From the pseudogap to the strange metal

S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB 94, 115147 (2016)

S. Sachdev and S. Chatterjee, arXiv:1703.00014

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Talk online: sachdev.physics.harvard.edu
$YBa_2Cu_3O_{6+x}$
A conventional metal: the Fermi liquid with Fermi surface of size $1 + p$. 

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size $p$ and not $1+p$. 

Begin with the “spin-fermion” model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = - \sum_{i,\rho} t_\rho \left( c_{i,\alpha}^\dagger c_{i+\rho,\alpha} + c_{i+\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$, $\ell = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_{\Phi}$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i) =$constant independent of $i$, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.
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When $\Phi^\ell(i) =$constant independent of $i$, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.
(A) Antiferromagnetic metal
\[ \langle \Phi \rangle \neq 0 \]

(B) Fermi liquid with large Fermi surface
\[ \langle \Phi \rangle = 0 \]
Antiferromagnetic metal

\[ \langle \Phi \rangle \neq 0 \]

Fermi liquid with large Fermi surface

\[ \langle \Phi \rangle = 0 \]

Criticality in Fe-based and electron-doped-cuprate materials

LGW-Hertz criticality of antiferromagnetism
Can we get a stable zero temperature state with “fluctuating antiferromagnetism” and a small Fermi surface (and so a gap near the anti-nodes)?
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Yes, provided the metal has topological order

For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation $R_i$

\[
\begin{pmatrix}
c_i^{\uparrow} \\
c_i^{\downarrow}
\end{pmatrix} = R_i \begin{pmatrix}
\psi_{i,+} \\
\psi_{i,-}
\end{pmatrix},
\]

in terms of fermionic “chargons” $\psi_s$ and a **Higgs field** $H^a(i)$

\[
\sigma^\ell \Phi^\ell(i) = R_i \sigma^a H^a(i) R_i^\dagger
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The Higgs field is the AFM order in the rotating reference frame.
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\sigma^\ell \Phi^\ell(i) = R_i \sigma^a H^a(i) R_i^\dagger
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The Higgs field is the AFM order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, $V_i$

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\begin{pmatrix}
 \psi_{i,+} \\
 \psi_{i,-}
\end{pmatrix} \rightarrow V_i \begin{pmatrix}
 \psi_{i,+} \\
 \psi_{i,-}
\end{pmatrix}
$$

$$
R_i \rightarrow R_i V_i^\dagger
$$

$$
\sigma^a H^a(i) \rightarrow V_i \sigma^b H^b(i) V_i^\dagger.
$$
Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = - \sum_{i, \rho} t_\rho \left( \psi_{i, s}^\dagger \psi_{i+\nu_\rho, s} + \psi_{i+\nu_\rho, s}^\dagger \psi_{i, s} \right) - \mu \sum_i \psi_{i, s}^\dagger \psi_{i, s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi_{i, s}^\dagger \sigma^a_{ss'} \psi_{i, s'} + V_H$$
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\]

**IF** we can transform to a rotating reference frame in which \(H^a(i)\) is a constant independent of \(i\) and time, **THEN** the \(\psi\) fermions in the presence of fluctuating AFM will inherit the anti-nodal gap of the electrons in the presence of static AFM.
Fluctuating antiferromagnetism

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Fluctuating antiferromagnetism

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A.V. Chubukov, T. Senthil and S. Sachdev, PRL 72, 2089 (1994);
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Topological order

We cannot always find a single-valued SU(2) rotation $R_i$ to make the Higgs field $H^a(i)$ a constant!

Vortices associated with $\pi_1(\text{SO}(3)) = \mathbb{Z}_2$ must be suppressed: such a metal with "fluctuating antiferromagnetism" has \textit{Z}_2 \hspace{1pt} \text{TOPOLOGICAL ORDER} and fermions which inherit the Fermi surfaces of the antiferromagnetic metal \textit{i.e.} a pseudogap.

