Holography of compressible quantum states

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Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

Compressible quantum matter

- Compressible systems must be gapless.
- Conformal systems are compressible in $d = 1$, but not for $d > 1$. Compressible quantum matter
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- Describe zero temperature phases where $d\langle Q\rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$. 

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- Conformal systems are compressible in $d = 1$, but not for $d > 1$. 
One compressible state is the **solid** (or “Wigner crystal” or “stripe”). This state breaks translational symmetry.
Another familiar compressible state is the **superfluid**. This state breaks the global $U(1)$ symmetry associated with $Q$.

Condensate of fermion pairs
Graphene
The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**.
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- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.
The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**

- **Luttinger relation:** The total “volume (area)” $A$ enclosed by the Fermi surface is equal to $\langle Q \rangle$. 
Exotic phases of compressible quantum matter

I. Field theory

II. Holography
Exotic phases of compressible quantum matter

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II. Holography
ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $U(N) \times U(N) \times SU(4)_R$.

- $4N^2$ complex bosons carrying fundamental charges of $U(N) \times U(N) \times SU(4)_R$.

- $\mathcal{N} = 6$ supersymmetry
**ABJM theory in D=2+1 dimensions**

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- $4N^2$ complex bosons carrying fundamental charges of $\text{U}(N) \times \text{U}(N) \times \text{SU}(4)_R$.
- $\mathcal{N} = 6$ supersymmetry

**Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance**
Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, $f_+$ and $f_-$ ("quarks"), carry U(1) gauge charges $\pm 1$, and global U(1) charge 1.
- Bosons, $b_+$ and $b_-$ ("squarks"), carry U(1) gauge charges $\pm 1$, and global U(1) charge 1.
- No supersymmetry

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- No supersymmetry

- Fermions, $c$ ("mesinos"), gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

Theory similar to ABJM

\[ \mathcal{L} = f_\sigma^\dagger \left[ (\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma A)^2}{2m} - \mu \right] f_\sigma \\
+ b_\sigma^\dagger \left[ (\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma A)^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\
+ \frac{u}{2} \left( b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left( b_+^\dagger b_- f_- f_+ + \text{H.c.} \right) \]

The index \( \sigma = \pm 1 \)

Theory similar to ABJM

\[ \mathcal{L} = f_\sigma^\dagger \left[ \left( \partial_\tau - i\sigma A_\tau \right) - \frac{\left( \nabla - i\sigma A \right)^2}{2m} - \mu \right] f_\sigma \\
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+ \frac{u}{2} \left( b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left( b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
+ c^\dagger \left[ \partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
- g_2 \left[ c^\dagger \left( f_+ b_- + f_- b_+ \right) + \text{H.c.} \right] \]

The index \( \sigma = \pm 1 \), and \( \epsilon_{1,2} \) are tuning parameters of phase diagram.

Conserved U(1) charge: \( Q = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c \)

Phases of ABJM-like theories

\langle b_{\pm} \rangle = 0

2A_c = \langle Q \rangle

Fermi liquid (FL)

with Fermi surface of gauge-neutral mesinos

U(1) gauge theory is in \textit{confining} phase
Phases of ABJM-like theories

\[ \langle b_\pm \rangle = 0 \]

\[ 2A_f = \langle Q \rangle \]

non-Fermi liquid (NFL)
with Fermi surface of gauge-charged quarks

U(1) gauge theory is in \textit{deconfined} phase
Phases of ABJM-like theories

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Fermi surface coupled to Abelian or non-Abelian gauge fields:

- Longitudinal gauge fluctuations are screened by the fermions.
- Transverse gauge fluctuations are unscreened, and Landau-damped. They are IR fluctuations with dynamic critical exponent \( z > 1 \).
- Theory is \textit{strongly coupled in two spatial dimensions}.
- “Non-Fermi liquid” broadening of the fermion quasiparticle pole.

