Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm

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Phys. Rev. Lett. **90**, 216403 (2003).
Science **303**, 1490 (2004).

Talk online:
Google Sachdev

$T=0$

SDW

$$\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$$

Collinear spins: $\mathbf{N}_1 \times \mathbf{N}_2 = 0$

Non-collinear spins: $\mathbf{N}_1 \times \mathbf{N}_2 \neq 0$

$$\langle \mathbf{S}_j \rangle = 0$$

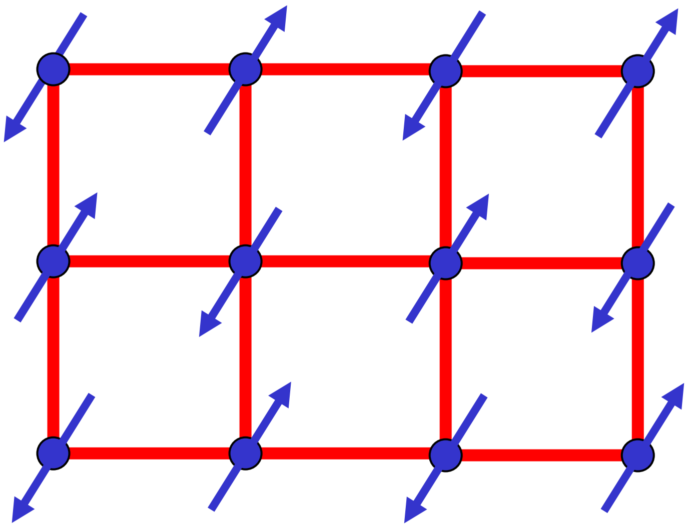
Pressure,
carrier concentration,….

Quantum critical point

States on both sides of critical point
could be either (A) Insulators
(B) Metals
(C) Superconductors

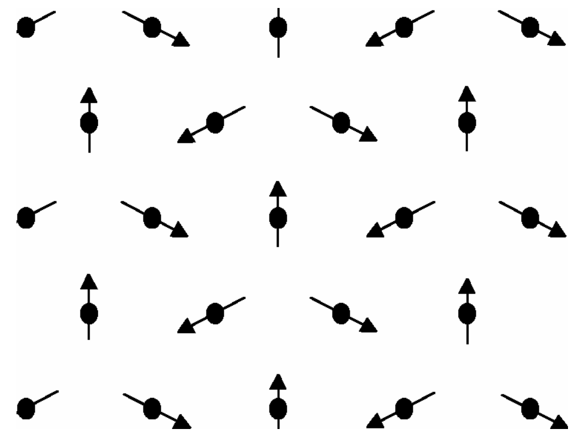
SDWs in Mott insulators

La_2CuO_4



$\vec{K} = (\pi, \pi)$

Collinear spins



$\vec{K} = \left(4\pi/3, 4\pi/\sqrt{3}\right)$

Non-collinear spins

"Disorder" the spins by enhancing quantum fluctuations in a variety of ways…..

Outline

A. "Dimerized" Mott insulators

Landau-Ginzburg-Wilson (LGW) theory.

B. Kondo lattice models

"Large" Fermi surfaces and the LGW SDW paramagnon theory.

C. Fractionalized Fermi liquids

Spin liquids and Fermi volume changing transitions with a topological order parameter.

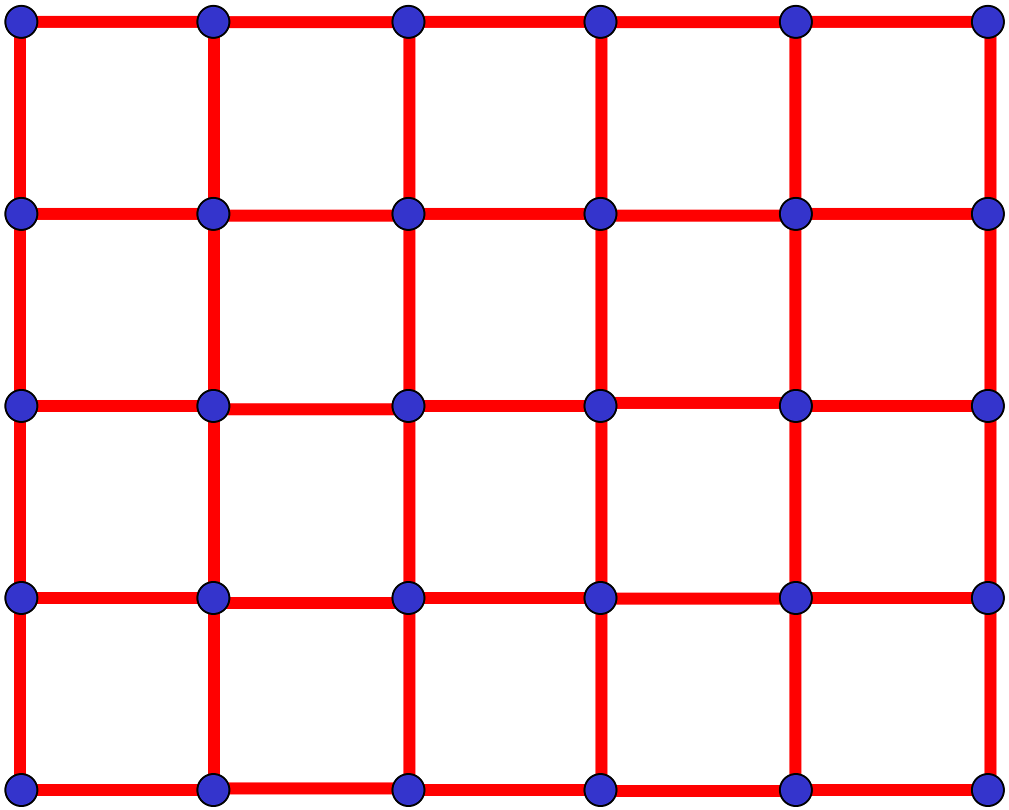
D. Deconfined quantum criticality

Berry phases and the transition from SDW to bond order. (Talks by T. Senthil (N20.008) and L. Balents (N20.009))

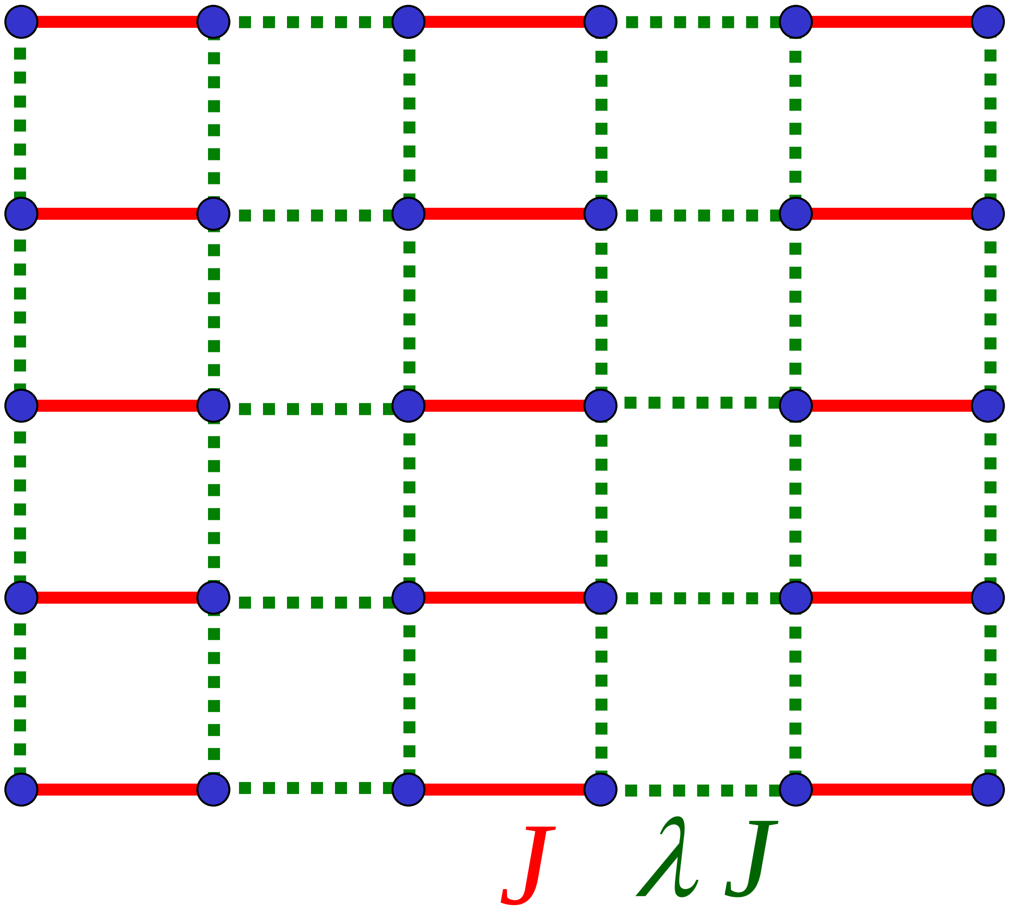
(A) Magnetic quantum phase tranitions in "dimerized" Mott insulators

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***



Coupled Dimer Antiferromagnet



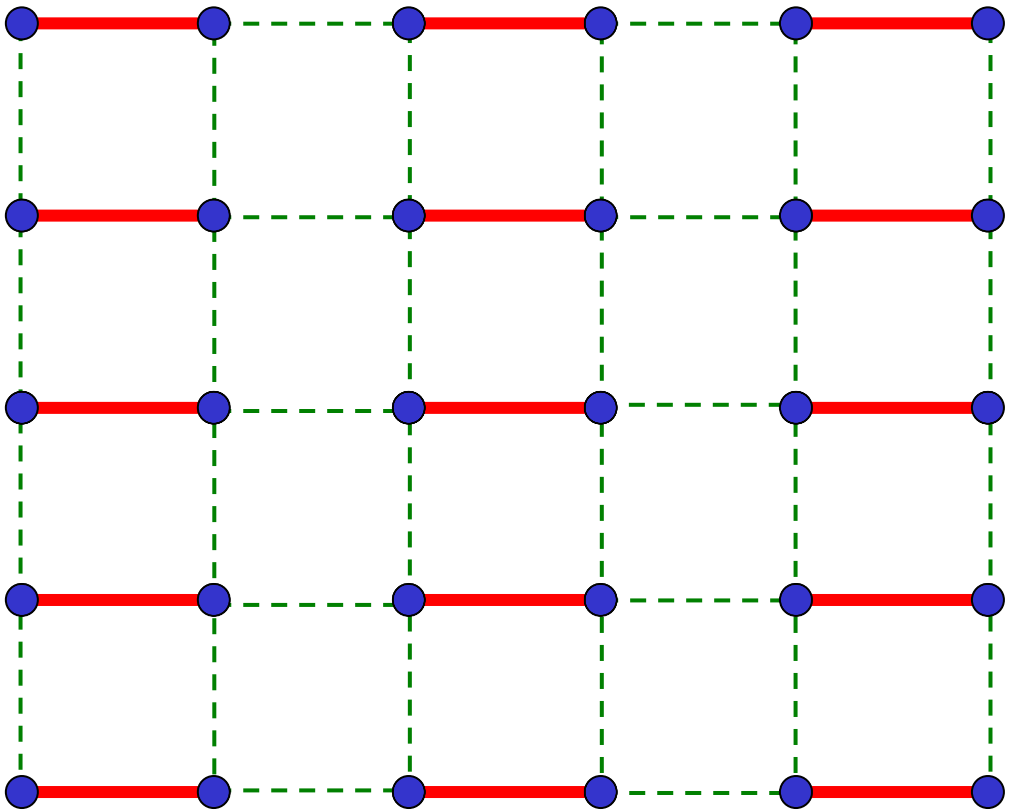
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

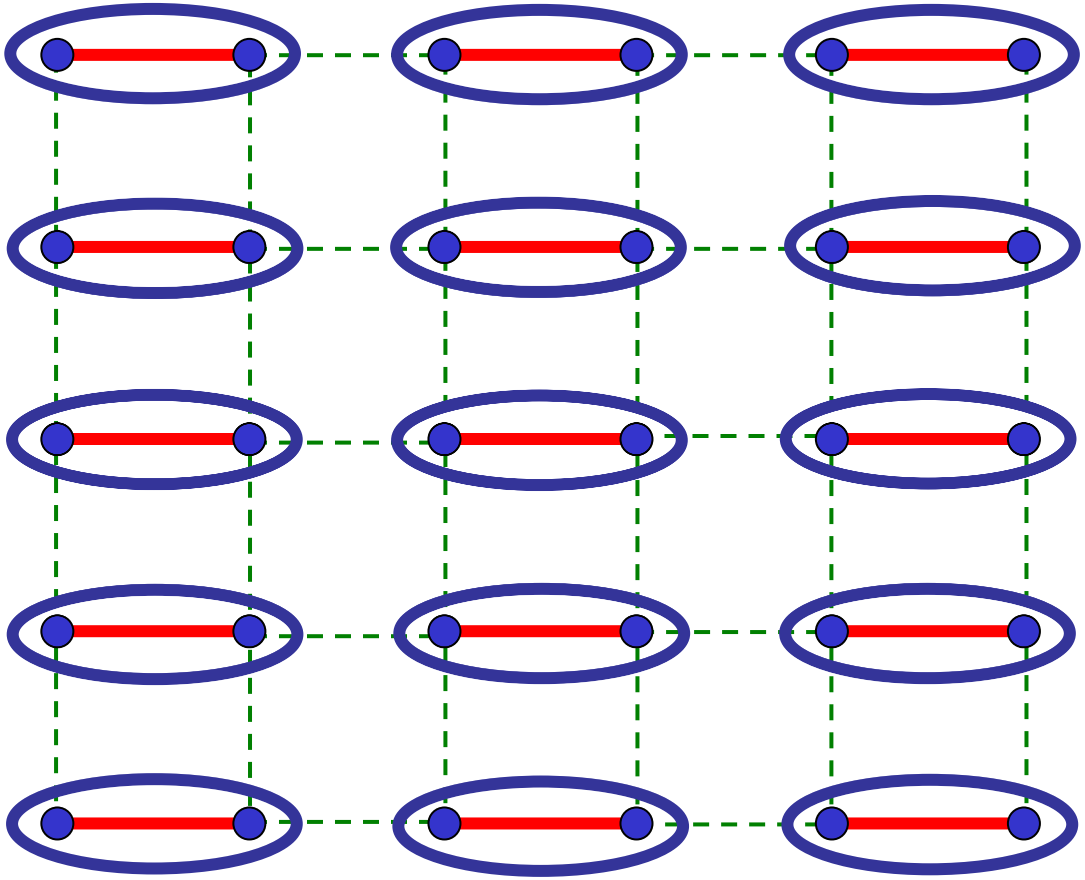
λ close to 0

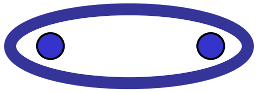
Weakly coupled dimers



λ close to 0

Weakly coupled dimers



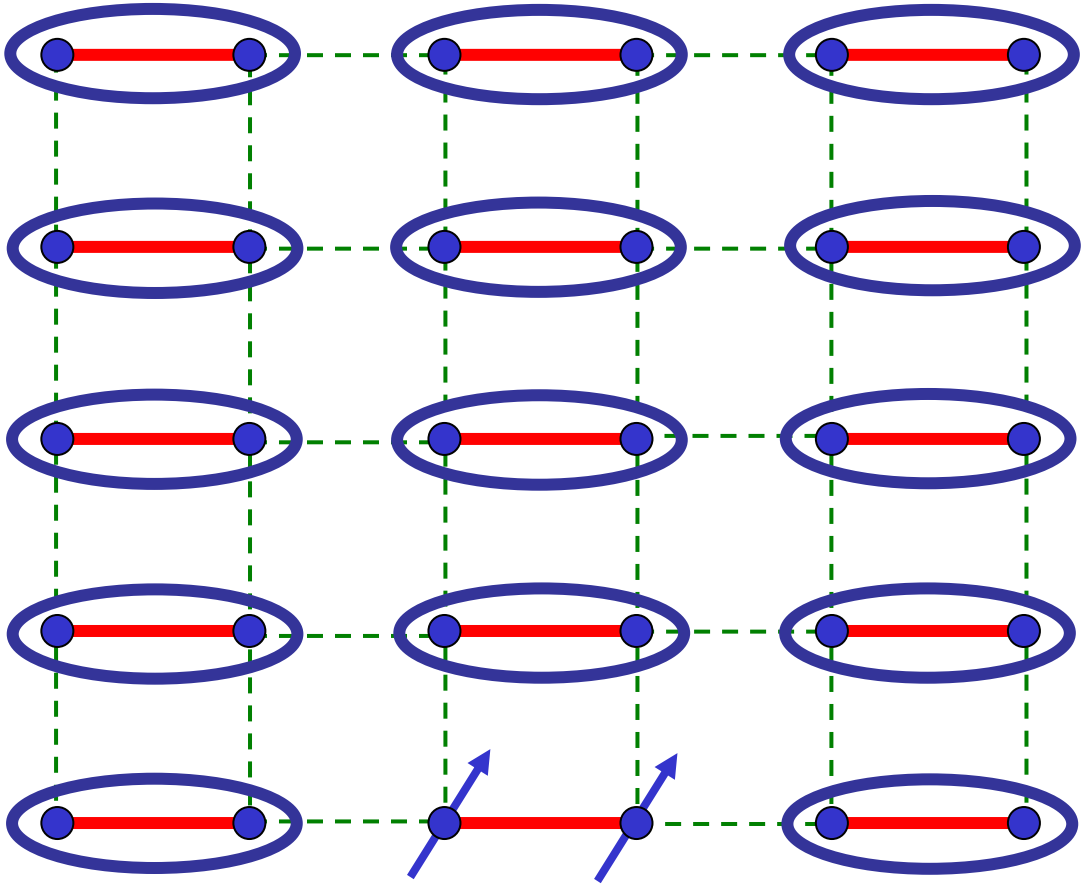
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

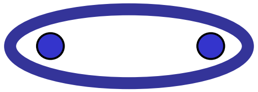
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0, \left\langle \vec{\varphi} \right\rangle = 0$$