(A) Antiferromagnetic metal

\[ \langle \Phi \rangle \neq 0 \]

(B) Fermi liquid with large Fermi surface

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Criticality in Fe-based and electron-doped-cuprate materials

LGW-Hertz criticality of antiferromagnetism
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LGW-Hertz criticality of antiferromagnetism

Criticality in Fe-based and electron-doped-cuprate materials
Global phase diagram

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(B) Fermi liquid with large Fermi surface
\[ \langle R \rangle \neq 0, \quad \langle H^a \rangle = 0 \]

(C) Metal with \( Z_2 \) topological order
\[ \langle R \rangle = 0, \quad \langle H^a \rangle \neq 0 \]

(D) SU(2) ACL eventually unstable to pairing and confinement
\[ \langle R \rangle = 0, \quad \langle H^a \rangle = 0 \]

Higgs criticality: Deconfined SU(2) gauge theory with large Fermi surface

LGW-Hertz criticality of antiferromagnetism

Increasing SDW order

Deconfined SU(2) gauge theory with large Fermi surface

Higgs criticality:

Global phase diagram

Increasing SDW order
Global phase diagram

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Higgs criticality: Deconfined SU(2) gauge theory with large Fermi surface

Proposal for optimal doping criticality in hole-doped cuprates
Topological order

More generally, the effective Hamiltonian for the fermionic chargons can also have non-trivial SU(2) gauge connections $U^\rho(i)$ along with the Higgs field $H^a(i)$.

$$\mathcal{H}_\psi = - \sum_{i,\rho} t_\rho \left( \psi_{i,s}^\dagger \psi_{i+\nu_{\rho},s'} + \text{H.c.} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\text{int}}$$

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Such a gauge-connection can induce various gauge-invariant fluxes which can break one or more of time-reversal, inversion, and lattice rotation symmetries.
Topological order

Gauge-invariant combinations of Higgs fields and gauge connections which are proportional to the electrical current on links

\[ O_{mj} = i \text{Tr} (\sigma^a U_{mj} U_{jk} U_{km}) H^a(m) \]
\[ - i \text{Tr} (\sigma^a U_{jm} U_{mn} U_{nj}) H^a(j) \]
\[ + i \text{Tr} (\sigma^a U_{mj} U_{ji} U_{im}) H^a(m) \]
\[ - i \text{Tr} (\sigma^a U_{jm} U_{ml} U_{lj}) H^a(j) \]

\[ O_{mk} = i \text{Tr} (\sigma^a U_{mj} U_{jk} U_{km}) H^a(m) \]
\[ - i \text{Tr} (\sigma^a U_{kj} U_{jm} U_{mk}) H^a(k) \]
\[ + i \text{Tr} (\sigma^a U_{mn} U_{nk} U_{km}) H^a(m) \]
\[ - i \text{Tr} (\sigma^a U_{kn} U_{nm} U_{mk}) H^a(k) \]

Such a gauge-connection can induce various gauge-invariant fluxes which can break one or more of time-reversal, inversion, and lattice rotation symmetries.

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States with topological order can have these patterns of spontaneous currents, while preserving translational symmetry. Both patterns are consistent with present neutron and light scattering experiments. Both patterns have Ising-nematic order: the Ising-nematic order of (a) is similar to that observed in the cuprates.

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(C) Metal with $Z_2$ topological order
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LGW-Hertz criticality of antiferromagnetism

Higgs criticality: Deconfined SU(2) gauge theory with large Fermi surface
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\[ \langle R \rangle \neq 0, \langle H^a \rangle \neq 0 \]

(B) Fermi liquid with large Fermi surface
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(C) Metal with $\mathbb{Z}_2$ topological order and discrete symmetry breaking
\[ \langle R \rangle = 0, \langle H^a \rangle \neq 0 \]

(D) SU(2) ACL eventually unstable to pairing and confinement
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LGW-Hertz criticality of antiferromagnetism

Higgs criticality: Deconfined SU(2) gauge theory with large Fermi surface
Lattice gauge theory for a metal with topological order co-existing with broken time-reversal and inversion symmetries, and Ising-nematic order.
Gauge theory for a topological phase transition, and for the strange metal (SM)