# Understanding correlated electron systems by a classification of Mott insulators

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Talk online at
http://pantheon.yale.edu/~subir

# Strategy for analyzing correlated electron systems

(cuprate superconductors, heavy fermion compounds ……..)

Standard paradigms of solid state physics (Bloch theory of metals, Landau Fermi liquid theory, BCS theory of electron pairing near Fermi surfaces) are very poor starting points.

So…….
Start from the point where the break down on Bloch theory is complete---
the Mott insulator.

Classify ground states of Mott insulators using conventional and topological order parameters.

Correlated electron systems are described by phases and quantum phase transitions associated with order parameters of Mott insulator and the "orders" of Landau/BCS theory. Expansion away from quantum critical points allows description of states in which the order of Mott insulator is "fluctuating".

# Outline

**I. Order in Mott insulators**

*Magnetic order*

**A. Collinear spins**

B. Non-collinear spins

*Paramagnetic states*

A. Bond order and confined spinons

B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order

A global phase diagram and applications to the cuprates

III. Doping Mott insulators with non-collinear spins and topological order

(A) A small Fermi surface state.

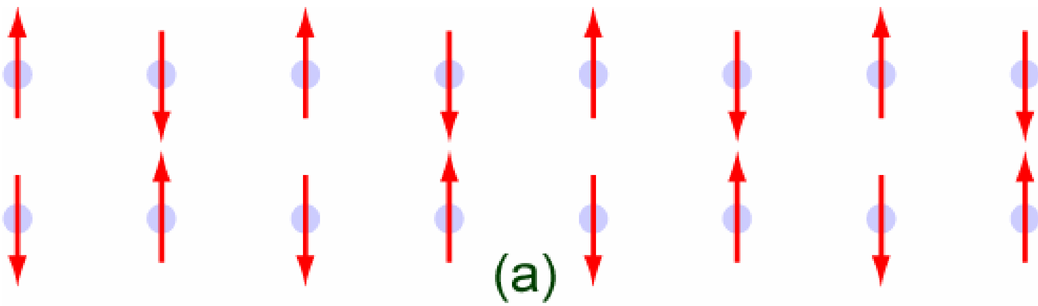
(B) Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments

IV. Conclusions

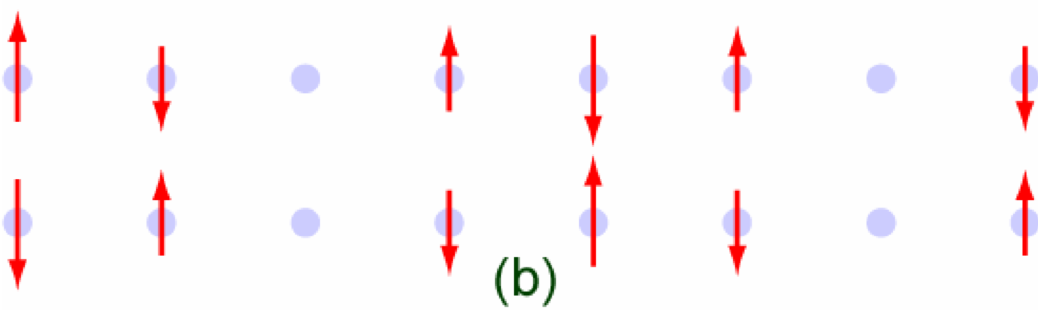
# I. Order in Mott insulators

*Magnetic order* $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

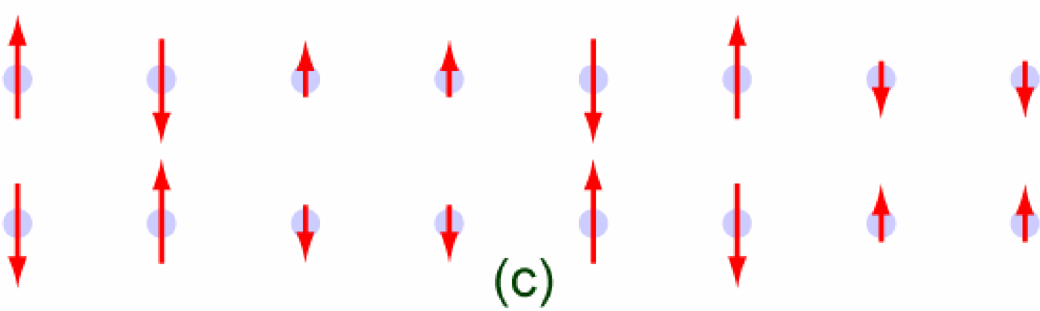
## A. Collinear spins




$$\vec{K} = (\pi, \pi) \; ; \; \boldsymbol{N}_2 = 0$$




$$\vec{K} = (3\pi/4, \pi) \; ; \; \boldsymbol{N}_2 = 0$$




$$\vec{K} = (3\pi/4, \pi) \; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2} - 1\right) \boldsymbol{N}_1$$

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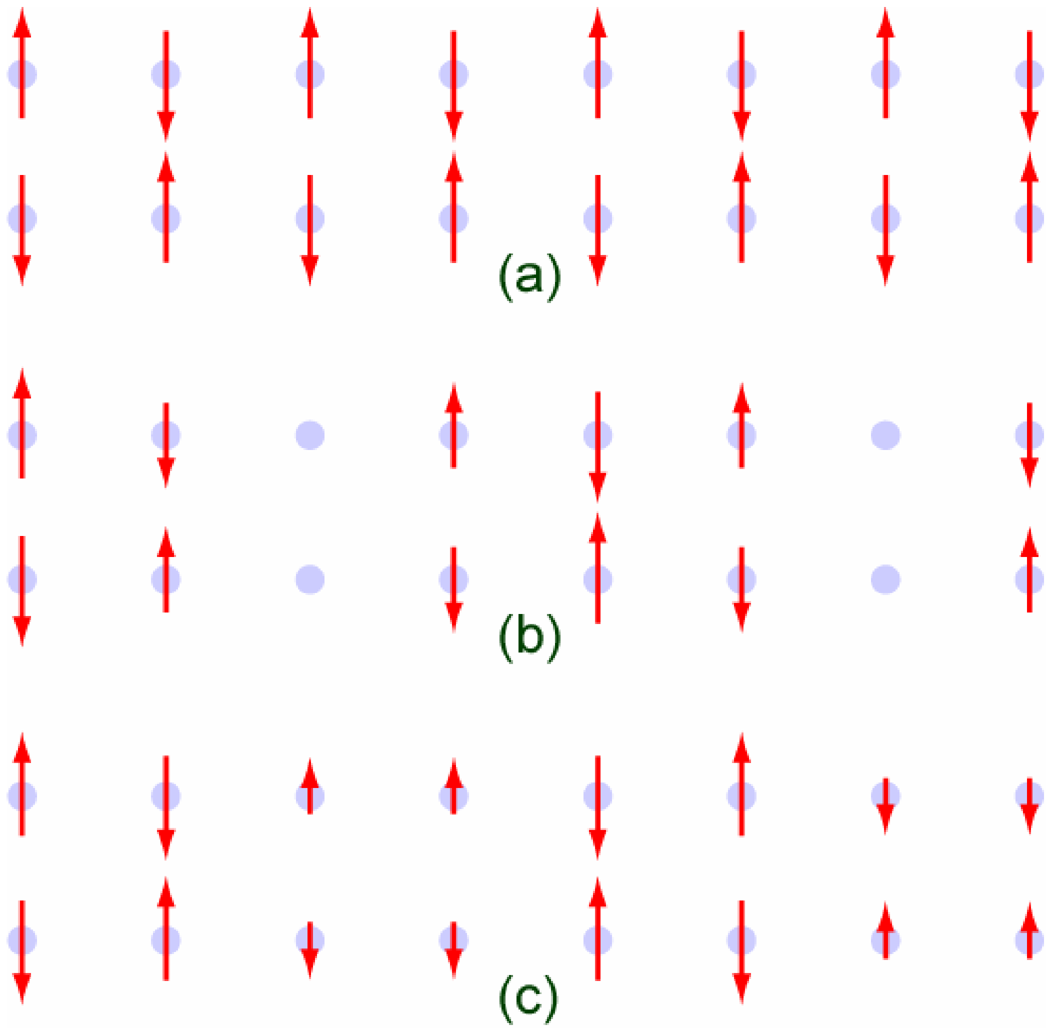
## A. Collinear spins




