Outline

1. Antiferromagnetism and quantum criticality in insulators

2. Onset of antiferromagnetism in metals, and d-wave superconductivity

3. Competing density wave order, and the pseudogap of the cuprate superconductors

4. Non-Fermi liquids
This specific $d$-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. 111, 027202 (2013).
A form-factor density wave has unidirectional domains.
dFF-DW Unidirectional Domains

\[ Z(r, 150\text{mV}) \]
\[ \frac{|O_y(r,q=Q_x)| - |O_x(r,q=Q_y)|}{|O_y(r,q=Q_x)| + |O_x(r,q=Q_y)|} \]

\[ +0.75 -0.75 \]

Primary DW direction Orange : // (1,0), Blue : // (0,1)
dFF-DW Unidirectional Domains

\[
\frac{|O_y(r,q=Q_x)| - |O_x(r,q=Q_y)|}{|O_y(r,q=Q_x)| + |O_x(r,q=Q_y)|}
\]

Primary DW direction Orange : // (1,0), Blue : // (0,1)
Phase-resolved Visualization of $d$-form factor DW in Cuprates

Quantum criticality of Ising-nematic ordering in a metal

A metal with a Fermi surface with full square lattice symmetry
Quantum criticality of Ising-nematic ordering in a metal

Pomeranchuk instability as a function of coupling $\lambda$

or

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$
Quantum criticality of Ising-nematic ordering in a metal

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Phase diagram as a function of $T$ and $\lambda$
Quantum criticality of Ising-nematic ordering in a metal

Quantum critical

Phase diagram as a function of $T$ and $\lambda$

Strongly-coupled “non-Fermi liquid” metal with no quasiparticles

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

Fermi liquid

Fermi liquid

$T_{l-n}$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

- Strongly-coupled “non-Fermi liquid” metal with no quasiparticles
- $\langle \phi \rangle \neq 0$
- $\langle \phi \rangle = 0$

Fermi liquid

$T_{l-n}$
Effective action for Ising order parameter

\[ S_\phi = \int d^2 r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u\phi^4 \right] \]
Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

\[ S_\phi = \int d^2 r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right] \]

Effective action for electrons:

\[ S_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[ \sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right] \]

\[ \equiv \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger (\partial_\tau + \varepsilon_k) c_{k\alpha} \]
Quantum criticality of Ising-nematic ordering in a metal

“Yukawa” coupling between Ising order and electrons

\[ S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q (\cos k_x - \cos k_y) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha} \]

for spatially dependent \( \phi \)

\[ \langle \phi \rangle > 0 \quad \text{and} \quad \langle \phi \rangle < 0 \]
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[ S_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + \kappa^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right] \]

\[ S_c = \sum_{\alpha=1}^{N_f} \sum_{k} \int d\tau c_{k\alpha}^\dagger (\partial_\tau + \varepsilon_k) c_{k\alpha} \]

\[ S_{\phi_c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q (\cos k_x - \cos k_y) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha} \]
Quantum criticality of Ising-nematic ordering in a metal

- \( \phi \) fluctuation at wavevector \( \vec{q} \) couples most efficiently to fermions near \( \pm \vec{k}_0 \).
Quantum criticality of Ising-nematic ordering in a metal

- $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm \vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm \vec{k}_0$ and boson ($\phi$) kinetic energy about $\vec{q} = 0$. 
Quantum criticality of Ising-nematic ordering in a metal

\[ \mathcal{L}[\psi_+, \phi] = \]

\[ \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]

\[ -\phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y \phi \right)^2 \]

Quantum criticality of Ising-nematic ordering in a metal

\[ \mathcal{L} = \psi_+^\dagger (\partial_\tau - i \partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i \partial_x - \partial_y^2) \psi_- \]

\[ - \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \]

One loop $\phi$ self-energy with $N_f$ fermion flavors:

\[ \Sigma_\phi(q, \omega) = N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]} \]

\[ = \frac{N_f}{4\pi} \frac{\omega}{|q_y|} \]

Landau-damping
Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i \partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i \partial_x - \partial_y^2) \psi_-$$

$$- \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k}, \Omega) = -\frac{1}{N_f} \int \frac{d^2q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[ \frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]}$$

$$= -i \frac{2}{\sqrt{3}N_f} \left( \frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega)|\Omega|^{2/3} \sim |\Omega|^{d/3} \text{ in dimension } d.$$
Quantum criticality of Ising-nematic ordering in a metal

\[ \mathcal{L} = \psi_+^\dagger \left( \partial_\tau - i\partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i\partial_x - \partial_y^2 \right) \psi_- - \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y \phi \right)^2 \]

Schematic form of \( \phi \) and fermion Green’s functions in \( d \) dimensions

\[
D(\vec{q}, \omega) = \frac{1/N_f}{q_\parallel^2 + \frac{1}{|\omega|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_\parallel^2 - i\text{sgn}(\omega)|\omega|^{d/3}/N_f}
\]

In the boson case, \( q_\parallel^2 \sim \omega^{1/z_b} \) with \( z_b = 3/2 \).