λ close to 0

Weakly coupled dimers



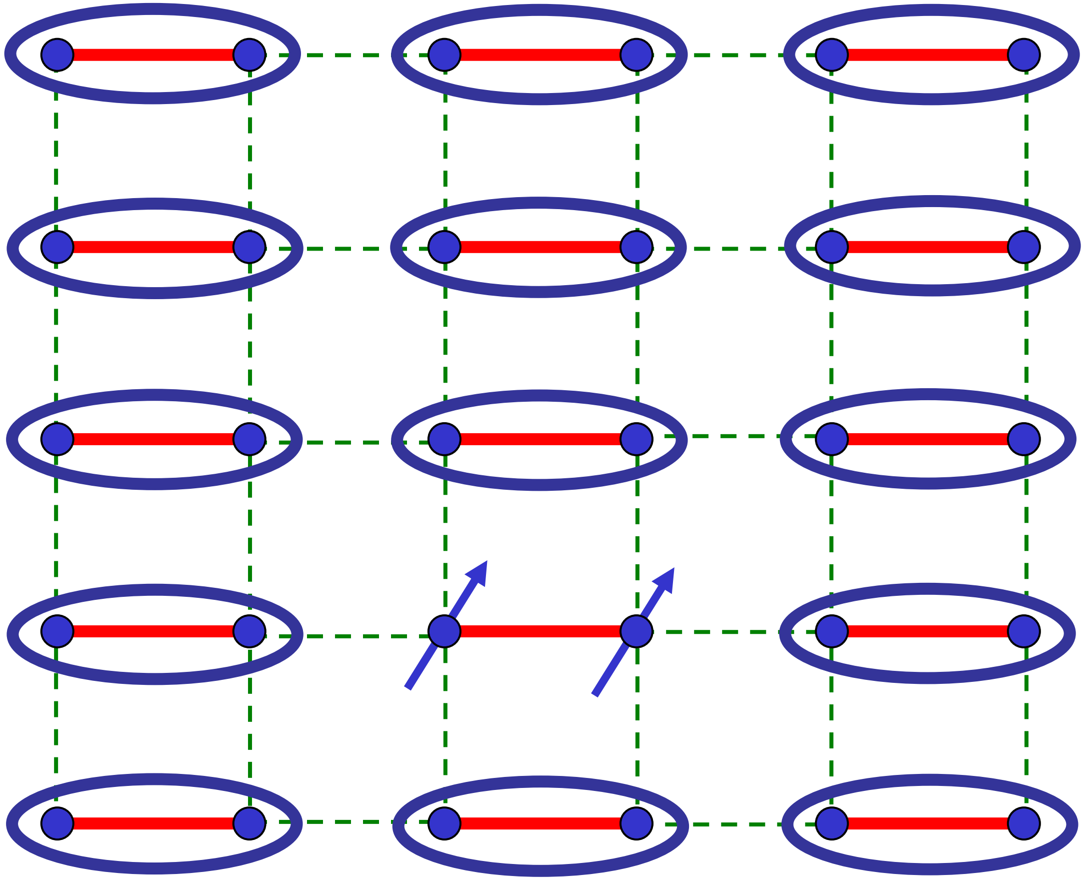


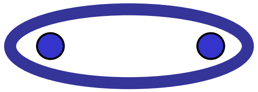
$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





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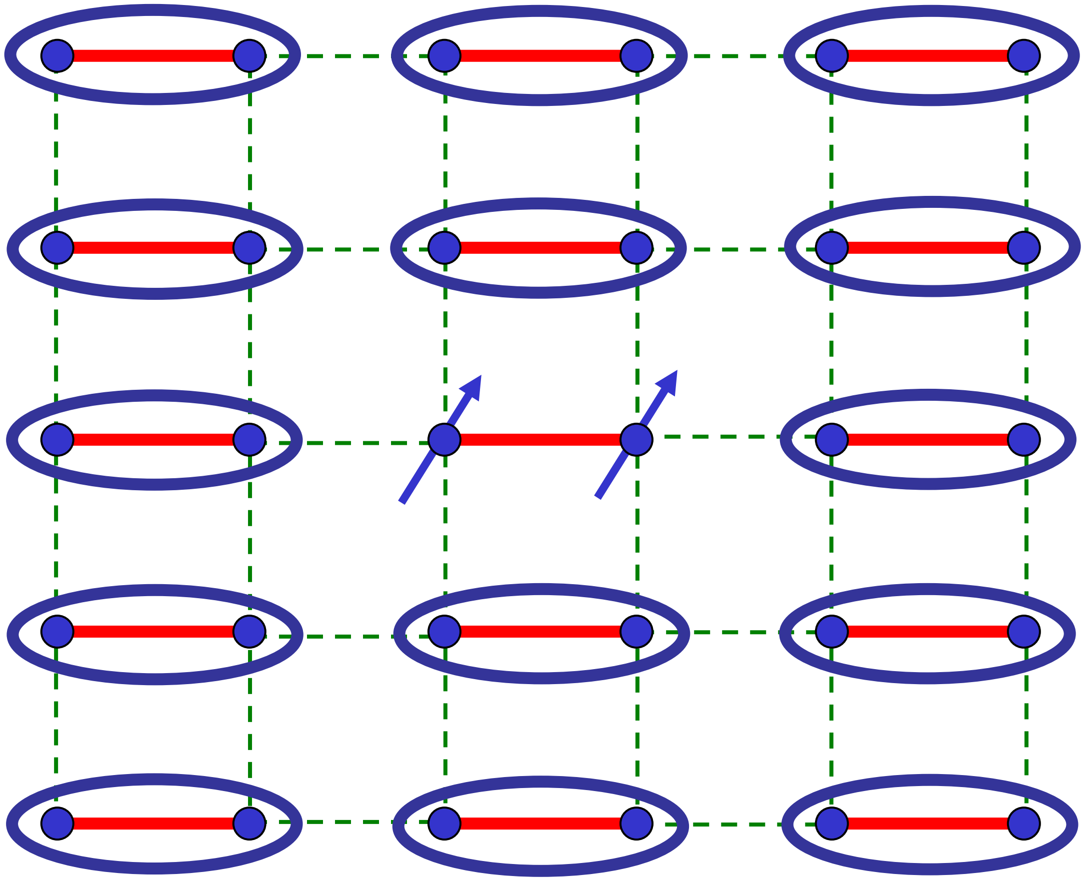

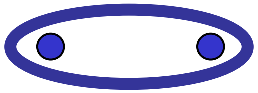 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$

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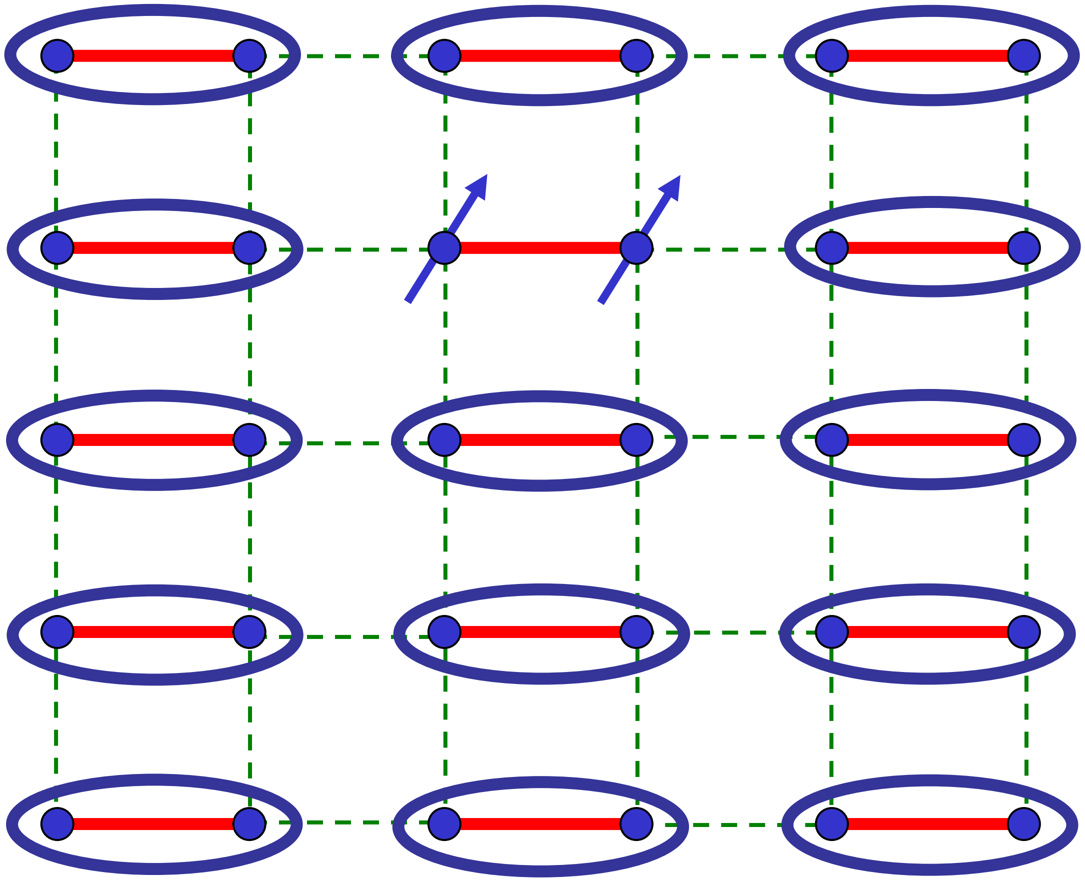

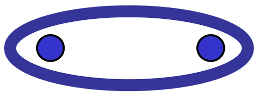

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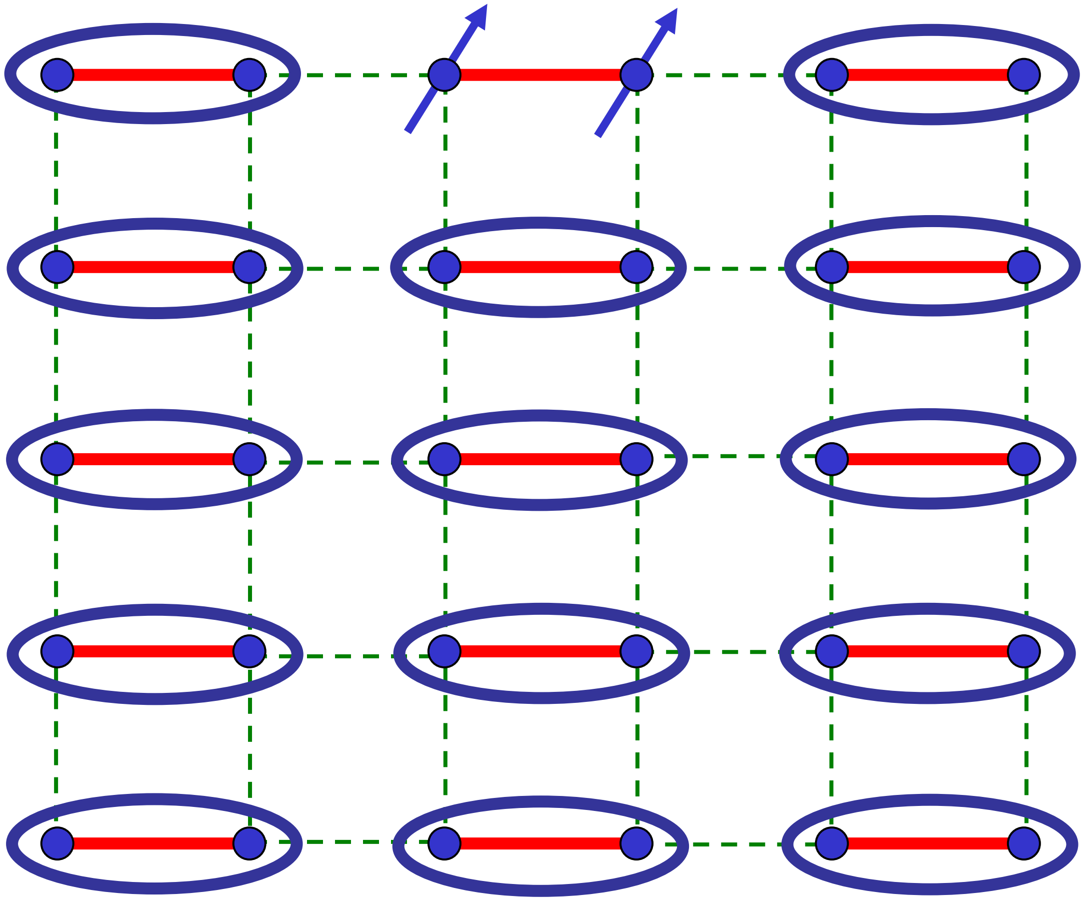

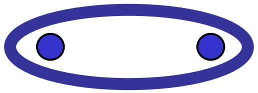

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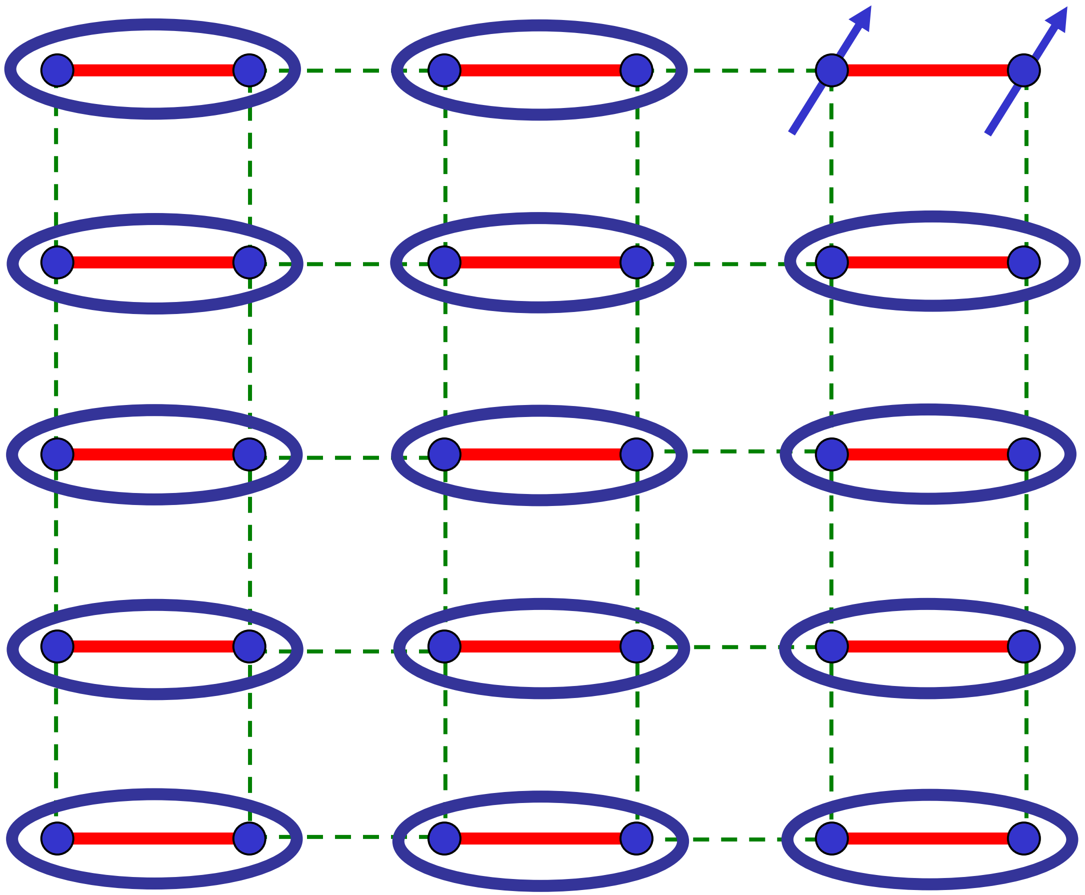
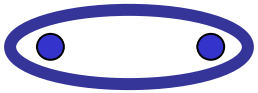




$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Excitation: S=1 ***triplon***

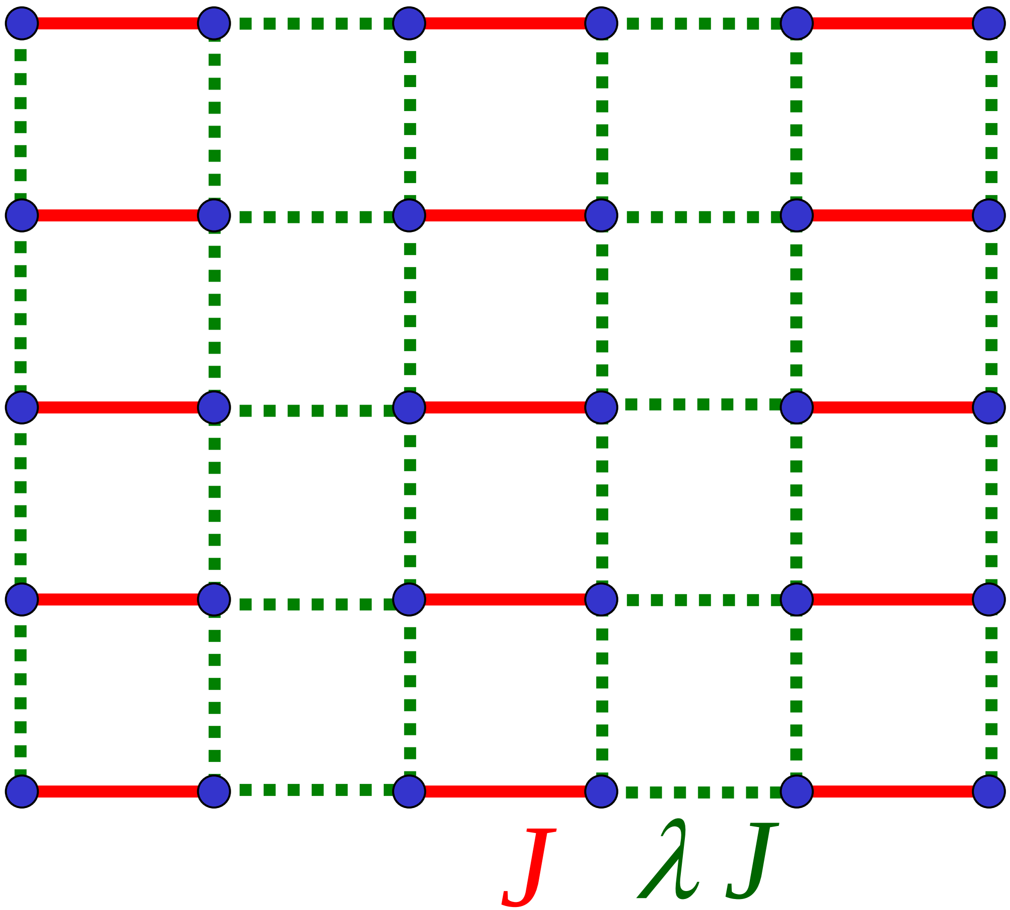
(***exciton***, spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

Coupled Dimer Antiferromagnet



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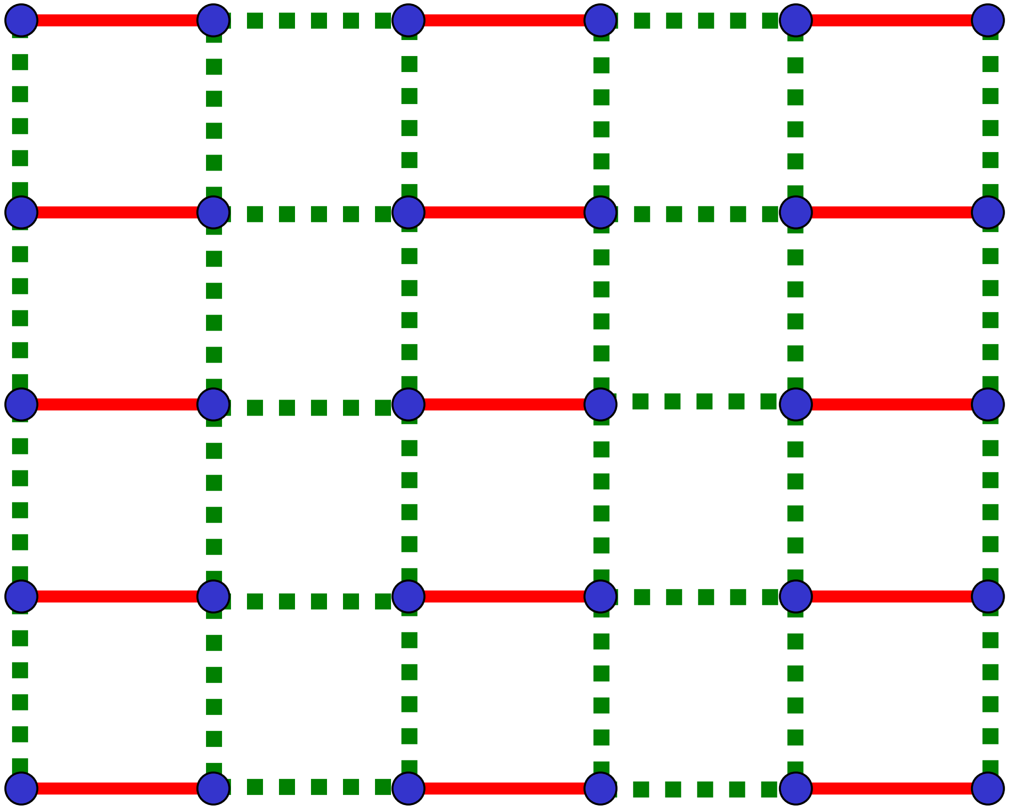
λ close to 1

