Dynamics of Mott insulators in strong potential gradients

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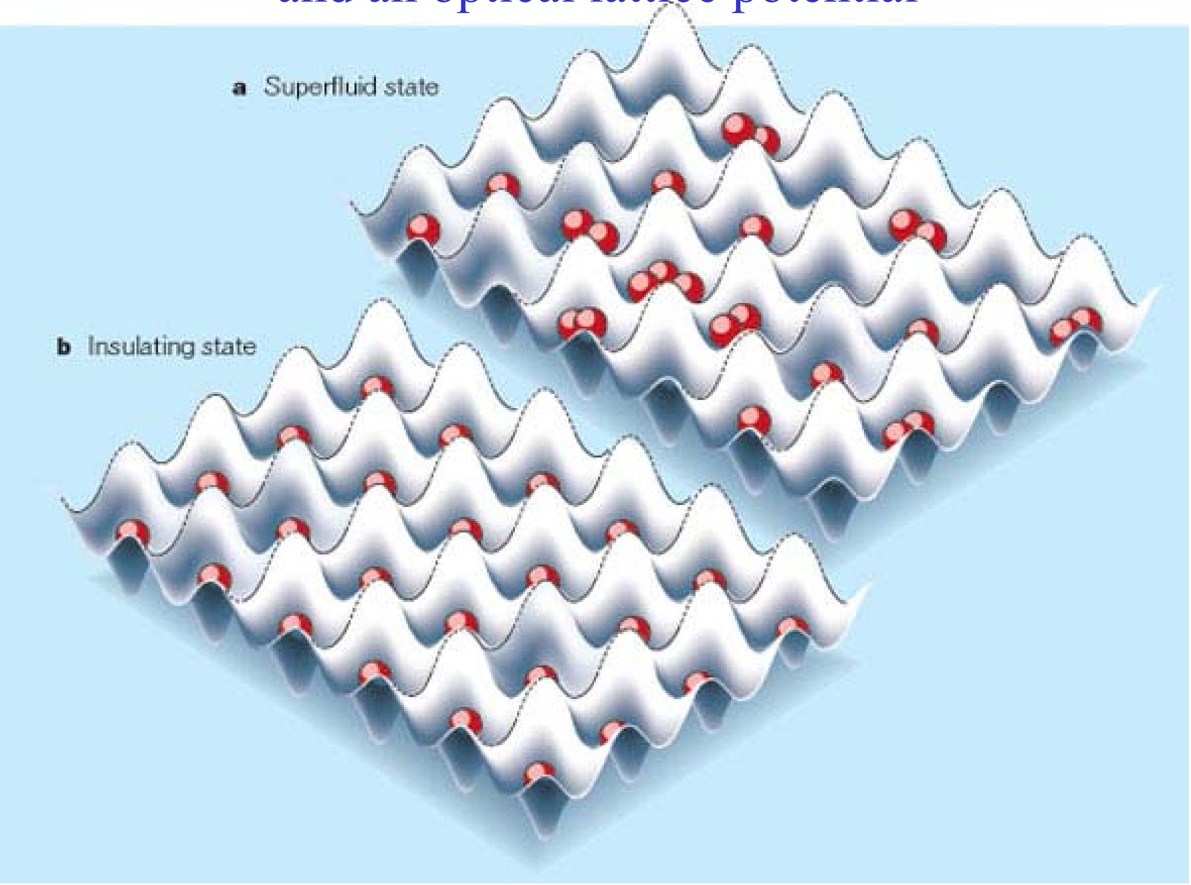
Physical Review B **66**, 075128 (2002).
Physical Review A **66**, 053607 (2002).

Phase oscillations and "cat" states in an optical lattice

Transparencies online at
http://pantheon.yale.edu/~subir

Superfluid-insulator transition of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Related earlier work by C. Orzel, A.K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, *Science* **291**, 2386 (2001).

Detection method

Trap is released and atoms expand to a distance far larger than original trap dimension

$$\psi(\boldsymbol{R},T) = \exp\left(i\frac{m\boldsymbol{R}^2}{2\hbar T}\right)\psi(\boldsymbol{0},0) \approx \exp\left(i\frac{m\boldsymbol{R}_0^2}{2\hbar T} + i\frac{m\boldsymbol{R}_0\cdot\boldsymbol{r}}{\hbar T}\right)\psi(\boldsymbol{0},0)$$

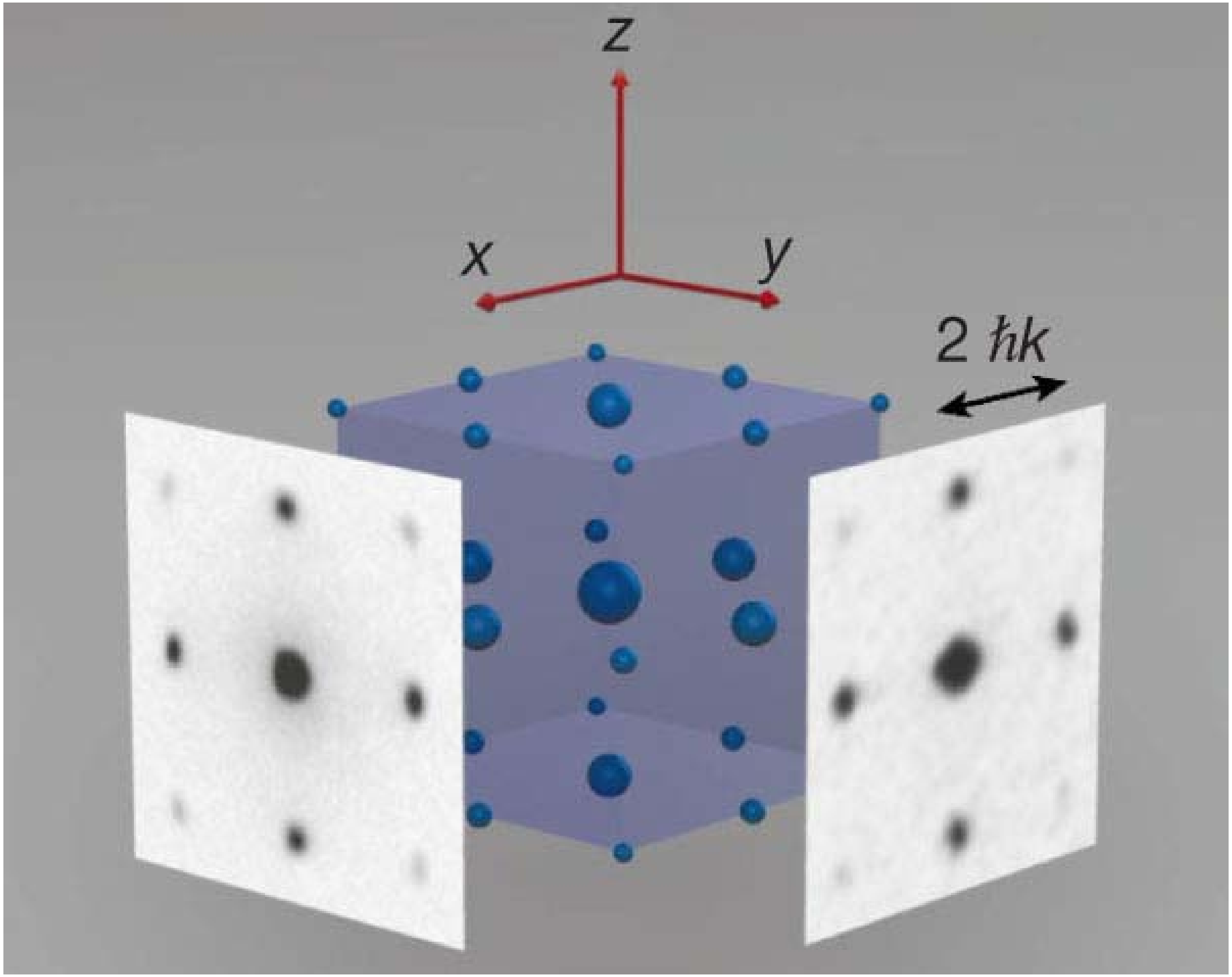
where $\boldsymbol{R} = \boldsymbol{R}_0 + \boldsymbol{r}$, with $\boldsymbol{R}_0$ = the expansion distance, and $\boldsymbol{r}$ = position within trap

In tight-binding model of lattice bosons b_i,

detection probability $\propto \sum_{i,j}\left\langle b_i^\dagger b_j \right\rangle \exp\left(i\boldsymbol{q}\cdot\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right)\right)$ with $\boldsymbol{q} = \dfrac{m\boldsymbol{R}_0}{\hbar T}$

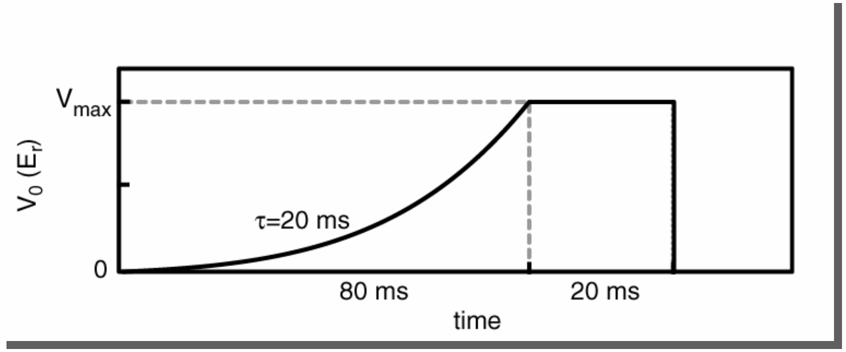
Measurement of momentum distribution function

