xy}$ pairing
- (G) $p$ excitons

fermion spectrum fully gapped

superconducting nematics

Nodal points
Main results

Only cases

(A) \( d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is \) pairing and

(B) \( d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy} \) pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions.

Only cases (A) and (B) satisfy conditions 1,2,3

\( d_{xy} \) pairing vanishes along the (1,0),(0,1) directions, and so only case (B) does not strongly scatter the anti-nodal quasiparticles

Transition to \( d_{xy} \) pairing is expected with increasing \( J_2 \)
Conclusions

1. Evidence from neutron scattering and NMR for a $z=1$ quantum phase transition at which magnetic order vanishes.

2. Argued that many properties of the superconductor can be understood by adiabatic continuity from a reference paramagnetic Mott insulator with confinement – such a state requires $S=1$ spin resonance, broken translational symmetry (stripe order), and moments near non-magnetic impurities.

3. Evidence for theoretically predicted bond-centered stripes in paramagnetic phase with $d$-wave superconductivity.

4. Damping of nodal quasiparticles may be associated with proximity to a quantum critical point to a $d_{x^2-y^2} + id_{xy}$ superconductor. Such a state is expected at larger second neighbor exchange.