Weakly dimerized square lattice



λ close to 1

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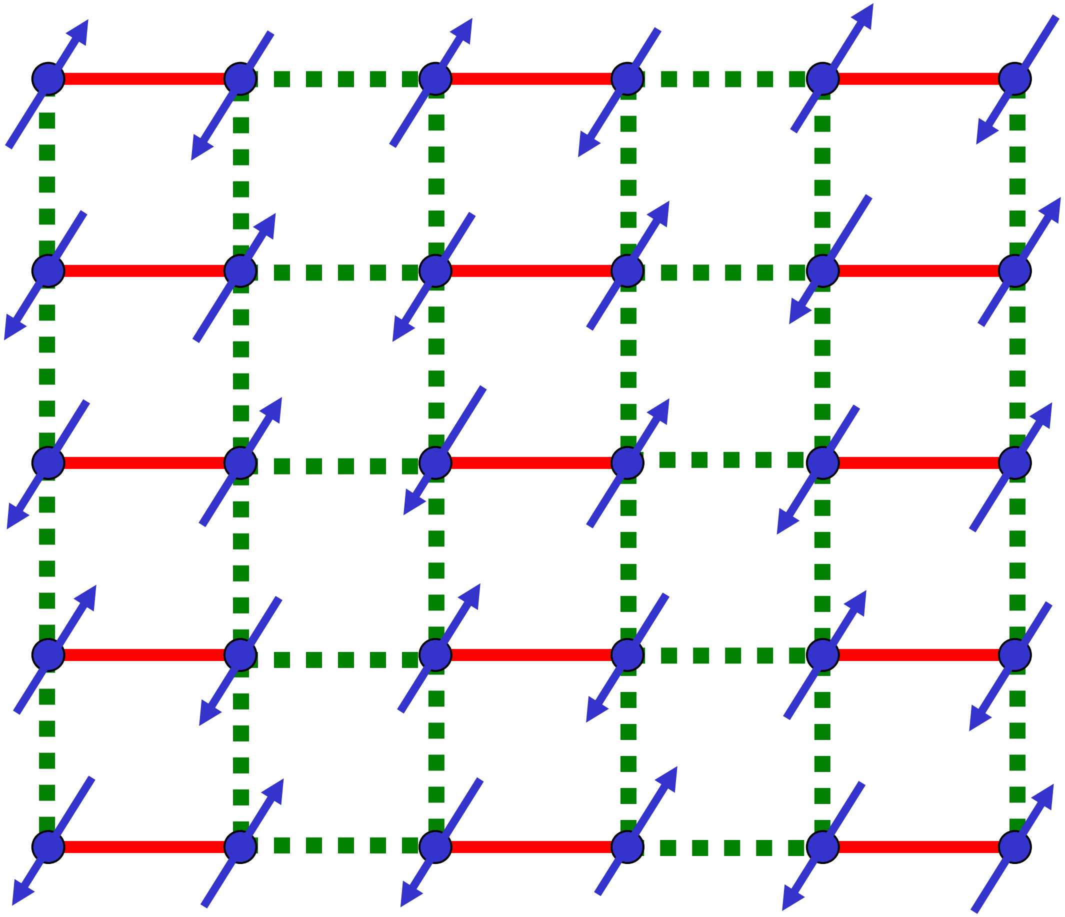


Excitations:

2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

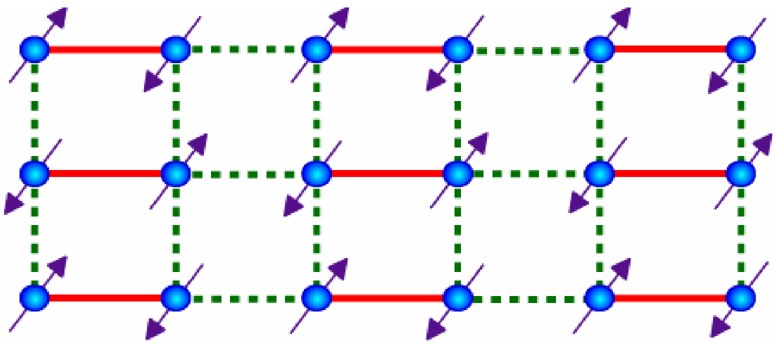
$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \dfrac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

T=0

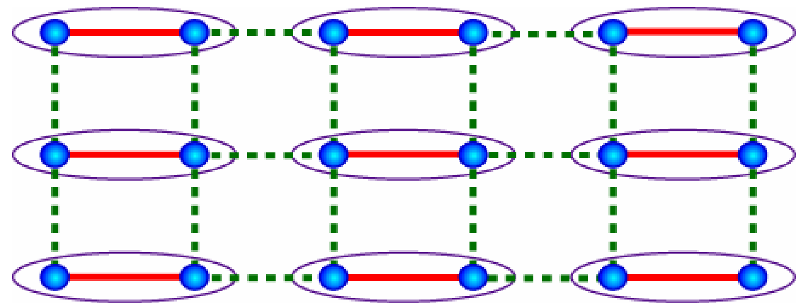
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



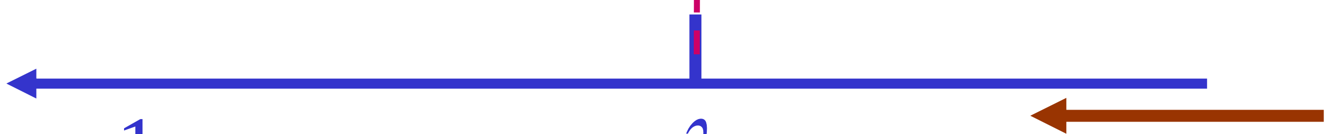
Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$



λ 1 λ_c

Pressure in $TlCuCl_3$

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

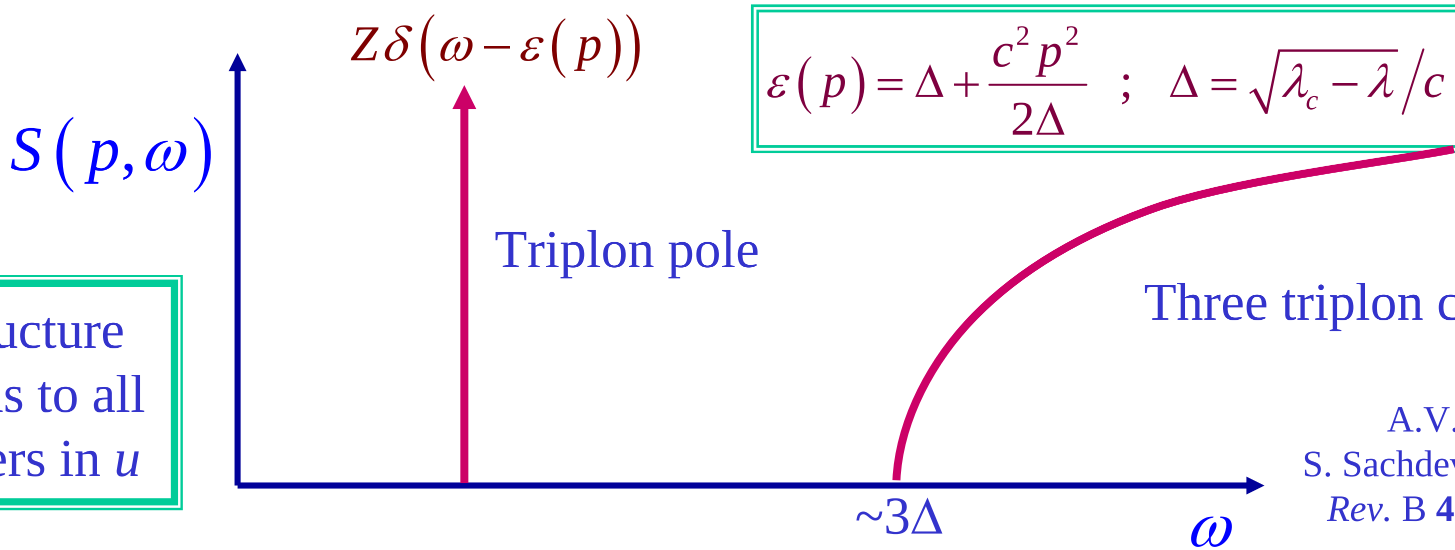
LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$



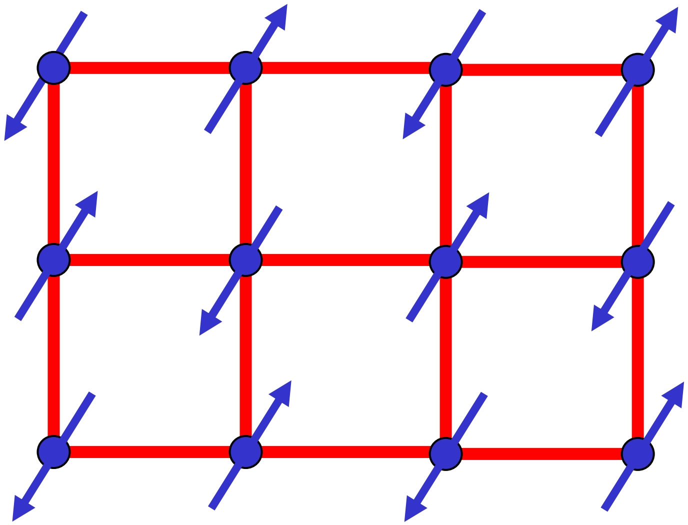
Structure holds to all orders in u

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

(B) Kondo lattice models

"Large" Fermi surfaces and the Landau-Ginzburg-Wilson spin-density-wave paramagnon theory

Kondo lattice



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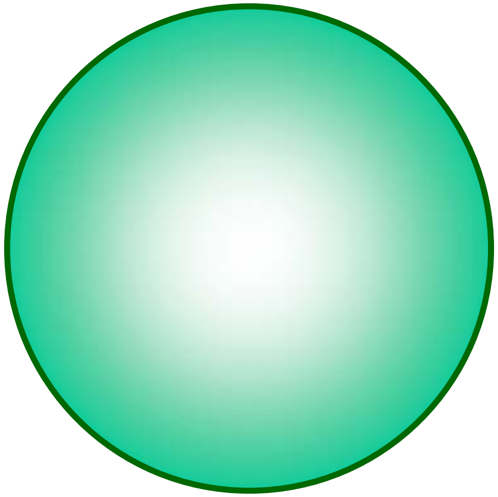


Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

At large J_K, magnetic order is destroyed, and we obtain a non-magnetic Fermi liquid (FL) ground state

S. Doniach, *Physica* B **91**, 231 (1977).

Luttinger's Fermi volume on a *d*-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

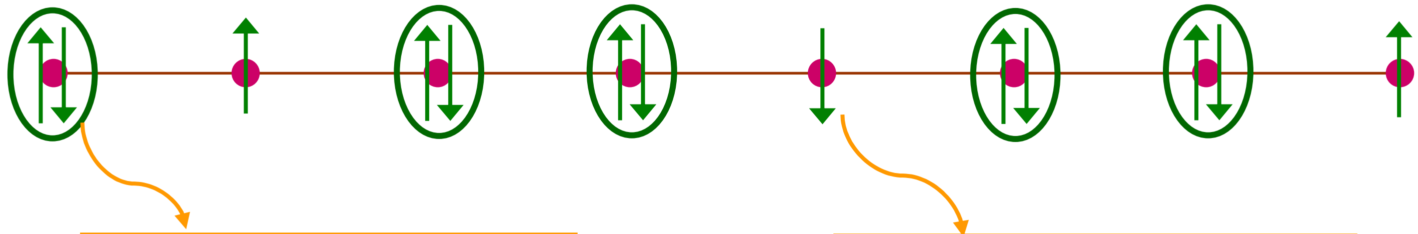
$$2 \times \frac{v_0}{(2\pi)^d}\left(\text{Volume enclosed by Fermi surface}\right)$$

$$= n_T \left(\text{mod } 2\right)$$

A "large" Fermi surface

Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



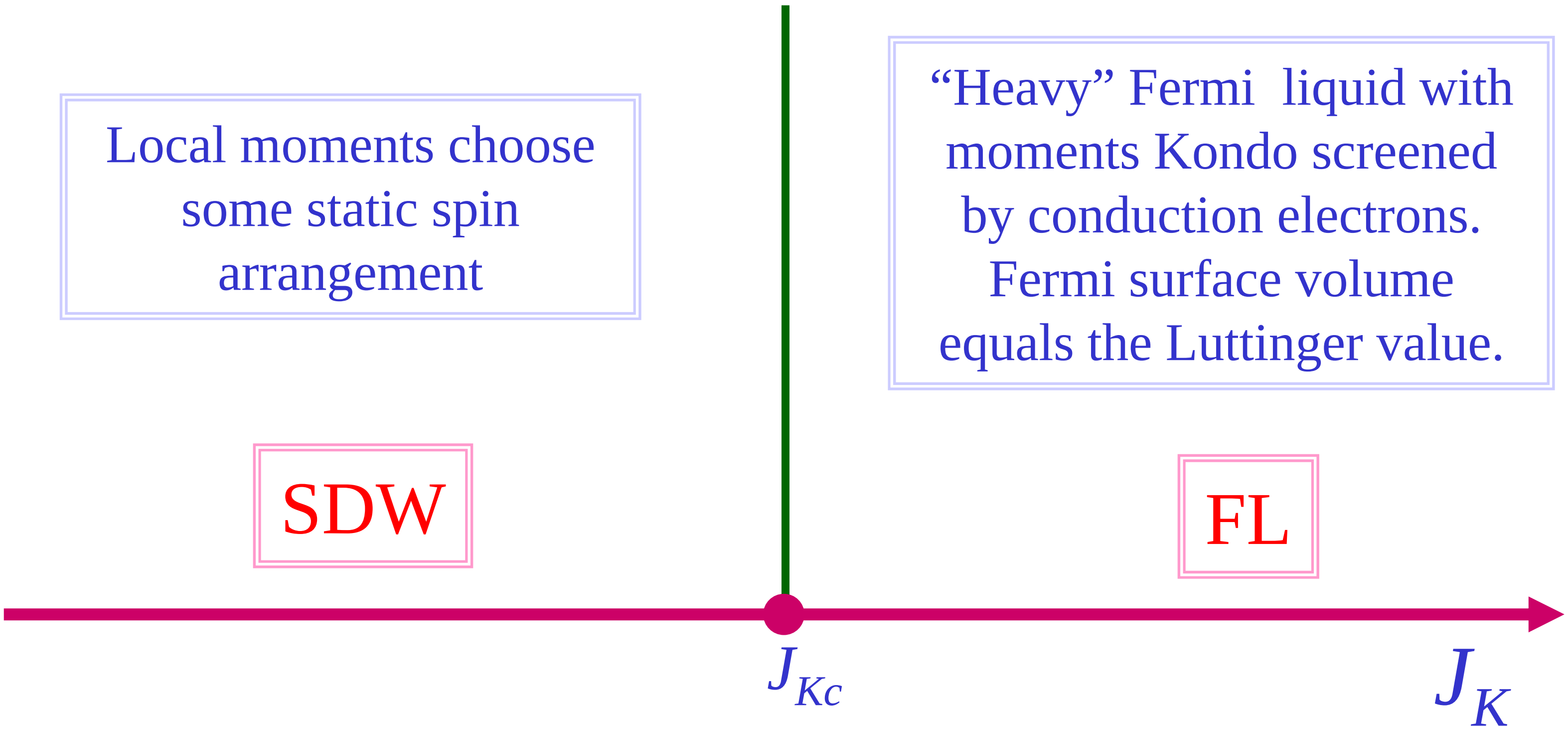
$$\left(c_{i\uparrow}^\dagger f_{i\downarrow}^\dagger - c_{i\downarrow}^\dagger f_{i\uparrow}^\dagger\right)\left|0\right\rangle$$

$$f_{i\downarrow}^\dagger \left|0\right\rangle, \; S = 1/2 \text{ hole}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

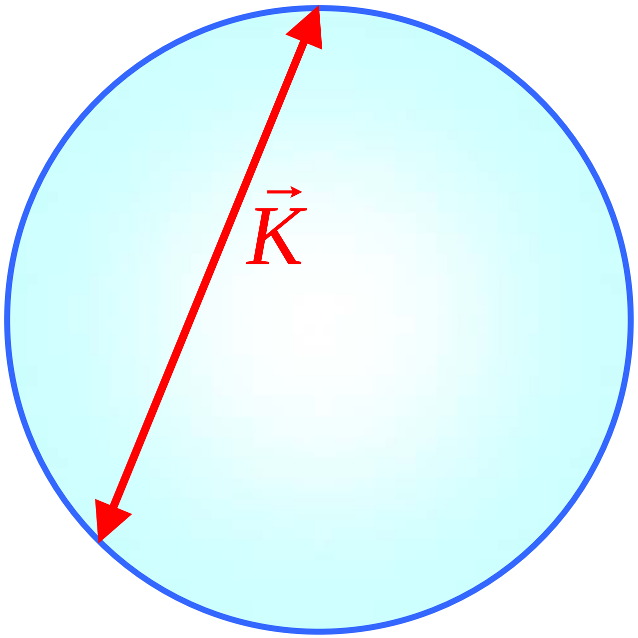
$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1-n_c\right) = \left(1+n_c\right) \bmod 2\end{aligned}$$

Doniach's $T=0$ phase diagram for the Kondo lattice



LGW theory for quantum critical point

Write down effective action for SDW order parameter $\vec{\varphi}$



$\vec{\varphi}$ fluctuations are damped by mixing with fermionic quasiparticles near the Fermi surface

$$S_\varphi = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\varphi}(q,\omega)\right|^2 \left(q^2 + |\omega| + (J_K - J_{Kc})\right) + \frac{u}{4}\int d^d r d\tau \left(\vec{\varphi}^2\right)^2$$

Fluctuations of $\vec{\varphi}$ about $\vec{\varphi}=0 \Rightarrow$ the triplon is now a paramagnon

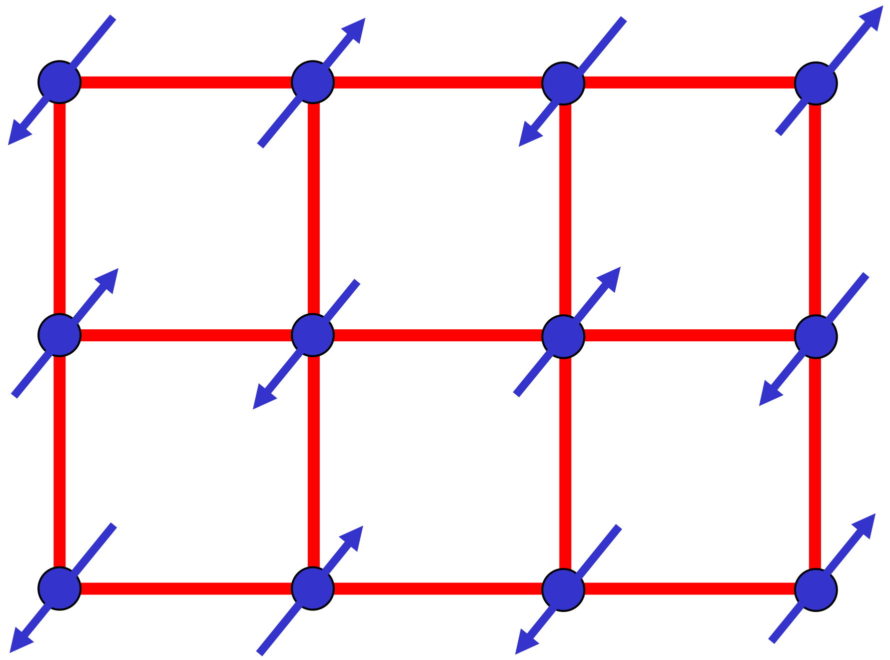
J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
M. T. Beal-Monod and K. Maki, *Phys. Rev. Lett.* **34**, 1461 (1975); J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985);
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985); A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