In the fermion case, \( q_x \sim q_\parallel^2 \sim \omega^{1/z_f} \) with \( z_f = 3/d \).

Note \( z_f < z_b \) for \( d > 2 \) \( \Rightarrow \) Fermions have higher energy than bosons, and perturbation theory in \( g \) is OK.
Quantum criticality of Ising-nematic ordering in a metal

\[ L = \psi_+^\dagger (\partial_\tau - i \partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i \partial_x - \partial_y^2) \psi_- 
- \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \]

Schematic form of \( \phi \) and fermion Green’s functions in \( d = 2 \)

\[ D(\vec{q}, \omega) = \frac{1/N_f}{q_x^2 + \frac{\omega}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i \text{sgn}(\omega) |\omega|^{2/3} / N_f} \]

In both cases \( q_x \sim q_y^2 \sim \omega^{1/z} \), with \( z = 3/2 \). Note that the bare term \( \sim \omega \) in \( G_f^{-1} \) is irrelevant.

Strongly-coupled theory without quasiparticles.
Quantum criticality of Ising-nematic ordering in a metal

\[ \mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \]
\[ - \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \]

Simple scaling argument for \( z = 3/2 \).
Quantum criticality of Ising-nematic ordering in a metal

\[ \mathcal{L} = \psi_+^\dagger (\mathcal{X} - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\mathcal{X} + i\partial_x - \partial_y^2) \psi_- \\
- \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \]

Simple scaling argument for \( z = 3/2 \).
Quantum criticality of Ising-nematic ordering in a metal

\( \mathcal{L} = \psi_+^\dagger \left( \partial_x - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_x + i \partial_x - \partial_y^2 \right) \psi_- \\
- \phi \left( \psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y \phi \right)^2 \)

Simple scaling argument for \( z = 3/2 \).

Under the rescaling \( x \rightarrow x/s, \ y \rightarrow y/s^{1/2}, \ \text{and} \ \tau \rightarrow \tau/s^z \), we find invariance provided

\[
\begin{align*}
\phi & \rightarrow \phi s \\
\psi & \rightarrow \psi s^{(2z+1)/4} \\
g & \rightarrow g s^{(3-2z)/4}
\end{align*}
\]

So the action is invariant provided \( z = 3/2 \).
• \( \kappa_F^q \sim Q \), the fermion density

• Sharp fermionic excitations near Fermi surface with \( \omega \sim |q|^z \), and \( z = 1 \).

• Entropy density \( S \sim T^{(d-\theta)/z} \) with violation of hyperscaling exponent \( \theta = d - 1 \).

• Entanglement entropy \( S_E \sim \kappa_F^{d-1} P \ln P \).
- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$. 
**FL Fermi liquid**

- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

**NFL Nematic QCP**

- Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.
• $k_F^d \sim Q$, the fermion density

• Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

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• Fermi surface with $k_F^d \sim Q$.

• Diffuse fermionic excitations with $z = 3/2$ to three loops.

• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$. 
• $k_F^d \sim Q$, the fermion density

• Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

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• Fermi surface with $k_F^d \sim Q$.

• Diffuse fermionic excitations with $z = 3/2$ to three loops.

• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

• $S_E \sim k_F^{d-1} P \ln P$. 
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

Strongly-coupled “non-Fermi liquid” metal with no quasiparticles

$T_{I-n}$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$
Quantum criticality of Ising-nematic ordering in a metal

Common theoretical belief from an analysis of scattering of charged electronic quasiparticles off bosonic $\phi$ fluctuations: resistivity of strange metal $\rho(T) \sim T^{4/3}$. 

Strongly-coupled “non-Fermi liquid” metal with no quasiparticles
Quantum criticality of Ising-nematic ordering in a metal

This ignores constraints arising from conservation of total momentum.


Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic $\phi$ fluctuations.

- Analogous to electron-phonon scattering in metals, where we have "Bloch's law": a resistivity $\rho \sim T^{4/3}$.

- "Bloch's law" for the Ising-nematic critical point yields $\rho \sim T^{2/3}$.

- However, Bloch's law ignores conservation of total momentum, or phonon drag.

- The field theory for the Ising-nematic critical point has strong electron scattering, and no quasi-particle excitations. Nevertheless, because of the central importance of the analog of phonon drag, it has $\rho \sim T^0$. 

