Strange metals and black holes

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Ordinary metals:
quasiparticles

Strange metals:
no quasiparticles

Black holes
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
Quasiparticles are additive excitations:
The low-lying excitations of the many-body system can be identified as a set \( \{ n_\alpha \} \) of quasiparticles with energy \( e_\alpha \)

\[
E = \sum_\alpha n_\alpha e_\alpha + \sum_\alpha,\beta F_{\alpha\beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly(\( N \)) numbers.
What are quasiparticles?

- Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{eq} \sim \frac{\hbar U^2}{E_F (k_B T)^2}$$

as $$T \to 0$$,

where $$U$$ is the strength of interactions and $$E_F$$ is the Fermi energy.
What are quasiparticles?

- Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

\[ \tau_{\text{eq}} \sim \frac{\hbar U^2}{(k_B T)^2} \quad \text{as } T \to 0, \]

where \( U \) is the strength of interactions and \( E_F \) is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

\[ \rho = \frac{m^*}{ne^2 \tau} \sim T^2 \quad \text{with} \quad \tau \sim \tau_{\text{eq}} \]
What are quasiparticles?

- These times are much longer than the ‘Planckian time’ $\frac{\hbar}{k_B T}$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{eq} \gg \frac{\hbar}{k_B T}, \quad \text{as } T \rightarrow 0.$$
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
Twisted bilayer graphene

Cao et al., Nature 556, 80 (2018)
Twisted bilayer graphene

Cao et al., Nature 556, 80 (2018)
Twisted bilayer graphene

Cao et al., arXiv:1901.03710
High temperature superconductors

YBa$_2$Cu$_3$O$_{6+x}$
Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, $\rho$, is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}$$

independent of the strength of interactions!
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The SYK model

Place electrons randomly on some sites
Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites
Entangle electrons pairwise randomly

The SYK model
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This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij; k\ell} c_i^\dagger c_j^\dagger c_k c_\ell + e \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij; k\ell}$ are independent random variables with $\overline{U_{ij; k\ell}} = 0$ and $|U_{ij; k\ell}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

The SYK model

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\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;\ell}^{\dagger} c_i^\dagger c_j^\dagger c_k^\dagger c_\ell + e \sum_i c_i^\dagger c_i \]

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S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

The large $N$ limit is given by the sum of “melon” Feynman graphs

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The complex SYK model

There is a one-parameter family of critical solutions with varying $e/U$, yielding different $0 < Q < 1$.

For long (imaginary) times $\tau > 0$

$$\langle c_i(\tau)c_i^{\dagger}(0) \rangle \sim e^{-2\pi \varepsilon T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}$$

In a Fermi liquid, \(\langle c_i(\tau)c_i^{\dagger}(0) \rangle \sim \frac{T}{\sin(\pi T\tau)}\)

S. Sachdev and J.Ye, PRL 70, 3339 (1993)
A. Georges and O. Parcollet PRB 59, 5341 (1999)
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Characteristic Planckian time scale $\sim \hbar/(k_B T)$ for ‘dissipation’ of excitations.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB 96, 205123 (2017)

S. Sachdev and J.Ye, PRL 70, 3339 (1993)

A. Georges and O. Parcollet PRB 59, 5341 (1999)
The complex SYK model

There is a one-parameter family of critical solutions with varying $e/U$, yielding different $0 < Q < 1$.

For long (imaginary) times $\tau > 0$

$$\langle c_i(\tau)c_i^{\dagger}(0) \rangle \sim e^{-2\pi\mathcal{E}T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}$$

The exponential pre-factor determines the particle-hole asymmetry, and $\mathcal{E} = 0.41(e/U)$ from a numerical solution.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
A. Georges and O. Parcollet, PRB 59, 5341 (1999)
The SYK model

\[ \varepsilon = \mathbb{C} \frac{e}{U} \]

Planckian dynamics with peak width \( \sim k_B T / \hbar \)
and independent of \( U \)

A. Georges and O. Parcollet PRB 59, 5341 (1999)
S. Sachdev, PRX 5, 041025 (2015)
The SYK model

There are $2^N$ many body levels with energy $E$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi \mathcal{E}.$$ 

At $Q = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where $G$ is Catalan’s constant.

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$
The SYK model

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where $G$ is Catalan’s constant.
All electrons in the (flat band) SYK model have the same $e$

In a more realistic metal, the electrons have a dispersion $e_k$ ($k$ is momentum), and $e_k = 0$ is the Fermi surface.
Flat band metal

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of $U$

All electrons have the same $e$

$$\mathcal{E} = \frac{e}{U}$$

$A. \text{Georges and O. Parcollet PRB} \textbf{59}, 5341 (1999)$

$S. \text{Sachdev, PRX} \textbf{5}, 041025 (2015)$
Planckian metal ansatz with some dispersion

Electrons ‘remember’ their momentum, and have a SYK spectral function according to their $e_k$.

$$\mathcal{E}_k = \mathcal{C} \frac{e_k}{U}$$

$$\mathcal{E}_k = 0$$

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of $U$.