**Key property**

Order specified by a single vector $\boldsymbol{N}$.

Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector ($S$=1) quasiparticle excitation.

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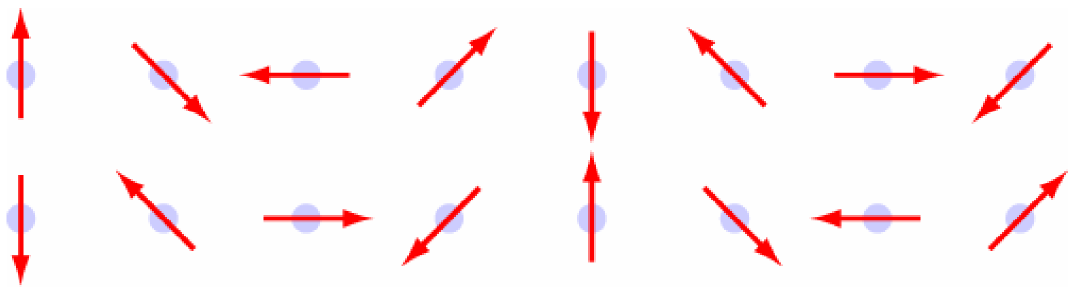
*Magnetic order* $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K}\,.\,\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}\,.\,\vec{r}_j\right)$

## B. Noncollinear spins

(B.I. Shraiman and E.D. Siggia, *Phys. Rev. Lett.* **61**, 467 (1988))



$\vec{K} = \left(3\pi/4, \pi\right)$ ;

$\boldsymbol{N}_2^2 = \boldsymbol{N}_1^2 \,,\; \boldsymbol{N}_1 \cdot \boldsymbol{N}_2 = 0$

Solve constraints by expressing $\boldsymbol{N}_{1,2}$ in terms of two complex numbers $z_\uparrow\,,\, z_\downarrow$

$$\boldsymbol{N}_1 + i\boldsymbol{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2 z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign).

This spinor could become a $S$=1/2 spinon in a quantum "disordered" state.

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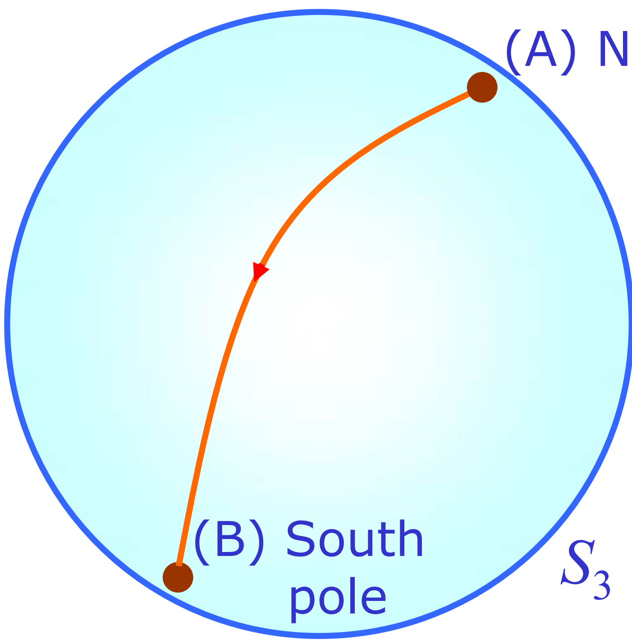
## B. Noncollinear spins

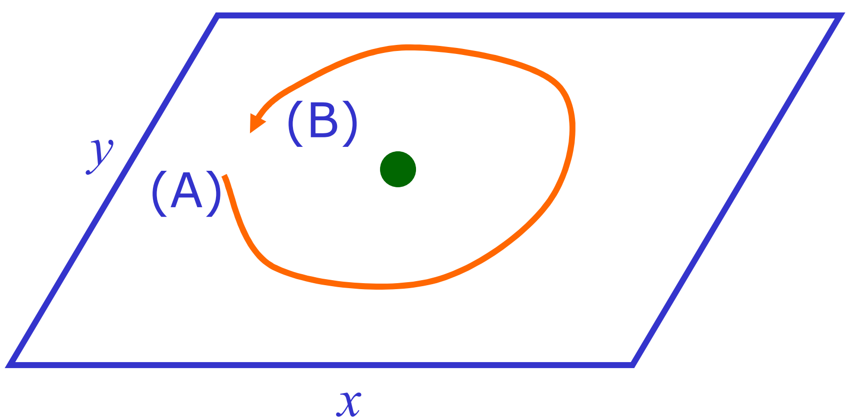
$$\boldsymbol{N}_1 + i\boldsymbol{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Order parameter space: $S_3/Z_2$

Physical observables are invariant under the $Z_2$ gauge transformation $z_a \to \pm z_a$

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$



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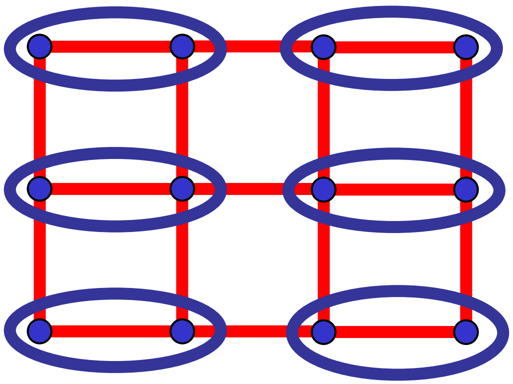
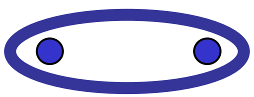
# I. Order in Mott insulators