- Boltzmann view of electrical transport:
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic $\phi$ fluctuations.

- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$. 

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic $\phi$ fluctuations.

- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$.

- “Bloch’s law” for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$. 
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations with bosonic fluctuations.
- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$.
- “Bloch’s law” for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

However, this ignores “phonon drag”

**PHONON DRAG**

Peierls$^{28}$ pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$.

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations $\phi$ fluctuations.
- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$.
- “Bloch’s law” for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.
- However, Bloch’s law ignores conservation of total momentum, or phonon drag.
- The field theory for the Ising-nematic critical point has strong electron scattering, and no quasi-particle excitations. Nevertheless, because of the central importance of the analog of phonon drag, it has $\rho(T) = 0$.

However, this ignores “phonon drag”

PHONON DRAG

Peierls\textsuperscript{28} pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$. This behavior has yet to be observed.

\textsuperscript{28} R. E. Peierls, Ann. Phys. (5) 12, 154 (1932).
Rates of Momentum Flow

Electrons

SLOW

Phonons
Rates of Momentum Flow

Electrons → Phonons
SLOW

Phonons → Defects
FAST
Electrons

SLOW

Process controlling resistivity

Defects

Phonons

Rates of Momentum Flow
**Rates of Momentum Flow**

- Electrons
- Nematic boson $\phi$
- FAST
Rates of Momentum Flow

Electrons

LESS
FAST

FAST

Nematic boson $\phi$
Rates of Momentum Flow

Electrons

LESS
FAST

FAST

Nematic
boson $\phi$

Defects

SLOW
Rates of Momentum Flow

Electrons

LESS FAST

FAST

Nematic boson $\phi$

SLOW

Defects

EVEN SLOWER

Defects
Rates of Momentum Flow

Electrons

LESS FAST

FAST

Nematic boson $\phi$

Defects

EVEN SLOWER

Defects

Process controlling resistivity
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[ S_\phi = \int d^2r d\tau \left[ \left( \partial_\tau \phi \right)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right] \]

\[ S_c = \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger \left( \partial_\tau + \varepsilon_k \right) c_{k\alpha} \]

\[ S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q \left( \cos k_x - \cos k_y \right) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha} \]
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[ S_{\phi} = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right] \]

\[ S_c = \sum_{\alpha=1}^{N_f} \int d^2r d\tau c_\alpha^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} + \frac{\nabla^4}{2m'} + \ldots - \mu \right) c_\alpha \]

\[ S_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[ c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha \right\} \right. \]

\[ \left. + \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha^\dagger \right\} c_\alpha \right] \]

This continuum theory has strong electron–\(\phi\) scattering, and no quasi-particle excitations. But it has a conserved momentum \(\mathbf{P}\), and \(\chi_{\mathbf{J},\mathbf{P}} \neq 0\) (“phonon drag”), and so the resistivity \(\rho(T) = 0\).
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

The most-probable state with a non-zero current \( J \) has a non-zero momentum \( P \) (and vice versa).

At non-zero density, \( J \) “drags” \( P \).

- Focus on the interplay between \( J_\mu \) and \( T_{\mu\nu} \)!
The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic.

The most-probable state with a non-zero current $J$ has a non-zero momentum $P$ (and vice versa). At non-zero density, $J$ “drags” $P$.
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:


- Focus on the interplay between $J_\mu$ and $T_{\mu\nu}$!

The dominant momentum loss occurs via the scattering of the neutral bosonic $\phi$ excitations off random fields. This is good news for the AdS/CMT approaches, which do not capture the Fermi surface of most of the charged carriers.
dFF-DW Unidirectional Domains

\[
\frac{|O_y(r, q=Q_x)| - |O_x(r, q=Q_y)|}{|O_y(r, q=Q_x)| + |O_x(r, q=Q_y)|} + 0.75 - 0.75
\]

Primary DW direction
- Orange: // (1,0)
- Blue: // (0,1)
Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[ S_{\text{dis}} = \int d^2r d\tau \left[ V(\mathbf{r}) \, c^\dagger c + h(\mathbf{r}) \phi \right], \]

\[ \frac{\overline{V(\mathbf{r})}}{2\pi} = 0 \quad ; \quad \frac{\overline{V(\mathbf{r})V(\mathbf{r}')}}{2\pi} = V_0^2 \, \delta(\mathbf{r} - \mathbf{r}'), \]

\[ \frac{\overline{h(\mathbf{r})}}{2\pi} = 0 \quad ; \quad \frac{\overline{h(\mathbf{r})h(\mathbf{r}')}}{2\pi} = h_0^2 \, \delta(\mathbf{r} - \mathbf{r}'), \]

we use the memory-function approach to obtain the resistivity for current along angle \( \vartheta \)

\[ \rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} \, k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \, \Pi_{c^\dagger c}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} \, D^R(\omega, \mathbf{k})}{\omega} \right). \]
Resistivity of strange metal

