Quantum criticality in the high temperature superconductors

Institut d’Etudes Scientifiques de Cargese
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Talk online: sachdev.physics.harvard.edu
Strange Metal

K.M. Shen et al., Science 2005

M. Plaë et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface
Quantum oscillations and the Fermi surface in an underdoped high-$T_c$ superconductor

Nicolas Doiron-Leyraud$^1$, Cyril Proust$^2$, David LeBoeuf$^1$, Julien Levallois$^2$, Jean-Baptiste Bonnemaison$^1$, Ruixing Liang$^{3,4}$, D. A. Bonn$^{3,4}$, W. N. Hardy$^{3,4}$ & Louis Taillefer$^{1,4}$

Twofold twisted Fermi surface from staggered order in an underdoped high $T_c$ superconductor

Suchitra E. Sebastian,$^1$ N. Harrison,$^2$ F. F. Balakirev,$^2$ M. M. Altarawneh,$^{2,3}$ Ruixing Liang,$^{4,5}$ D. A. Bonn,$^{4,5}$ W. N. Hardy,$^{4,5}$ G. G. Lonzarich,$^1$

APS March meeting 2013
B2.00004
Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa$_2$Cu$_3$O$_y$

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8 September 2011 | Vol 477 | Nature | 191
Resonant X-Ray Scattering Measurements of a Spatial Modulation of the Cu 3d and O 2p Energies in Stripe-Ordered Cuprate Superconductors

A. J. Achkar, ² F. He, ³ J. Geck, ⁴ H. Zhang, ⁵ Y.-J. Kim, ⁵ and D. G. Hawthorn ¹

PRL 110, 017001 (2013)

may point to a valence-bond-solid interpretation of the stripe phase.
Long-Range Incommensurate Charge Fluctuations in (Y,Nd)Ba$_2$Cu$_3$O$_{6+x}$

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M. Moretti Sala,$^3$ D. C. Peets,$^2$ M. Salluzzo,$^4$ E. Schierle,$^5$ R. Sutarto,$^7,8$ G. A. Sawatzky,$^8$
E. Weschke,$^5$ B. Keimer,$^{2,*}$ L. Braicovich$^1$

**Fig. 3.** Dependence of the CDW signal at 15 K on the hole doping level $p$. The CDW signal is present in several YBa$_2$Cu$_3$O$_{6+x}$ and Nd$_{x+y}$Ba$_2$Cu$_3$O$_7$ samples, but only for $0.09 \leq p \leq 0.13$. In this doping range (shaded in the central panel), the $T_c$-versus-$p$ relation exhibits a plateau. The CDW peak position does not change with $p$ outside of the experimental error, but its intensity is maximum at $p \simeq 0.11$. 

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*SCIENCE* VOL 337 17 AUGUST 2012

resonant soft x-ray scattering

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*References (et al)*
Direct observation of competition between superconductivity and charge density wave order in YBa$_2$Cu$_3$O$_{6.67}$

J. Chang$^{1,2*}$, E. Blackburn$^3$, A. T. Holmes$^3$, N. B. Christensen$^4$, J. Larsen$^{4,5}$, J. Mesot$^{1,2}$, Ruixing Liang$^{6,7}$, D. A. Bonn$^{6,7}$, W. N. Hardy$^{6,7}$, A. Watenphul$^8$, M. v. Zimmermann$^8$, E. M. Forgan$^3$ and S. M. Hayden$^9$

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**Diagram A**: Diagram showing the relationship between temperature and doping. The region marked by YBCO indicates superconductivity, while the region marked by SC indicates charge density wave (CDW) order.

**Diagram B**: Diagrams illustrating the intensity (I) of the signal as a function of doping (p) at different temperatures (T) and magnetic fields (H). The intensity peaks at certain doping levels and temperatures, indicating the presence of CDW order.

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**Equation**: The order parameter $<\delta n^2>$ is plotted as a function of temperature and doping, showing a peak at a certain critical temperature $T_C$ and a peak at a critical doping $\rho_C$. The peak position shifts with changes in magnetic field $H$ and temperature $T$.

