Transport without quasiparticles in graphene and Weyl semi-metals

X05:3
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Talk online: sachdev.physics.harvard.edu
Graphene

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Graphene

\[ \varepsilon_{\vec{k}} = \hbar v_F |\vec{k}| \]
Graphene at half-filling; no impurities

Low energy theory has 4 two-component Dirac fermions, $\psi_\sigma$, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$S = \int d^2r d\tau \psi_\sigma^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma$$

$$+ \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_\sigma^\dagger \psi_\sigma'(r')$$

Dimensionless “fine-structure” constant $\alpha = e^2/(\hbar v_F)$.

RG flow of $\alpha$:

$$\frac{d\alpha}{dl} = -\alpha^2 + \ldots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$
Conductivity is finite without impurities and with particle-hole symmetry, but thermal conductivity is infinite.
Density correlations in CFTs at \( T > 0 \)

Two-point density correlator, \( \chi(k, \omega) \)

Kubo formula for conductivity \( \sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega) \)

For all CFT3s, at \( \hbar \omega \gg k_B T \)

\[
\chi(k, \omega) = K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} ; \quad \sigma(\omega) = K
\]

where \( K \) is a universal dimensionless number (in units of \( e^2 / \hbar \)) characterizing the CFT3, and \( v \) is the velocity of “light”.
Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

However, for all CFT3s, at $\hbar \omega \ll k_B T$, we have “phase” randomizing collisions and relaxation to local thermodynamic equilibrium. This leads to the hydrodynamic behavior

$$\chi(k, \omega) = \chi_c \frac{Dk^2}{Dk^2 - i\omega} ; \quad \sigma(\omega) = D\chi_c \equiv \sigma_Q$$

where $\chi_c$ is the compressibility and $D$ is the diffusion constant, and $\sigma_Q$ is the conductivity = a dimensionless number times $(e^2 / \hbar)$. (We are ignoring logarithmic corrections from ‘long-time tails’ present when momentum is exactly conserved.)

Collisionless-hydrodynamic crossover in graphene

\[
\sigma_Q(\omega) = \begin{cases} 
\frac{e^2}{h} \left[ \frac{\pi}{2} + \mathcal{O} \left( \frac{1}{\ln(\Lambda/\omega)} \right) \right] & , \quad \hbar \omega \gg k_B T \\
\frac{e^2}{\hbar \alpha^2(T)} \left[ 0.760 + \mathcal{O} \left( \frac{1}{|\ln(\alpha(T))|} \right) \right] & , \quad \hbar \omega \ll k_B T \alpha^2(T)
\end{cases}
\]

where \( \alpha(T) \) is the \( T \)-dependent fine structure constant which obeys

\[
\alpha(T) = \frac{\alpha}{1 + (\alpha/4) \ln(\Lambda/T)} \sim 0 \to 0 \frac{4}{\ln(\Lambda/T)}
\]


Graphene
Electron
Fermi surface

μ > 0

Graphene
Graphene

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

Prediction: hydrodynamic theory without quasiparticles, similar to a perturbed CFT

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
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Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities
Graphene

Prediction: hydrodynamic theory without quasiparticles, similar to a perturbed CFT

Quantum critical Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

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Prediction: hydrodynamic theory without quasiparticles, similar to a perturbed CFT

\[ n \sim \sqrt{n} \left( 1 + \lambda \ln \Lambda \sqrt{n} \right) \]

\[ T(K) \]

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Wiedemann-Franz law in a Fermi liquid:

\[ L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{W \cdot \Omega}{K^2}. \]

Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{ee}, t \gg t_{ee}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left( F^{\text{ext}} \right)^{\mu\nu}, \quad \partial_\mu J^\mu = 0.$$ 

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}, J^\mu$ in perturbative parameter $l_{ee} \partial_\mu: n$ in terms of the velocity $u^\mu(x)$, the chemical potential $\mu(x)$ and the temperature $T(x)$

$$T^{\mu\nu} = P \eta^{\mu\nu} + (\epsilon + P) u^\mu u^\nu$$

$$J^\mu = Q u^\mu - \sigma_Q P^{\mu\rho} \left( \partial_\rho \mu - \frac{\mu}{T} \partial_\rho T - u^\nu F^{\text{ext}}_{\rho\nu} \right) + \cdots,$$

$$P^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

New (and only) transport co-efficient, $\sigma_Q: \text{“quantum critical” conductivity at } Q = 0.$
From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} , \quad \gamma = \sigma Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

**Longitudinal conductivity**

$$\sigma_{xx} = \sigma Q \left[ \frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$
Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa / (T \sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2 / (3e^2)$.

