Compressible quantum matter and gauge-gravity duality

Review: arXiv:1203.4565

Gravity, black holes, and condensed matter,
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Talk online at sachdev.physics.harvard.edu
anti-de Sitter space
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J. McGreevy, arXiv0909.0518
Consider the metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i \]
\[ t \rightarrow \zeta^z t \]
\[ ds \rightarrow \zeta^{\theta/d} ds. \]

This identifies \( z \) as the dynamic critical exponent (\( z = 1 \) for “relativistic” quantum critical points).

\( \theta \) is the violation of hyperscaling exponent.
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This identifies \( z \) as the dynamic critical exponent (\( z = 1 \) for "relativistic" quantum critical points).

\( \theta \) is the violation of hyperscaling exponent. The most general choice of such a metric is

\[
d s^2 = \frac{1}{r^2} \left( -\frac{d t^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} d r^2 + d x_i^2 \right)
\]

We have used reparametrization invariance in \( r \) to choose so that it scales as \( r \rightarrow \zeta^{(d-\theta)/d} r \).
At $T > 0$, there is a “black-brane” at $r = r_h$. The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, $S$, is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$
At $T > 0$, there is a “black-brane” at $r = r_h$.

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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.
The thermal entropy density scales as

\[ S \sim T^{(d-\theta)/z}. \]

The third law of thermodynamics requires \( \theta < d \).

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]
Entanglement entropy

Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region A}$$

Entanglement entropy $$S_{EE} = - \text{Tr} (\rho_A \ln \rho_A)$$
Holographic entanglement entropy

Emergent holographic direction

$r$
Holographic entanglement entropy

Area of minimal surface equals entanglement entropy

Emergent holographic direction

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The entanglement entropy, \( S_E \), of an entangling region with boundary surface ‘area’ \( \Sigma \) scales as

\[ S_E \sim \begin{cases} 
\Sigma, & \text{for } \theta < d - 1 \\
\Sigma \ln \Sigma, & \text{for } \theta = d - 1 \\
\Sigma^{\theta/(d-1)}, & \text{for } \theta > d - 1
\end{cases} \]

All local quantum field theories obey the “area law” (upto log violations) and so \( \theta \leq d - 1 \).
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The null energy condition implies \( z \geq 1 + \frac{\theta}{d} \).
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The null energy condition implies \( z \geq 1 + \frac{\theta}{d} \).
Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

• Compressible systems must be gapless.

• Conformal quantum matter is compressible in $d = 1$, but not for $d > 1$. Compressible quantum matter
Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

- Describe zero temperature phases where $d\langle Q\rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$. 

Monday, April 23, 2012
The only compressible phase of traditional condensed matter physics which does not break the translational or $U(1)$ symmetries is the Landau Fermi liquid.
Compressible quantum matter

**Challenge to string theory:**

Classify and understand non-Fermi liquid phases of compressible quantum matter,
i.e. *strange metals*
Strange metals

A. Field theory

B. Holography
Strange metals

A. Field theory

B. Holography
The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field $A_\mu$.

$$
L = f_\sigma^\dagger \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_\sigma
$$

$$
= \frac{1}{\sqrt{2}} \left( \langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle \right)
$$
Fermi surface of an ordinary metal

\[ \mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma \]
Fermions coupled to a gauge field

\[ \mathcal{L} = f_\sigma^+ \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_\sigma \]
There is a sharp Fermi surface defined by the (gauge-dependent) fermion Green’s function: \( G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0 \). This Green’s function is not measurable, and so the Fermi surface is “hidden”.

\[
\mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_\sigma
\]
Properties of this strange metal

\[ \mathcal{L} = f^\dagger_\sigma \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_\sigma \]

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- Area enclosed by the Fermi surface \( \mathcal{A} = Q \), the fermion density

• There is a sharp Fermi surface defined by the (gauge-dependent) fermion Green’s function: $G_f^{-1}(|k| = k_F, \omega = 0) = 0$. This Green’s function is not measurable, and so the Fermi surface is “hidden”.

• Area enclosed by the Fermi surface $A = Q$, the fermion density $\rho_f$.

• Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |k| - k_F$ is the distance from the Fermi surface and $z$ is the dynamic critical exponent.

\[ \mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - i A_\tau - \frac{(\nabla - i A)^2}{2m} - \mu \right) f_\sigma \]

\[ A \quad \rightarrow \quad |q| \quad \leftarrow \]


Properties of this strange metal

\[ \mathcal{L} = f^+_\sigma \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_\sigma \]

- Gauge-dependent Green’s function \( G_f^{-1} = q^{1-\eta} F(\omega/q^z) \).
  Three-loop computation shows \( \eta \neq 0 \) and \( z = 3/2 \).
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- Gauge-dependent Green’s function \( G_f^{-1} = q^{1-\eta} F(\omega/q^z) \). Three-loop computation shows \( \eta \neq 0 \) and \( z = 3/2 \).