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“Hidden” Fermi surface
Phases of ABJM-like theories

\[ \langle b_{\pm} \rangle = 0 \]

Fractionalized Fermi liquid (FL*)
with Fermi surfaces of both quarks and mesinos

\[ 2A_c + 2A_f = \langle Q \rangle \]

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Fractionalized Fermi liquid (FL*) with Fermi surfaces of both quarks and mesinos

U(1) gauge theory is in deconfined phase

“Hidden” and visible Fermi surfaces co-exist
Key question:

How do we detect the “hidden Fermi surfaces” of fermions with gauge charges in the non-Fermi liquid phases?

These are not directly visible in the gauge-invariant fermion correlations computable via holography.
How do we detect the “hidden Fermi surfaces” of fermions with gauge charges in the non-Fermi liquid phases?

One promising answer:

Compute entanglement entropy

Entanglement entropy of Fermi surfaces

\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

Entanglement entropy \( S_{EE} = -\text{Tr} (\rho_A \ln \rho_A) \)
Logarithmic violation of “area law”: \( S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P) \)

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape.


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Non-Fermi liquids have, at most, the “1/12” prefactor modified.

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II. Holography
Begin with a CFT

Dirac fermions + gauge field + ......
$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$
Apply a chemical potential to the “deconfined” CFT

\[ \mu > 0 \]
The Maxwell-Einstein theory of the applied chemical potential yields a AdS$_4$-Reissner-Nordström black-brane

\[ \mathcal{E}_r = \langle Q \rangle \]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right] \]

The Maxwell-Einstein theory of the applied chemical potential yields a AdS$_4$-Reissner-Nordström black-brane

At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of AdS$_2 \times R^2$

$$ds^2 = \frac{L^2}{6} \left( \frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$
• Non-zero entropy density at $T = 0$

• Green’s function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface

• Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with “hidden Fermi surfaces” of gauge-charged particles (the *quarks*).

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field

\[ r \]

Electric flux

\[ E_r = \langle Q \rangle \]

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R - 2(\nabla \Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right] \]

with \( Z(\Phi) = Z_0 e^{\alpha\Phi} \), \( V(\Phi) = -V_0 e^{\delta\Phi} \), as \( \Phi \to \infty \).

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Leads to metric

\[ ds^2 = L^2 \left( -f(r) dt^2 + g(r) dr^2 + \frac{dx^2 + dy^2}{r^2} \right) \]

with \( f(r) \sim r^{-\gamma} \), \( g(r) \sim r^{\beta} \) as \( r \to \infty \).

With the choice of the exponents, $\alpha$, $\delta$, a large zoo of NFL phases appear possible. But the fate of the Luttinger count of Fermi surfaces seems unclear.
Holographic theory of a non-Fermi liquid (NFL)

Key idea:

Restrict attention to those models in which there is logarithmic violation of area law in the entanglement entropy. This restricts \( \delta = -\alpha/3 \) and \( g(r) \sim \text{constant} \), as \( r \to \infty \).

\[
ds^2 = L^2 \left( -\frac{dt^2}{r \gamma} + dr^2 + \frac{dx^2 + dy^2}{r^2} \right)
\]

Holographic theory of a non-Fermi liquid (NFL)

Evidence for “hidden” Fermi surface:

\[ S_{EE} = F(\alpha)(P\sqrt{Q}) \ln(P\sqrt{Q}) \]

where \( P \) is the perimeter of the entangling region, and \( F(\alpha) \) is a known function of \( \alpha \) only.

- The dependence on \( Q \) and the shape of the entangling region is just as expected for a Fermi surface.

Logarithmic violation of “area law”: \[ S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P) \]

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape.

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Holographic theory of a non-Fermi liquid (NFL) and a fractionalized Fermi liquid (FL*)

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where \( P \) is the perimeter of the entangling region, and \( F(\alpha) \) is a known function of \( \alpha \) only.