For a strange metal with a “relativistic” Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H} \sigma_Q}\right), \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T} \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H} \sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H} \sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\text{imp}}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, “quantum critical” conductivity.
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where $\mathcal{H}$ is the enthalpy density, $\tau_{\text{imp}}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, “quantum critical” conductivity.

Note that the limits $Q \to 0$ and $\tau_{\text{imp}} \to \infty$ do not commute. For $Q = 0$ ($Q \neq 0$) when we take the clean limit $\tau_{\text{imp}} \to \infty$, the electrical conductivity is finite (diverges), and the thermal conductivity diverges (is finite).

M. Müller and S. Sachdev, PRB 78, 115419 (2008)
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$$L = \frac{v_F^2 \mathcal{H} \tau_{imp}}{T^2 \sigma_Q} \left( 1 + \frac{e^2 v_F^2 Q^2 \tau_{imp}}{\mathcal{H} \sigma_Q} \right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, “quantum critical” conductivity.

Note that for a clean system ($\tau_{imp} \to \infty$ first), the Lorentz ratio diverges $L \sim 1/Q^4$, as we approach “zero” electron density at the Dirac point.

M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Thermal Conductivity (nW/K)

Blue line: value for $L = L_0$

Red dots: data

J. Crossno et al., Science 351, 1058 (2016)
Red dots: data
Blue line: value for $L = L_0$
Temperature and density dependent electrical and thermal conductivity. (A) Electrical conductivity (blue) as a function of the charge density set by the back gate for different temperatures. At low temperature and/or high doping, the minimum conductivity (green) aligns with the temperature axis to the right. Solid black lines correspond to a linear fit of log(σ). At the CNP, the residual carrier density at the neutrality point (green) is estimated by the intersection of the minimum conductivity with the x-axis.

Red dots: data
Blue line: value for $L = L_0$
Graphene

Predicted strange metal

\[ T(K) \]

Quantum critical
Dirac liquid

\[ \sim \sqrt{n}(1 + \lambda \ln \Lambda \sqrt{n}) \]

\[ n = \frac{10^{12}}{m^2} \]

Hole Fermi liquid

Electron Fermi liquid

Impurity scattering dominates

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Strange metal in graphene

Wiedemann-Franz Law Violations in Experiment

J. Crossno et al., Science 351, 1058 (2016)
Strange metal in graphene

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Wiedemann-Franz Law Violations in Experiment

Wiedemann-Franz violated!
Lorentz ratio $L = \frac{\kappa}{(T\sigma)}$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}$$

$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities


J. Crossno et al., Science 351, 1058 (2016)
Comparison to theory with a single momentum relaxation time $\tau_{\text{imp}}$. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity.
Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential \( \mu(x) \) always obeys \( |\mu| \ll k_B T \), and so the entropy density \( s/k_B \) is much larger than the charge density \( |n| \); both electrons and holes are everywhere excited, and the energy density \( \epsilon \) does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder \( \xi \) is much larger than \( l_{ee} \), the electron-electron interaction length.

Note \( n \equiv Q \).

Numerically solve the hydrodynamic equations in the presence of a \( x \)-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, \( \eta \).
Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.

Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.


See also models with separately conserved particle and hole densities: Yunseok Seo, Geunho Song, Philip Kim, S. Sachdev, and Sang-Jin Sin, PRL 118, 036601 (2017)
Weyl semi-metals

Andrew Lucas

Richard Davison
Quasiparticle transport in Weyl metals

- **Weyl Hamiltonian:**

\[
H = \pm \hbar v_F \left( \sigma_x q_x + \sigma_y q_y + \sigma_z q_z \right).
\]

- **Berry flux** \( k = \pm 1 \) associated with this Hamiltonian:

\[
\mathcal{A}_i = i \langle q | \frac{\partial}{\partial q_i} | q \rangle, \quad \frac{1}{2\pi} \int d^3q \ \epsilon_{ijk} \mathcal{A}_i \partial_j \mathcal{A}_k = \pm 1
\]

- **Theorem:** net Berry flux must vanish on a lattice (BZ is compact) [Nielsen, Ninomiya (1983)]
Quasiparticle transport in Weyl metals

- consider applying $\mathbf{B} = B\mathbf{\hat{z}}$ to $k = 1$ Weyl fermion:

![Graph showing quasiparticle transport]

- quantum mechanical effect spoils current conservation: electromagnetic anomaly
Transport in Weyl metals

- consider applying $\mathbf{B} = B\mathbf{\hat{z}}$ to $k = 1$ Weyl fermion:

- quantum mechanical effect spoils current conservation: electromagnetic anomaly

- effect on classical hydrodynamics: [Son, Surówka (2009)]

$$\partial_\mu \langle J^\mu \rangle = -\frac{C}{8} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = C \mathbf{E} \cdot \mathbf{B}, \quad C = \frac{k}{4\pi^2}.$$
There is a similar anomaly in the flow of heat:

- **generation of heat:** [Lucas, Davison, Sachdev (2016)]

\[
\partial_i Q^i_a = 2G_a T \nabla T \cdot B.
\]

The heat flow can be accounted for by considering a background gravitational field (Luttinger 1964)
Hydrodynamics transport in Weyl metals

- In (3+1)d, an anomalous U(1) current in background fields generally obeys

\[ \nabla_\mu J^\mu = -\frac{C}{8} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{G}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} R^\alpha_\beta \mu \nu R^\beta_\alpha \rho \sigma, \]

\[ \nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu - \frac{G}{16\pi^2} \nabla_\mu \left( \varepsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^{\nu\mu}_{\alpha\beta} \right) \]

- C is the chiral anomaly coefficient.

- G is the mixed chiral-gravitational anomaly coefficient.

- The values of each coefficient depends on the matter content.

- For a single Weyl fermion,

\[ C = \pm \frac{1}{4\pi^2} \quad G = \pm \frac{1}{24} \]
Hydrodynamics transport in Weyl metals

• Assume that there is fast equilibration at each Weyl node, and slow scattering between Weyl nodes:

\[ \nabla_\mu J_\mu^a = -\frac{C_a}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{G_a}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\sigma} - \sum_b \left( R_{ab} \frac{\mu_b}{T_b} + S_{ab} \frac{1}{T_b} \right) \]

\[ \nabla_\mu T_{\mu\nu} = F_{\mu\nu} J_{\mu a} - \frac{G_a}{16\pi^2} \nabla_\mu \left( \epsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^\nu_{\alpha\beta} \right) + u^\nu \sum_b \left( U_{ab} \frac{\mu_b}{T_b} + V_{ab} \frac{1}{T_b} \right) \]

• The indices a,b label the different Weyl nodes.

\[ \sum_a C_a = 0 \quad \sum_a G_a = 0 \]

• R, S, U, V account for transfer of charge, energy and momentum between nodes. Their values depend on microscopic details.

• Express \( T_{\mu\nu} \) and \( J_\mu \) in terms of the velocity \( u^\mu(x) \), the chemical potential \( \mu(x) \) and the temperature \( T(x) \), constrained by positivity of entropy production.
Hydrodynamics transport in Weyl metals

- There are anomalous contributions to three conductivities
  \[ \sigma_{zz} = \sum_{a} \frac{n_{a}^2}{\Gamma_{a}} + s B^2 \]
  \[ \bar{\kappa}_{zz} = \sum_{a} \frac{T s_{a}^2}{\Gamma_{a}} + \hbar B^2 \]
  \[ \alpha_{zz} = \sum_{a} \frac{n_{a} s_{a}}{\Gamma_{a}} + a B^2 \]
- The 1st term in each is due to momentum relaxation at rate \( \Gamma_{a} \)
  \[ \Gamma_{a} = \frac{T_{0}^2 (s_{a} (\partial n_{a}/\partial \mu) - n_{a} (\partial s_{a}/\partial \mu))^{2}}{3 \sigma_{Q} (\epsilon_{a} + P_{a})^{2}} \]
  R. Davison, Schalm, Zaanen, 2013
- The 2nd is due to anomalies
  \[ s = T (C_{a} \ C_{a} \mu) \left( \begin{array}{cc} R_{ab} & -S_{ab} \\ -U_{ab} & V_{ab} \end{array} \right)^{-1} \left( \begin{array}{c} C_{b} \\ C_{b} \mu \end{array} \right) \]
  \[ a = 2 T^2 (0 \ G_{a}) \left( \begin{array}{cc} R_{ab} & -S_{ab} \\ -U_{ab} & V_{ab} \end{array} \right)^{-1} \left( \begin{array}{c} C_{b} \\ C_{b} \mu \end{array} \right) \]
- Negative electrical magnetoresistance due to the chiral anomaly.
- The mixed chiral-gravitational anomaly produces negative thermal magnetoresistance and anomalous thermoelectric resistance.
Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

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Conclusions:

- Dirac materials with Coulomb interactions have interactions of strength unity, $e^2/(\hbar v_F)$, leading to an inelastic scattering time $\sim \hbar/(k_B T)$.

- In sufficiently clean Dirac materials, there is hydrodynamic flow of the electron fluid.

- Evidence for such flow in graphene, and in other materials.