- The phase space density of fermions is effectively one-dimensional, so the entropy density \( S \sim T^{d_{\text{eff}}/z} \) with \( d_{\text{eff}} = 1 \).


Field theory of this strange metal

- Gauge fluctuation at wavevector \( \vec{q} \) couples most efficiently to fermions near \( \pm \vec{k}_0 \).

- Expand fermion kinetic energy at wavevectors about \( \vec{k}_0 \).

- In Landau gauge, only need the component of the gauge field, \( a \), orthogonal to \( \vec{q} \).
Field theory of this strange metal

\[
\mathcal{L}[\psi_\pm, a] = \\
\psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \\
-a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2 g^2} \left( \partial_y a \right)^2
\]

Field theory of this strange metal

\[ \mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- 
- a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2g^2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).
Field theory of this strange metal

\[ \mathcal{L} = \psi_+^\dagger (\mathbf{X} - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\mathbf{X} + i\partial_x - \partial_y^2) \psi_- \\
- a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2g^2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).

Perturbative computations show that the \( \psi_\pm^\dagger \partial_\tau \psi_\pm \) terms are irrelevant.
Field theory of this strange metal

\[ \mathcal{L}_{\text{scaling}} = \psi_+^\dagger (-i \partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i \partial_x - \partial_y^2) \psi_- \\
- g a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).
Field theory of this strange metal

\[ L_{\text{scaling}} = \psi_+^\dagger (-i \partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i \partial_x - \partial_y^2) \psi_- \]

\[ - g a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).

Under the rescaling \( x \to x/s, y \to y/s^{1/2}, \) and \( \tau \to \tau/s^z \), we find invariance provided

\[ a \to a s^{(2z+1)/4} \]
\[ \psi \to \psi s^{(2z+1)/4} \]
\[ g \to g s^{(3-2z)/4} \]

So the action is invariant provided \( z = 3/2 \).
Fermions and bosons coupled to a gauge field

\[ \mathcal{L} = f^\dagger \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f \\
+ b^\dagger \left( \partial_\tau + iA_\tau - \frac{(\nabla + iA)^2}{2m_b} - \mu_b \right) b + s|b|^2 - gb^\dagger f^\dagger fb + \ldots \]
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Another strange metal: the fractionalized Fermi liquid (FL*)

Bosons can bind with fermions to form a gauge-neutral fermion \( c \sim b f \). The result FL* phase has *partial confinement* and 2 Fermi surfaces: the gauge-neutral Fermi surface of \( c \), and the gauge-charged Fermi surface of \( f \). They enclose a *combined* area equal to \( \langle Q \rangle \).

\[ A_c = \langle Q_b \rangle \]
\[ A_f = \langle Q - Q_b \rangle \]

Fermions and bosons coupled to a gauge field

\[ \mathcal{L} = f^\dagger \left( \partial_\tau - i A_\tau - \frac{(\nabla - i A)^2}{2m} - \mu \right) f + b^\dagger \left( \partial_\tau + i A_\tau - \frac{(\nabla + i A)^2}{2m_b} - \mu_b \right) b + s |b|^2 - g b^\dagger f^\dagger f b + \ldots \]

Another strange metal: the fractionalized Fermi liquid (FL*)

In holography:
the \( c \) Fermi surface is that of the "probe" fermion;
the fractionalized \( f \) Fermi surface is "hidden" past the horizon.

\[ A_c = \langle Q_b \rangle \]
\[ A_f = \langle Q - Q_b \rangle \]

Kondo lattice model

Another strange metal: the fractionalized Fermi liquid (FL*)

Spin liquid of \( f \) electrons

Fermi surface of \( c \) conduction electrons

Strange metals

A. Field theory

B. Holography
The value of $\theta$ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general $d$. Conjecture: this metric then describes a compressible state with a hidden Fermi surface.
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The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops!
The entanglement entropy exhibits logarithmic violation of the area law only for this value of $\theta$!!

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2d(z-1)/(d-\theta)} + \frac{r^2\theta/(d-\theta)}{d-\theta} dr^2 + dx_i^2 \right) \]

$\theta = d - 1$

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The logarithmic violation is of the form $P \ln P$, where $P$ is the perimeter of the entangling region. This form is independent of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

\[
\theta = d - 1
\]
Begin with a CFT

Dirac fermions + gauge field + ......
Holographic representation: \( \text{AdS}_4 \)

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
Apply a chemical potential to the “deconfined” CFT

$\mu > 0$
The Maxwell-Einstein theory of the applied chemical potential yields a AdS$_4$-Reissner-Nordström black-brane.

\[ \mathcal{E}_r = \langle Q \rangle \]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right] \]

The Maxwell-Einstein theory of the applied chemical potential yields a AdS$_4$-Reissner-Nordström black-brane.