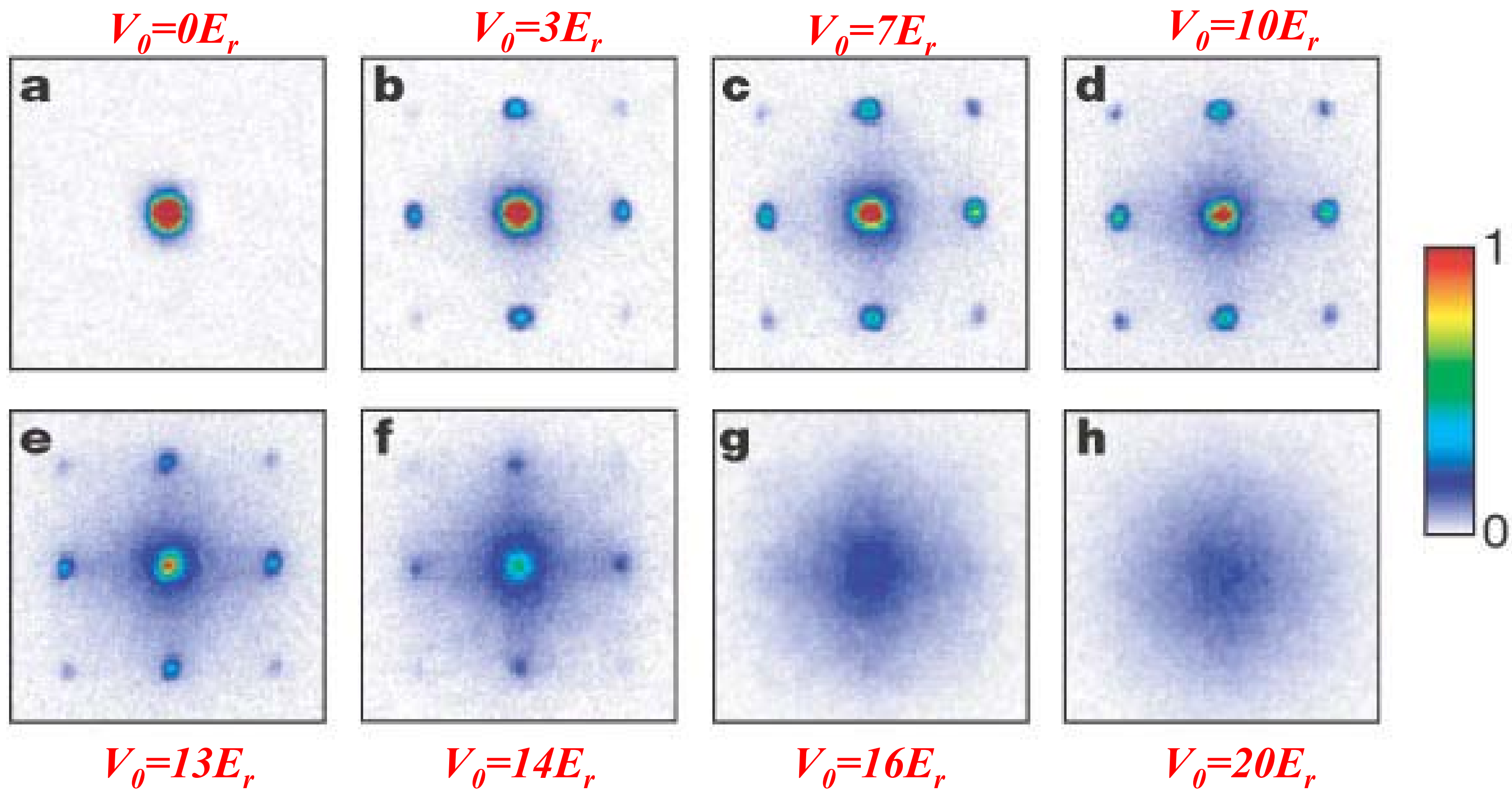
Superfluid state



Schematic three-dimensional interference pattern with measured absorption images taken along two orthogonal directions. The absorption images were obtained after ballistic expansion from a lattice with a potential depth of $V_0 = 10\ E_r$ and a time of flight of 15 ms.

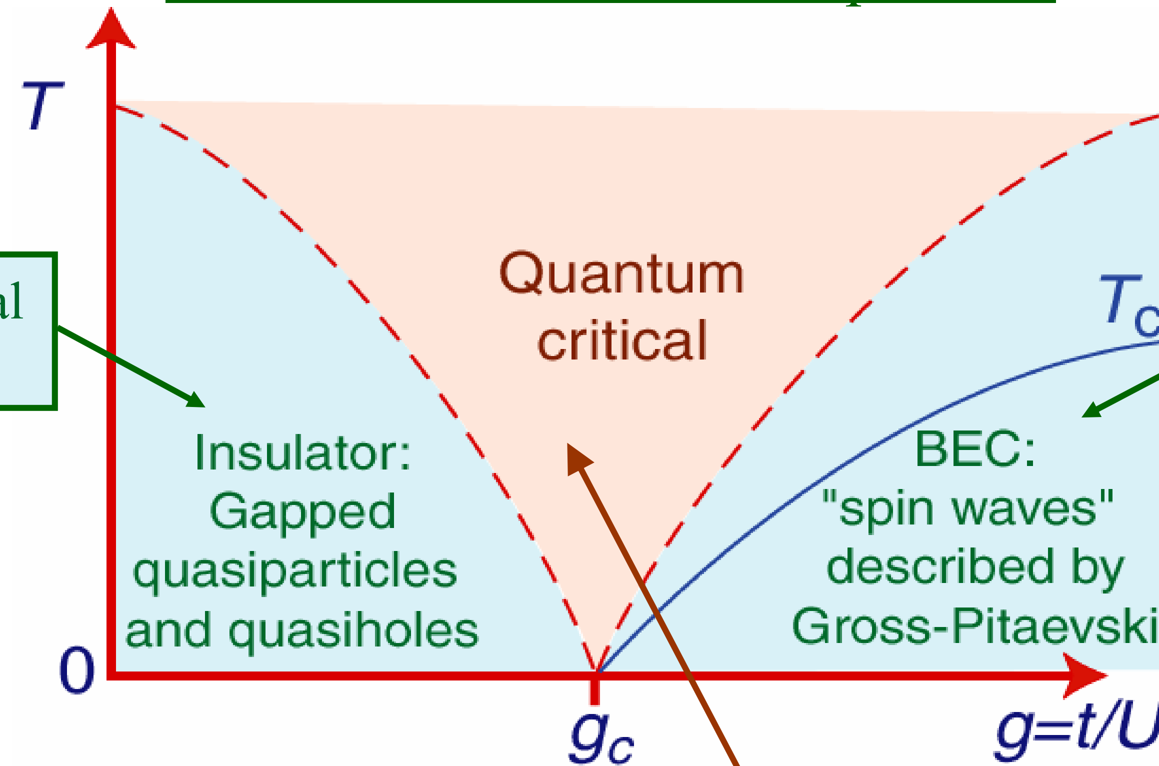
Superfluid-insulator transition





Crossovers at nonzero temperature

Quasiclassical dynamics



Quasiclassical dynamics

Relaxational dynamics ("Bose molasses") with phase coherence/relaxation time τ_φ given by

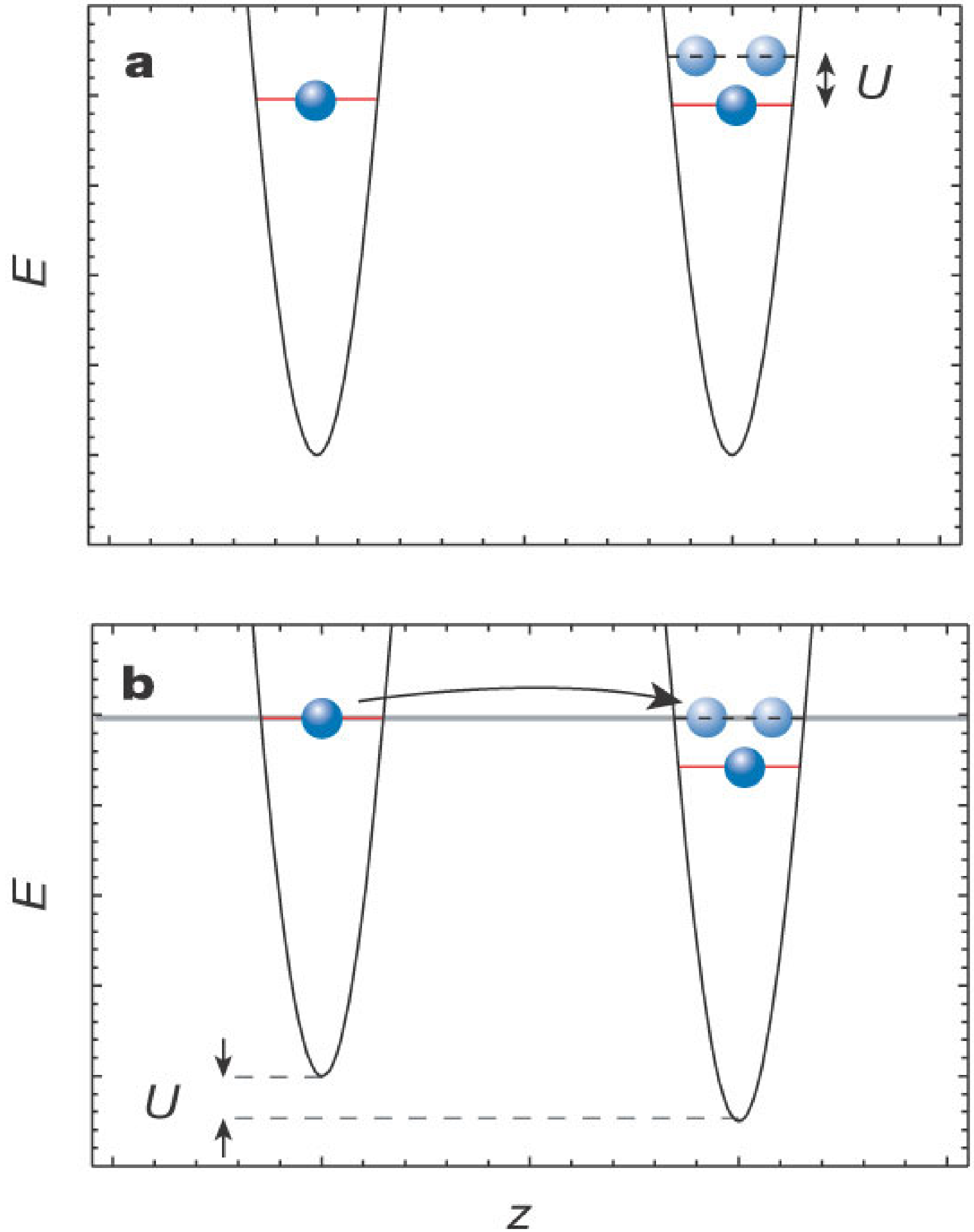
$$\frac{1}{\tau_\varphi} = (\text{Universal number})\frac{k_B T}{\hbar}$$