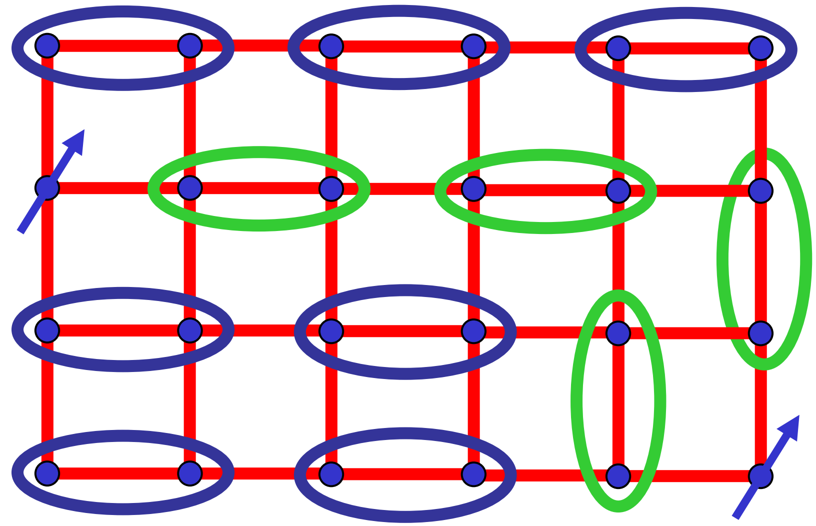
*Paramagnetic states* $\langle \mathbf{S}_j \rangle = 0$

## A. Bond order and spin excitons





$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$



*S*=1/2 spinons are ***confined*** by a linear potential into a *S*=1 spin exciton

Such a state is obtained by quantum-``disordering" collinear state with $\vec{K} = (\pi, \pi)$: fluctuating ***N*** becomes the *S*=1 spin exciton and Berry phases induce bond order

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

# Collinear spins, Berry phases, and bond-order

*S=1/2* square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left( \frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

***Small*** $g$ $\rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

---

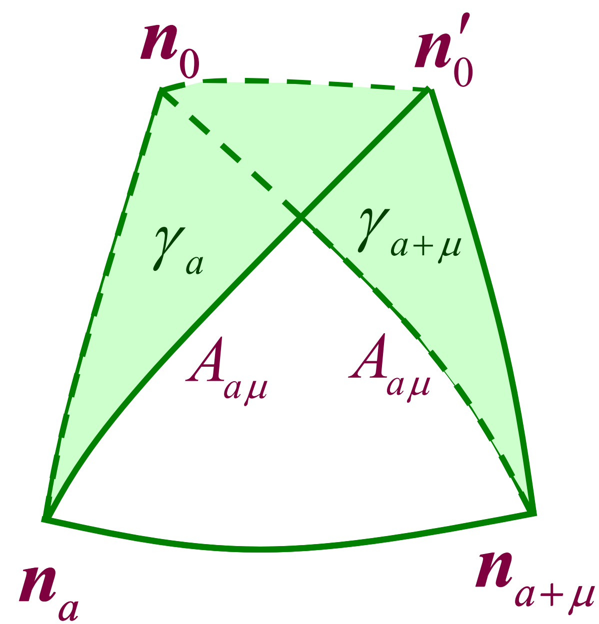
***Large*** $g$ $\rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \mathbf{n}_a \rangle = 0$

***Integrate out $\mathbf{n}_a$ to obtain effective action for $A_{a\mu}$***

Change in choice of $\mathbf{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

($\gamma_a$ is the oriented area of the spherical triangle formed by $\mathbf{n}_a$ and the two choices for $\mathbf{n}_0$).




The area of the triangle is uncertain modulo $4\pi$, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large $g$ effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left( -\frac{1}{2e^2} \sum_{\square} \cos\left( \frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 2+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is ***always*** in a *confining* phase:

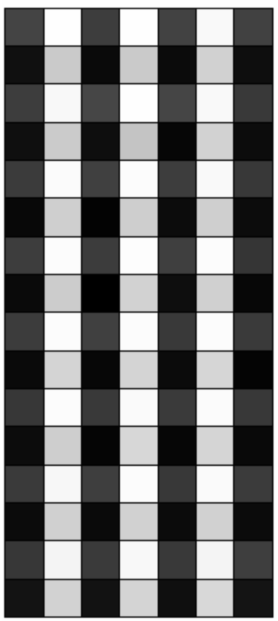
There is an energy gap and the ground state has a **bond order**.

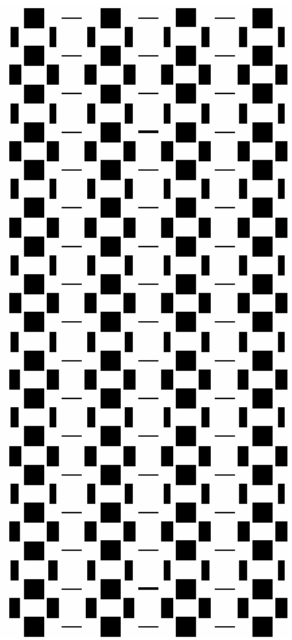
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

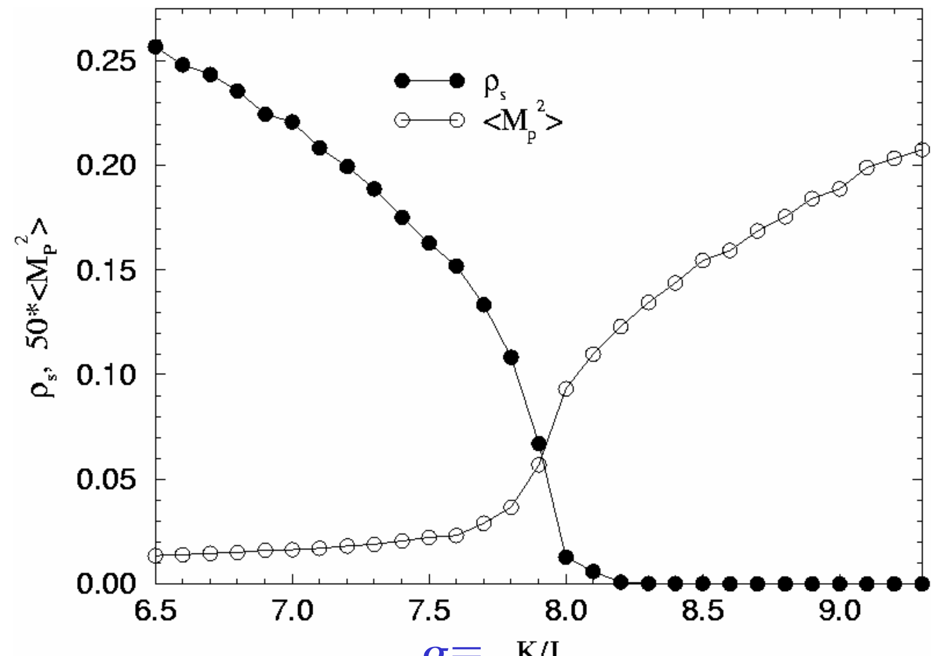
# Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First large scale numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry








$g=$

$$H = 2J \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle \subset \square} \left( S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