A. A. Patel and S. Sachdev, PRL 123, 066601 (2019)
Planckian metal ansatz with some dispersion

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$$\mathcal{E}_k = C \frac{e_k}{U}$$

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A. A. Patel and S. Sachdev, PRL 123, 066601 (2019)
Flat band metal

For a dispersionless SYK model

\[ \langle c_i(\tau)c_i^\dagger(0) \rangle \sim e^{-(e/U)2\pi C T\tau} \times \left( \frac{T/U}{\sin(\pi T\tau)} \right)^{1/2} \]

S. Sachdev and J. Ye,
PRL 70, 3339 (1993)

A. Georges and O. Parcollet
PRB 59, 5341 (1999)
Planckian metal ansatz with some dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion $e_k$ ($k$ is the momentum)

$$\left\langle c_k(\tau)c_k^\dagger(0)\right\rangle \sim e^{-(e_k/U)2\pi CT\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}$$

A. A. Patel and S. Sachdev, PRL 123, 066601 (2019)
Planckian metal ansatz with some dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion $e_k$ ($k$ is the momentum)

\[
\left\langle c_k(\tau)c_k^\dagger(0)\right\rangle \sim e^{-(e_k/U)2\pi\xi T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}
\]

At $e_k = 0$ we have a ‘remnant Fermi surface’ with a particle-hole symmetric spectral function.
Planckian metal ansatz with some dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion $e_k$ ($k$ is the momentum)

$$\left\langle c_k(\tau)c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U)2\pi CT\tau} \times \left( \frac{T/U}{\sin(\pi T\tau)} \right)^{1/2}$$

No free parameters—everything is determined by the (underlying) quasiparticle dispersion $e_k$, and the interaction strength $U$.

A.A. Patel and S. Sachdev, PRL 123, 066601 (2019)
Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{de}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[ \text{Im} G_{\text{SYK}}^R \left( e, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left( \frac{\omega}{2T} \right)$$

where the Fermi surface is defined by $e_k = 0$, $v_F = \nabla_k e_k$ on the Fermi surface, and

$$m^* = \frac{dV_{FS}}{\oint_{FS} |\mathbf{v}_F|},$$

with $d$ the spatial dimensionality, and $V_{FS}$ is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual $m^*$. 

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Evaluating the integrals, we find

$$\rho = \frac{m^*}{ne^2} \frac{2.71 \zeta}{2\pi^d} \frac{k_B T}{\hbar}, \quad \text{using } \mathcal{E} = \frac{\zeta e}{U},$$

where $n = V_{FS}/(2\pi)^d$ is the density.
Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2} 2.71 C \frac{k_B T}{\hbar}$$

Note that all explicit dependence on $U$ has cancelled out!

Choosing $C = 0.41$ as in the SYK model, we have the prefactor $2.71 C = 1.11$.  

Aavishkar Patel

A.A. Patel and S. Sachdev, PRL 123, 066601 (2019)
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
• The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds.

• The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$; so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!
Quantum Black holes

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- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$. 
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- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Holography:
Quantum black holes “look like” quantum many-particle systems without quasiparticle excitations, residing “on” the surface of the black hole

Work with a theory of Maxwell’s electromagnetism and Einstein’s general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge.
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Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space ($\zeta$) and one time dimension.
The near-horizon region of a charged black hole has the geometry of $(1+1)$-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model.
**SYK model and charged black holes**

Black hole horizon

\[
\text{AdS}_2 \times S^2
\]

\[
ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2
\]

Gauge field: \( A = (\mathcal{E}/\zeta)dt \)

\[\zeta = \infty\]

\[\vec{x}\]

\[S^2\]

Bekenstein-Hawking entropy of AdS\(_2\) horizon at \(T = 0\) \(\iff\) \(N s_0\) entropy of SYK model.

\[
\frac{ds_0}{dQ} = 2\pi \mathcal{E}
\]
can be obtained from the Einstein equations for the black hole, and the quantum theory of the SYK model, and \(\mathcal{E}\) determines identical fermion spectral functions.

Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a ‘Schwarzian’ theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS$_2$ and AdS$_4$. 

\[
\begin{align*}
\text{AdS}_2 \times S^2 \\
\text{Gauge field: } A &= \frac{\mathcal{E}}{\zeta} dt \\
\end{align*}
\]
Main result

A. Kitaev (2015)
J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)
J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
S. Sachdev, arXiv:1902.04078
Main result

SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$
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SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius $R_h$, with total charge $Q$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in 0 + 1 dimensions
Main result

The common low $T$ path integral is $Z = \int \mathcal{D}f \mathcal{D}\phi e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left( \frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \text{Sch}[^\tau, T^T \phi(\tau), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with $f(\tau + 1/T) = f(\tau) + 1/T$,

$\phi$ is a phase conjugate to the charge density with $\phi(\tau + 1/T) = \phi(\tau) + 2\pi n$, $n$ integer,

$\text{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

The couplings are related to the entropy $S(T, Q)$ and the chemical potential $\mu$ via

$$S(T \rightarrow 0, Q) = s_0 + \gamma T, \quad K = \left( \frac{dQ}{d\mu} \right)_{T \rightarrow 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{dQ}.$$
Main result

- Closely related to, but not the usual AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.
- Unlike the AdS/CFT correspondence, both sides of the duality are fully solvable. This has enabled numerous recent studies of black holes quantum information.
Quantum matter without quasiparticles

- Planckian dynamics (i.e. fastest possible local thermalization in a time $\frac{\hbar}{k_B T}$) is realized in the ‘solvable’ SYK models.
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- Planckian SYK ansatz yields resistivity $\rho \approx (m^*/(ne^2))(k_B T/\hbar)$; this ansatz is realized by a ‘resonant’ SYK model in momentum space.
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- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.
**Quantum matter without quasiparticles**

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- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS$_2$ horizons