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**Discussion**: The competition between superconductivity and charge density wave order is evident in the temperature-doping phase diagram. Above $T_C$, the system exhibits superconductivity; below $T_C$, the charge density wave order is observed. The peak at $T_C$ indicates the critical temperature for superconductivity, while the peak at $\rho_C$ indicates the critical doping for CDW order. The magnetic field and temperature dependence of these peaks provide insights into the underlying physics of the system.
The comparison of different acoustic modes indicates that the charge modulation is biaxial, which differs from a uniaxial stripe charge order.
Outline

1. Pseudogap: angular fluctuations of a multi-component order parameter

2. Instabilities of a two-dimensional metal with antiferromagnetic exchange interactions: $d$-wave superconductivity and bond order
Outline

1. Pseudogap: angular fluctuations of a multi-component order parameter

2. Instabilities of a two-dimensional metal with antiferromagnetic exchange interactions: $d$-wave superconductivity and bond order
Competing orders in thermally fluctuating superconductors in two dimensions

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(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.
Competing orders in thermally fluctuating superconductors in two dimensions

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Mean-field theory of charge/bond order and \(d\)-wave superconductivity

Both orders are induced by a “glue” provided by the antiferromagnetic exchange interaction (details in part 2)
Needed: a theory to explain the maximum in the charge order at the superconducting $T_c$, and its gradual onset over a very wide range of $T$: unlike precursor critical fluctuations above a finite temperature ordering transition.
**Key idea:** analogy with the onset of antiferromagnetism in the *insulator* $\text{La}_2\text{CuO}_4$

Below $T_{\text{Néel}}$

$$T_{\text{Néel}} = 325\text{K}$$


Above $T_{\text{Néel}}$


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Key idea: analogy with the onset of antiferromagnetism in the insulator $La_2CuO_4$

Gradual onset of intensity over a wide range of $T$ is a consequence of angular thermal fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975

$T_{\text{Néel}} = 325\text{K}$

B. Keimer et al.,

Above $T_{\text{Néel}}$
O(3) non-linear sigma model

\[ Z = \int \mathcal{D}\vec{n}(x) \delta (\vec{n}^2(x) - 1) \exp \left( -\frac{\rho_s}{2T} \int d^2x (\nabla_x \vec{n})^2 \right) \]
O(3) non-linear sigma model

\[
Z = \int \mathcal{D}\vec{n}(x) \delta (\vec{n}^2(x) - 1) \exp \left( -\frac{\rho_s}{2T} \int d^2x (\nabla_x \vec{n})^2 \right)
\]

Generalize \( \vec{n} \) to a \( N \)-component vector \( n_\alpha, \alpha = 1 \ldots N \), and take the \( N \to \infty \) limit while taking \( \rho_s \propto N \). This is implemented by a Lagrange multiplier \( \lambda \)

\[
Z = \int \mathcal{D}\lambda(x) \mathcal{D}n_\alpha(x) \exp \left( -\frac{\rho_s}{2T} \int d^2x \left[ (\nabla_x n_\alpha)^2 + i\lambda (n_\alpha^2 - 1) \right] \right)
\]
Generalize $\vec{n}$ to a $N$-component vector $n_\alpha$, $\alpha = 1 \ldots N$, and take the $N \to \infty$ limit while taking $\rho_s \propto N$. This is implemented by a Lagrange multiplier $\lambda$

$$Z = \int \mathcal{D} \lambda(x) \mathcal{D} n_\alpha(x) \exp \left( -\frac{\rho_s}{2T} \int d^2 x \left[ (\nabla_x n_\alpha)^2 + i\lambda (n_\alpha^2 - 1) \right] \right)$$

We can now perform the Gaussian integral over $n_\alpha$

$$Z = \int \mathcal{D} \lambda(x) \exp \left( -\frac{N}{2} \text{Tr} \ln \left( -\nabla_x^2 + i\lambda \right) + \frac{\rho_s}{2T} \int d^2 x i\lambda \right)$$

Because $\rho_s \propto N$, in the $N \to \infty$ limit the partition function is dominated by the saddle point.
At the saddle point, we set $i\lambda(x) = \xi^{-1}$, and then the “structure factor” $S(k)$ of the order parameter is

$$S(k) = \int d^2x \langle n_{\alpha}(x)n_{\alpha}(0) \rangle e^{ikx} = \frac{NT}{\rho_s} \frac{1}{(k^2 + \xi^{-2})}$$