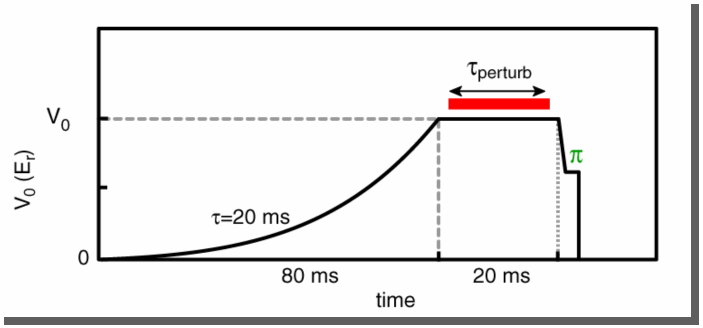
S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
K. Damle and S. Sachdev *Phys. Rev.* B **56**, 8714 (1997).

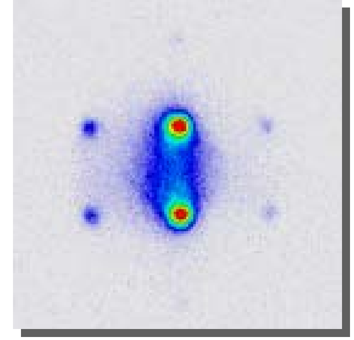
$$\text{Conductivity (in d=2)} = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \qquad \Sigma \rightarrow \text{universal function}$$

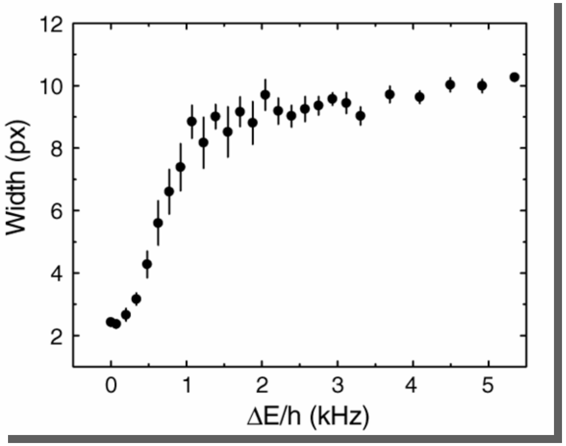
M.P.A. Fisher, G. Girvin, and G. Grinstein, *Phys. Rev. Lett.* **64**, 587 (1990).
K. Damle and S. Sachdev *Phys. Rev.* B **56**, 8714 (1997).

Applying an "electric" field to the Mott insulator

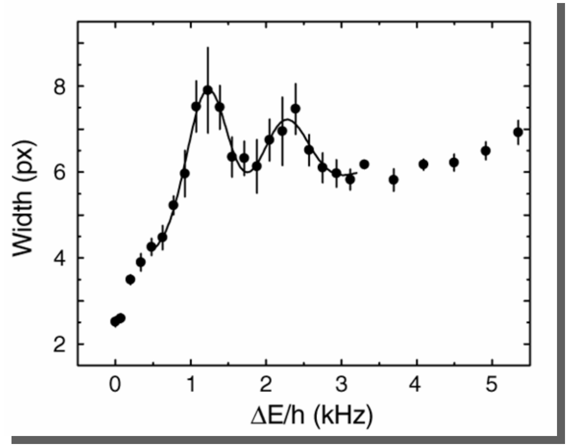






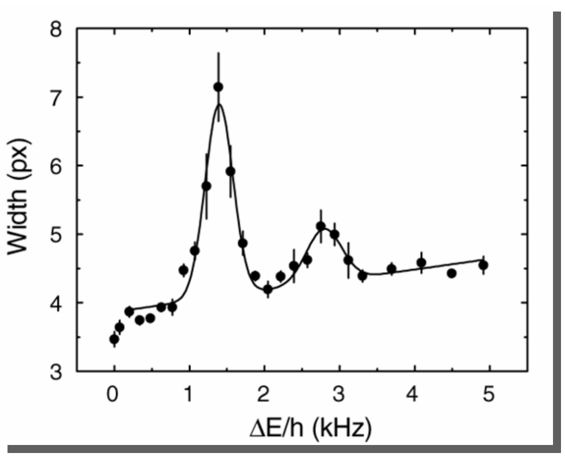


V_0=10 E_{recoil} $\tau_{perturb}$ = 2 ms

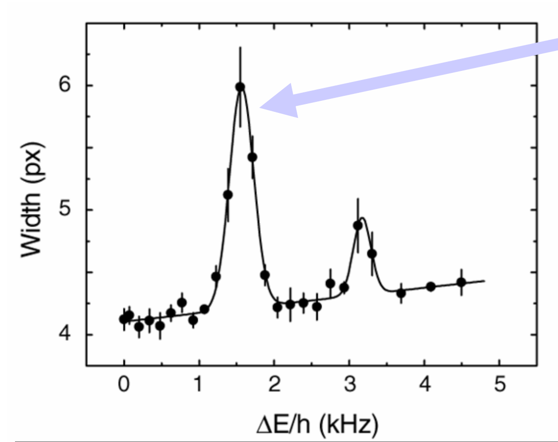


V_0= 13 E_{recoil} $\tau_{perturb}$ = 4 ms

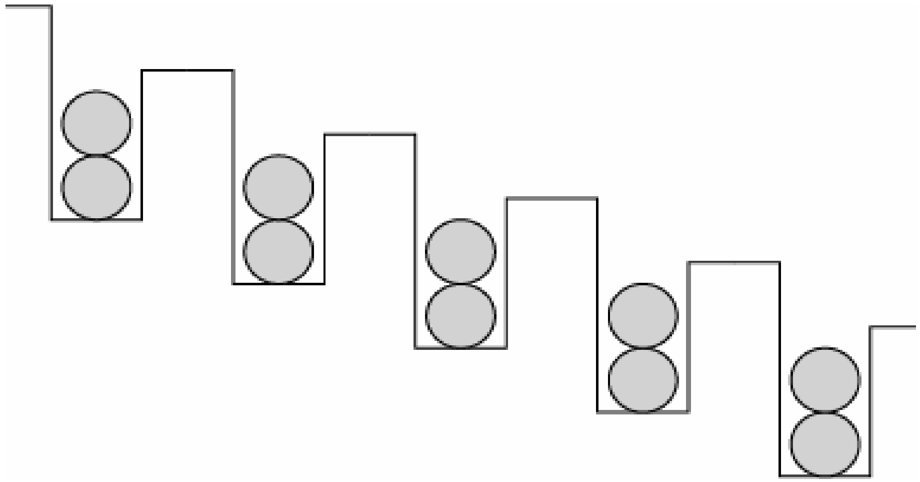
What is the quantum state here ?



V_0= 16 E_{recoil} $\tau_{perturb}$ = 9 ms



V_0= 20 E_{recoil} $\tau_{perturb}$ = 20 ms



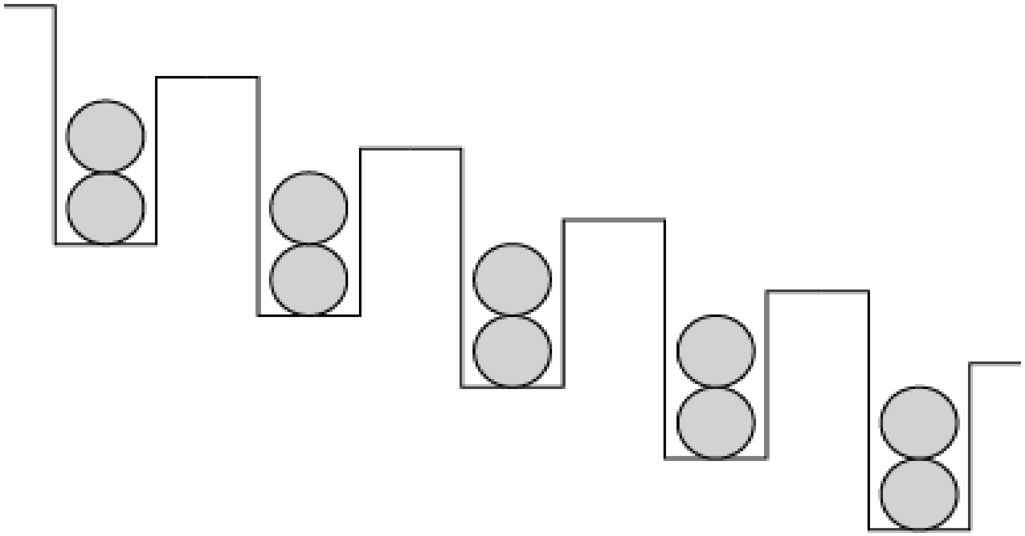
$$H = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