- Adding probe fermions leads to a (visible) Fermi surface of “mesinos” as in a FL* phase, and a holographic entanglement entropy in which

\[ Q \rightarrow Q - Q_{\text{mesinos}}. \]

So only the “hidden” Fermi surfaces of “quarks” are measured by the holographic minimal area formula.
In a deconfined NFL phase, the metric extends to infinity (representing critical IR modes), and all of the electric flux “leaks out”.
In a deconfined FL* phase, the metric extends to infinity, there is a mesino charge density in the bulk, and **only part** of the electric flux “leaks out”.

\[
\mathcal{E}_r = \langle Q \rangle - \langle Q_{\text{mesino}} \rangle
\]

Holographic theory of a fractionalized Fermi liquid (FL*)
In a confining FL phase, the metric terminates, all of the mesino density is in the bulk spacetime, and none of the electric flux “leaks out”.
Gauss Law in the bulk ⇔ Luttinger theorem on the boundary
Consider QED$_4$, with full quantum fluctuations,

$$ S = \int d^4 x \sqrt{g} \left[ \frac{1}{4e^2} F_{ab} F^{ab} + i (\bar{\psi} \Gamma^M D_M \psi + m \bar{\psi} \psi) \right]. $$

in a metric which is AdS$_4$ in the UV, and confining in the IR. A simple model

$$ ds^2 = \frac{1}{r^2} (dr^2 - dt^2 + dx^2 + dy^2) \quad , \quad r < r_m $$

with $r_m$ determined by the confining scale.
Massive Dirac fermions at zero chemical potential

Dispersion \( E_{\ell}(k) = \sqrt{k^2 + M_{\ell}^2} \)

Masses \( M_{\ell} \sim 1/r_m \)
Holographic theory of a Fermi liquid (FL)

Massive Dirac fermions at zero chemical potential

Dispersion $E_\ell(k) = \sqrt{k^2 + M_\ell^2}$
Masses $M_\ell \sim 1/r_m$

Almost all previous holographic theories have considered the situation where the spacing between the $E_\ell(k)$ vanishes, and an infinite number of $E_\ell(k)$ are relevant.
The spectrum at non-zero chemical potential is determined by self-consistently solving the Dirac equation and Gauss’s law:

$$
\left( \bar{\Gamma} \cdot \vec{D} + m \right) \Psi_\ell = E_\ell \Psi_\ell \ ; \ \nabla_r \mathcal{E}_r = \sum_\ell \int \frac{d^2k}{4\pi^2} \Psi^\dagger_\ell(k, z) \Psi_\ell(k, z) f(E_\ell(k))
$$

where \( \mathcal{E} \) is the electric field, and \( f(E) \) is the Fermi function.
• The confining geometry implies that all gauge and graviton modes are gapped (modulo Landau damping from the Fermi surface).

\begin{equation}
E_r^{\text{boundary}} - E_r^{\text{IR}} = A
\end{equation}

But \( E_r^{\text{boundary}} = \frac{Q}{4\pi} \), by the rules of AdS/CFT. So we obtain the usual Luttinger theorem of a Landau Fermi liquid, \( A = \frac{Q}{4\pi} \) provided \( E_r^{\text{IR}} = 0 \).
- The confining geometry implies that all gauge and graviton modes are gapped (modulo Landau damping from the Fermi surface).

- We can apply standard many body theory results, treating this multi-band system in 2 dimensions, like a 2DEG at a semiconductor surface.
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Integrating Gauss’s Law, we obtain

\[ \mathcal{E}_r(\text{boundary}) - \mathcal{E}_r(\text{IR}) = A \]

But \( \mathcal{E}_r(\text{boundary}) = \langle Q \rangle \), by the rules of AdS/CFT. So we obtain the usual Luttinger theorem of a Landau Fermi liquid,

\[ A = \langle Q \rangle \]

provided \( \mathcal{E}_r(\text{IR}) = 0 \).
Holographic theory of a Fermi liquid (FL)

Electric flux

Gauss Law in the bulk
\[ \mathcal{E}_r = \langle Q \rangle \]

\[ \mathcal{E}_r = 0 \]

In a confining FL phase, the metric terminates, all of the mesino density is in the bulk spacetime, and \textit{none} of the electric flux “leaks out”.

Gauss Law in the bulk
\[ \Rightarrow \text{Luttinger theorem on the boundary} \]
Conclusions

Compressible quantum matter

Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.
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Fermi liquid (FL) state described by a confining holographic geometry
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Fermi liquid (FL) state described by a confining holographic geometry.

Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a fractionalized Fermi liquid (FL*)