This identifies $\xi$ as the correlation length.
At the saddle point, we set $i\lambda(x) = \xi^{-1}$, and then the “structure factor” $S(k)$ of the order parameter is

$$S(k) = \int d^2x \langle n_\alpha(x)n_\alpha(0) \rangle e^{ikx} = \frac{NT}{\rho_s} \frac{1}{(k^2 + \xi^{-2})}$$

This identifies $\xi$ as the correlation length. The value of $\xi$ is determined by the saddle-point equation, which simply enforces the constraint $n_\alpha^2(x) = 1$. So we have

$$\frac{NT}{\rho_s} \int \frac{d^2k}{4\pi^2} \frac{1}{k^2 + \xi^{-2}} = 1$$

Performing the $k$ integral with a momentum cutoff $\Lambda$ we obtain

$$\frac{NT}{4\pi \rho_s} \ln \left(1 + \Lambda^2 \xi^2\right) = 1 \quad \Rightarrow \quad \xi = \Lambda^{-1} \exp \left(\frac{2\pi \rho_s}{NT}\right)$$

So $\xi$ is finite at all non-zero $T$ (no LRO), and diverges exponentially as $T \to 0$ (consistent with Mermin-Wagner theorem).
The exact result (for the exponential) at finite $N$ is

$$\xi = \Lambda^{-1} \exp \left( \frac{2\pi \rho_s}{(N - 2)T} \right)$$
**O(3) non-linear sigma model**

The *exact* result (for the exponential) at finite $N$ is

$$\xi = \Lambda^{-1} \exp \left( \frac{2\pi \rho_s}{(N-2)T} \right)$$

Neutron scattering measures the structure factor, and the peak value is $S(0)$

$$S(0) = \frac{NT}{\rho_s} \xi^2 = \frac{NT}{\Lambda^2 \rho_s} \exp \left( \frac{4\pi \rho_s}{(N-2)T} \right)$$

So there is no Bragg peak at the ordering wavevector for any two-dimensional antiferromagnet.

$\text{La}_2\text{CuO}_4$ has a non-zero ordering temperature $T_N = 325\text{K}$, and this arises solely from the *inter-layer* coupling.
Key idea: analogy with the onset of antiferromagnetism in the insulator \( \text{La}_2\text{CuO}_4 \)

Below \( T_{\text{Néel}} \)

\[ T_{\text{Néel}} = 325 \text{K} \]


Key idea: analogy with the onset of antiferromagnetism in the insulator $La_2CuO_4$

Gradual onset of intensity over a wide range of $T$ is a consequence of angular thermal fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

$T_{\text{Néel}} = 325$K


Polyakov, 1975
**Multi-component order parameter**

**Superconducting order $\Psi(\mathbf{r})$:**

$$\left\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \right\rangle = \varepsilon_{\alpha\beta} \left[ \sum_{\mathbf{k}} \Delta_0(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi (\mathbf{r}_i + \mathbf{r}_j)/2$$

**Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:**

$$\left\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \right\rangle = \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_x(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x ((\mathbf{r}_i + \mathbf{r}_j)/2)$$

$$+ \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_y(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y ((\mathbf{r}_i + \mathbf{r}_j)/2)$$
Symmetries:
Charge conservation (O(2)), $x$ translations (O(2)), $y$ translations (O(2)), $x \leftrightarrow y$ ($\mathbb{Z}_2$), inversion, time-reversal.
Landau-Ginzburg free energy:

$$F = \int d^2 r \left[ |\nabla \Psi|^2 + s_1 |\Psi|^2 + u_1 |\Psi|^4 
+ |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + s_2 (|\Phi_x|^2 + |\Phi_y|^2) 
+ u_2 (|\Phi_x|^2 + |\Phi_y|^2)^2 + w (|\Phi_x|^4 + |\Phi_y|^4) 
+ v |\Psi|^2 (|\Phi_x|^2 + |\Phi_y|^2) \right]$$

