Electrical transport near a pair-breaking superconductor-metal quantum phase transition

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See also talk by Daniel Podolsky,
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Talk online at http://sachdev.physics.harvard.edu
Standard Abrikosov-Gorkov theory for the suppression of the mean-field BCS critical temperature, $T_{c0}$, of a superconductor by a pair-breaking frequency $\alpha$:

$$\ln \left( \frac{T_c}{T_{c0}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{\hbar \alpha}{2\pi k_B T_c} \right)$$
Standard Abrikosov-Gorkov theory for the suppression of the mean-field BCS critical temperature, $T_{c_0}$, of a superconductor by a pair-breaking frequency $\alpha$:

$$\ln \left( \frac{T_c}{T_{c_0}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{\hbar \alpha}{2\pi k_B T_c} \right)$$

There is a critical $\alpha = \alpha_c$ such that $T_c = 0$ for $\alpha > \alpha_c$. We are interested in the nature of the crossovers near the quantum phase transition at $\alpha = \alpha_c$ especially in spatial dimensions $d = 1, 2$. 
Pairbreaking, $\alpha$ can be be due to a magnetic field, $H$, applied on a wire of radius $r$

$$\alpha = D(eHr/c)^2/4,$$

where $D$ is the Cooperon diffusion constant.

On a hollow cylinder with radii $r_1$ and $r_2$:

$$\alpha = D \left[ \frac{eH}{4c} \left[ -4n + \frac{eH}{c} (r_1^2 + r_2^2) \right] + \frac{n^2 \ln(r_2/r_1)}{r_2^2 - r_1^2} \right]$$

where $n$ is an integer. (A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. 94, 037003 (2005)).

Other sources of pairbreaking

- Parallel magnetic field, $H$, on a film of thickness $t$, $\alpha = D(eHt/c)^2/6$. Experiments by K. A. Parendo, K. H. Sarwa, B. Tan, and A. M. Goldman, cond-mat/0512704.


- Magnetic impurities.
I. Theory for the superconductor-metal quantum phase transition
At $T = 0$, the Maki-Thomson and density of states corrections to the conductivity, $\delta \sigma$, increase with increasing $\alpha$ (negative magnetoresistance):

$$\delta \sigma \sim (\alpha - \alpha_c)$$

Computation of fluctuation conductivity in metal at low temperatures

At $T = 0$, the Maki-Thomson and density of states corrections to the conductivity, $\delta \sigma$, increase with increasing $\alpha$ (negative magnetoresistance):

$$\delta \sigma \sim (\alpha - \alpha_c)$$


We will argue that these are corrections to scaling to the theory of the quantum critical point. These corrections are dangerously irrelevant, because they dominate at low $T$. 
Computation of fluctuation conductivity in metal at low temperatures

At $T > 0$, Aslamazov-Larkin corrections lead to

$$\delta \sigma \sim \frac{4e^2}{h} \frac{D^{2-d}(k_B T/\hbar)^2}{(\alpha - \alpha_c)^{6-d/2}}$$

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At $T > 0$, Aslamazov-Larkin corrections lead to

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We will argue these are contained in the quantum critical theory. Note, however, the leading critical fluctuations vanish at $T = 0$ for $\alpha > \alpha_c$. This leads to a non-monotonic $T$ dependence in critical theory.
Cooperon fluctuations have propagator $\sim 1/(Dq^2 + |\omega| + \alpha)$. Self-interactions between such fluctuations are described by

$$S_{\text{bulk}} = \int d^d x \left[ \int \frac{d\omega}{2\pi} \left( D |\nabla_x \psi(x, \omega)|^2 + (|\omega| + \alpha) |\psi(x, \omega)|^2 \right) + \frac{u}{2} \int d\tau |\psi(x, \tau)|^4 \right],$$

In one dimension, theory reduces to the Langer-Ambegaokar-McCumber-Halperin theory (Model A dynamics), near mean-field $T_c$.

$$\frac{\partial \psi}{\partial t} = - \left[ -D \frac{\partial^2 \psi}{\partial x^2} + \alpha \psi + u |\psi|^2 \psi \right]$$

+ thermal Langevin noise
Role of charge conservation in quantum critical theory

(related to the question of why dissipation is not $|\omega|q^2$)

Dynamics of quantum theory (and model A) does not conserve total charge.

Analogous the Fermi-liquid/spin-density-wave transition (Hertz theory), where dynamics of critical theory does not conserve total spin.
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Conservation laws place strong constraints for \(\omega/q \to \infty\), but can be ignored in the critical regime, where \(\omega/q \to 0\).