(C) Fractionalized Fermi liquids (FL*)

Spin liquids and Fermi volume changing transitions with a topological order parameter

Beyond LGW: quantum phases and phase transitions with emergent gauge excitations and fractionalization

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments



Ground state has Neel order with $\vec{\varphi} \neq 0$

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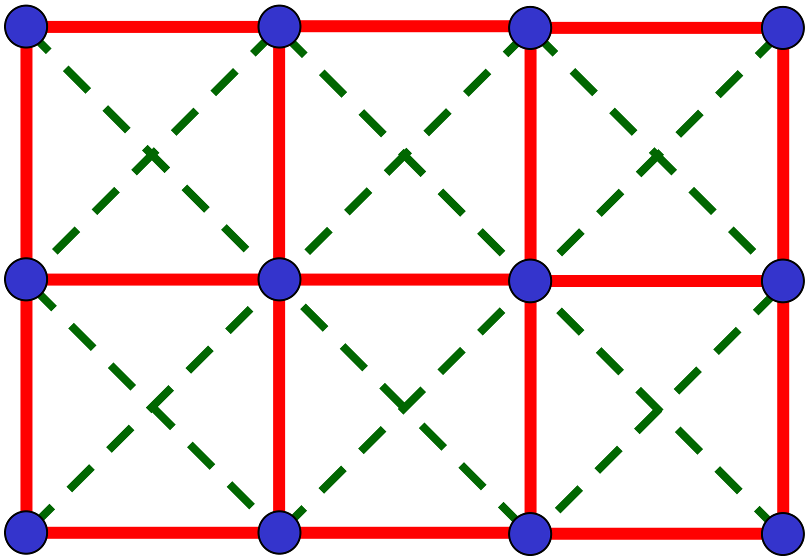

Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

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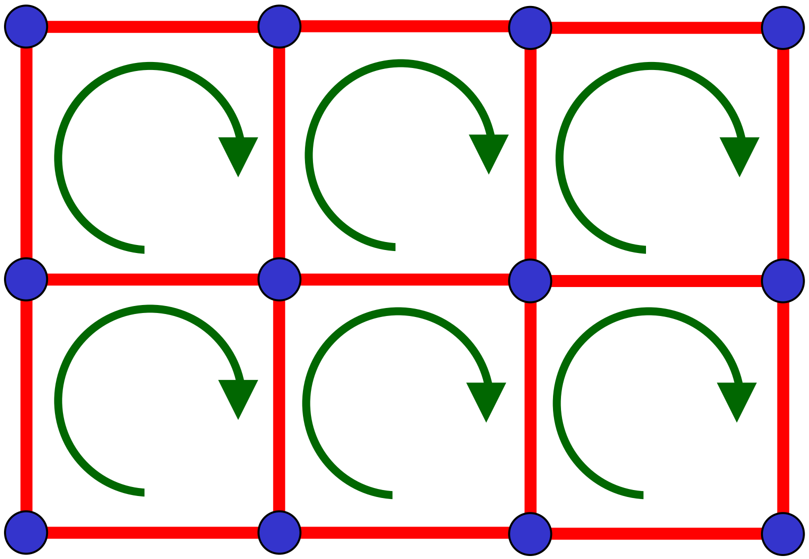

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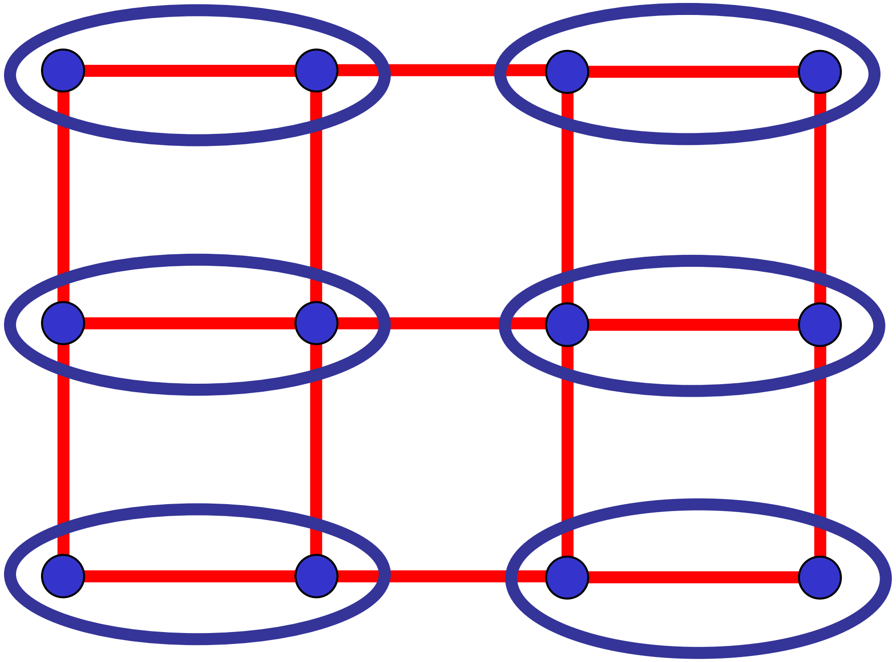
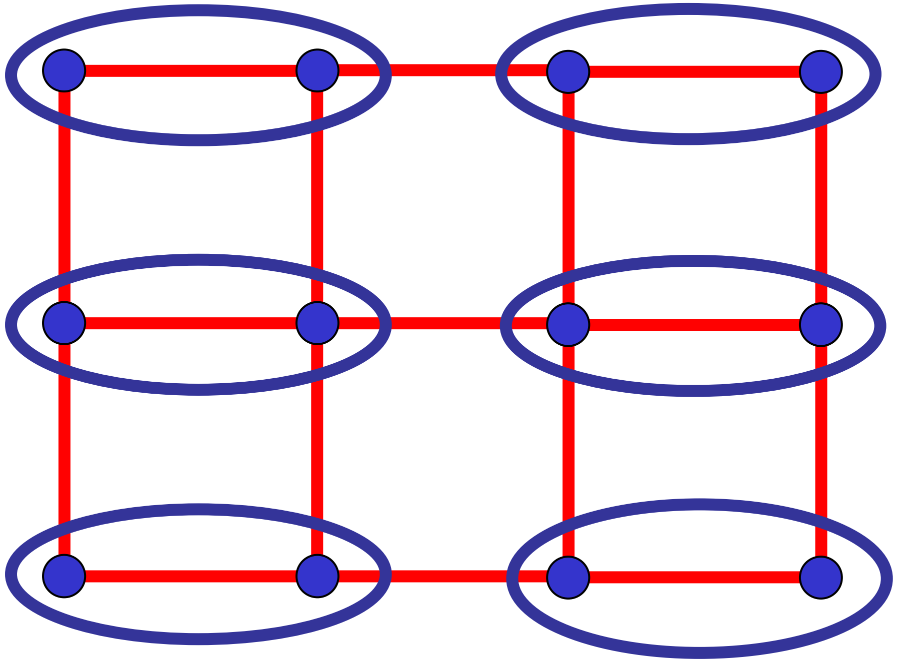
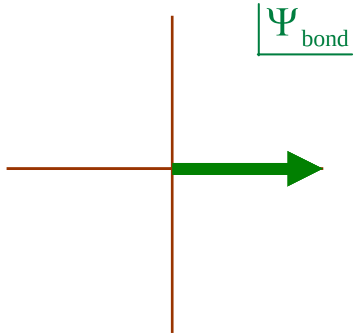


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments





Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{bond}} \rangle \neq 0$,
where Ψ_{bond} is the ***bond order parameter***

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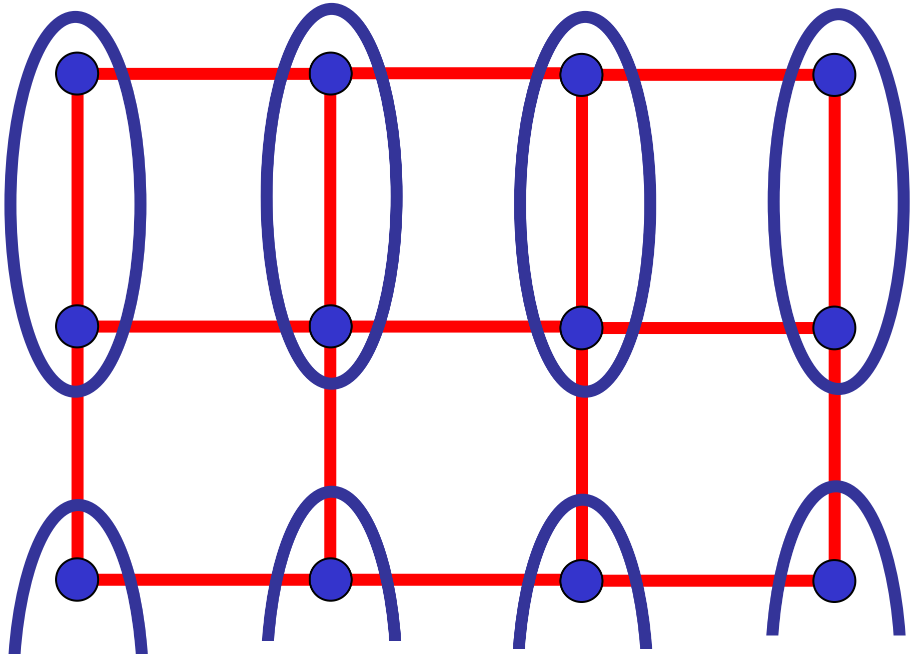

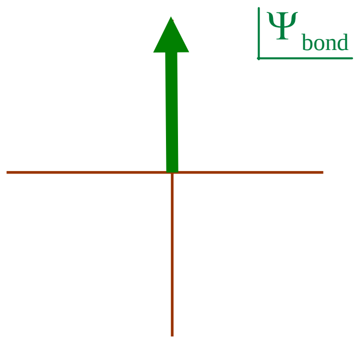

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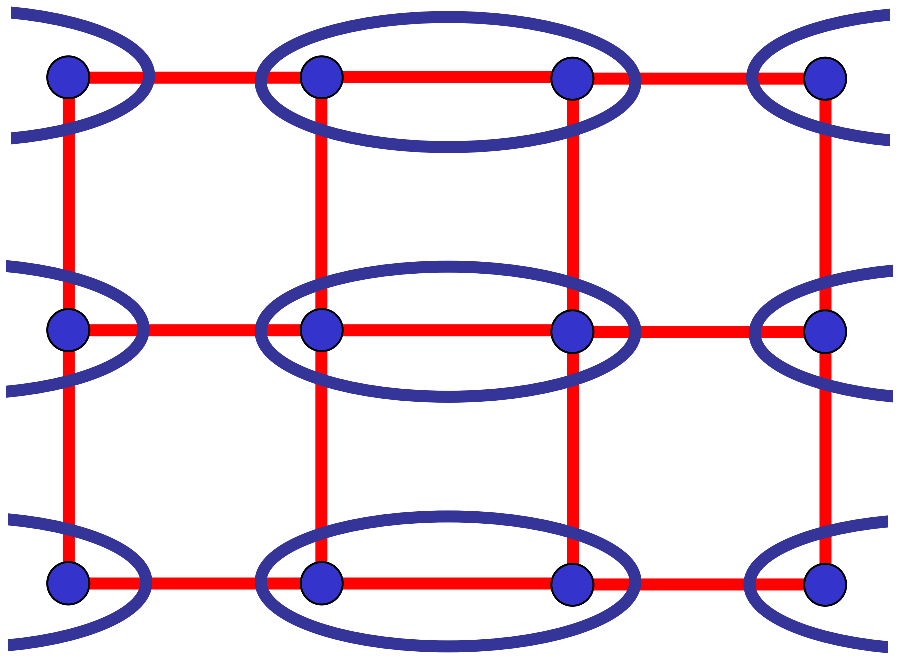

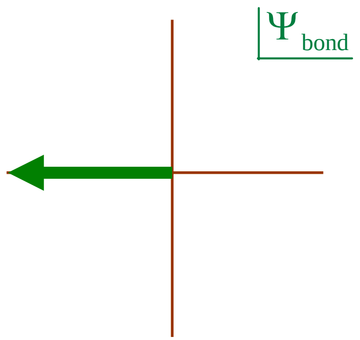

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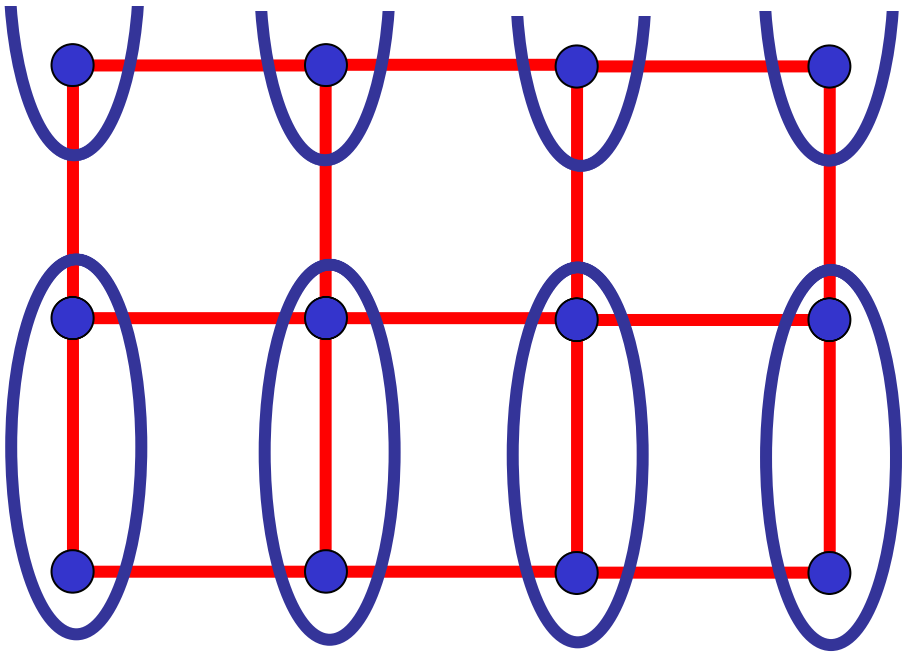
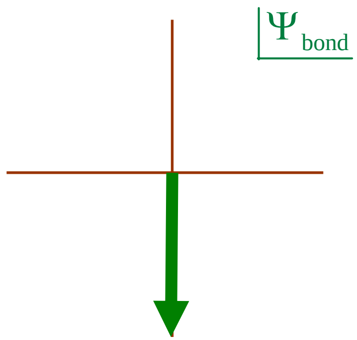


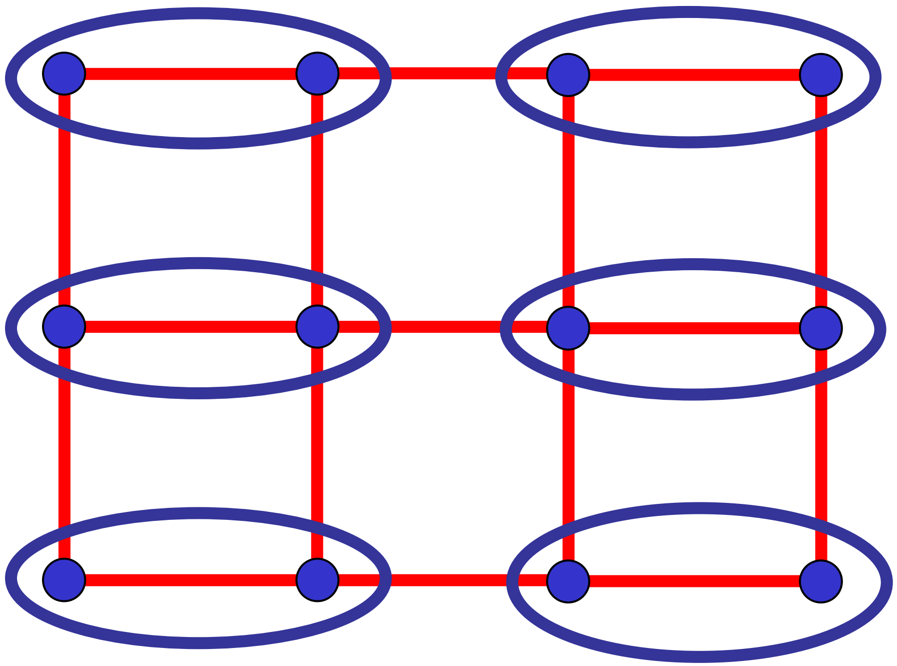


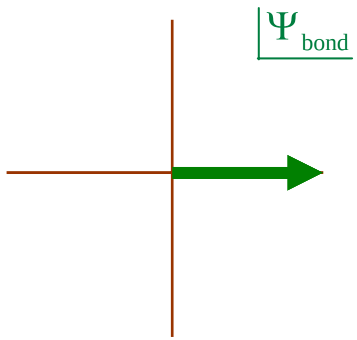
Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{bond}} \rangle \neq 0$,

where Ψ_{bond} is the ***bond order parameter***

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments





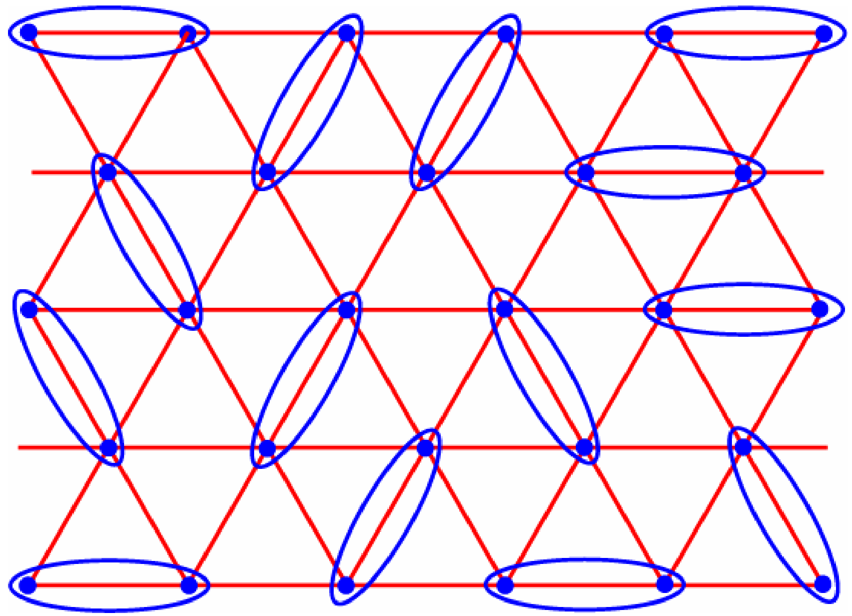
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

Bond order (and confinement) appear for collinear spins in $d=2$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments

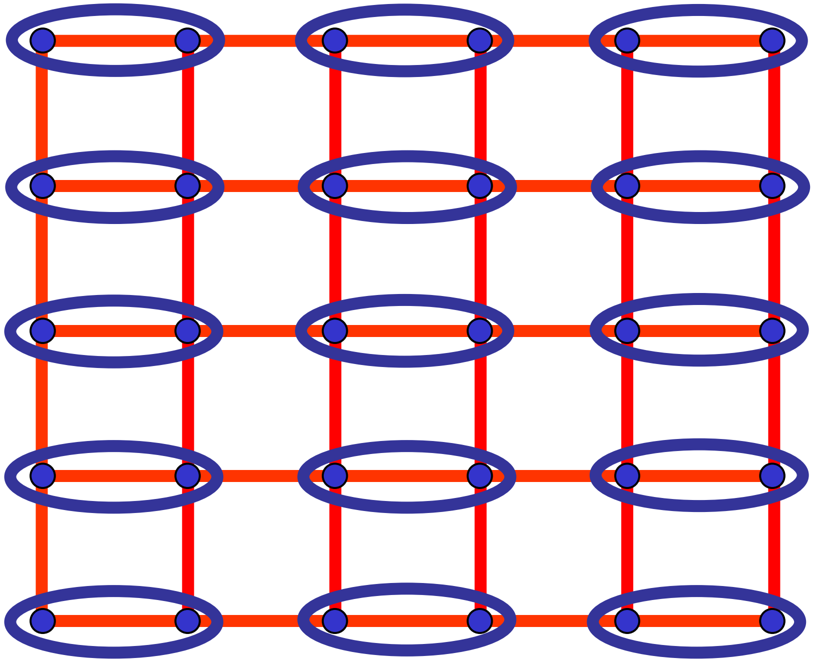


A *spin liquid* ground state with $\langle \vec{\varphi} \rangle = 0$ and $\langle \Psi_{\text{bond}} \rangle = 0$

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974);
P.W. Anderson 1987

Excitations of the paramagnet with non-zero spin

$$\langle \Psi_{\text{bond}} \rangle \neq 0$$



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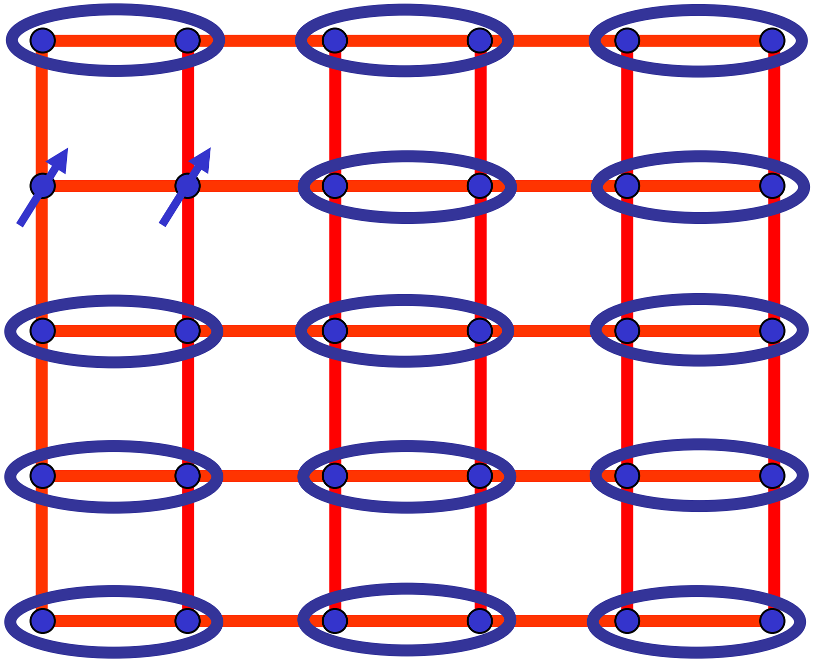

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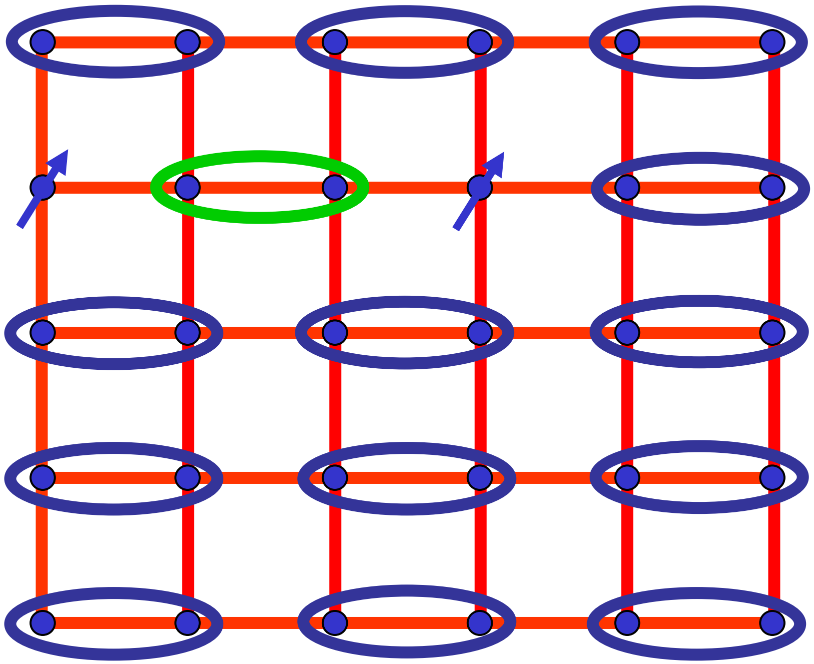

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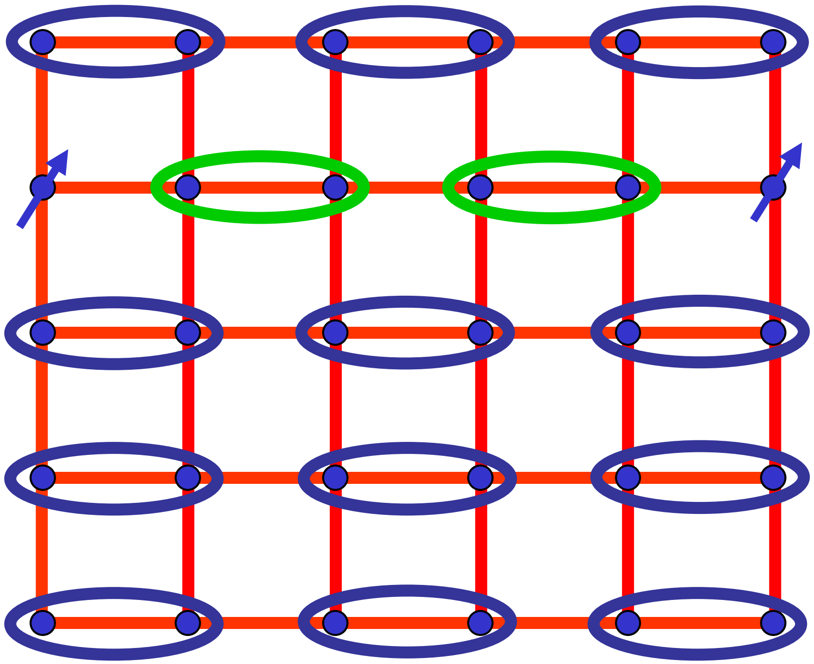

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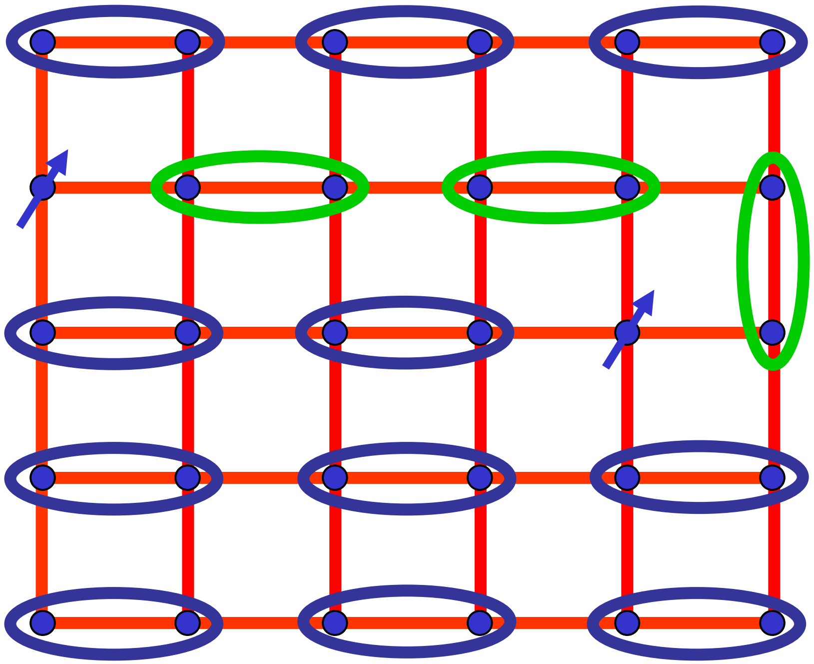

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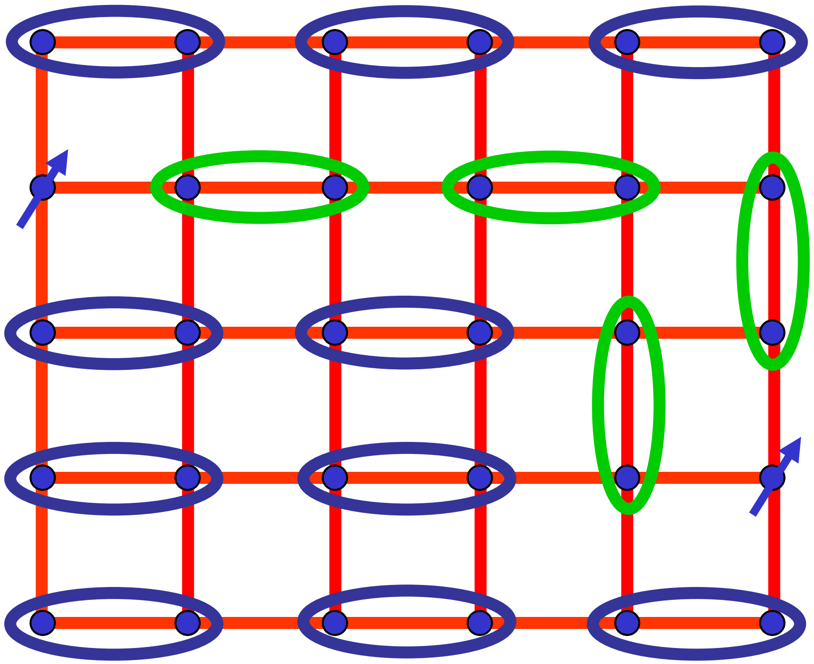

$S = 1$ **spinons** f_σ are confined into a $S = 1$ **triplon** $\vec{\varphi}$ by a confining compact U(1) gauge force

N. Read and S. Sachdev,
Phys. Rev. Lett. **62**, 1694 (1989).

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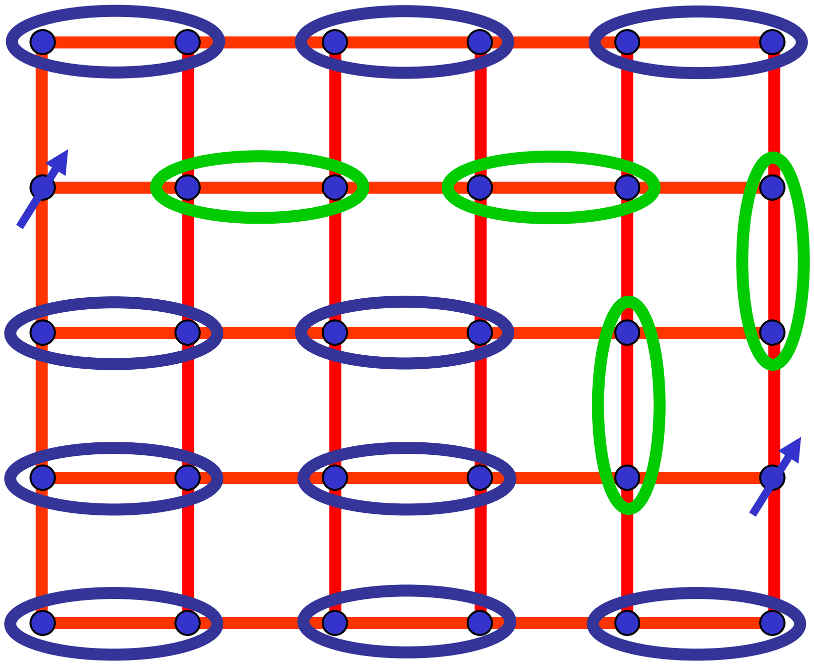

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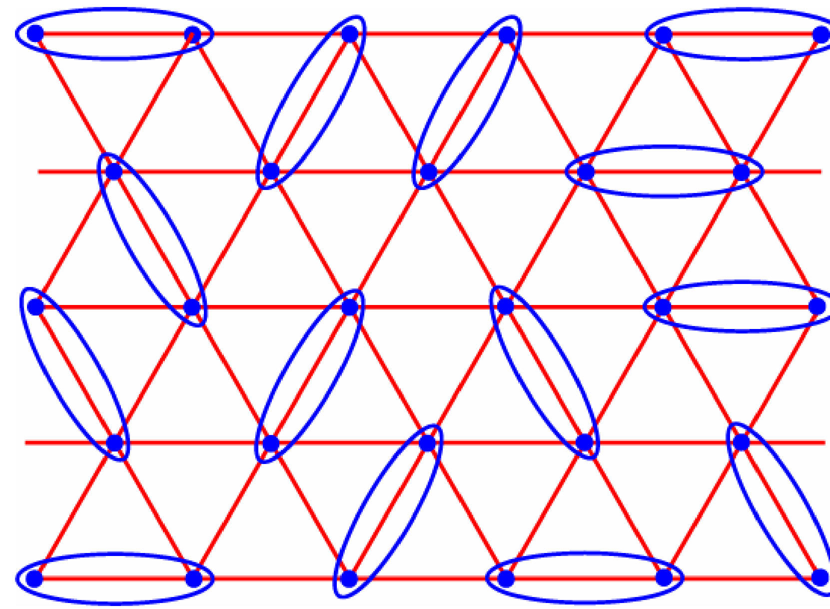

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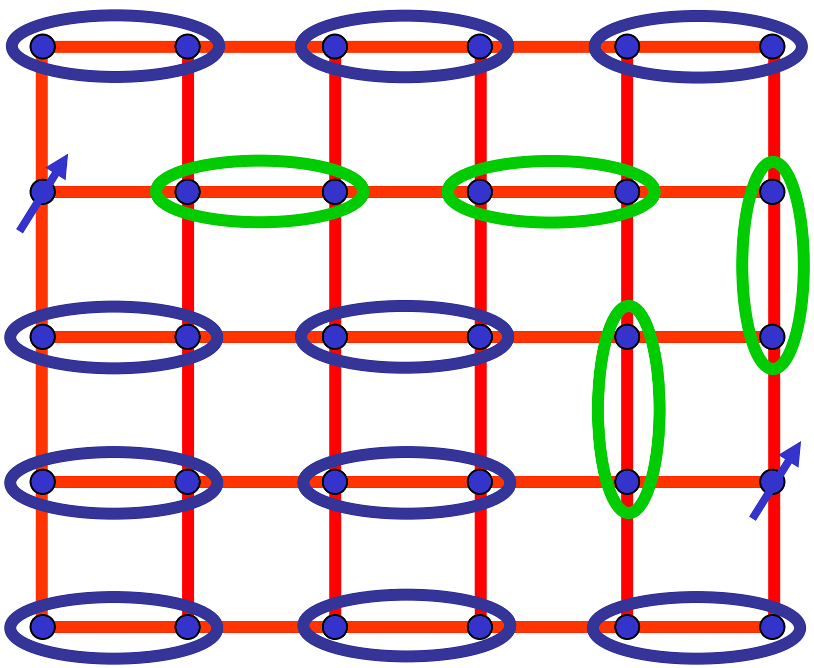

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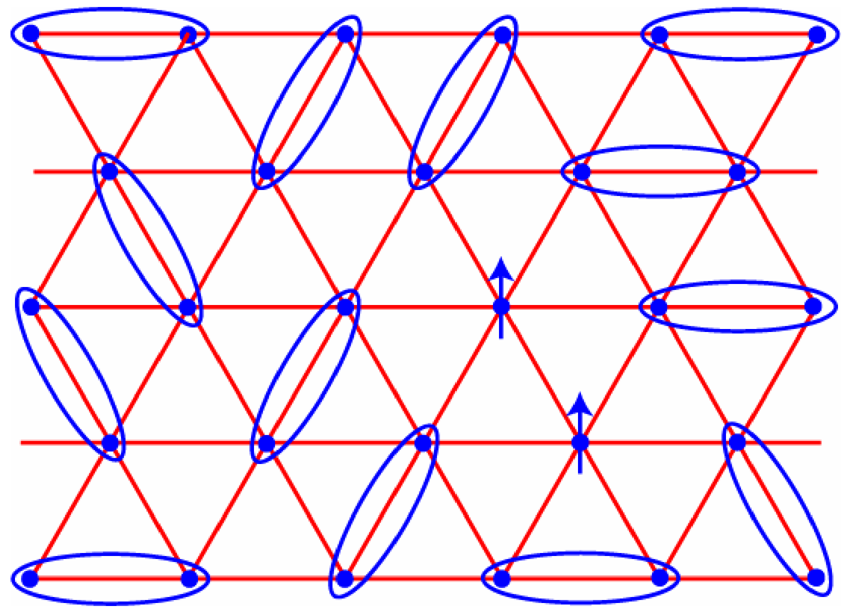

Excitations of the paramagnet with non-zero spin

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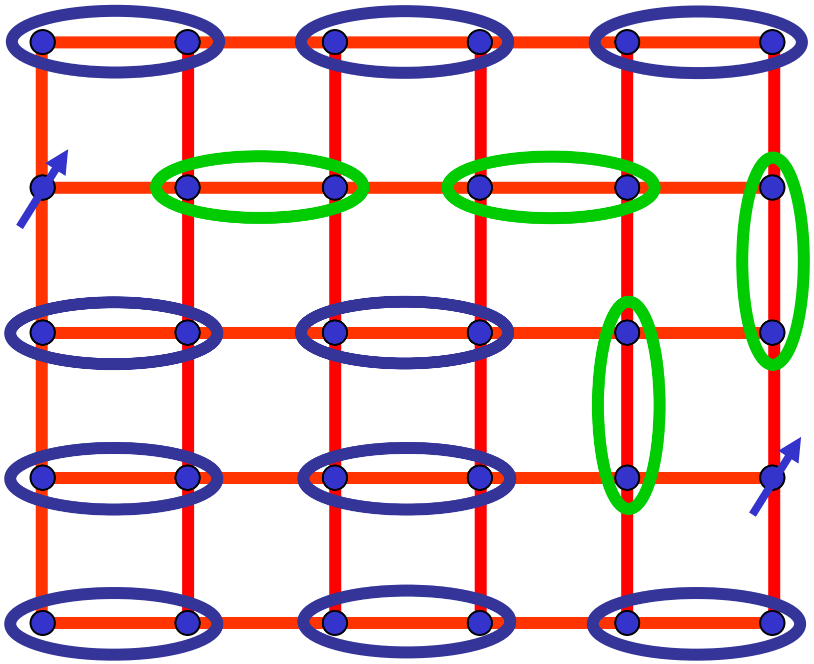