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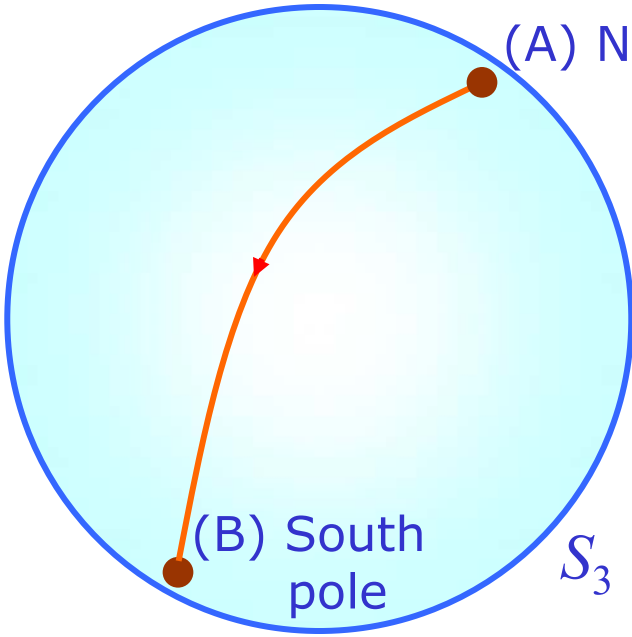
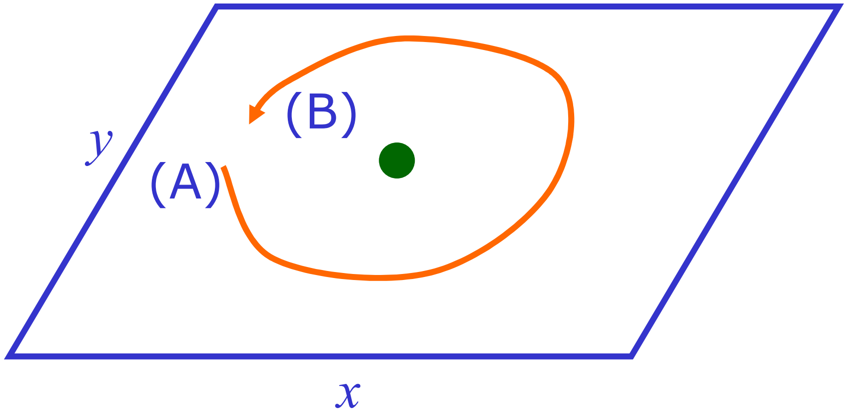
## B. Topological order and deconfined spinons

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (*visons*)




Such vortices (visons) can also be defined in the phase in which spins are "quantum disordered". A RVB state with deconfined spinons must have ***visons supressed***

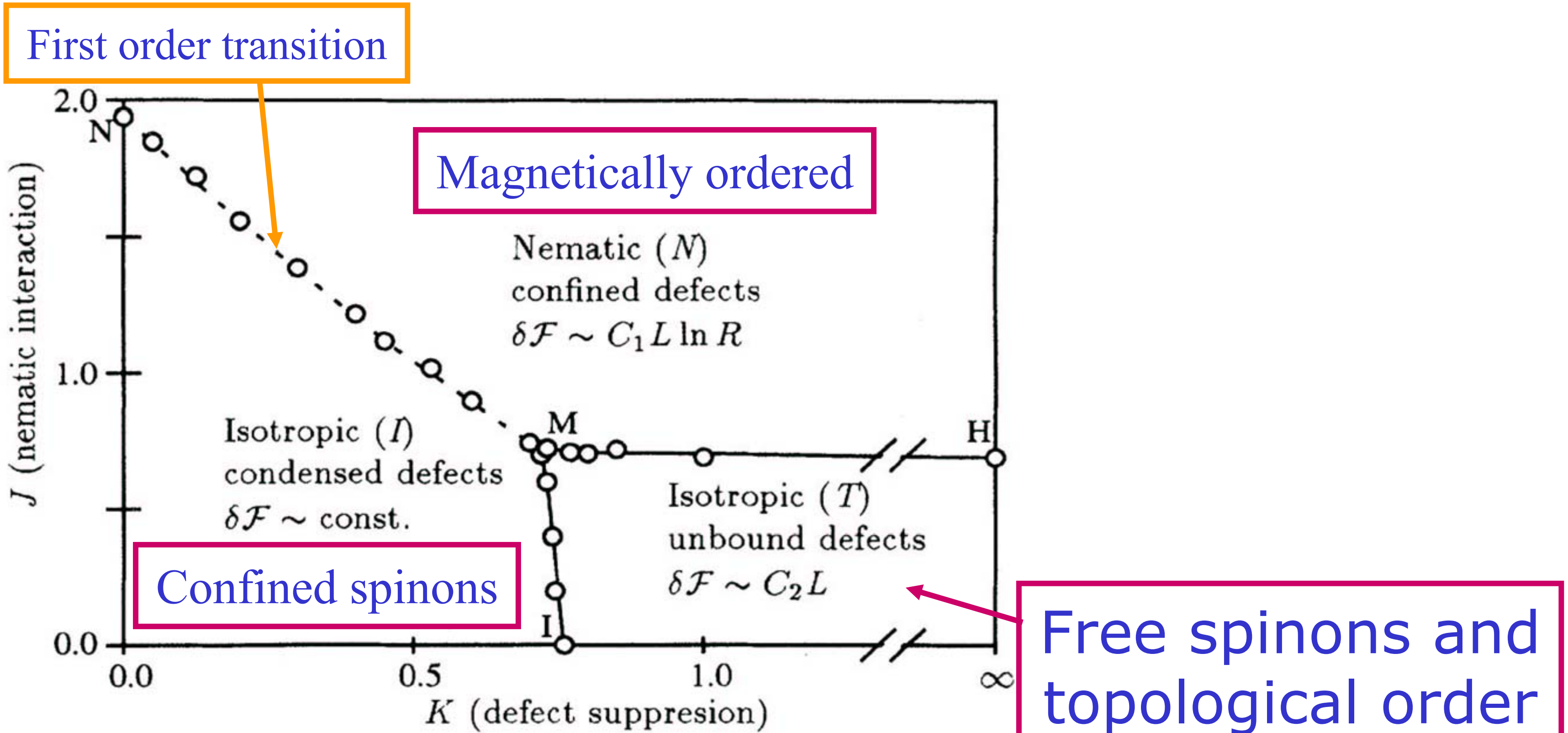
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

# Model effective action and phase diagram

$$S = -J\sum_{\langle ij\rangle}\sigma_{ij} z^*_{\alpha i} z_{\alpha j} + \text{h.c.} - K\sum_{\square}\prod_{\square}\sigma_{ij}$$

(Derivation using Schwinger bosons on a quantum antiferromagnet: S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991)).

$$\sigma_{ij} \to Z_2 \text{ gauge field}$$

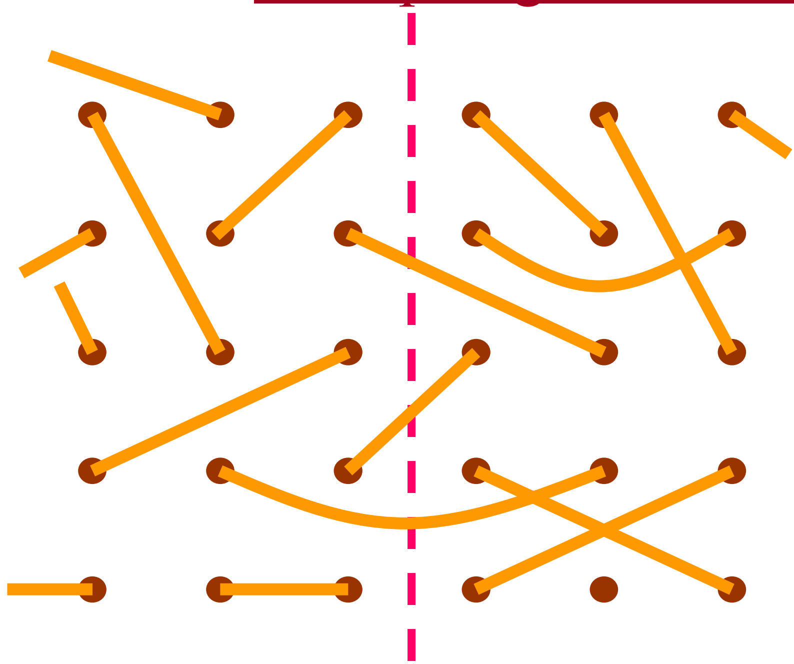



P. E. Lammert, D. S. Rokhsar, and J. Toner, *Phys. Rev. Lett*. **70**, 1650 (1993) ; *Phys. Rev.* E **52**, 1778 (1995). (For nematic liquid crystals)