Cooper pairs (SDW) fluctuations decay into fermionic excitations at a finite rate, before any appreciable phase precession due to changes in chemical potential (magnetic field).
II. Quantum criticality in $d=1$
Quantum critical theory obeys strong hyperscaling properties in spatial dimensions \( d < 2 \). Exponents can be determined by an expansion in \( \epsilon = 2 - d \) in a theory with \( n \)-component fields (\( n = 2 \) here).

\[
\begin{align*}
    z &= 2 - \eta \quad ; \quad \eta = \frac{(n + 2)(12 - \pi^2)}{4(n + 8)^2} \epsilon^2 \\
    \nu &= \frac{1}{2} + \frac{(n + 2)}{4(n + 8)} \epsilon + \frac{(n + 2)(n^2 + (38 - 7\pi^2/6)n + 132 - 19\pi^2/3)}{8(n + 8)^3} \epsilon^2
\end{align*}
\]

Results at \( \epsilon = 1 \) in very good agreement with QMC simulations.
Theory for quantum-critical region, and beyond in $d=1$

In $d = 1$, conductivity of critical theory obeys universal scaling form:

$$\delta\sigma = \frac{4e^2}{h} \left( \frac{\hbar D}{k_B T} \right)^{1/z} \Phi_{\sigma} \left( \frac{\alpha - \alpha_c}{T^{1/(zu)}} \right)$$

where $\Phi_{\sigma}$ is a scaling function.
Theory for quantum-critical region, and beyond in $d=1$

Quantum critical $T$ dependence in $d = 1$:

$$\delta \sigma \sim \begin{cases} 
\frac{1}{T^{1/z}} & \text{for } T > (\alpha - \alpha_c)^{z\nu} \\
\frac{T^2}{(\alpha - \alpha_c)^{(2z+1)\nu}} & \text{for } T < (\alpha - \alpha_c)^{z\nu}
\end{cases}$$

Non-monotonic dependence on $T$. 
III. Nanowires near the superconductor-metal quantum critical point
Now the conductance, $g$, of the wire is universal

$$g = \frac{4e^2}{\hbar} F(\omega L^{1/z})$$

where $L$ is the length of the wire, and $L < (\hbar D/k_B T)^{1/2}$.  

Nanowires near the quantum critical point in $d=1$
Effect of the leads

\[ S_{\text{lead}} = \int d\tau \left[ -H^* \psi(0, \tau) - H \psi^*(0, \tau) + C |\Psi(0, \tau)|^2 \right] \]

where \( H \neq 0 \) for a superconducting lead.

Both \( H \) and \( C \) scale to strong-coupling, and therefore we have Dirichlet boundary conditions (\( \Psi = 0 \)) for a N lead, and Fixed boundary conditions for a S lead.

Conductance is independent of the specific bare values of \( H \) and \( C \).
Large $n$ computation of conductance

\[ g = \frac{4e^2}{h} F_X(y) \quad ; \quad y = c_1 \omega L^z \]
Quantum Monte Carlo and large $n$ computation of d.c. conductance

$g = \frac{4e^2}{\hbar} C_{SN}$
IV. Quantum criticality in $d=2$
Theory for quantum-critical region, and beyond in $d=2$

To leading logarithmic accuracy, (nearly) all physical properties can be expressed in terms of computable universal functions of two energy scales, $R$, and $U$. $R$ measures distance from the quantum critical point, while $U$ is a quartic self-coupling. These are parameters in a classical theory of equal-time correlations with free energy

$$F = \int d^2 x \left[ |\nabla \Psi|^2 + \tilde{R} |\Psi|^2 + \frac{U}{2} |\Psi|^4 \right].$$

$R$ and $U$ depend upon the bare values of $\alpha$, $D$, $T$, and logarithmically on a cutoff energy scale $\Lambda$, and are determined by solving a simple integral equation.

The loci of points with a fixed $U/R$ has the same physical properties, upto a shift in the overall energy scale, $R$. 
Locus of points with U/R constant

The Kosterlitz-Thouless transition occurs at $T = T_{KT}$, where $U/R \approx 34$ (a universal number).
Locus of points with U/R constant

The conductivity obeys the scaling form

$$\sigma = \frac{4e^2}{\hbar} \frac{k_B T}{R} \Phi_A \left( \frac{U}{R} \right)$$

where $\Phi_A$ is a completely universal function which can be (numerically) determined by a classical, continuum Model A theory.
Locus of points with U/R constant

At the quantum-critical point

\[ R \sim \frac{k_B T}{\ln(\Lambda/(k_B T))} \]
\[ \frac{U}{R} \sim \frac{1}{\ln \ln(\Lambda/(k_B T))} \]
\[ \sigma \sim \frac{4e^2}{h} \ln(\Lambda/(k_B T)) \]
Conclusions

• Universal transport in wires near the superconductor-metal transition

• Theory includes contributions from thermal and quantum phase slips ---- reduces to the classical LAMH theory at high temperatures

• Sensitivity to leads should be a generic feature of the "coherent" transport regime of quantum critical points.

• Complete computation of electrical transport in $d=2$ to leading logarithmic accuracy.