At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of AdS$_2 \times R^2$

$$ds^2 = \frac{L^2}{6} \left( \frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

Artifacts of AdS$_2 \times \mathbb{R}^2$

- Corresponds to $\theta \to d$ and $z \to \infty$. This implies non-zero entropy density at $T = 0$, and “volume” law for entanglement entropy.

- Green’s function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface.

- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with “hidden Fermi surfaces” of gauge-charged particles (the *quarks*).

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field

\[ \mathcal{E}_r = \langle Q \rangle \]

\[
S = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R - 2(\nabla \Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right]
\]

with \( Z(\Phi) = Z_0 e^{\alpha \Phi} \), \( V(\Phi) = -V_0 e^{-\beta \Phi} \), as \( \Phi \to \infty \).

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This is a “bosonization” of the Fermi surface
Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field

\[ E_r = \langle Q \rangle \]

\[ \mathcal{E}_r = \langle Q \rangle \]

Leads to metric
\[ ds^2 = L^2 \left( -f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right) \]

with \( f(r) \sim r^{-\gamma} \), \( g(r) \sim r^\delta \), \( \Phi(r) \sim \ln(r) \) as \( r \to \infty \).

Holographic theory of a non-Fermi liquid (NFL)

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2 d(z-1)/(d-\theta)} + \frac{r^{2\theta}/(d-\theta) dr^2 + dx_i^2}{d^2} \right) \]

The \( r \to \infty \) metric has the above form with

\[
\theta = \frac{d^2 \beta}{\alpha + (d-1)\beta} \\
z = 1 + \frac{\theta}{d} + \frac{8(d(d - \theta) + \theta)^2}{d^2(d - \theta)\alpha^2}.
\]

Note \( z \geq 1 + \theta/d \).
The solution also specifies the missing numerical prefactors in the metric. In general, these depend upon the details on the UV boundary condition as \( r \to 0 \). However, the coefficient of \( dx_i^2/r^2 \) turns out to be independent of the UV boundary conditions, and proportional to \( Q^{2\theta}/(d(d-\theta)) \).

The square-root of this coefficient is the prefactor of the log divergence in the entanglement entropy for \( \theta = d - 1 \).
The entanglement entropy has log-violation of the area law

\[ S_E = \Xi Q^{(d-1)/d} \Sigma \ln \left( Q^{(d-1)/d} \Sigma \right), \]

where \( \Sigma \) is surface area of the entangling region, and \( \Xi \) is a dimensionless constant which is independent of all UV details, of \( Q \), and of any property of the entangling region. Note \( Q^{(d-1)/d} \sim k_F^{d-1} \) via the Luttinger relation, and then \( S_E \) is just as expected for a Fermi surface !!!!
Holographic theory of a non-Fermi liquid (NFL)

Gauss Law and the “attractor” mechanism

⇔ Luttinger theorem on the boundary

Hidden Fermi surfaces of “quarks”
Holographic theory of a fractionalized-Fermi liquid (FL*)

Hidden Fermi surfaces of "quarks"

Visible Fermi surfaces of "mesinos"

\[ \epsilon_r = Q - Q_{\text{mesino}} \]

\[ \epsilon_r = Q \]

A state with partial confinement


Now the entanglement entropy implies that the Fermi momentum of the hidden Fermi surface is given by \( k_F^d \sim Q - Q_{\text{mesino}} \), just as expected by the extended Luttinger relation. Also the probe fermion quasiparticles are sharp for \( \theta = d - 1 \), as expected for a FL* state.
Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

Visible Fermi surfaces of “mesinos”

Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D 84, 066009 (2011)
Conclusions

Compressible quantum matter

Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.
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After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, $S_E$. 
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- Leading-log $S_E$ independent of shape of entangling region.
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- Leading-log $S_E$ independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.
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- Leading-log $S_E$ independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.
- Evidence for Luttinger theorem in prefactor of $S_E$. 

Monday, April 23, 2012
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Fermi liquid (FL) state described by a confining holographic geometry
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Fermi liquid (FL) state described by a confining holographic geometry

Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a state with partial confinement: the fractionalized Fermi liquid (FL*)
Quantum phase transition with Fermi surface reconstruction

Metal with electron and hole pockets

Metal with "large" Fermi surface

$\langle \varphi \rangle \neq 0$

$\langle \varphi \rangle = 0$

Pnictides, electron-doped cuprates ....
**Proposed phase diagram for the hole-doped cuprates**

Metal with electron and hole pockets

- $\langle \varphi \rangle \neq 0$
- Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and “small” Fermi surface

Electron and/or hole Fermi pockets form in “local” SDW order, but quantum fluctuations destroy long-range SDW order

- $\langle \varphi \rangle = 0$
- Metal with “large” Fermi surface

M. Punk and S. Sachdev, arXiv:1202.4023

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