Competing orders: $v$ is positive (and large).
We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.
FIG. 2: Plots of the universal function \((T/T_c)\mathcal{D}(g,T/T_c)\) as a function of \(T/T_c\) for \(\rho_s(0)/(k_B T_c) = 6.8\). From (2.3) we see that \(\langle |\Psi|^2 \rangle_T = (T/T_c)\langle |\Psi|^2 \rangle_{T_c} + (m^* k_B T_c/\hbar^2)(T/T_c)\mathcal{D}(g,T/T_c); so \langle |\Psi|^2 \rangle_T\) is determined from the above plot up to an additive, non-singular, linear dependence on \(T\) determined by \(\langle |\Psi|^2 \rangle_{T_c}\). This linear \(T\) dependence can compensate for the the linear \(T\) dependence in the plot above so that \(\langle |\Psi|^2 \rangle_T\) saturates at high \(T\). Also, as noted in the text, the present theory breaks down at large enough \(T\), and its main utility is in capturing the singular increase in \(\langle |\Psi|^2 \rangle_T\) as \(T\) crosses \(T_c\). The solid line is the small \(\mathcal{G}\) approximation obtained by solving (3.5), (3.7), (3.10),

\[ S_{\Phi_x} \text{ is the charge order structure factor} \]

\(\sim -S_{\Phi_x} \)
Multi-component order parameter

\[ \Phi_x, \Phi_y \]

\[ \Psi \]
Multi-component order parameter

Support from electron theory:
Multi-component order parameter

Excluded region

Preferred configurations

\[ \Phi_x, \Phi_y \]

Support from electron theory:
Multi-component order parameter

Label order parameter by a 6-component unit vector $n_\alpha$ with $\sum_\alpha n_\alpha^2 = 1$
\( O(6) \) non-linear sigma model

\[
Z = \prod_i \left[ \int d\mathbf{n}_{i\alpha} \delta \left( \sum_{\alpha=1}^{6} n_{i\alpha}^2 - 1 \right) \right] \exp \left( - \frac{1}{2T} \sum_{\langle ij \rangle} \left[ \sum_{\alpha=1}^{2} (n_{i\alpha} - n_{j\alpha})^2 
+ \lambda \sum_{\alpha=3}^{6} (n_{i\alpha} - n_{j\alpha})^2 \right] \right. \\
\left. - \frac{g}{2T} \sum_i \sum_{\alpha=3}^{6} n_{i\alpha}^2 \right.
\left. - \frac{w}{2T} \sum_i \left[ (n_{i3}^2 + n_{i4}^2)^2 + (n_{i5}^2 + n_{i6}^2)^2 \right] \right)
\]

where \( \Psi \propto n_1 + i n_2, \Phi_x \propto n_3 + i n_4, \Phi_y \propto n_5 + i n_6 \).

Describes \( O(6) \Rightarrow O(2) \times O(2) \times O(2) \times \mathbb{Z}_2 \).

Solve by cluster Monte Carlo and \( 1/N \) expansion.
O(6) non-linear sigma model

Set B: $g=0.4$, $\lambda=1$

$I/N$ expansion
- $L=32$
- $L=40$
- $L=48$
- $L=56$
- $L=64$

Charge order structure factor $S_{\Phi x}$
**O(6) non-linear sigma model**

O(6) Model with \( g = 0.3 \), \( \lambda = 1 \) and \( w = 0 \)

Experimental Data: Timescan while cooling, moving avg

Monte Carlo on 72x72 lattice

Charge order structure factor \( S_{\Phi_x} \)

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O(6) non-linear sigma model

O(6) Model with g=0.3, λ=1 and w=0

Experimental Data: Timescan while cooling, moving avg
Monte Carlo on 72x72 lattice

Charge order structure factor $S_{\Phi x}$
O(6) non-linear sigma model

O(6) Model with $g=0.3$, $\lambda=1$ and $w=0$

Experimental Peak Amplitude

Charge order structure factor $S_{\Phi x}$

Experimental Data: Timescan while cooling, moving avg
Monte Carlo on 72x72 lattice

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In progress: possibility of Ising transition in \textit{nematic} order parameter $m = |\Phi_x|^2 - |\Phi_y|^2$.

This is expected to be present for $w < 0$ at sufficiently small $T$, and is found in the $N = \infty$ theory.
Pseudogap: Angular fluctuations of a multi-component order parameter