$$n_i = b_i^\dagger b_i$$

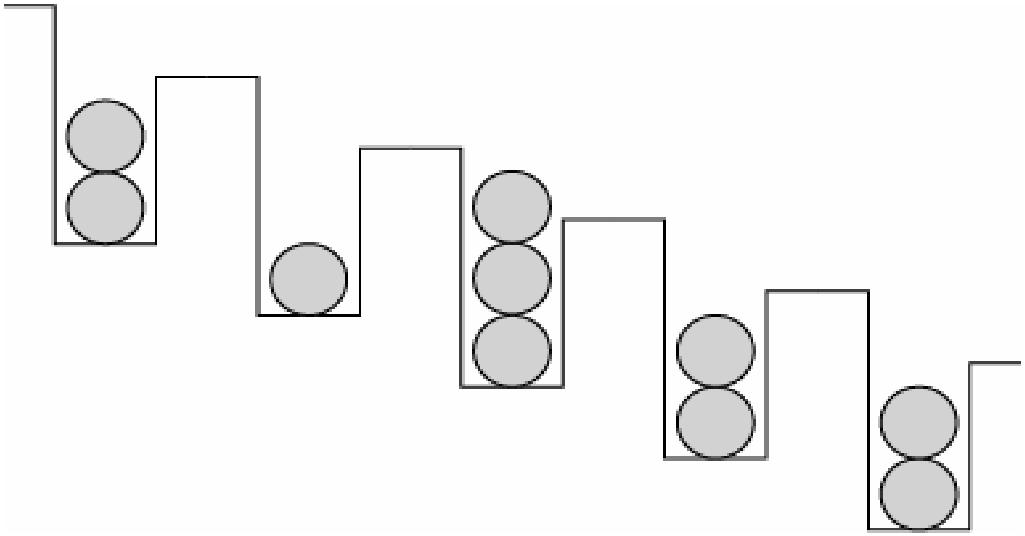
$$\left|U - E\right|, t \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator

Important neutral excitations (in one dimension)

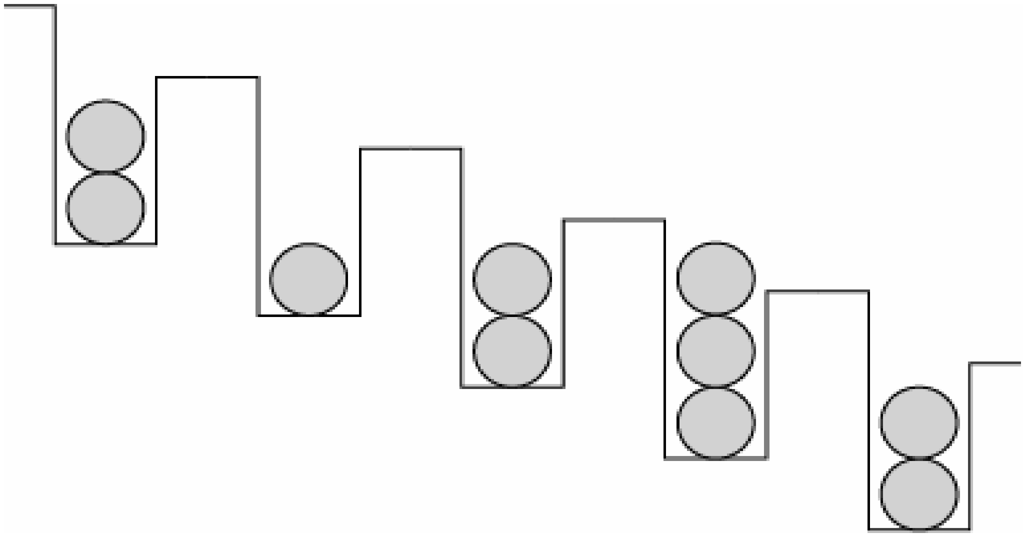


Important neutral excitations (in one dimension)



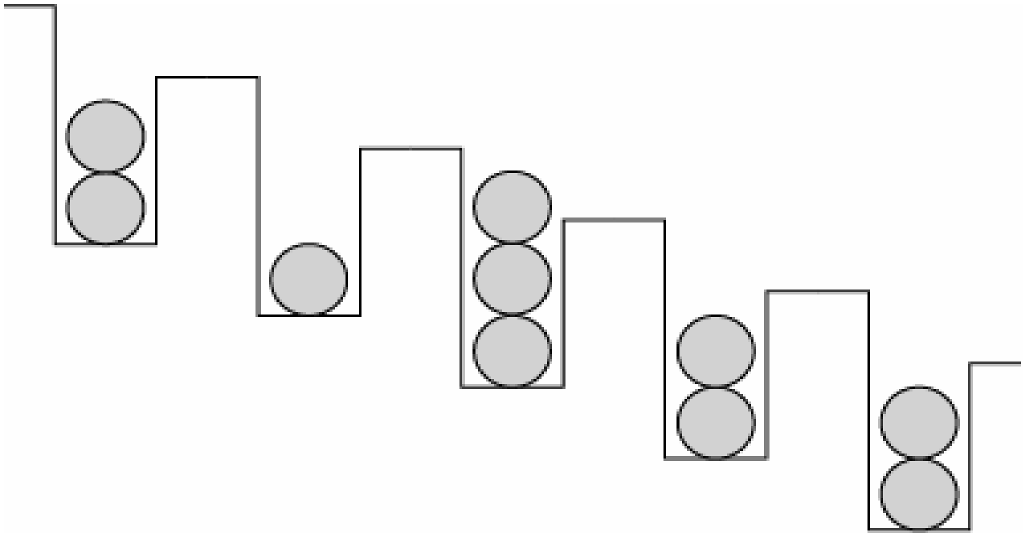
Nearest neighbor dipole

Important neutral excitations (in one dimension)



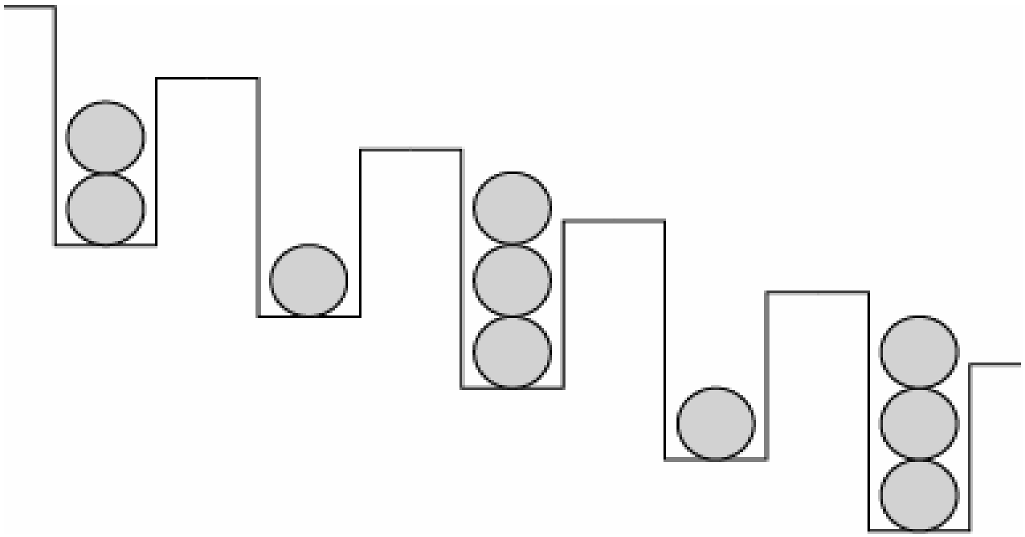
Creating dipoles on nearest neighbor links creates a state with relative energy *U-2E ;* such states are *not* part of the resonant manifold

Important neutral excitations (in one dimension)



Nearest neighbor dipole

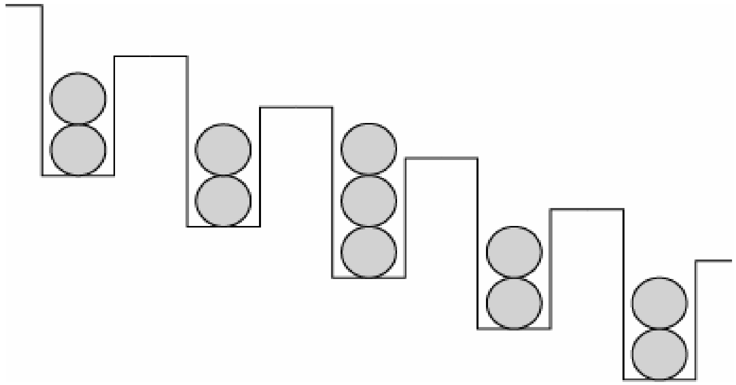
Important neutral excitations (in one dimension)

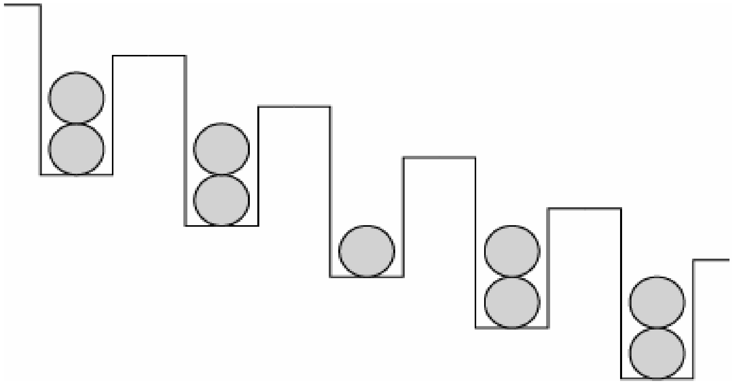


Nearest-neighbor dipoles

Dipoles can appear resonantly on non-nearest-neighbor links.
Within resonant manifold, dipoles have infinite on-link
and nearest-link repulsion

Charged excitations (in one dimension)





Effective Hamiltonian for a quasiparticle in one dimension (similar for a quasihole):

$$H_{\text{eff}} = -\sum_j \left[3t\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + Ejb_j^\dagger b_j \right]$$

