Quantum Criticality and Black Holes

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Three foci of modern physics

Quantum phase transitions
Three foci of modern physics

Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)
Three foci of modern physics

Quantum phase transitions

Black holes
Three foci of modern physics

Quantum phase transitions

Black holes

Bekenstein and Hawking originated the quantum theory, which has found fruition in string theory.
Three foci of modern physics

- Quantum phase transitions
- Hydrodynamics
- Black holes
Three foci of modern physics

Quantum phase transitions

Hydrodynamics
Universal description of fluids based upon conservation laws and positivity of entropy production

Black holes
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Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

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Hydrodynamics

Black holes
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \[ \vec{\varphi} = \eta_i \vec{S}_i \]

\[ \eta_i = \pm 1 \] on two sublattices

\[ \langle \vec{\varphi} \rangle \neq 0 \] in Néel state.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Quantum critical point with non-local entanglement in spin wavefunction

The $O(3)$ order parameter $\bar{\varphi}$ is given by:

$$S = \int d^2r d\tau \left[ (\partial_\tau \varphi)^2 + c^2(\nabla_r \bar{\varphi})^2 + s \bar{\varphi}^2 + u (\bar{\varphi}^2)^2 \right]$$

And it is related to the transformation:

$$\xi = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
Pressure in TlCuCl$_3$
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$ “triplon”
Excitation spectrum in the paramagnetic phase

$\vec{\phi}$

Spin $S = 1$

“triplon”

$V(\vec{\phi})$

$\lambda_c$

$\lambda$
Excitation spectrum in the paramagnetic phase

Spin $S = 1$ “triplon”
Excitation spectrum in the Néel phase

Spin waves ("Goldstone" modes) and a longitudinal "Higgs" particle
Observation of $3 \rightarrow 2$ low energy modes, emergence of new Higgs particle in the Néel phase, and vanishing of Néel temperature at the quantum critical point

**Prediction of quantum field theory**

\[
\frac{\text{Energy of “Higgs” particle}}{\text{Energy of triplon}} = \sqrt{2}
\]

![Graph showing the relationship between energy and pressure for TlCuCl₃](image)

TlCuCl₃

\(p_c = 1.07 \text{ kbar}\)

\(T = 1.85 \text{ K}\)

\[Q=(0 4 0)\]

\[E(p < p_c)\]

\[E(p > p_c)\]

\[\sqrt{2} \times E(p < p_c), E(p > p_c) \text{ [meV]}\]

Half-filled band $\rightarrow$ Mott insulator with spin $S = 1/2$

Triangular lattice of $[\text{Pd(dmit)}_2]_2$
$\rightarrow$ frustrated quantum spin system

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots \]

\( \vec{S}_i \Rightarrow \) spin operator with \( S = 1/2 \)
Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry

Neel ground state for small $J'/J$
Anisotropic triangular lattice antiferromagnet

Possible ground state for intermediate $J'/J$

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry

\[ \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \]

Valence bond solid (VBS)

Possible ground state for intermediate \( J'/J \)

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry

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\text{Possible ground state for intermediate } J'/J
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Possible ground state for intermediate $J'/J$

Magnetic Criticality

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Observation of a valence bond solid (VBS) in ETMe$_3$P[Pd(dmit)$_2$]$_2$

X-ray scattering

Spin gap $\sim$ 40 K
$J \sim$ 250 K

Magnetic Criticality

Theoretical global phase diagram

Valence bond solid (VBS)

Z\textsubscript{2} spin liquid

Neel antiferromagnet

Spiral antiferromagnet

Cenke Xu and S. Sachdev, arXiv:0811.1220
Theoretical global phase diagram

Valence bond solid (VBS)

$Z_2$ spin liquid

Neel antiferromagnet

Spiral antiferromagnet

CFTs


Cenke Xu and S. Sachdev, arXiv:0811.1220
Theoretical global phase diagram

CFTs described by a doubled U(1) Chern Simons theory of spinons $z_\alpha$ and visons $v_a$. (Structure similar to the supersymmetric ABJM theory of the M2 brane)

$$\mathcal{L} = \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - ia_\mu)z_\alpha|^2 + s_z|z_\alpha|^2 \right\}$$

$$+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - ib_\mu)v_a|^2 + s_v|v_a|^2 \right\}$$

$$+ \frac{i}{\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots$$

Cenke Xu and S. Sachdev, arXiv:0811.1220
Magnetic Criticality

$T_N (K)$

$\sqrt{J'/J}$

Magnetic Criticality

Quantum criticality described by CFT

Neel order

Spin gap

VBS order

$X\text{[Pd(dmit)$_2$]}_2$

$\text{Et}_2\text{Me}_2\text{Sb (CO)}$

$\sqrt{J'/J}$

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Black holes
Superfluid-insulator transition

Indium Oxide films

Superfluid-insulator transition

Ultracold $^{87}$Rb atoms - bosons

Superfluid

Insulator

0 \rightarrow g_c \rightarrow g
Graph showing a phase transition between a Superfluid phase and an Insulator phase, with a Quantum critical region defined by the $T_{KT}$ and $g_c$ axes.
Classical vortices and wave oscillations of the condensate

Dilute Boltzmann/Landau gas of particle and holes

Quantum critical

Superfluid

Insulator

$T_{KT}$

$T_c$
CFT at $T>0$

Quantum critical

Superfluid

Insulator

$T_{KT}$

$g_c$
Quantum critical transport

Quantum “perfect fluid” with shortest possible relaxation time, $\tau_R$

\[ \tau_R \gtrsim \frac{\hbar}{k_B T} \]