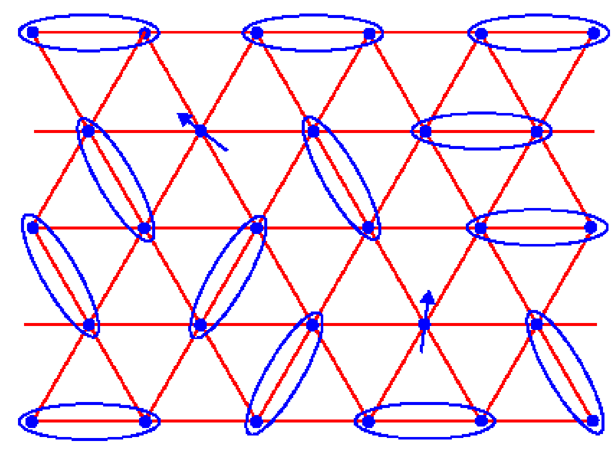
# I. Order in Mott insulators

*Paramagnetic states* $\left\langle \boldsymbol{S}_j \right\rangle = 0$

## B. Topological order and deconfined spinons



Number of valence bonds cutting line is conserved modulo 2 – this is described by the same $Z_2$ gauge theory as non-collinear spins



RVB state with free spinons

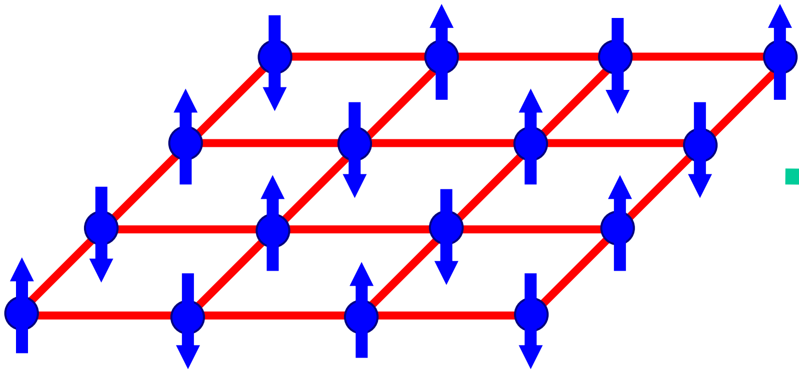
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

D.S. Rokhsar and S. Kivelson, *Phys. Rev. Lett*. **61**, 2376 (1988)
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
R. Jalabert and S. Sachdev, *Phys. Rev*. B **44**, 686 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).

# Orders of Mott insulators in two dimensions

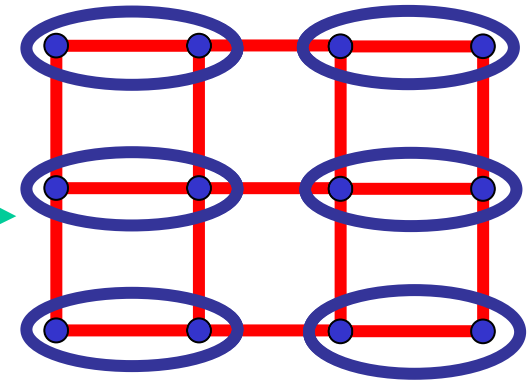
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991); S.S. and N.R. *Int. J. Mod. Phys.* B **5**, 219 (1991).

## A. Collinear spins, Berry phases, and bond order



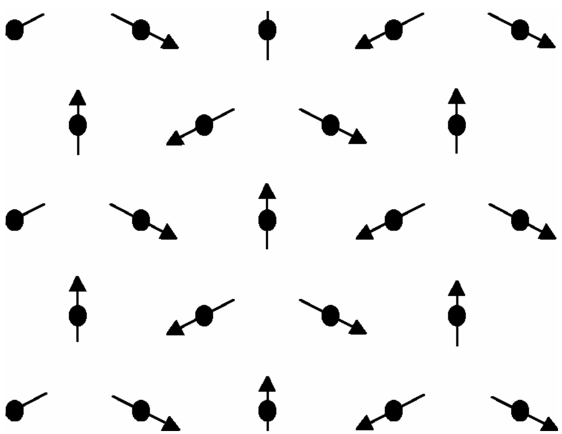
Néel ordered state

Quantum transition restoring spin rotation invariance



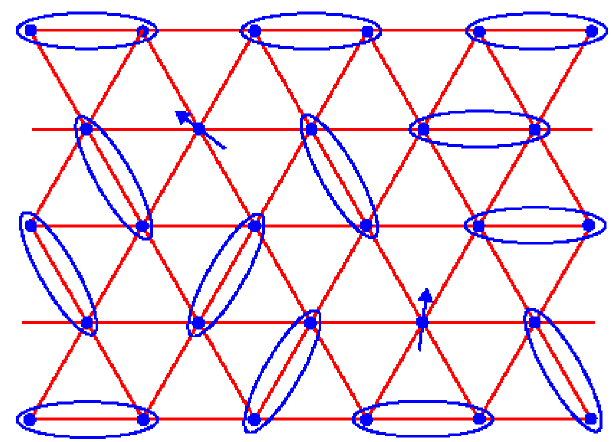
Bond order and $S=1$ spin exciton ***required***

## B. Non-collinear spins and deconfined spinons.



Non-collinear ordered antiferromagnet

Quantum transition restoring spin rotation invariance



Topological order ***may*** be present: RVB state with $Z_2$ gauge visons, $S=1/2$ spinons