Exact eigenvalues $\varepsilon_m = Em \quad ; \quad m = -\infty \cdots \infty$

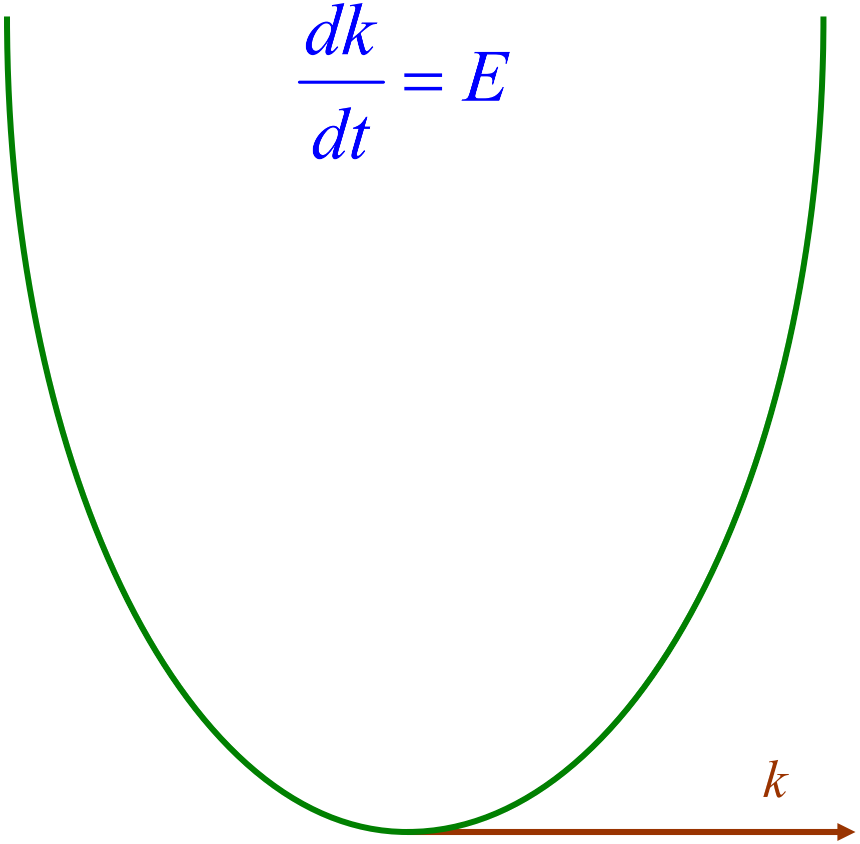
Exact eigenvectors $\psi_m(j) = J_{j-m}(6t/E)$

All charged excitations are strongly localized in the plane perpendicular electric field.
Wavefunction is periodic in time, with period *h/E* (Bloch oscillations)
Quasiparticles and quasiholes are not accelerated out to infinity

Charged excitations (in one dimension)

Semiclassical picture

$$\frac{dk}{dt} = E$$

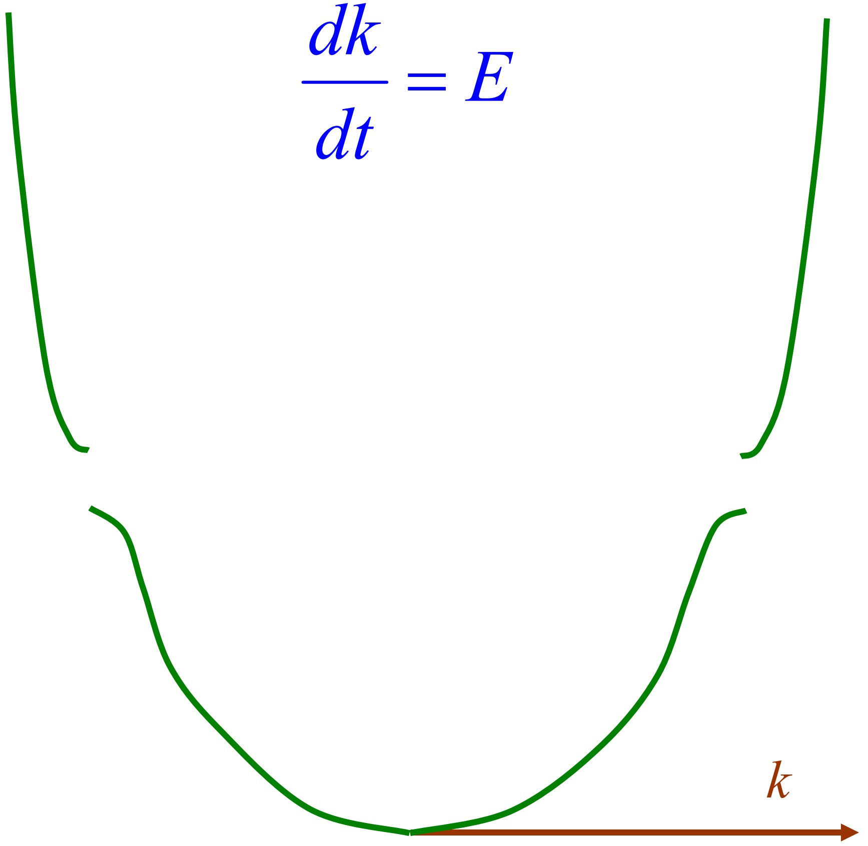


Free particle is accelerated out to infinity

Charged excitations (in one dimension)

Semiclassical picture

$$\frac{dk}{dt} = E$$

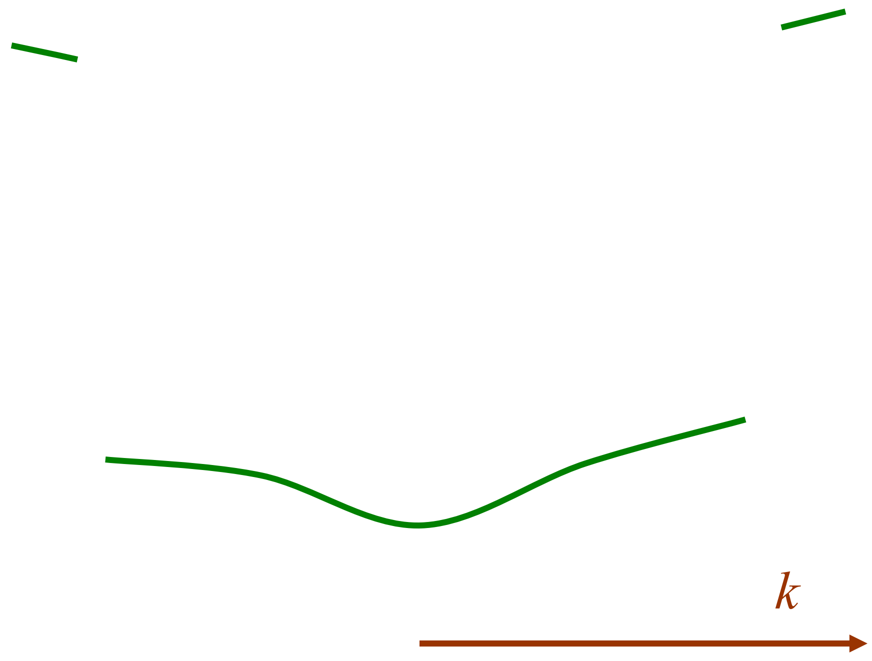


In a weak periodic potential, escape to infinity occurs via Zener tunneling across band gaps

Charged excitations (in one dimension)

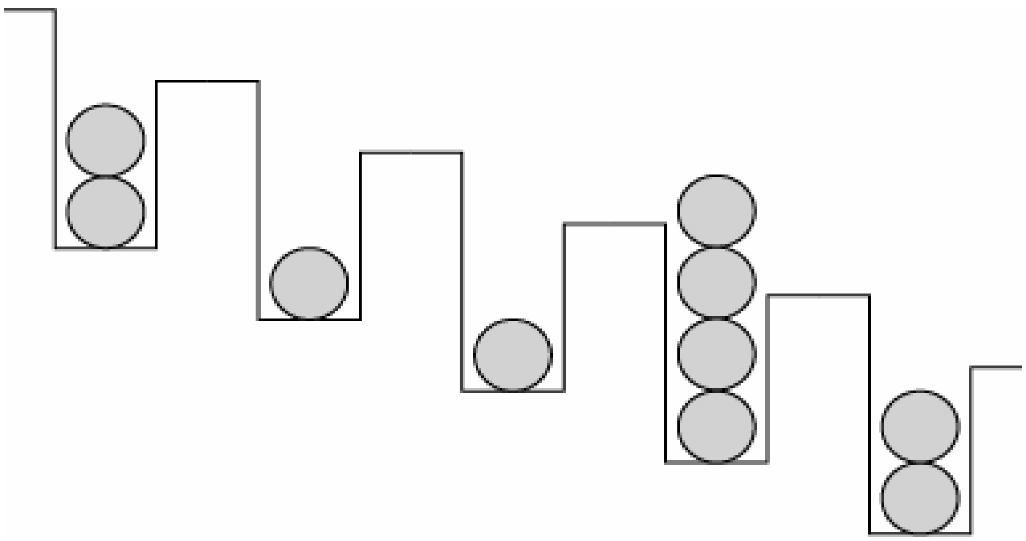
Semiclassical picture

$$\frac{dk}{dt} = E$$



Experimental situation: Strong periodic potential in which there is negligible Zener tunneling, and the particle undergoes Bloch oscillations

A non-dipole state



State has energy *3(U-E)* but is connected to resonant state by a matrix element smaller than t^2/U

State is not part of resonant manifold

Hamiltonian for resonant dipole states (in one dimension)

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}t\sum_\ell \left(d_\ell^\dagger + d_\ell\right) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $d_\ell^\dagger d_\ell \leq 1 \quad ; \quad d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of *(U-E)/t*

Note: there is no explicit dipole hopping term.