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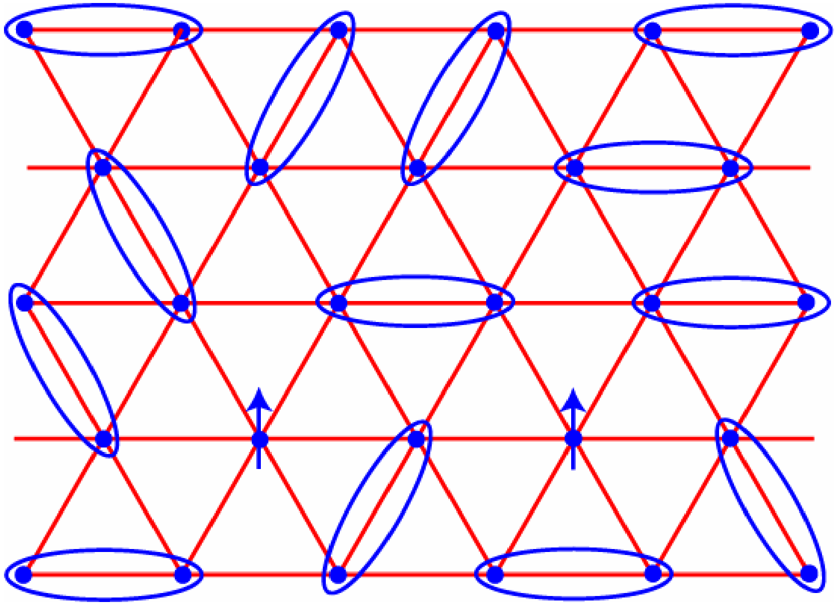

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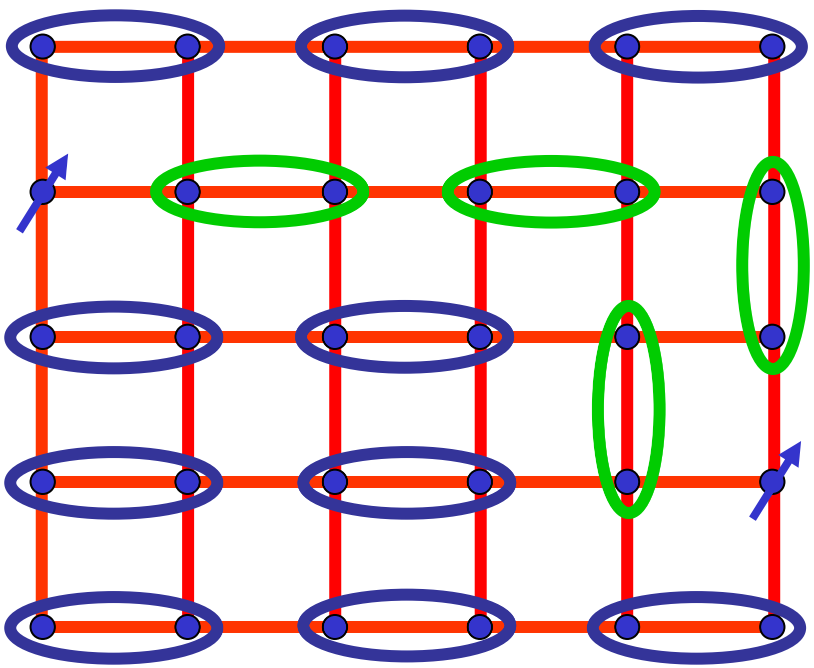

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N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

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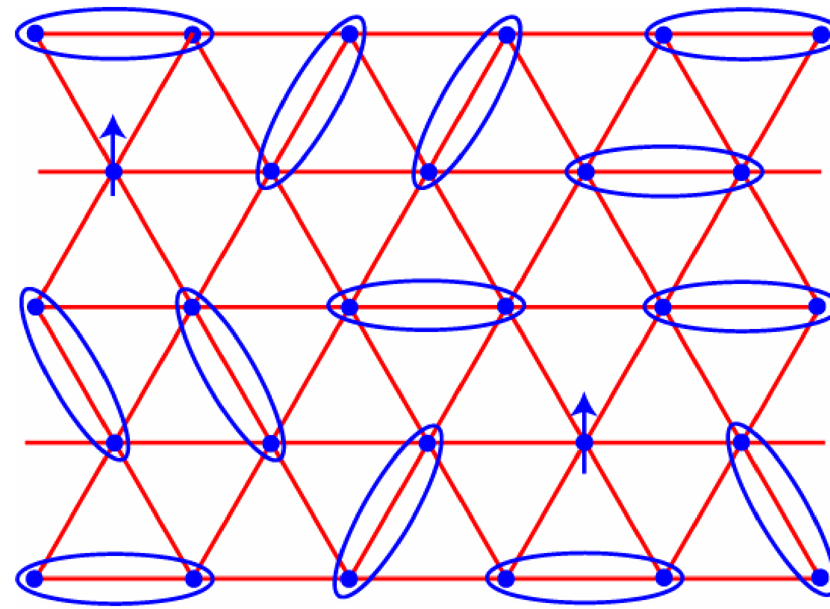
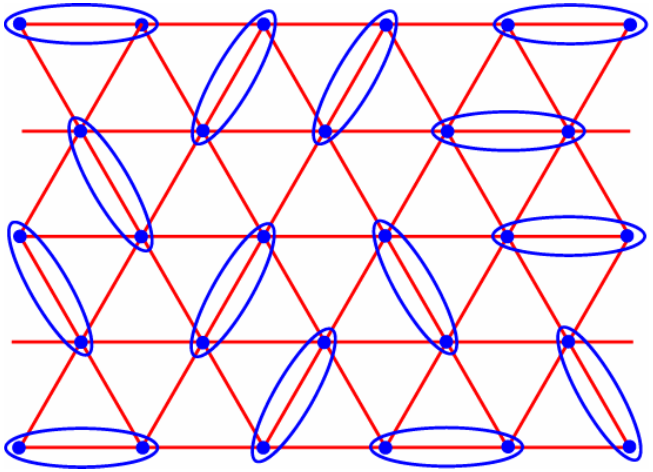


$S = 1/2$ **spinons** f_σ are deconfined and interact with Z_2 (non-collinear spins, $d = 2,3$) or U(1) (collinear spins, $d = 3$) gauge forces

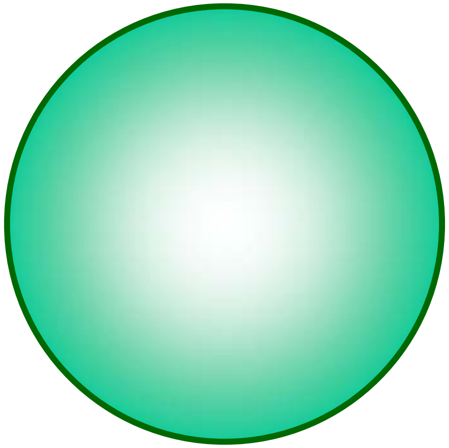
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991); X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).

Influence of conduction electrons



Local moments f_σ

+



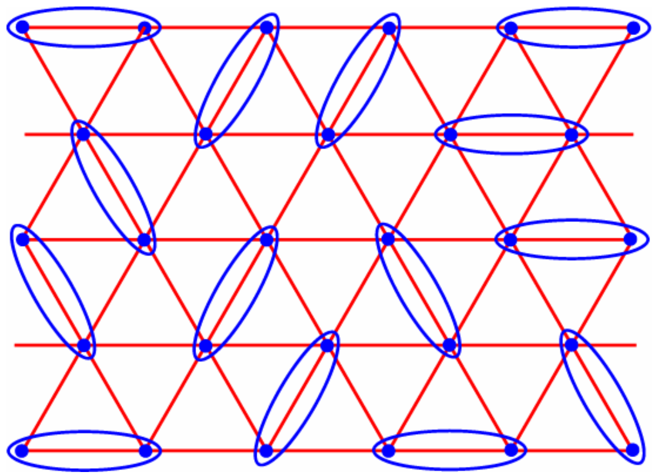
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



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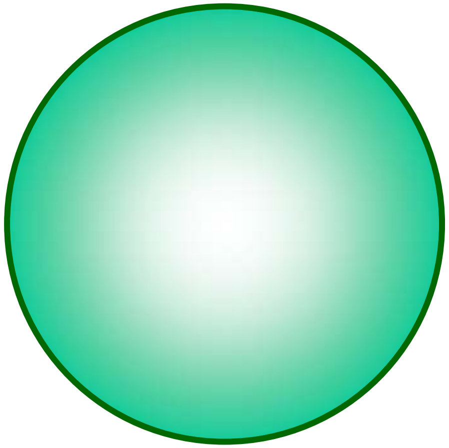


Conduction electrons c_σ

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by **$(n_T - 1) = n_c (\mathrm{mod}\ 2)$,** and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

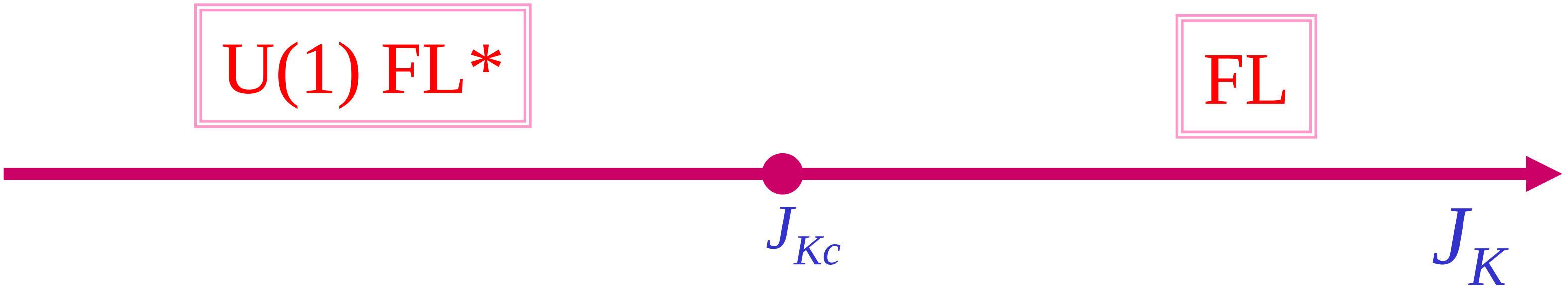
The state has "*topological order*" and associated neutral excitations. The topological order can be detected by the violation of Luttinger's Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= \left(n_T - 1 \right) \left(\text{mod } 2 \right)$$

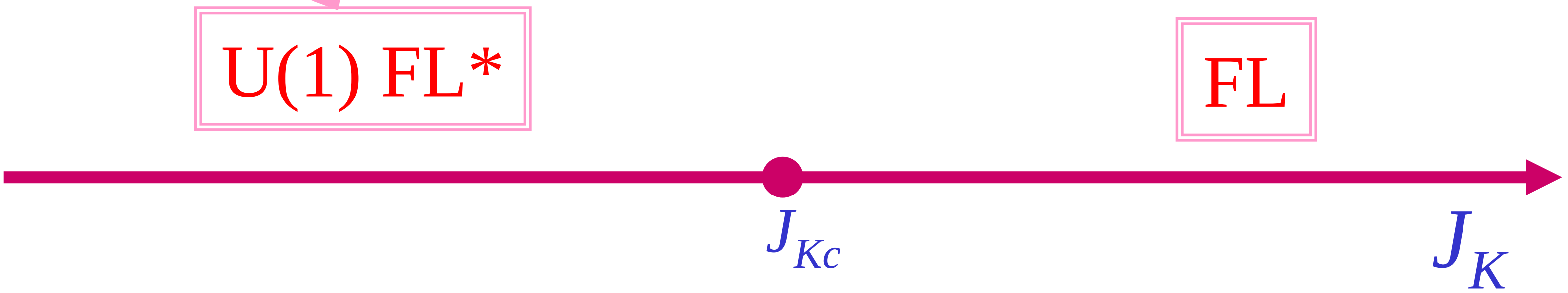
Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett*. **62**, 595 (1989).
Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica* B **182**, 201 (1992).
Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev*. B 66, 045111 (2002).
L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev*. B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).
F. H. L. Essler and A. M. Tsvelik, *Phys. Rev*. B **65**, 115117 (2002).

Phase diagram (U(1), d=3)



Phase diagram (U(1), *d*=3)

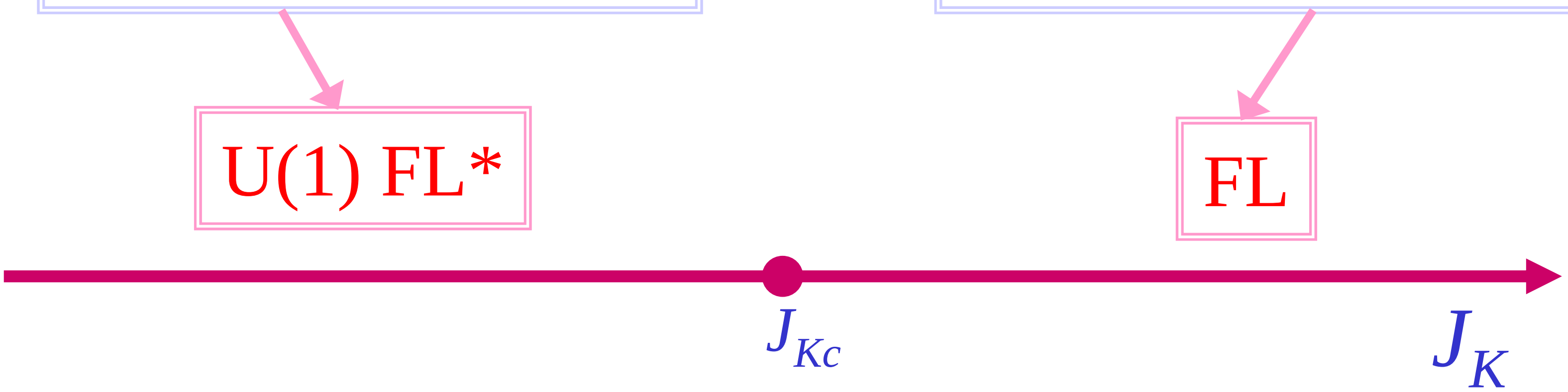
Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.



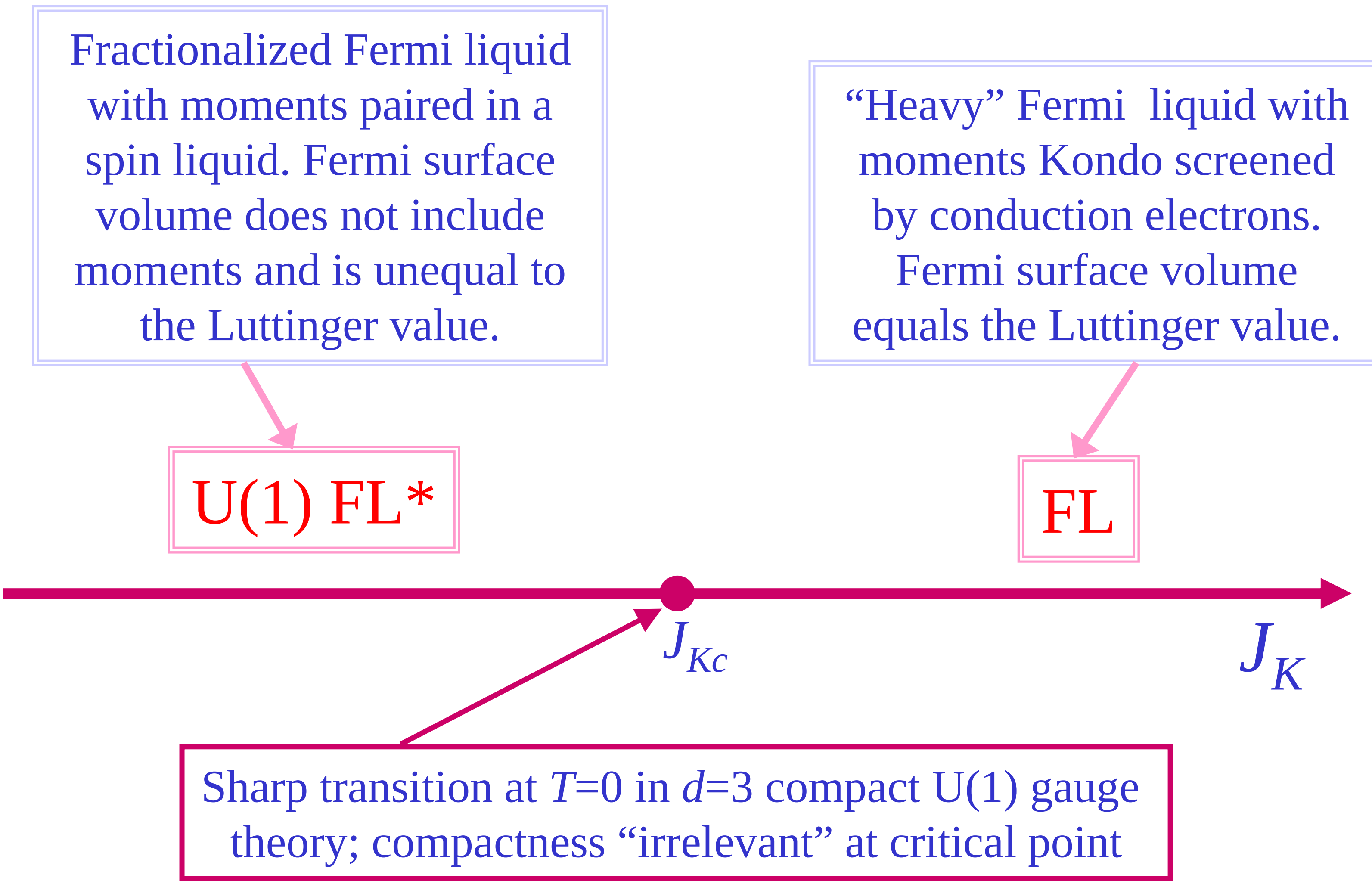
Phase diagram (U(1), d=3)

Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

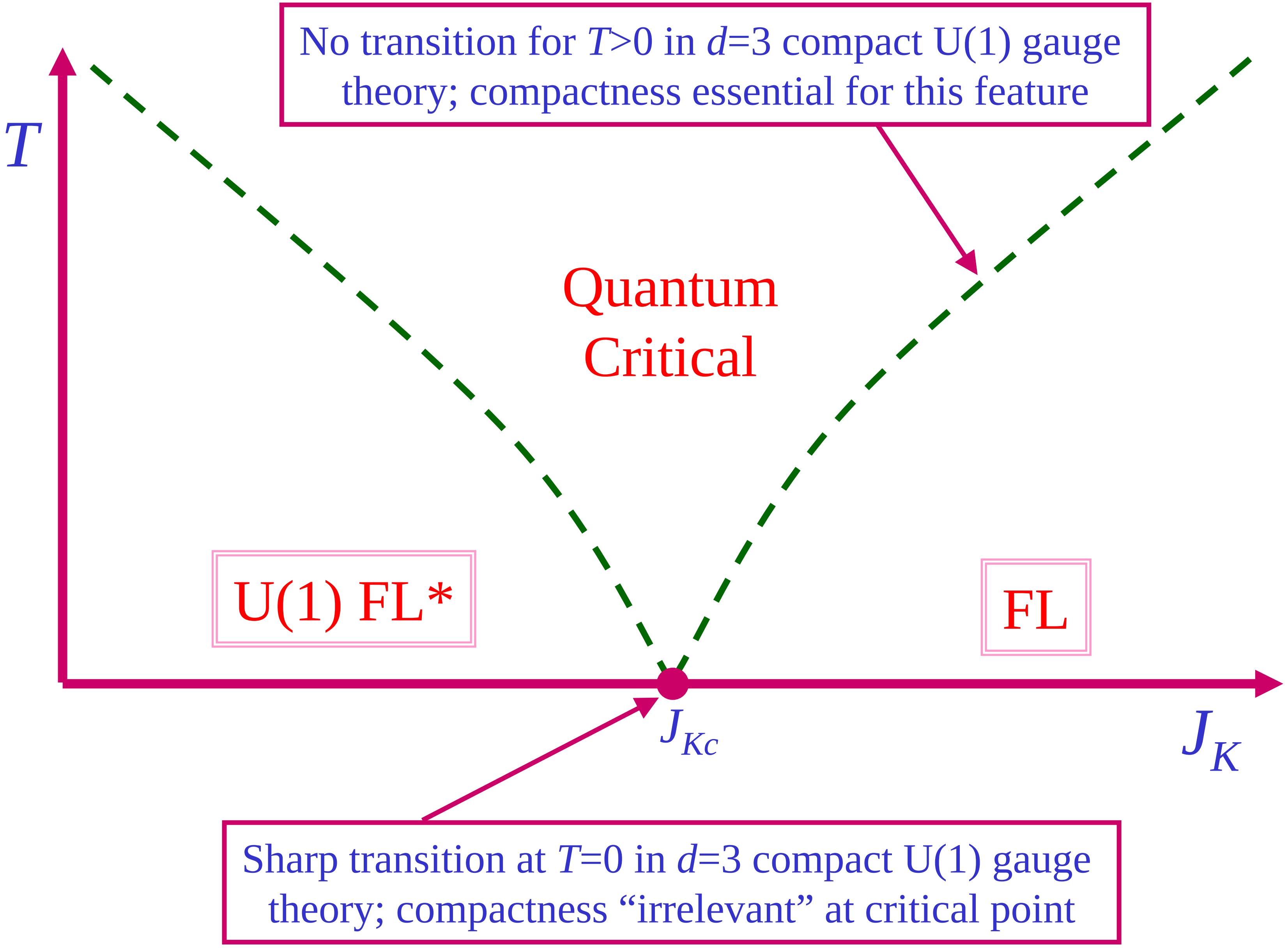
"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.