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\[ S_{\text{dis}} = \int d^2 r d\tau \left[ V(r) c^\dagger c + h(r) \phi \right] , \]

\[ V(r) = 0 \quad ; \quad V(r)V(r') = V_0^2 \delta(r - r') , \]

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**Fermi surface term:** Obtain \( T \)-dependent corrections to residual resistivity similar to earlier work


Resistivity of strange metal

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**Bosonic term: Dominant contribution:**

\[ \rho(T) \sim h_0^2 T^{(d-z+\eta)/z} \]

Crosses over from the “relativistic” form \((z = 1, \eta \approx 0)\) with \(\rho(T) \sim h_0^2 T\) at higher \(T\), to the “Landau-damped” form \((z = 3, \eta = 0)\) with \(\rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2}\) at lower \(T\) (subtle corrections to scaling specific to this field theory).

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[
S_{\text{dis}} = \int d^2 r d \tau \left[ V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi \right],
\]

\[
V(\mathbf{r}) = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2 \delta(\mathbf{r} - \mathbf{r}'),
\]

\[
h(\mathbf{r}) = 0 \quad ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2 \delta(\mathbf{r} - \mathbf{r}'),
\]

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\[
\rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi_{c\dagger c}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D_{\phi}^R(\omega, \mathbf{k})}{\omega} \right).
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\]

Crosses over from the “relativistic” form \((z = 1, \eta \approx 0)\) with \(\rho(T) \sim h_0^2 T\) at higher \(T\), to the “Landau-damped” form \((z = 3, \eta = 0)\) with \(\rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2}\) at lower \(T\) (subtle corrections to scaling specific to this field theory).

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[
S_{\text{dis}} = \int d^2 r d\tau \left[ V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi \right],
\]

\[
V(\mathbf{r}) = 0 \quad ; \quad V(\mathbf{r})V(\mathbf{r}') = V_0^2 \delta(\mathbf{r} - \mathbf{r}'), \quad \bar{V}(\mathbf{r}) = \bar{h}(\mathbf{r}) = 0 \quad ; \quad \bar{h}(\mathbf{r})\bar{h}(\mathbf{r}') = h_0^2 \delta(\mathbf{r} - \mathbf{r}'),
\]

we use the memory-function approach to obtain the resistivity for current along angle \( \vartheta \)

\[
\rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi_{c\dagger c}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D_\phi^R(\omega, \mathbf{k})}{\omega} \right).
\]

**Bosonic term: Dominant contribution:**

\[
\rho(T) \sim h_0^2 T^{(d-z+\eta)/z}
\]

Crosses over from the “relativistic” form \((z = 1, \eta \approx 0)\) with \(\rho(T) \sim h_0^2 T\) at higher \(T\), to the “Landau-damped” form \((z = 3, \eta = 0)\) with \(\rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2}\) at lower \(T\) (subtle corrections to scaling specific to this field theory).

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

Resistivity from random-field disorder
Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

\[ \rho(T) \sim h_0^2 T \] in region with ‘relativistic’ criticality of \( \phi \), with dynamic critical exponent \( z = 1 \).

Obtained by “memory function” and by holography.

Transport without quasiparticles:

\[ \rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2} \] in region with Landau-damped criticality of \( \phi \), with dynamic critical exponent \( z = 3 \).

\[ \rho(T) \sim h_0^2 T \] in region with ‘relativistic’ criticality of \( \phi \), with dynamic critical exponent \( z = 1 \).

Obtained by “memory function” and by holography.

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

\[ \rho(T) \sim h_0^2 (T \ln(1/T))^{-1/2} \] in region with Landau-damped criticality of $\phi$, with dynamic critical exponent $z = 3$.

\[ \rho(T) \sim h_0^2 T \] in region with ‘relativistic’ criticality of $\phi$, with dynamic critical exponent $z = 1$.

Obtained by “memory function” and by holography.

Conclusions

1. Antiferromagnetism and quantum criticality in insulators: triplons, spin-waves, and “Higgs” in TlCuCl$_3$

2. Onset of antiferromagnetism in metals, and $d$-wave superconductivity

3. Experimental evidence for $d$-form factor density wave order, linked to the pseudogap, in the cuprate superconductors

4. Non-Fermi liquid at the Ising-nematic quantum critical point in a two-dimensional metal, and its transport properties