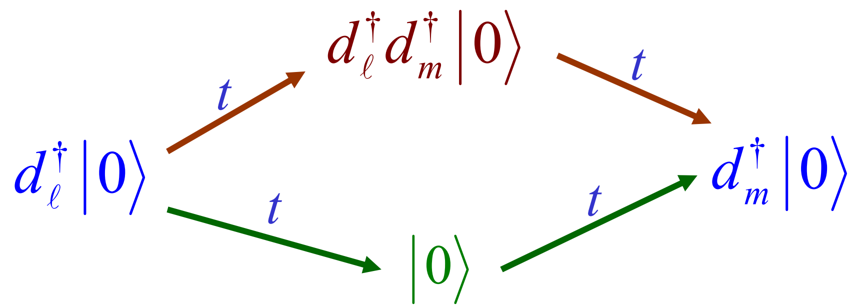
However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak electric fields: *(U-E)*$\gg t$

Ground state is dipole vacuum (Mott insulator) $|0\rangle$

First excited levels: single dipole states $d_\ell^\dagger|0\rangle$

Effective hopping between dipole states



If both processes are permitted, they exactly cancel each other.
The top processes is blocked when ℓ, m are nearest neighbors

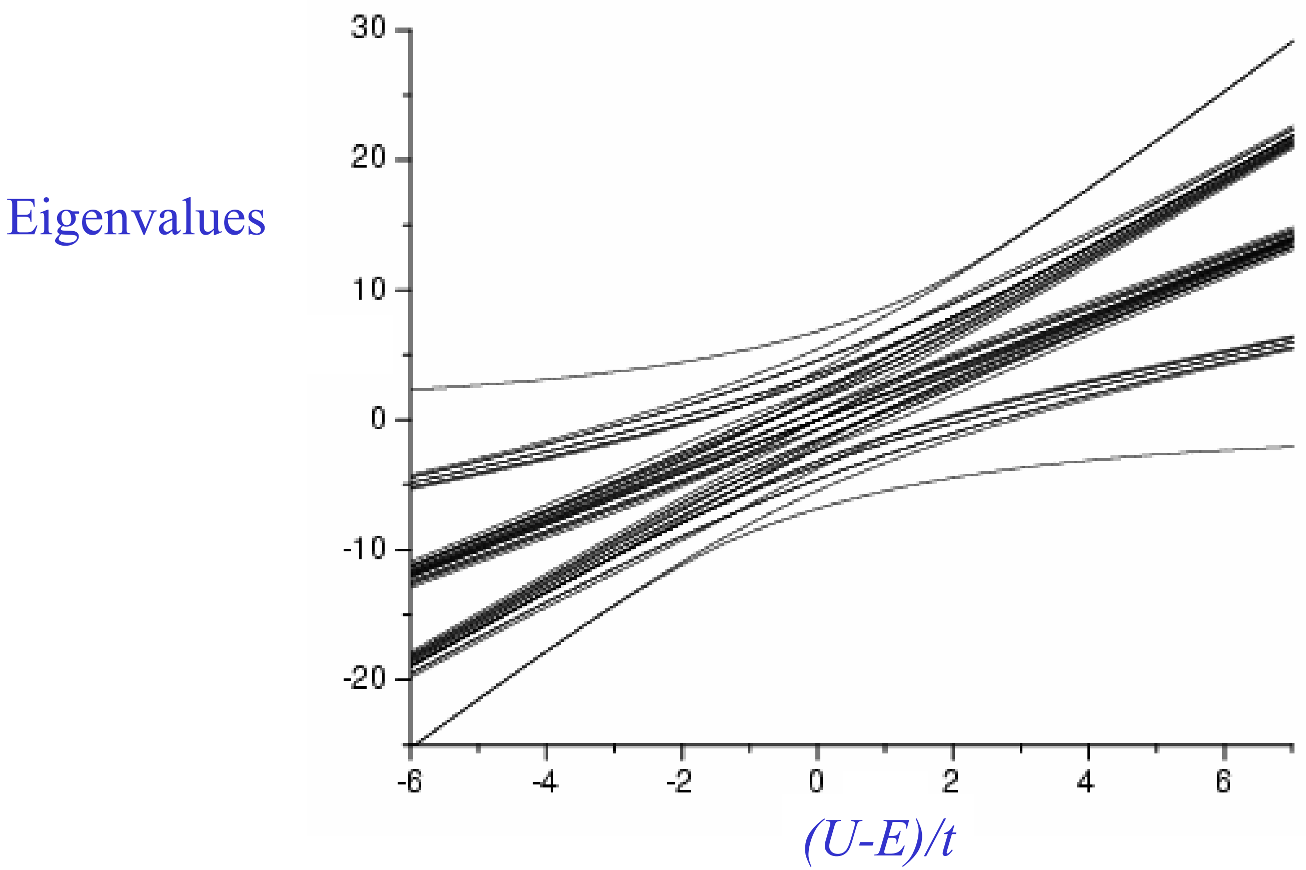
$\Rightarrow$ A nearest-neighbor dipole hopping term $\sim \dfrac{t^2}{U-E}$ is generated

Strong electric fields: $(E\text{-}U) \gg t$

Ground state has maximal dipole number.

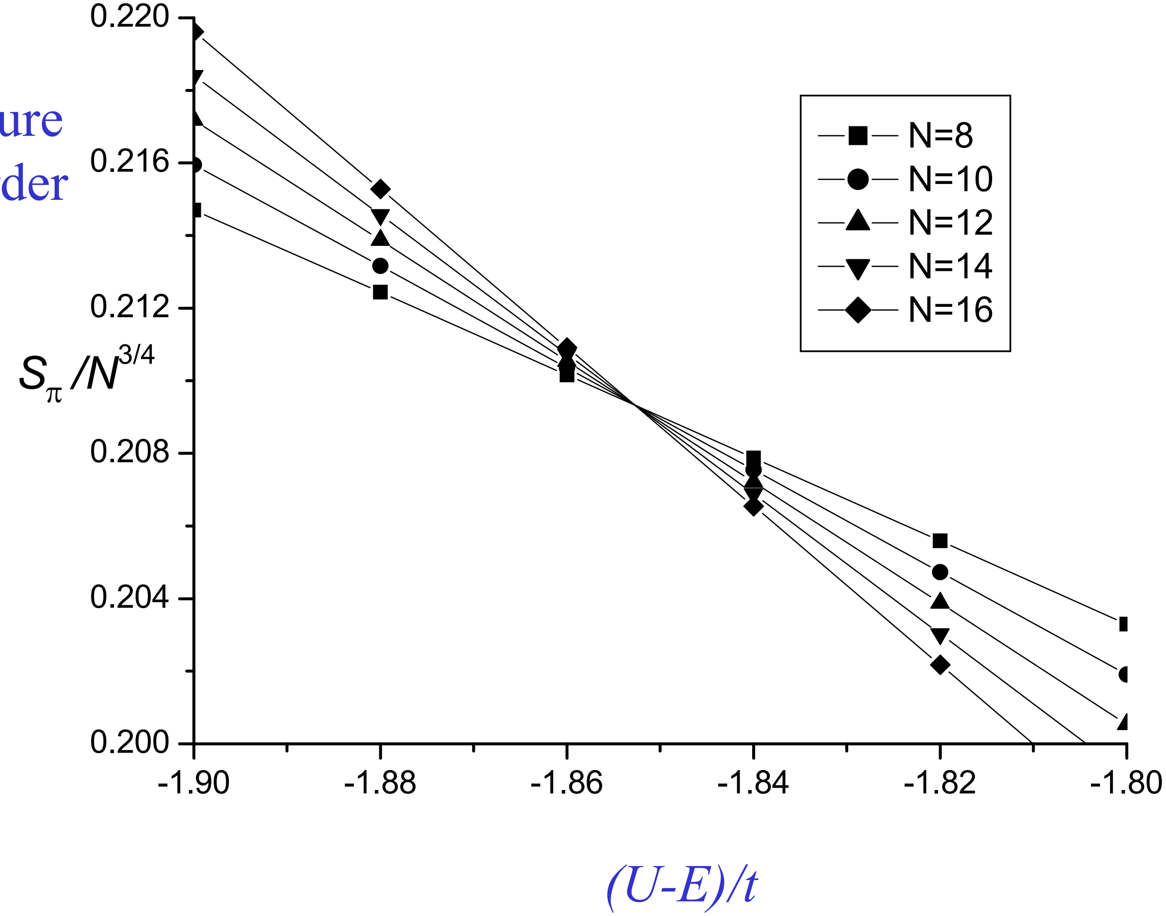
Two-fold degeneracy associated with Ising density wave order:

$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \qquad or \qquad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$