Phase diagram (U(1), d=3)



Phase diagram (U(1), *d*=3)



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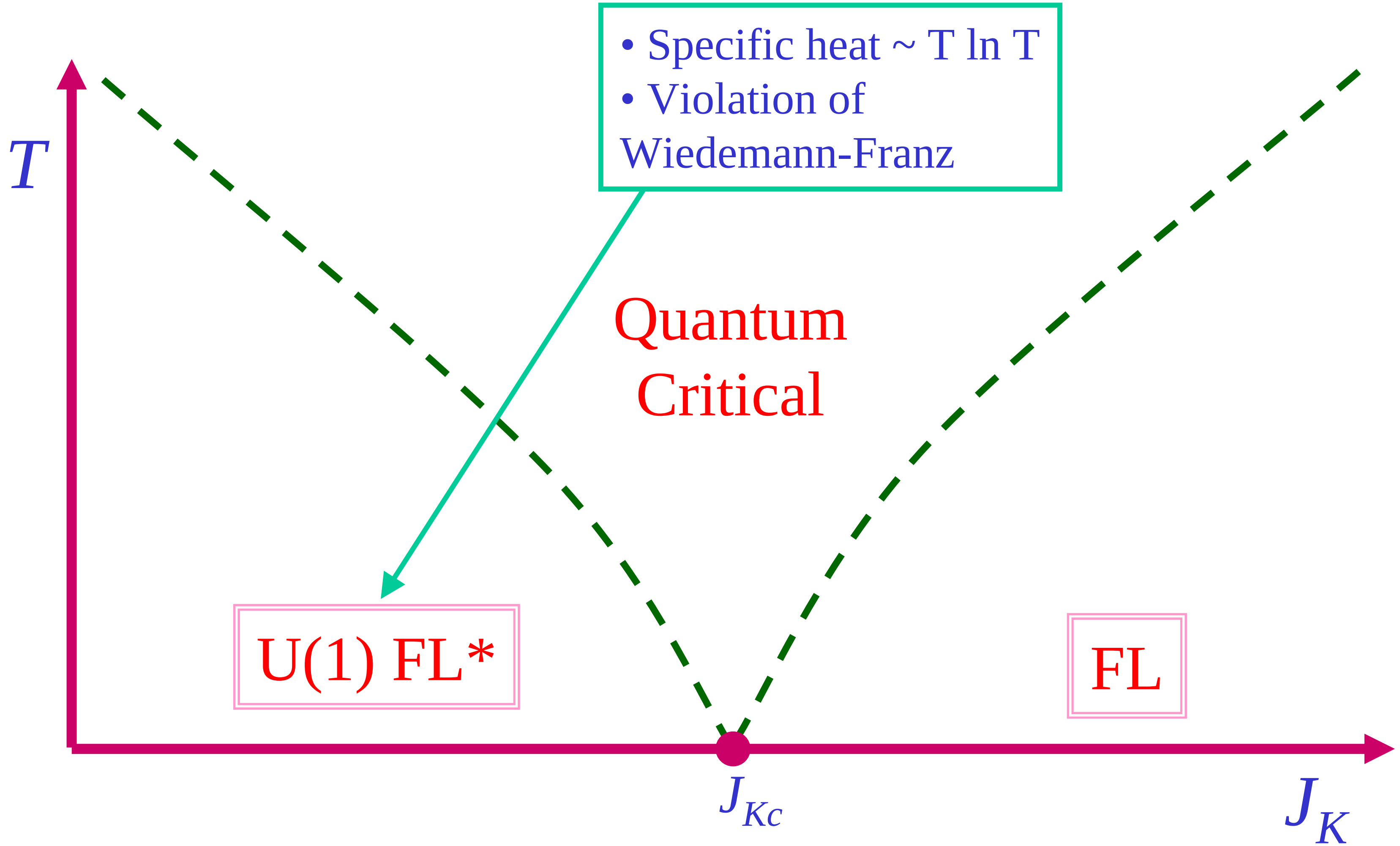
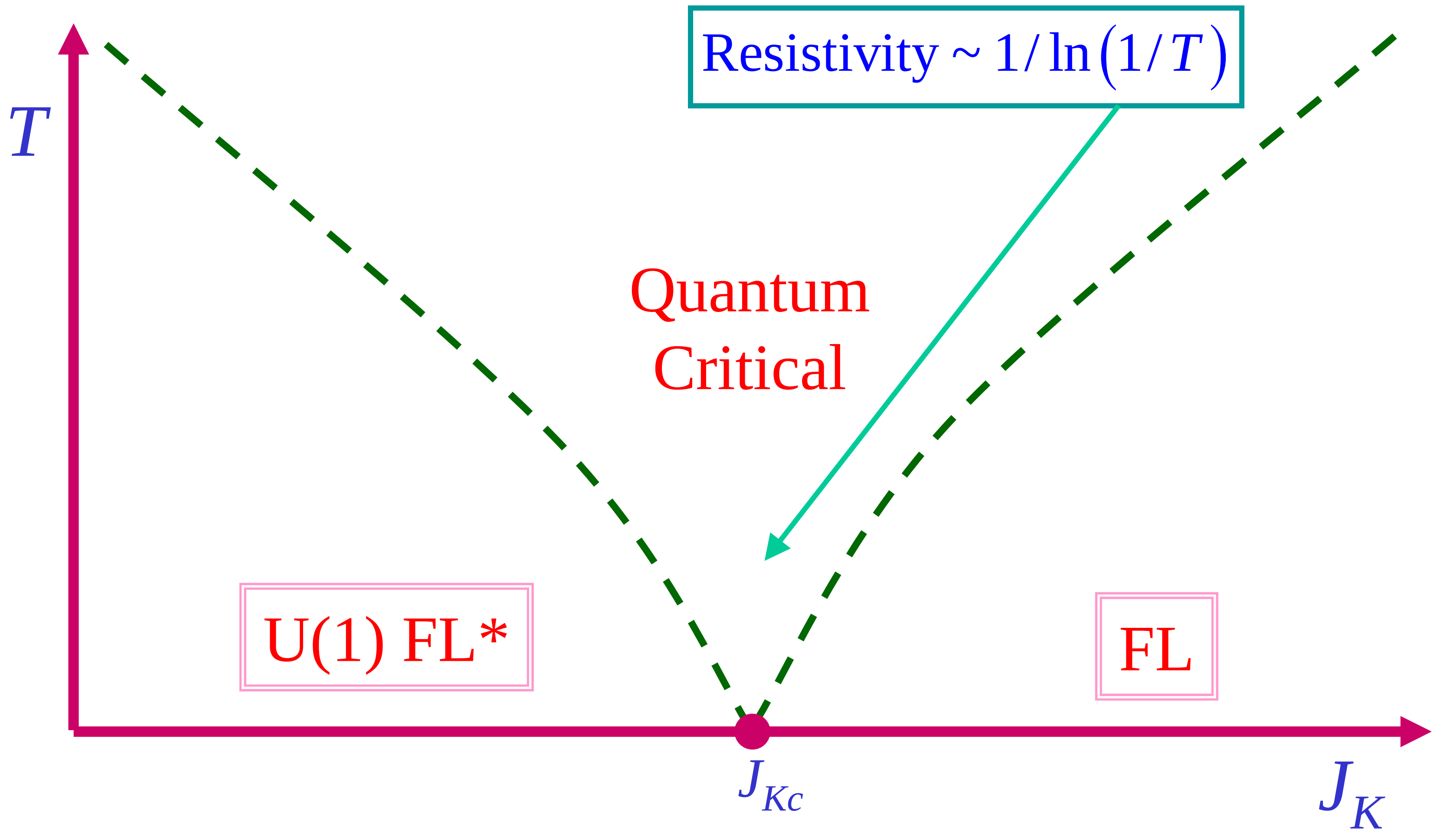


Phase diagram (U(1), *d*=3)



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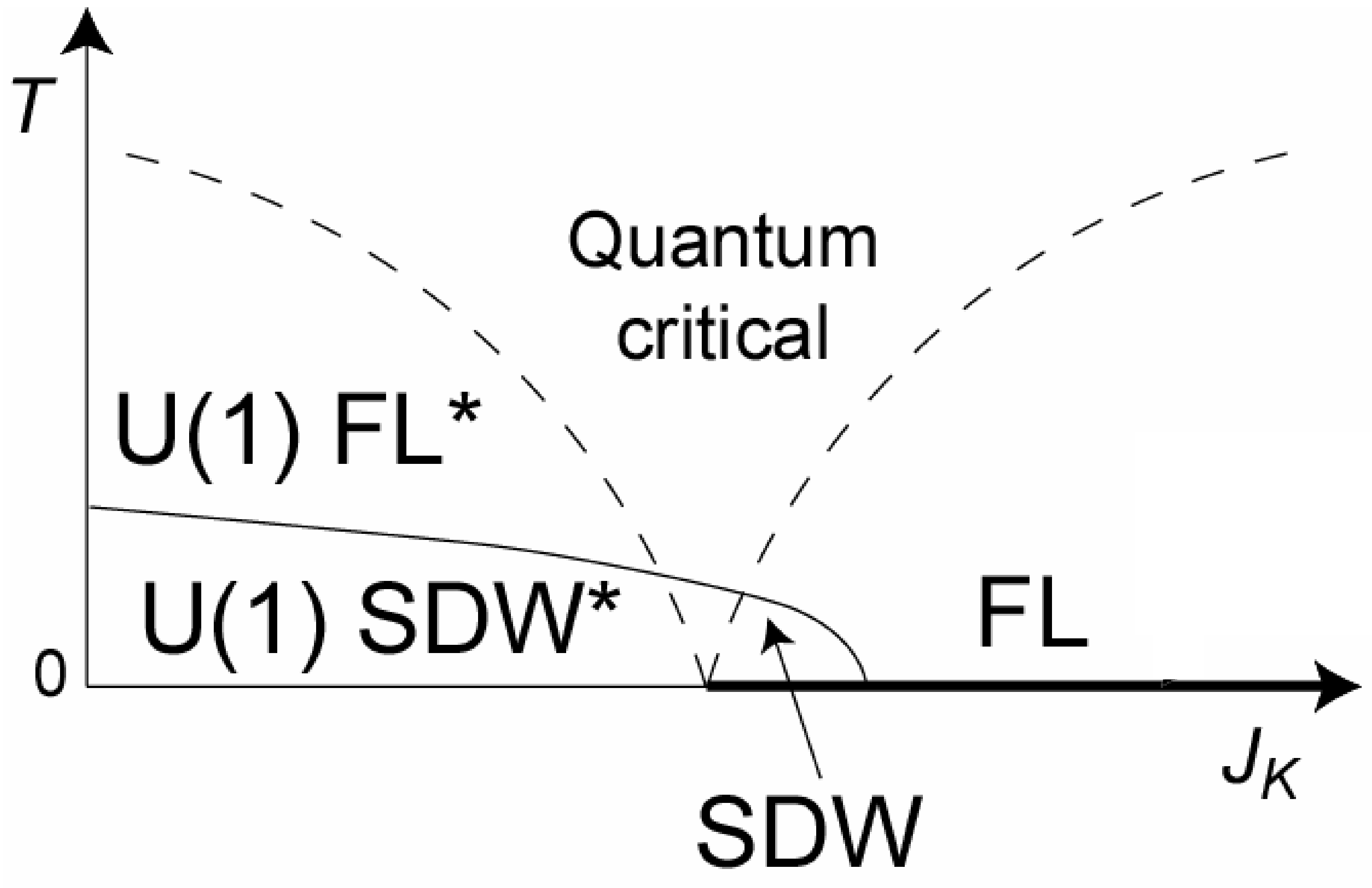


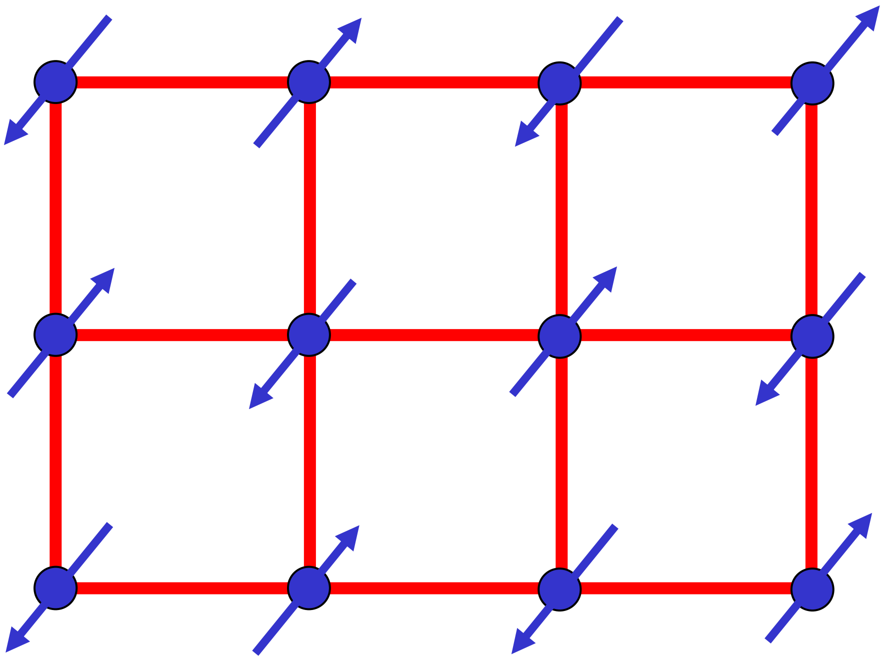
(D) Deconfined quantum criticality

Berry phases, bond order, and the breakdown of the LGW paradigm

All phases have conventional order, but gauge excitations and fractionalizion emerge at the quantum critical point.

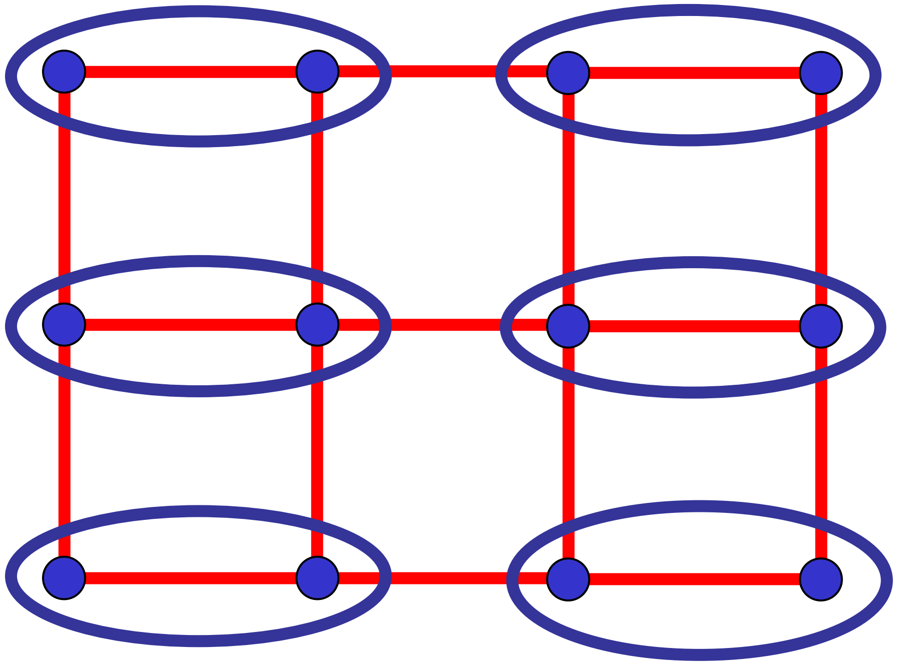
Talks by T. Senthil (N20.008) and L. Balents (N20.009)

Mott insulator with one S=1/2 spin per unit cell



Ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Mott insulator with one S=1/2 spin per unit cell



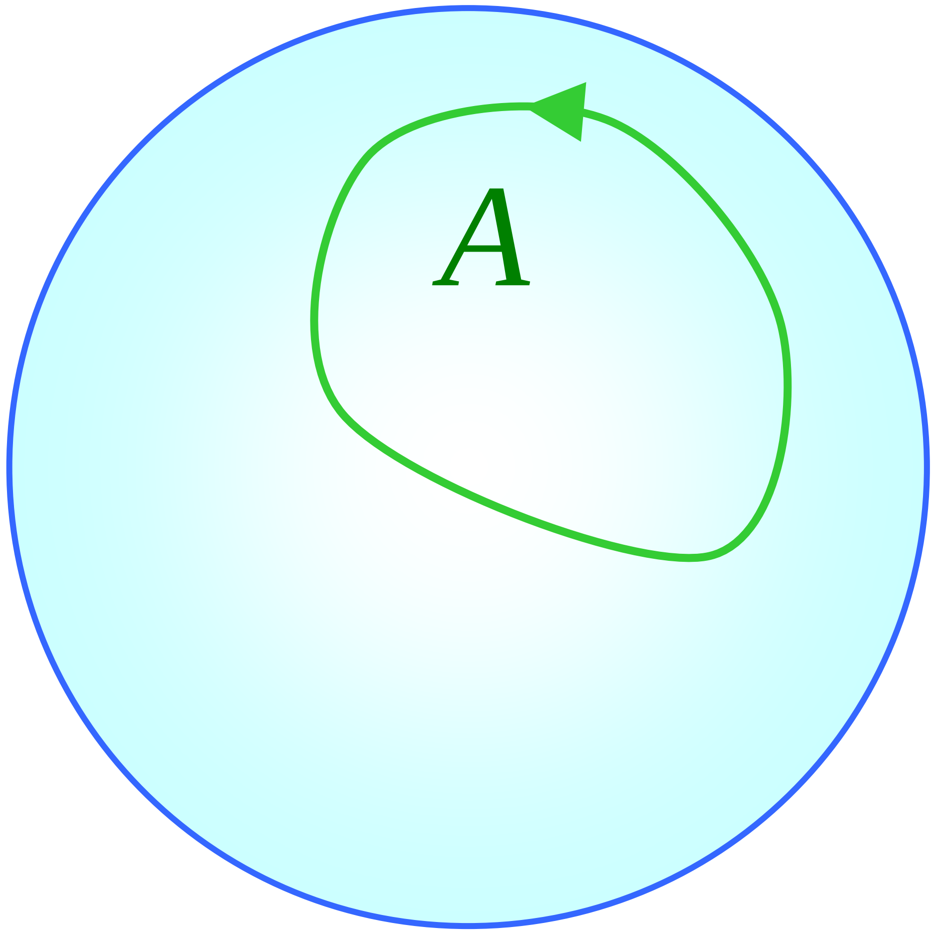
Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange. The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0, \ \langle \Psi_{\text{bond}} \rangle \neq 0$