Quantum critical transport

Transport co-efficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

\[ \sigma = \frac{e^2}{\hbar} \times [\text{Universal constant } O(1)] \]

Quantum critical transport

Transport co-efficients not determined by collision rate, but by universal constants of nature

Momentum transport

\[ \frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}} = \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)] \]
Superfluid-insulator transition

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Quantum phase transitions → Hydrodynamics

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Black holes
Objects so massive that light is gravitationally bound to them.
Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

Horizon radius $R = \frac{2GM}{c^2}$
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Entropy of a black hole $S = \frac{k_B A}{4\ell_P^2}$

where $A$ is the area of the horizon, and $\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$
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Horizon temperature: $4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
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**AdS/CFT correspondence**

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

- **3+1 dimensional AdS space**
- **Quantum criticality in 2+1 dimensions**
- **Black hole entropy = entropy of quantum criticality**

Strominger, Vafa
AdS/CFT correspondence

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- **3+1 dimensional AdS space**
- **Quantum criticality in 2+1 dimensions**
- **Friction of quantum criticality = waves falling into black hole**

Kovtun, Policastro, Son
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1. Quantum phase transitions

2. Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

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New insights and results from detour unifies disparate fields of physics
Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics
   
   Uses physical model but strong-coupling makes explicit solution difficult
Three foci of modern physics

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Canonical problem in condensed matter: transport properties of a correlated electron system

New insights and results from detour unifies disparate fields of physics.
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2. Canonical problem in condensed matter: transport properties of a correlated electron system
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Black holes
Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics
   *Uses physical model but strong-coupling makes explicit solution difficult*

2. Solve Einstein-Maxwell equations in the background of a black hole in AdS space
   *Yields hydrodynamic relations which apply to general classes of quantum critical systems. First exact numerical results for transport co-efficients (for supersymmetric systems).*
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Find perfect agreement between 1. and 2. In some cases, results were obtained by 2. earlier !!
Applications:

1. Magneto-thermo-electric transport near the superconductor-insulator transition and in graphene
   *Hydrodynamic cyclotron resonance*
   *Nernst effect*

2. Quark-gluon plasma
   *Low viscosity fluid*

3. Fermi gas at unitarity
   *Non-relativistic AdS/CFT*
Applications:

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The cuprate superconductors
The cuprate superconductors

Proximity to an insulator at 12.5% hole concentration
Cuprates

Superconductor

Insulator $x = 1/8$
STM observations of VBS modulations by Y. Kohsaka et al., Nature 454, 1072 (2008)

Insulator $x=1/8$
Cuprates

Superconductor

Insulator $x=1/8$

CFT?
Cuprates

Thermoelectric measurements

Superconductor

Insulator $x = 1/8$

CFT?
Cuprates

Thermoelectric measurements

$g$

$T$

Superconductor

Insulator $x=1/8$

CFT?
Hydrodynamic cyclotron resonance at a frequency

\[ \omega_c = \frac{e^* B \rho v^2}{c (\varepsilon + P)} \]

and with width

\[ \gamma = \sigma_Q \frac{B^2 v^2}{c^2 (\varepsilon + P)} \]

where \( B = \) magnetic field, \( \rho = \) charge density away from density of CFT, \( \varepsilon = \) energy density, \( P = \) pressure, \( v = \) velocity of “light” in CFT, and \( \sigma_Q e^2 / h \) is the universal conductivity of the CFT.

“Wiedemann-Franz”-like relation for thermal conductivity, \( \kappa \) at \( B = 0 \)

\[
\kappa = \sigma Q \left( \frac{k_B^2 T}{e^{\ast 2}} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2.
\]

At \( B \neq 0 \) and \( \rho = 0 \) we have a “Wiedemann-Franz” relation for “vortices”

\[
\kappa = \frac{1}{\sigma Q} k_B^2 T \left( \frac{\nu(\varepsilon + P)}{k_B T B} \right)^2.
\]

Nernst experiment
Nernst signal (transverse thermoelectric response)

\[ e_N = \left( \frac{k_B}{e^*} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right) \left[ \frac{\omega_c/\tau_{\text{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \]

where \( \tau_{\text{imp}} \) is the momentum relaxation time due to impurities or umklapp scattering.

LSCO Experiments

$B$ and $T$ dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, $\tau_{\text{imp}}$ and $v$. 

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Similar results apply to electronic transport in graphene, where the relativistic Dirac spectrum of the electrons leads to analogies with the hydrodynamics of CFTs. We have made specific quantitative predictions for THz experiments on graphene at room temperature in the presence of a modest applied magnetic field.
Applications:

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Au+Au collisions at RHIC

Quark-gluon plasma can be described as “quantum critical QCD”
Phases of nuclear matter
S=1/2 Fermi gas at a Feshbach resonance
detuning from Feshbach resonance
detuning from Feshbach resonance

RG fixed point described by a “non-relativistic” CFT

= detuning from Feshbach resonance
CFT is dual to quantum gravity models on AdS space. Explicit solutions of such gravity models with supersymmetry have been obtained

\[ \eta / s \equiv \text{viscosity} / \text{entropy density} \]

Ultracold $^6$Li gas at Feshbach resonance

\[ \frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}} \]

Ultracold $^6$Li gas at Feshbach resonance

Quark gluon plasma


\frac{\eta}{s} \equiv \text{viscosity} \over \text{entropy density}

Ultracold $^6$Li gas at Feshbach resonance

Quark gluon plasma

Supersymmetric black hole theory

\[ \eta \equiv \frac{\text{viscosity}}{\text{entropy density}} \]

- Ultracold $^6$Li gas at Feshbach resonance
- He near $\lambda$-transition
- Quark gluon plasma
- Supersymmetric black hole theory

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.