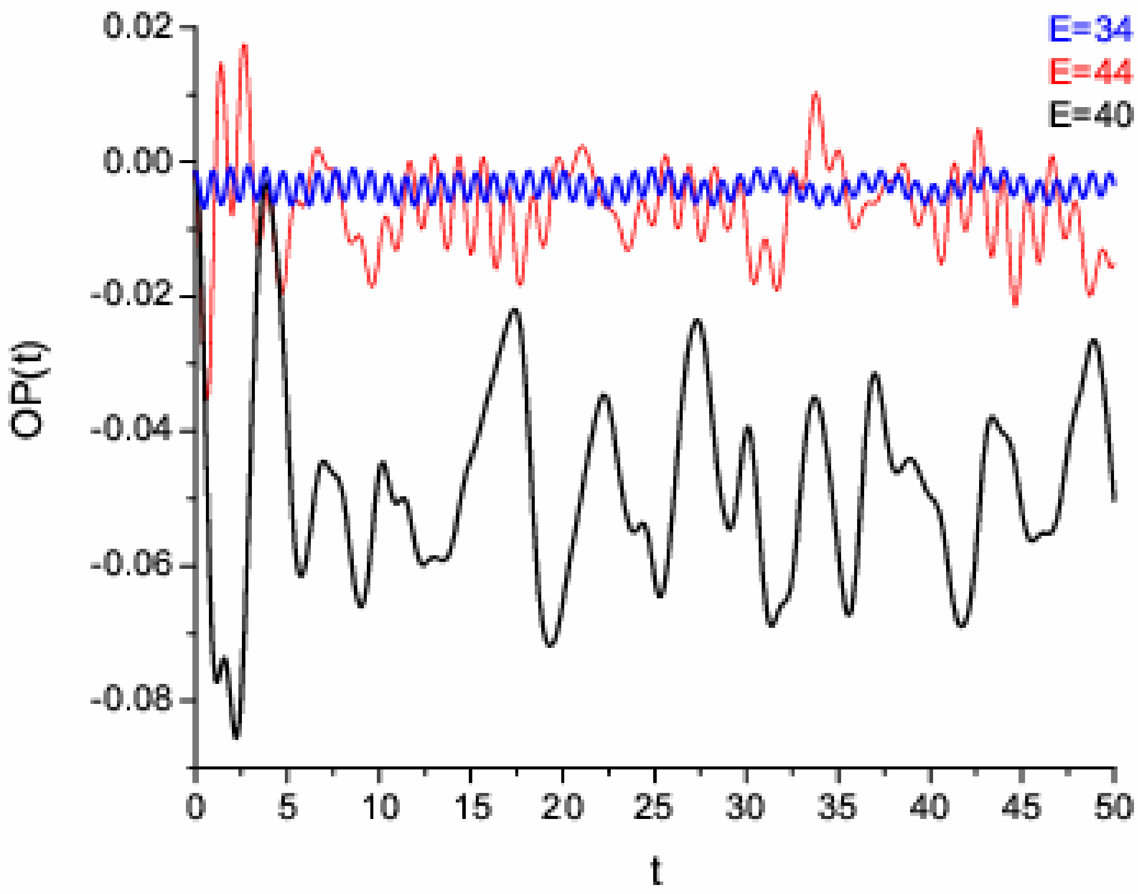
Ising quantum critical point at *E-U=1.08 t*

Equal-time structure factor for Ising order parameter



Non-equilibrium dynamics in one dimension

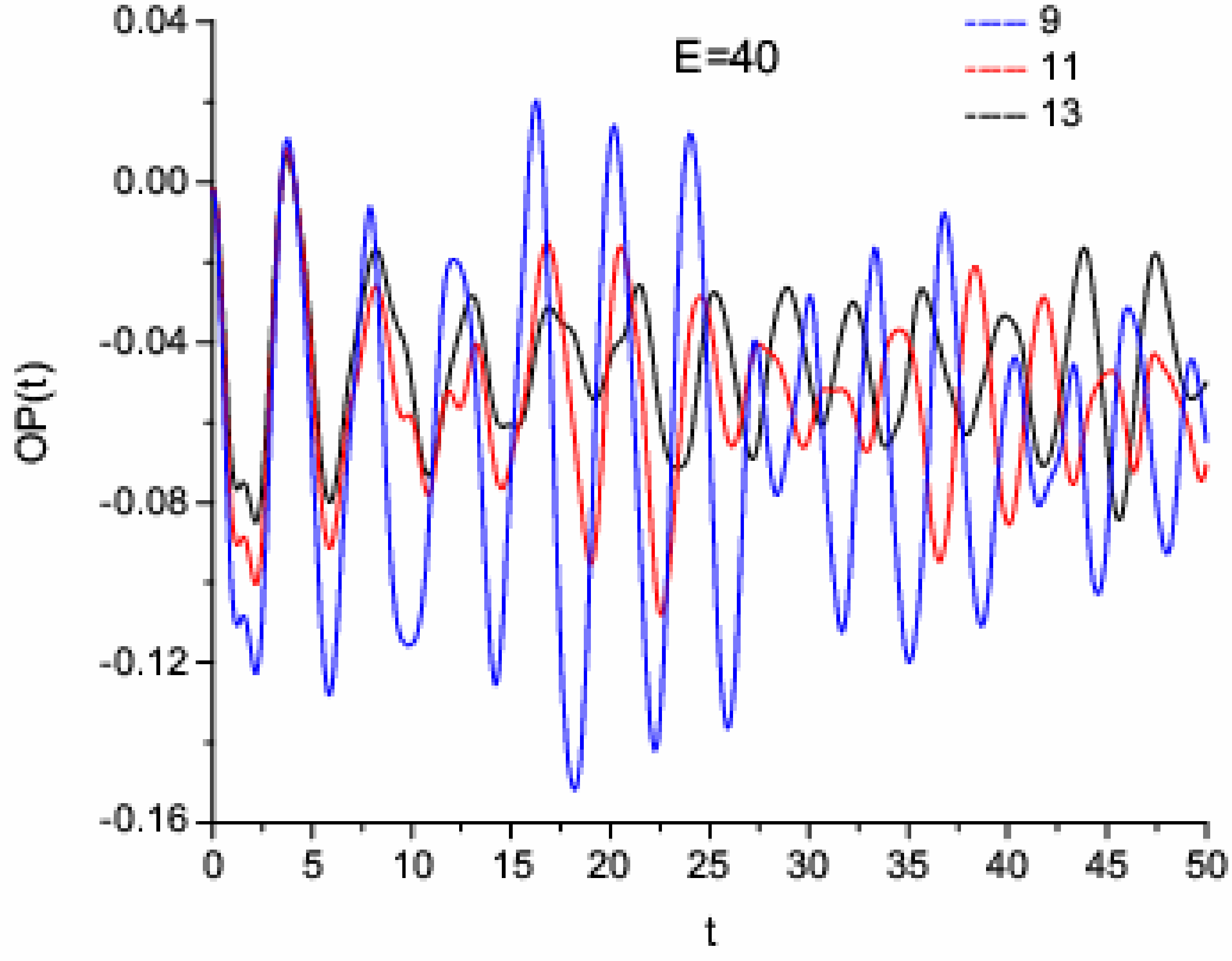
Start with the ground state at *E=32* on a chain with open boundaries.
Suddenly change the value of *E* and follow the evolution of the wavefunction