Quantum theory for destruction of Neel order

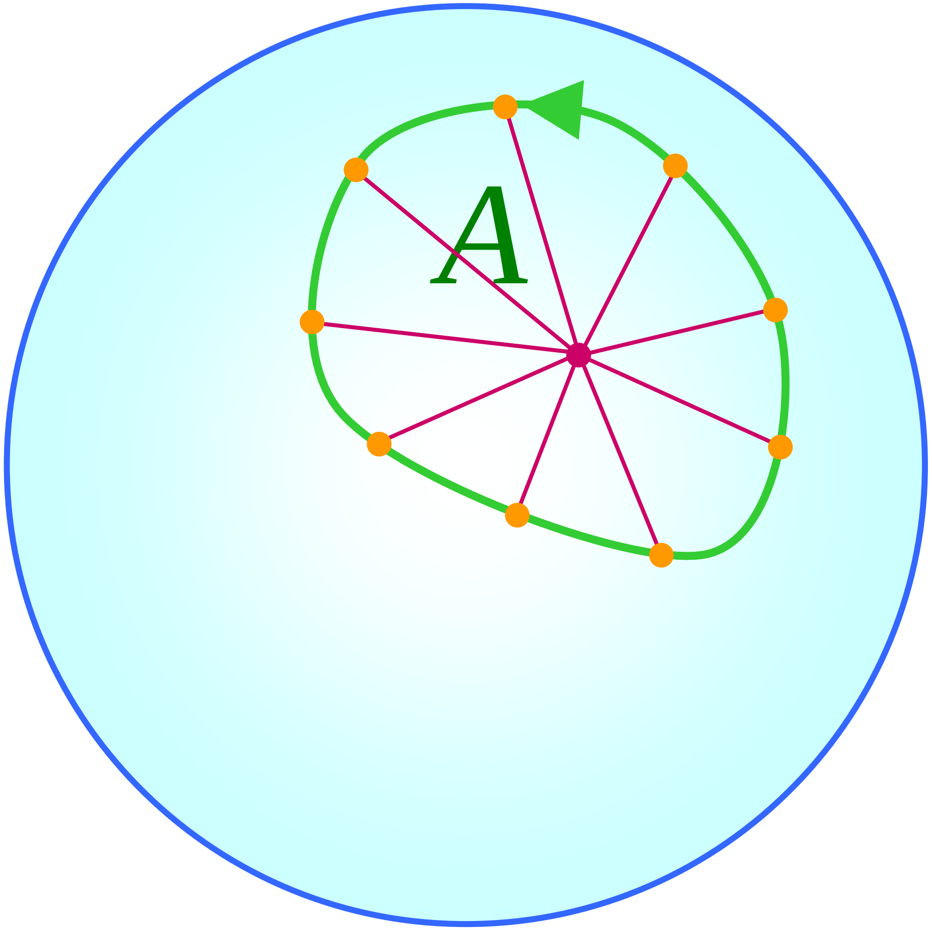
Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

Quantum theory for destruction of Neel order

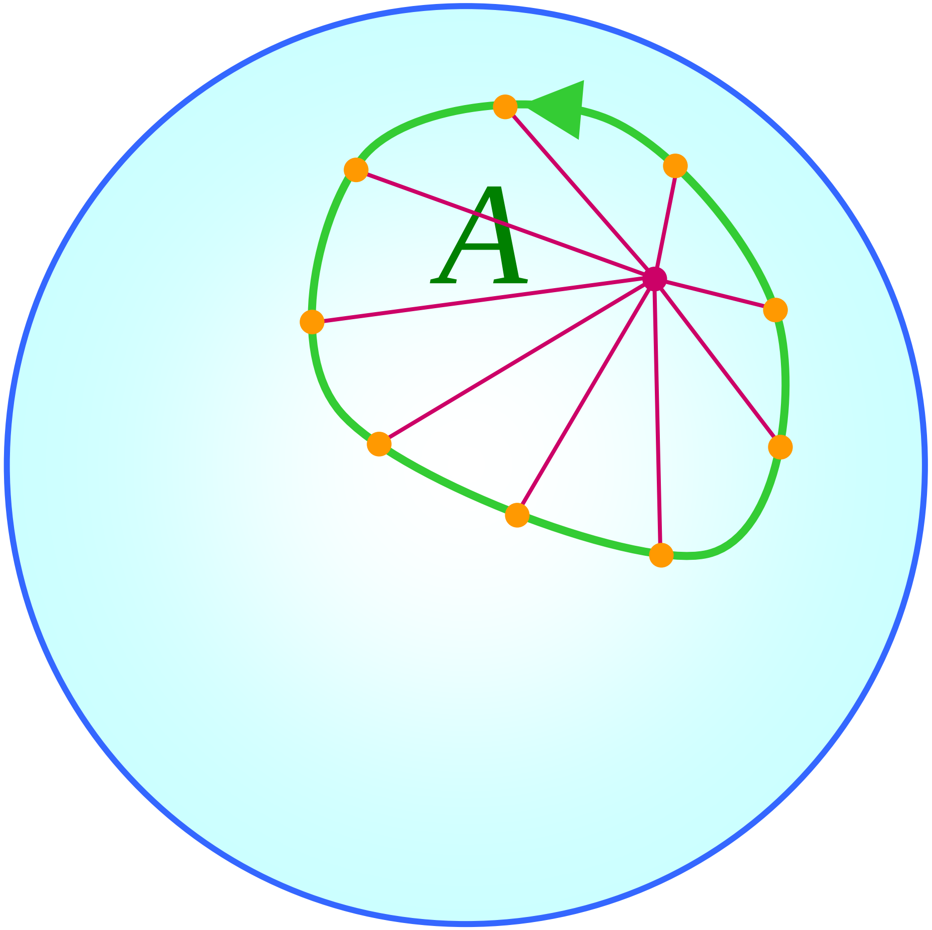
Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

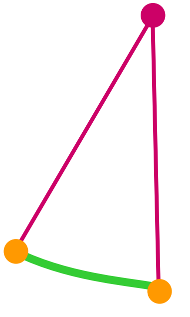
Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



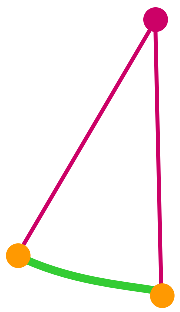
$$e^{iSA}$$

Quantum theory for destruction of Neel order



Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

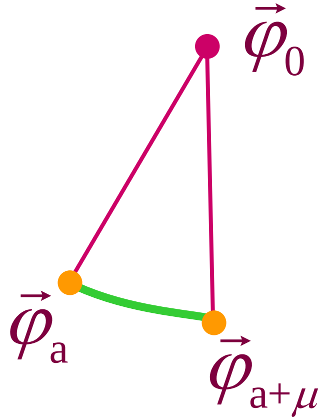


Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\mathrm{a}} = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_{\mathrm{a}} = (0,0,1)$ in classical Neel state;

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices ;



$(\mu = x, y, \tau)$

Quantum theory for destruction of Neel order

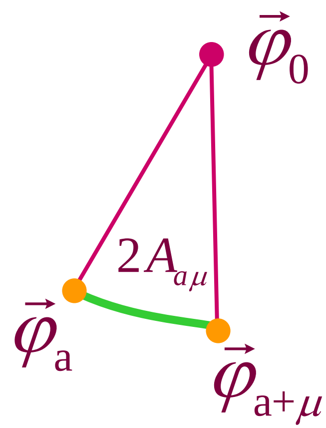
Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_a = (0,0,1)$ in classical Neel state;

$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

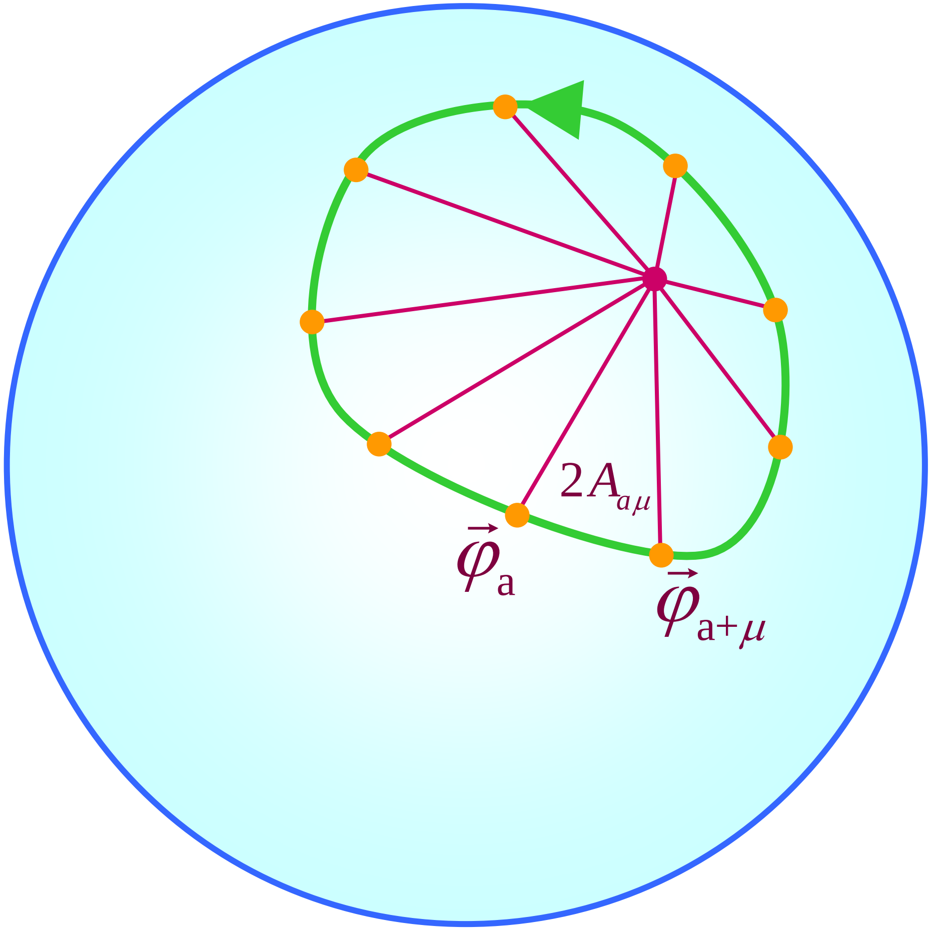


$A_{a\mu}$ transforms like a compact U(1) gauge field

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$\exp\left(i \sum_a \eta_a A_{a\tau} \right)$$

Sum of Berry phases of all spins on the square lattice.

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

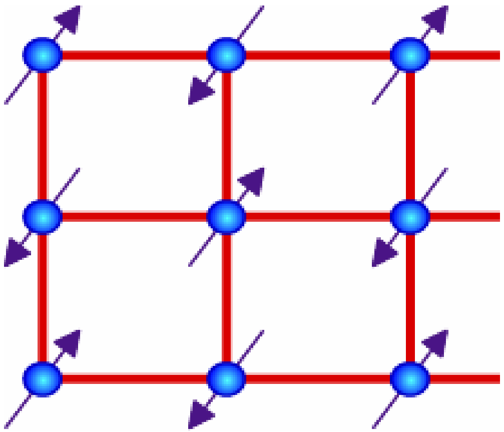
Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

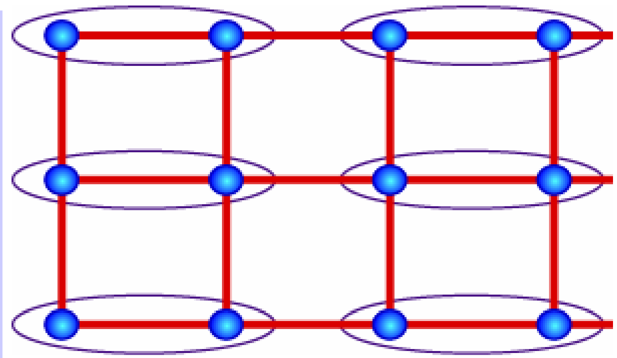
$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$



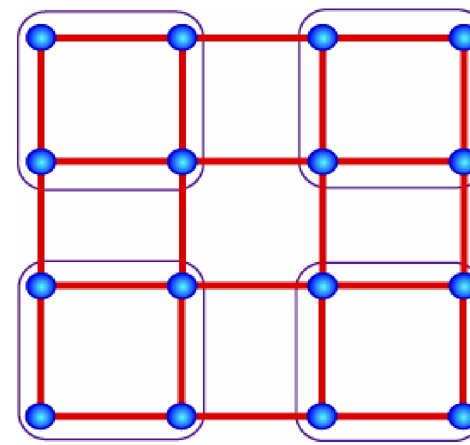
Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



or



Bond order

$\langle \Psi_{\text{bond}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0 $\longrightarrow$ g

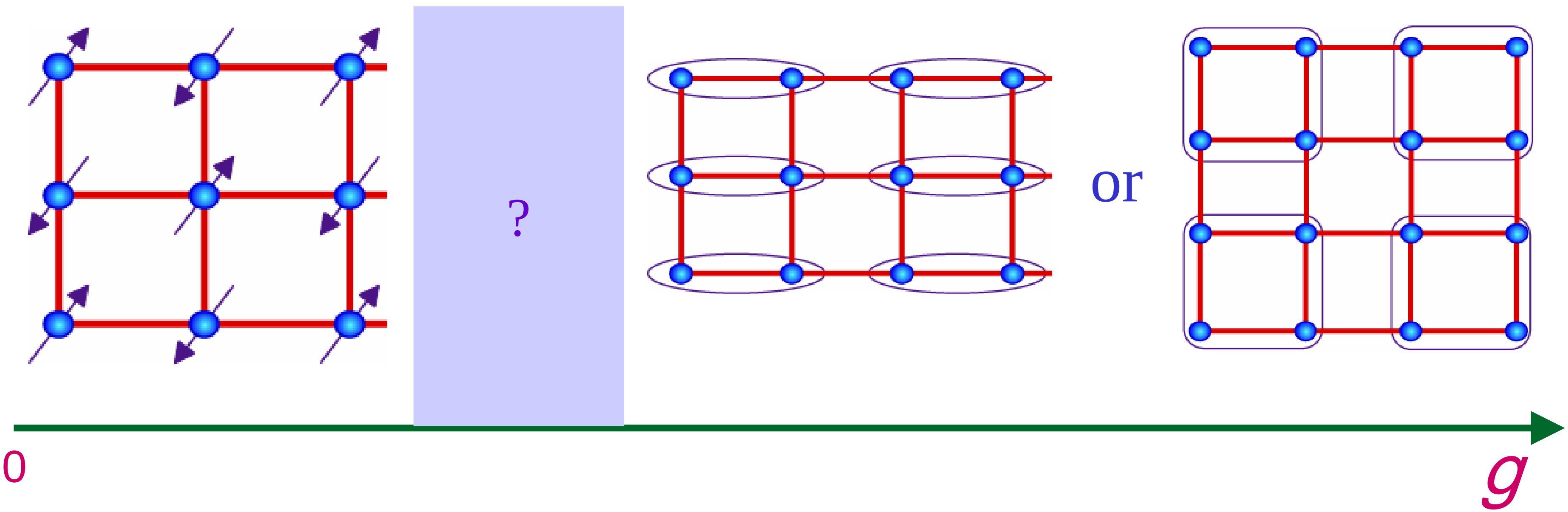
Alternative formulation to describe transition:

Express theory in terms of a complex spinor $z_{a\alpha}$, $\alpha = \uparrow, \downarrow$, with

$$\vec{\varphi}_a = z^*_{a\alpha} \vec{\sigma}_{\alpha\beta} z_{a\beta}$$

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} \right)$$



S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

Theory of a second-order quantum phase transition between Neel and bond-ordered phases

At the quantum critical point:

- $A_\mu \to A_\mu + 2\pi$ periodicity can be ignored

(Monopoles interfere destructively and are dangerously irrelevant).

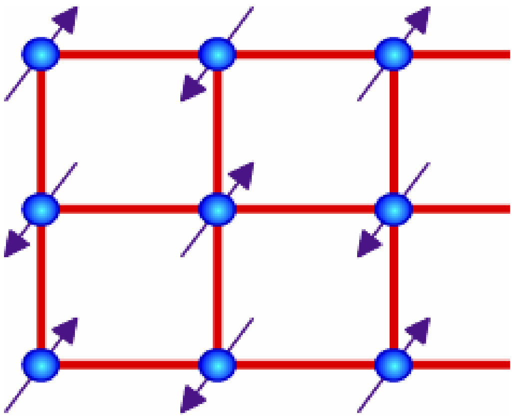
- $S = 1/2$ *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters ($\vec{\varphi}$ and Ψ_{bond}) are "composites" and of secondary importance

S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990); G. Murthy and S. Sachdev, *Nuclear Physics* B **344**, 557 (1990); C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001); S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002); O. Motrunich and A. Vishwanath, cond-mat/0311222.

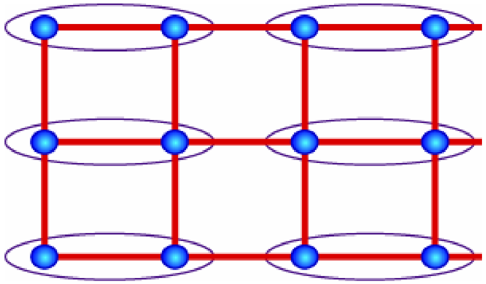
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of S=1/2 square lattice antiferromagnet

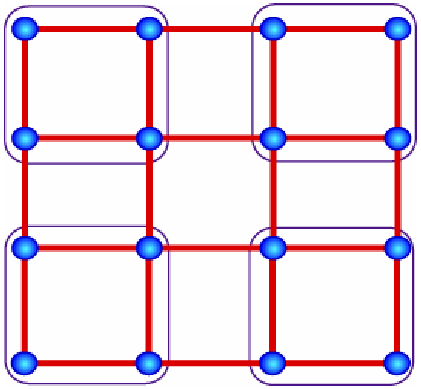


Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



or



Bond order $\langle \Psi_{\text{bond}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left| (\partial_\mu - i A_\mu) z_\alpha \right|^2 + r \left| z_\alpha \right|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Conclusions

I. New FL* phase with a Fermi surface of electron-like quasiparticles (whose volume violates the Luttinger theorem), topological order, emergent gauge excitations, and neutral fractionalized quasiparticles.

Novel quantum criticality in the transition between the FL and FL* phases (and associated SDW and SDW* phases)

Conclusions

II. Theory of quantum phase transitions between magnetically ordered and paramagnetic states of Mott insulators:

A. *Dimerized Mott insulators*: Landau-Ginzburg-Wilson theory of fluctuating magnetic order parameter.

B. S=1/2 square lattice: Berry phases induce bond order, and LGW theory breaks down. Critical theory is expressed in terms of emergent fractionalized modes, and the *order parameters are secondary*.

Conclusions

III. Deconfined quantum criticality in conducting systems ?

Theory for FL-FL* transition could also apply to the FL-SDW transition between *conventional* phases.