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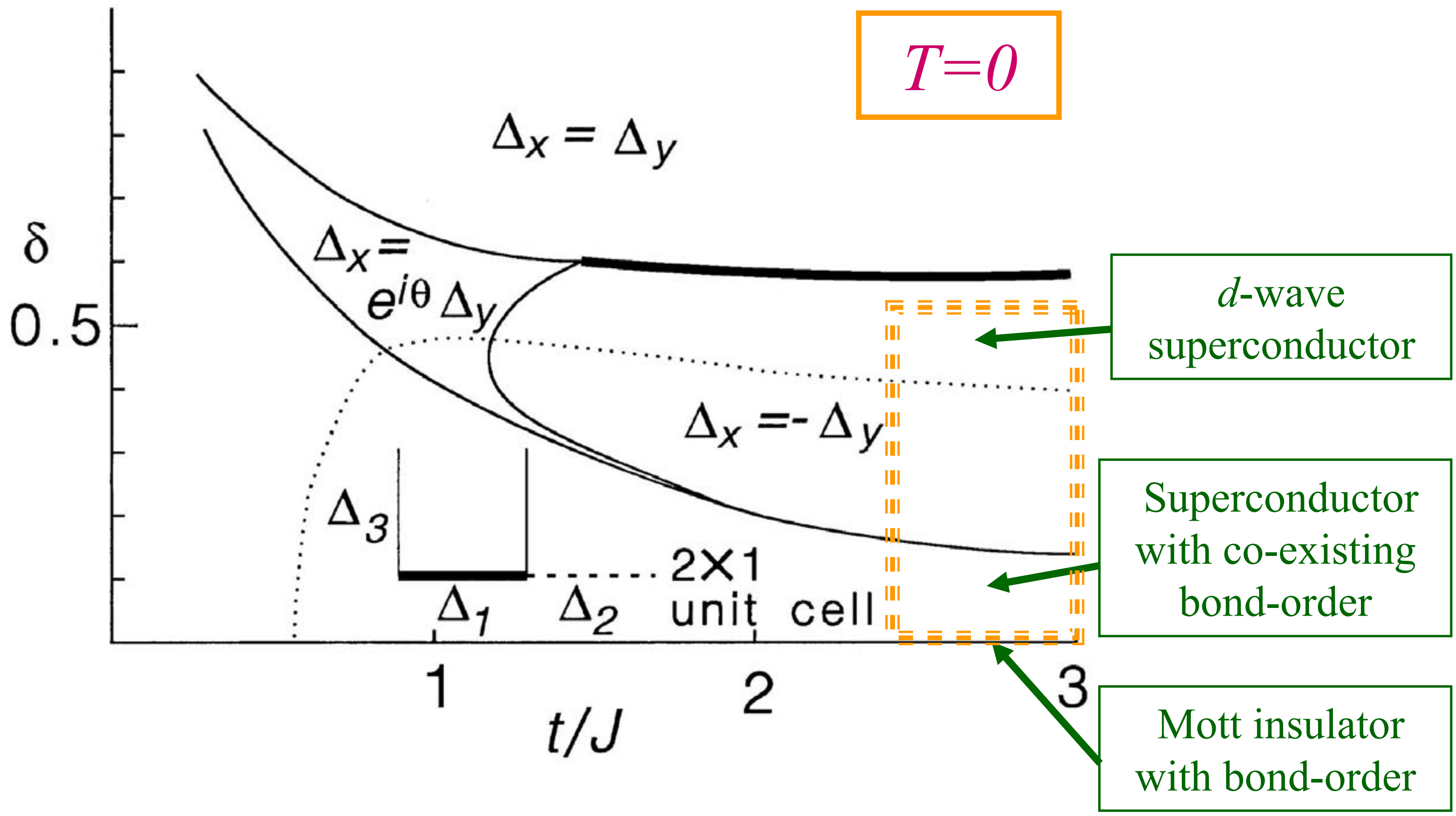
# II. Doping Mott insulators with collinear spins and bons order

## Doping a paramagnetic bond-ordered Mott insulator

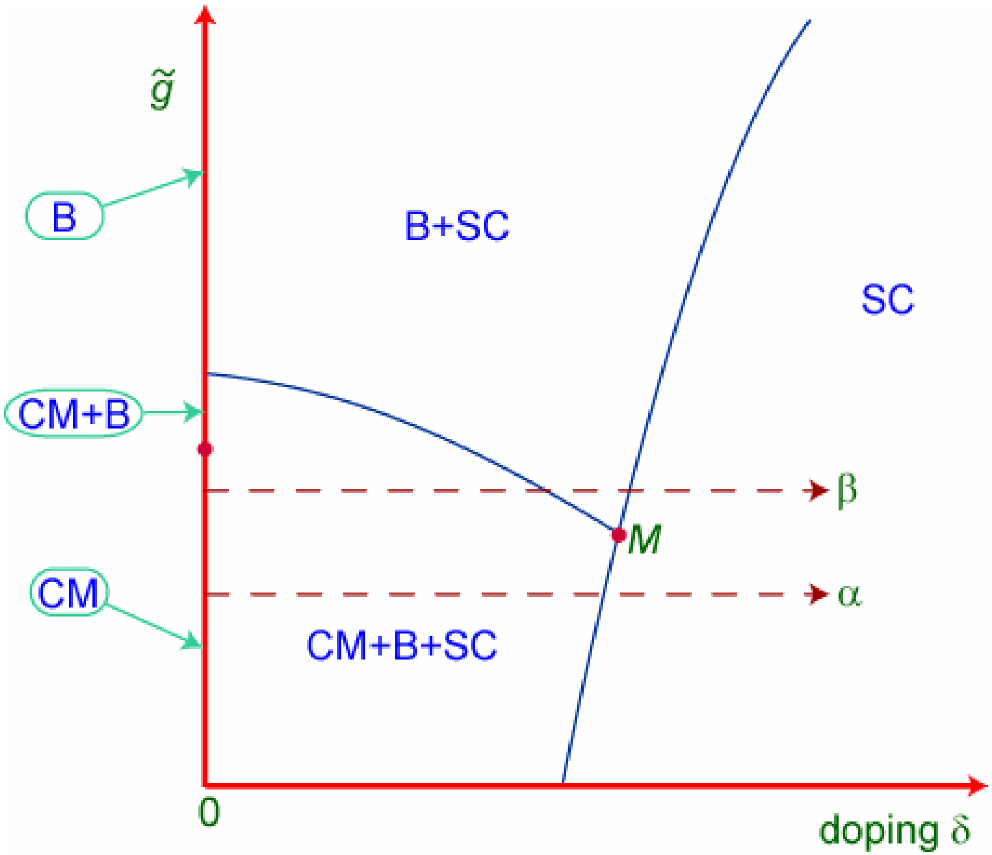
systematic Sp($N$) theory of translational symmetry breaking, while preserving spin rotation invariance.

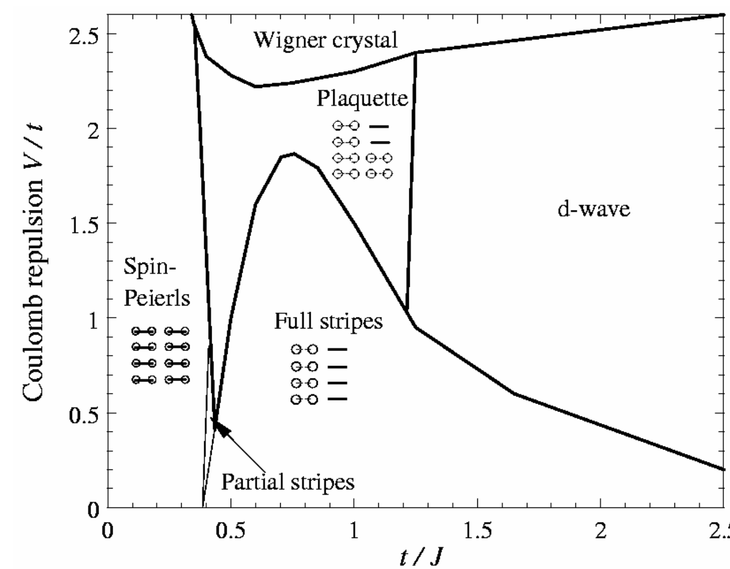



S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

# A phase diagram

Vertical axis is any microscopic parameter which suppresses CM order




Microscopic theory for the interplay of bond (B) and *d*-wave superconducting (SC) order

- **Pairing order of BCS theory (SC)**
- **Collinear magnetic order (CM)**
- **Bond order (B)**

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000); M. Vojta, Phys. Rev. B **66**, 104505 (2002).