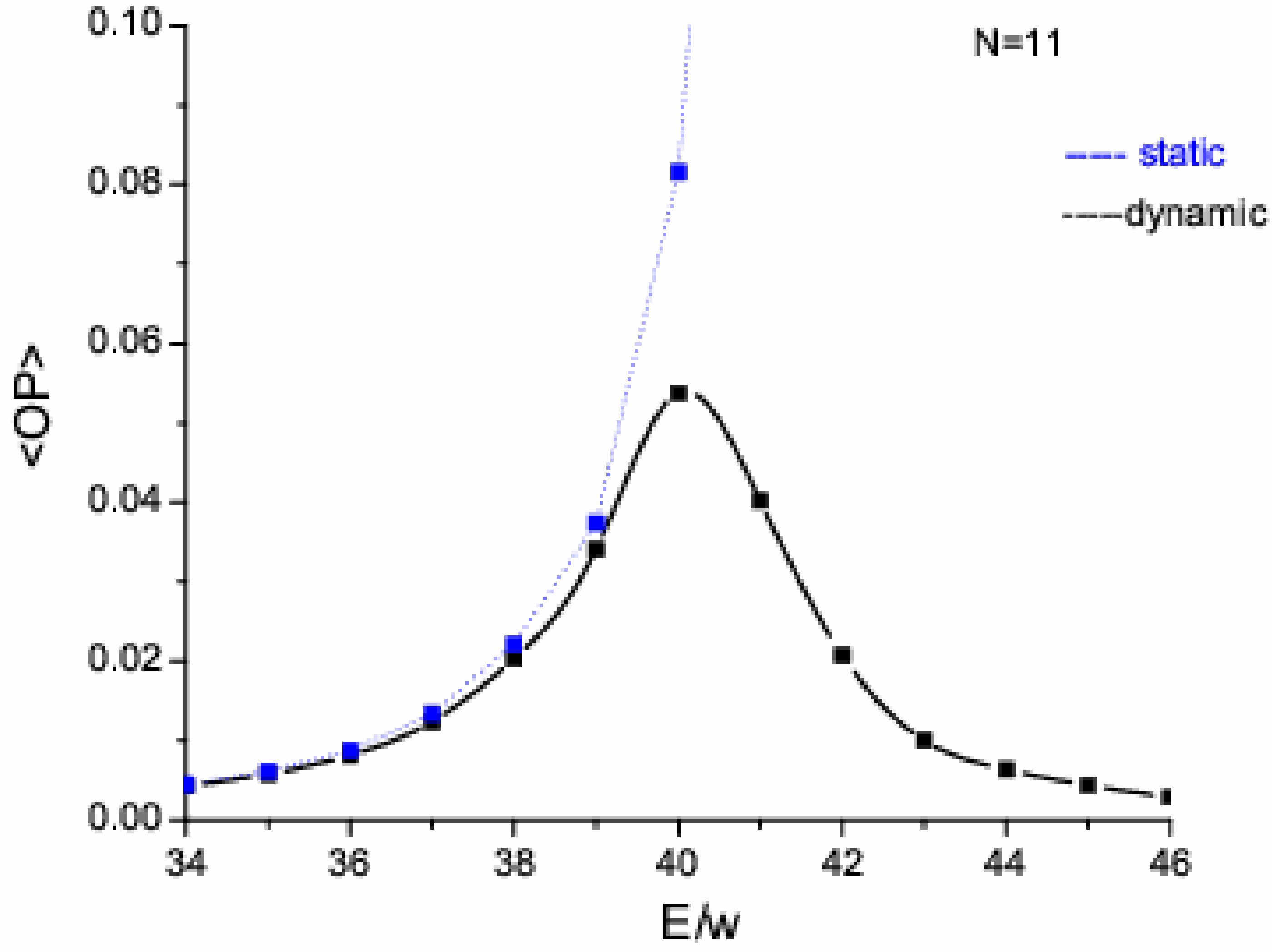
Critical point at *E*=41.85

Non-equilibrium dynamics in one dimension

Dependence on chain length

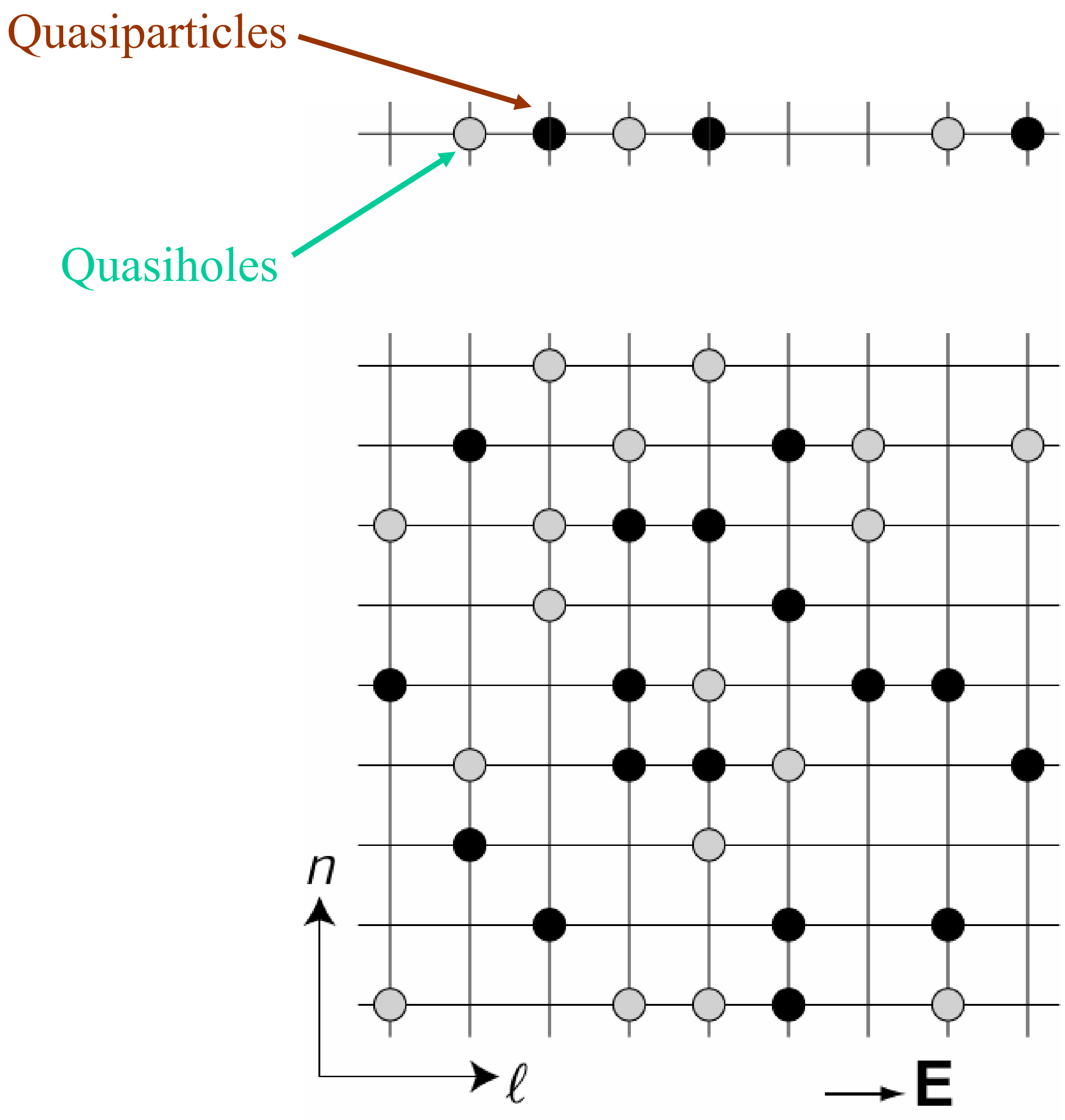


Non-equilibrium dynamics in one dimension



Non-equilibrium response is maximal near the Ising critical point

Resonant states in higher dimensions



Dipole states in one dimension

Quasiparticles and quasiholes can move resonantly in the transverse directions in higher dimensions.

Constraint: number of quasiparticles in any column = number of quasiholes in column to its left.

Hamiltonian for resonant states in higher dimensions

$p^{\dagger}_{\ell,n} \Rightarrow$ Creates quasiparticle in column ℓ and transverse position n

$h^{\dagger}_{\ell,n} \Rightarrow$ Creates quasihole in column ℓ and transverse position n

$$
\begin{aligned}
H_{ph} = & -\sqrt{6}t \sum_{\ell,n} \left(p_{\ell+1,n} h_{\ell,n} + p^{\dagger}_{\ell+1,n} h^{\dagger}_{\ell,n} \right) \\
& + \frac{(U-E)}{2} \sum_{\ell,n} \left(p^{\dagger}_{\ell,n} p_{\ell,n} + h^{\dagger}_{\ell,n} h_{\ell,n} \right) \\
& - t \sum_{\ell,\langle nm \rangle} \left(2 h^{\dagger}_{\ell,n} h_{\ell,m} + 3 p^{\dagger}_{\ell,n} p_{\ell,m} + \text{H.c.} \right)
\end{aligned}
$$

Terms as in one dimension

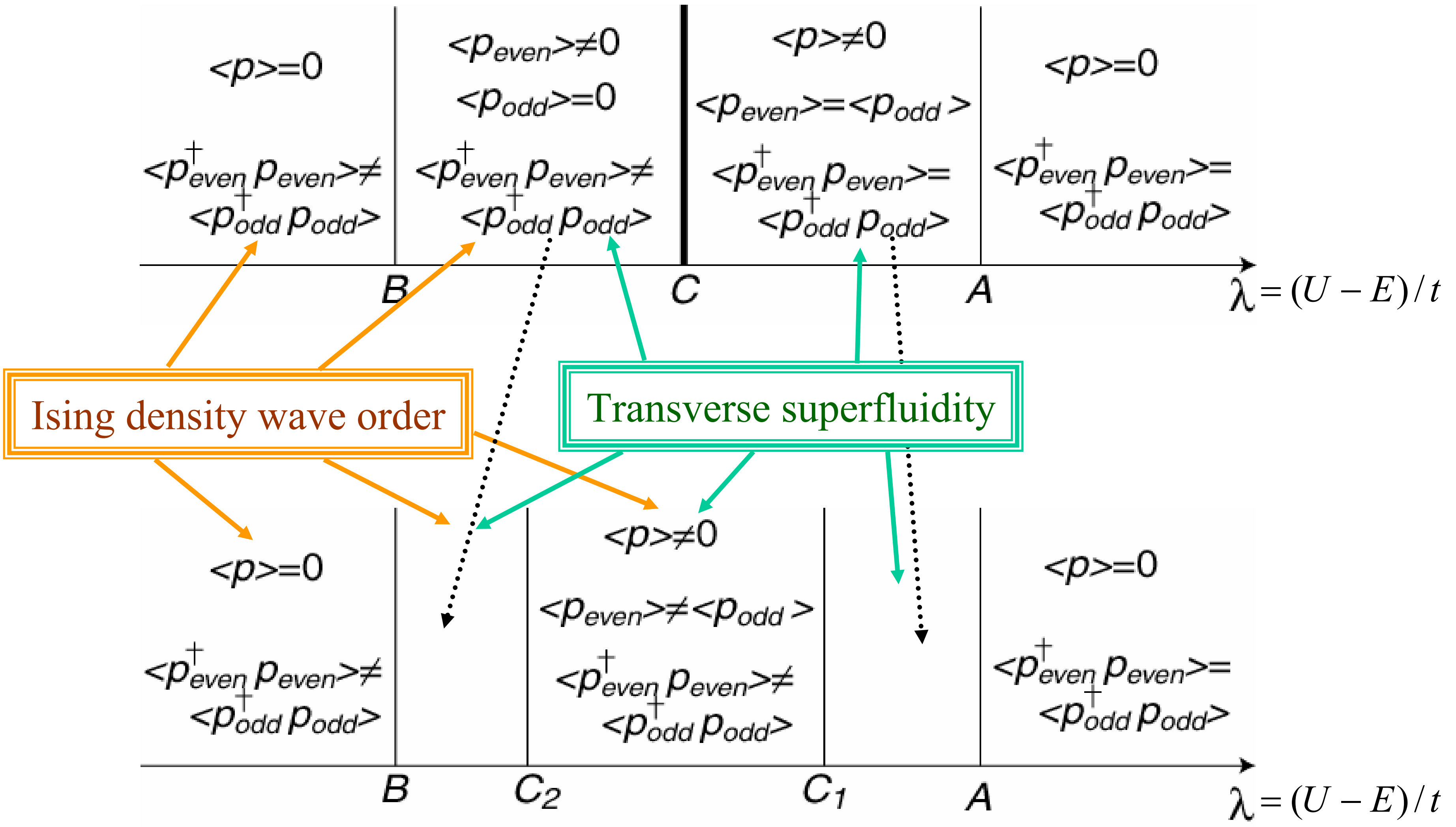
Transverse hopping

$$
p^{\dagger}_{\ell,n} p_{\ell,n} \leq 1 \quad ; \quad h^{\dagger}_{\ell,n} h_{\ell,n} \leq 1 \quad ; \quad p^{\dagger}_{\ell,n} p_{\ell,n} h^{\dagger}_{\ell,n} h_{\ell,n} = 0
$$

Constraints

New possibility: superfluidity in transverse direction (a smectic)

Possible phase diagrams in higher dimensions



Implications for experiments

- Observed resonant response is due to gapless spectrum near quantum critical point(s).
- Transverse superfluidity (smectic order) can be detected by looking for “Bragg lines” in momentum distribution function---bosons are phase coherent in the transverse direction.
- Furture experiments to probe for Ising density wave order?

Conclusions

I. Study of quantum phase transitions offers a controlled and systematic method of understanding many-body systems in a region of strong entanglement.

II. Atomic gases offer many exciting opportunities to study quantum phase transitions because of ease by which system parameters can be continuously tuned.