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(B) Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments.

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## III. Doping topologically ordered Mott insulators (RVB)

### A likely possibility:

Added electrons do *not* fractionalize, but retain their bare quantum numbers.
Spinons and vison states of the insulator survive unscathed.
There is a Fermi surface of *sharp electron-like* quasiparticles, enclosing a volume determined by the dopant electron alone.

This is a "Fermi liquid" state which violates Luttinger's theorem

A Fractionalized Fermi Liquid

T. Senthil, S. Sachdev, and M. Vojta, cond-mat/0209144

# Luttinger's theorem on a $d$-dimensional lattice

For simplicity, we consider systems with SU(2) spin rotation invariance, which is preserved in the ground state.

Let $v_0$ be the volume of the unit cell of the ground state,
$n_T$ be the total number density of electrons per volume $v_0$.
(need not be an integer)

Then, in a metallic Fermi liquid state with a sharp electron-like Fermi surface:

$$2 \times \frac{v_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \left( \text{mod } 2 \right)$$

A Fermi liquid

# <u>Our claim</u>

There exist "topologically ordered" ground states in dimensions $d > 1$ with a Fermi surface of **<u>sharp</u>** electron-like quasiparticles for which

$$2 \times \frac{v_0}{(2\pi)^d}\left(\text{Volume enclosed by Fermi surface}\right)$$

$$= \left(n_T - 1\right)\left(\text{mod } 2\right)$$

A Fractionalized Fermi Liquid

Kondo lattice model:

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left( J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Consider, first the case $J_K$=0 and $J_H$ chosen so that the $f$ spins form a *topologically ordered* paramagnet

This system has a Fermi surface of conduction electrons with volume $n_c$ (mod 2)

Now $n_f$=1 (per unit cell of ground state)

$$n_T = n_f + n_c \neq n_c \left(\text{mod } 2\right)$$

This state, and its Fermi volume, survive for a finite range of $J_K$

Perturbation theory is $J_K$ is free of infrared divergences, and the topological ground state degeneracy is protected.

A fractionalized Fermi liquid which violates the Luttinger theorem

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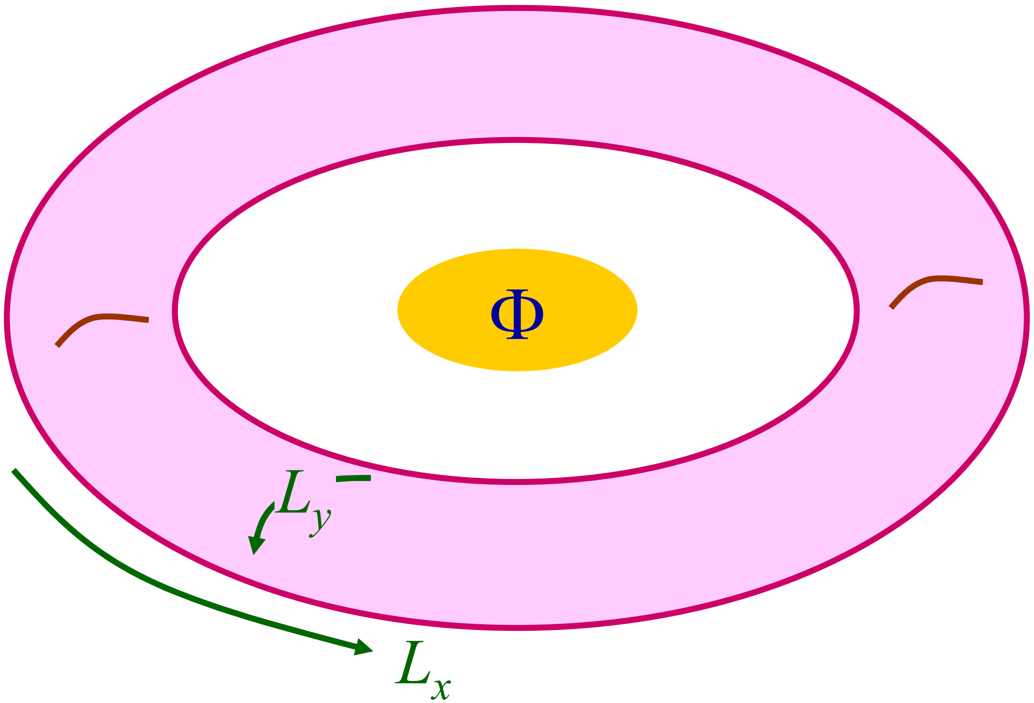
# III.B Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments




Unit cell $a_x$, $a_y$.
$L_x/a_x$, $L_y/a_y$
coprime integers

Adiabatically insert flux $\Phi=2\pi$ (units $\hbar$=c=e=1) acting on $\uparrow$ electrons.

State changes from $|\Psi\rangle$ to $|\Psi'\rangle$, and $UH(0)U^{-1} = H(\Phi)$, where

$$U = \exp\left[\frac{2\pi i}{L_x}\sum_{\boldsymbol{r}} x\,\hat{n}_{\boldsymbol{r}\uparrow}\right].$$

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000).

Adiabatic process commutes with the translation operator $T_x$, so momentum $P_x$ is conserved.

$$\text{However } U^{-1}T_x U = T_x \exp\left[\frac{2\pi i}{L_x}\sum_r \hat{n}_{Tr\uparrow}\right];$$

so shift in momentum $\Delta P_x$ between states $U|\Psi'\rangle$ and $|\Psi\rangle$ is

$$\Delta P_x = \frac{\pi L_y}{v_0} n_T \left(\mathrm{mod}\,\frac{2\pi}{a_x}\right) \qquad (1).$$

Alternatively, we can compute $\Delta P_x$ by assuming it is absorbed by quasiparticles of a Fermi liquid. Each quasiparticle has its momentum shifted by $2\pi/L_x$, and so

$$\Delta P_x = \frac{2\pi}{L_x}\frac{(\text{Volume enclosed by Fermi surface})}{(2\pi)^2/(L_x L_y)}\left(\mathrm{mod}\,\frac{2\pi}{a_x}\right) \qquad (2).$$

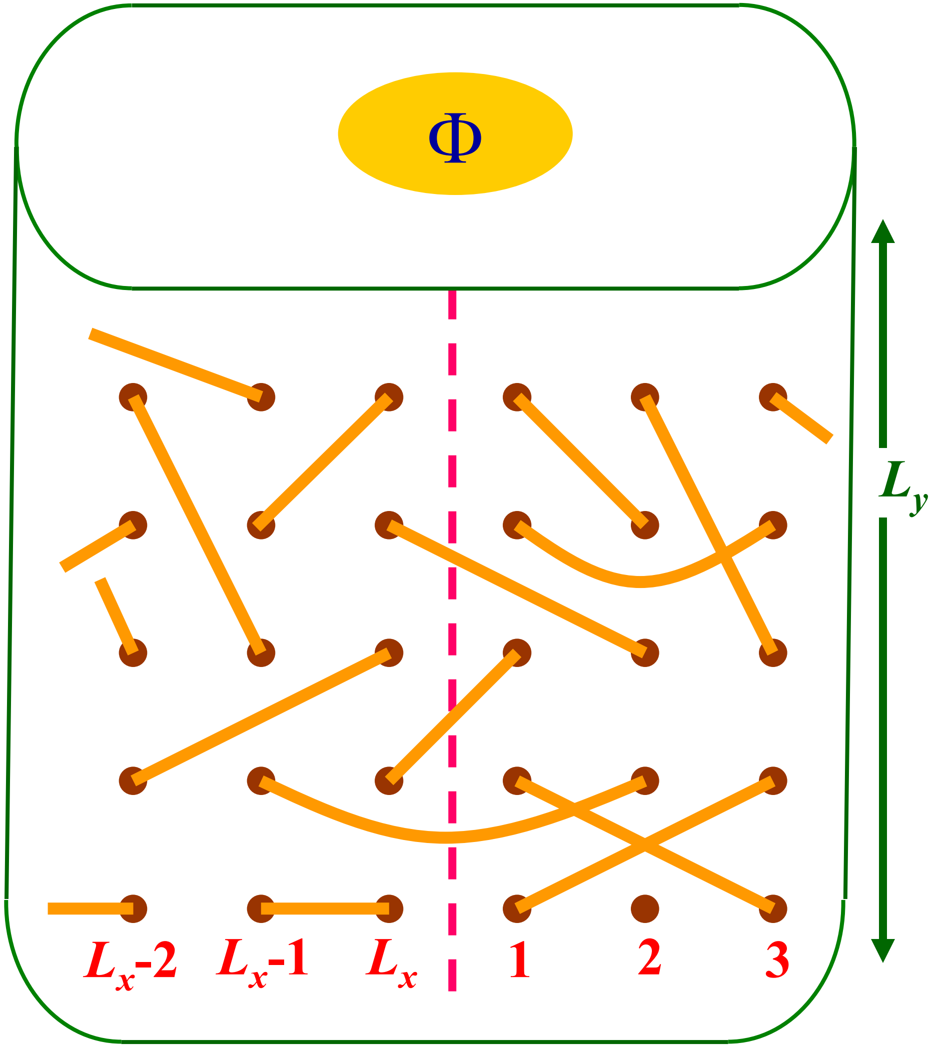
From (1) and (2), same argument in $y$ direction, using coprime $L_x/a_x, L_y/a_y$:

$$2\times\frac{v_0}{(2\pi)^2}(\text{Volume enclosed by Fermi surface}) = n_T \,(\mathrm{mod}\ 2)$$

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000).

# Effect of flux-piercing on a topologically ordered quantum paramagnet

$|D\rangle =$




N. E. Bonesteel, *Phys. Rev.* B **40**, 8954 (1989).
G. Misguich, C. Lhuillier, M. Mambrini, and P. Sindzingre, *Eur. Phys. J.* B **26**, 167 (2002).

$$|\Psi\rangle = \sum_D a_D |D\rangle$$

# Effect of flux-piercing on a topologically ordered quantum paramagnet

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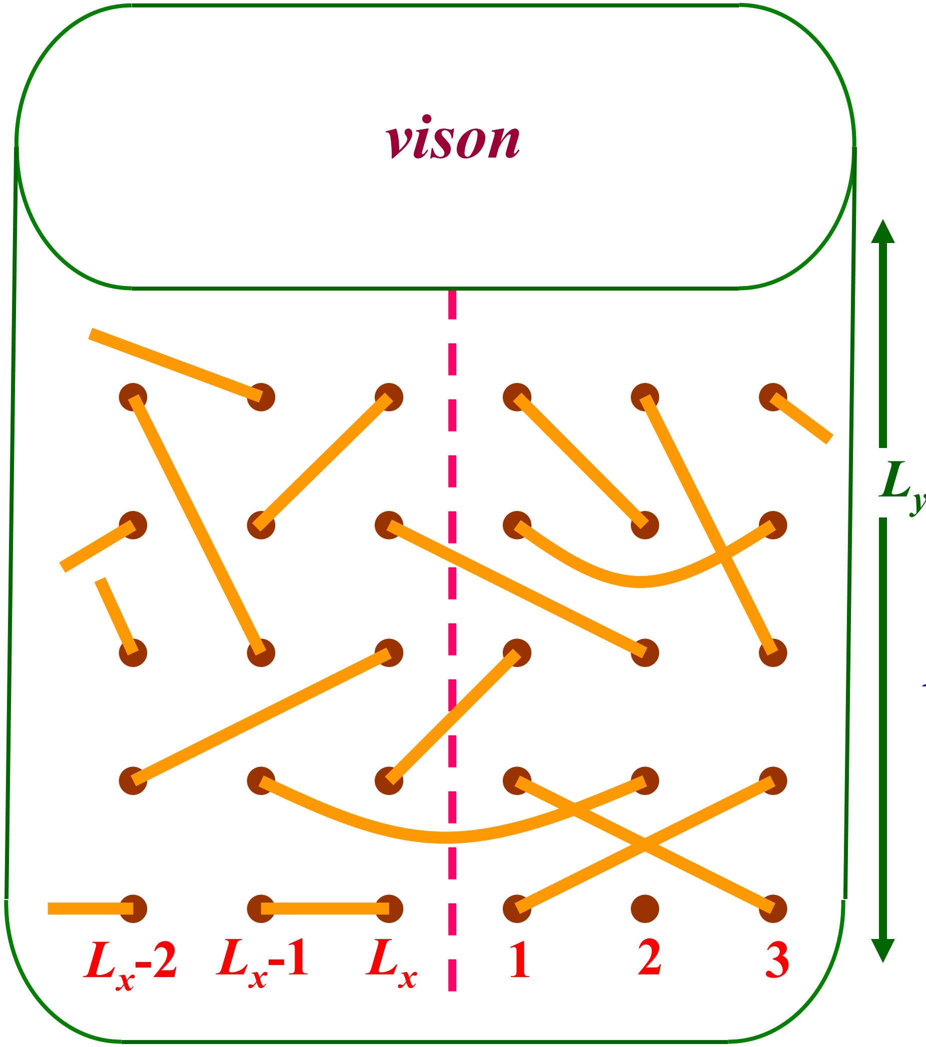


$$|\Psi\rangle = \sum_D a_D |D\rangle$$

After flux insertion $|D\rangle \Rightarrow$

$$(-1)^{\substack{\text{Number of bonds} \\ \text{cutting dashed line}}} |D\rangle;$$

Equivalent to inserting a ***vison*** inside hole of the torus.

***Vison*** carries momentum $\pi L_y / v_0$

# Flux piercing argument in Kondo lattice

Shift in momentum is carried by $n_T$ electrons, where

$$n_T = n_f + n_c$$

In topologically ordered, state, momentum associated with $n_f$=1 electron is absorbed by creation of vison. The remaining momentum is absorbed by Fermi surface quasiparticles, which enclose a volume associated with $n_c$ electrons.

A Fractionalized Fermi liquid.

cond-mat/0209144

# Conclusions

I. Two classes of Mott insulators:

(A) Collinear spins, bond order, confinements of spinons.

(B) Non-collinear spins, topological order, free spinons

II. Doping Class (A)

Magnetic/bond order co-exist with superconductivity at low doping

Cuprates most likely in this class.

Theory of quantum phase transitions provides a description of "fluctuating order" in the superconductor.

III. Doping Class (B)

New "Fractionalized Fermi liquid" state.