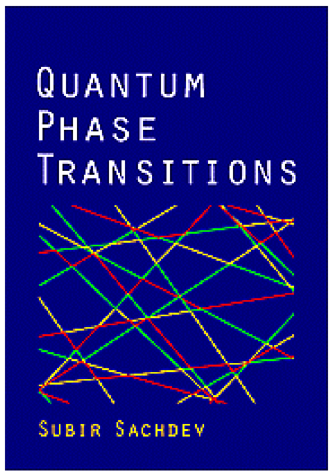
Quantum phase transitions of correlated electrons in two dimensions

Subir Sachdev

Science **286**, 2479 (1999).



Quantum Phase Transitions
Cambridge University Press

Transparencies online at
http://pantheon.yale.edu/~subir

1701 2001
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Outline

I. Coupled Ladder Antiferromagnet
- A. Ground states in limiting regimes
- B. Coherent state path integral
- C. Quantum field theory for critical point

II. Berry phases in one dimension
- S=1/2 quantum XY model.

III. Berry phases in two dimensions
- Bond-centered charge ("spin-Peierls") order.

IV. Magnetic transitions in *d*-wave superconductors
- Effect of an applied magnetic field

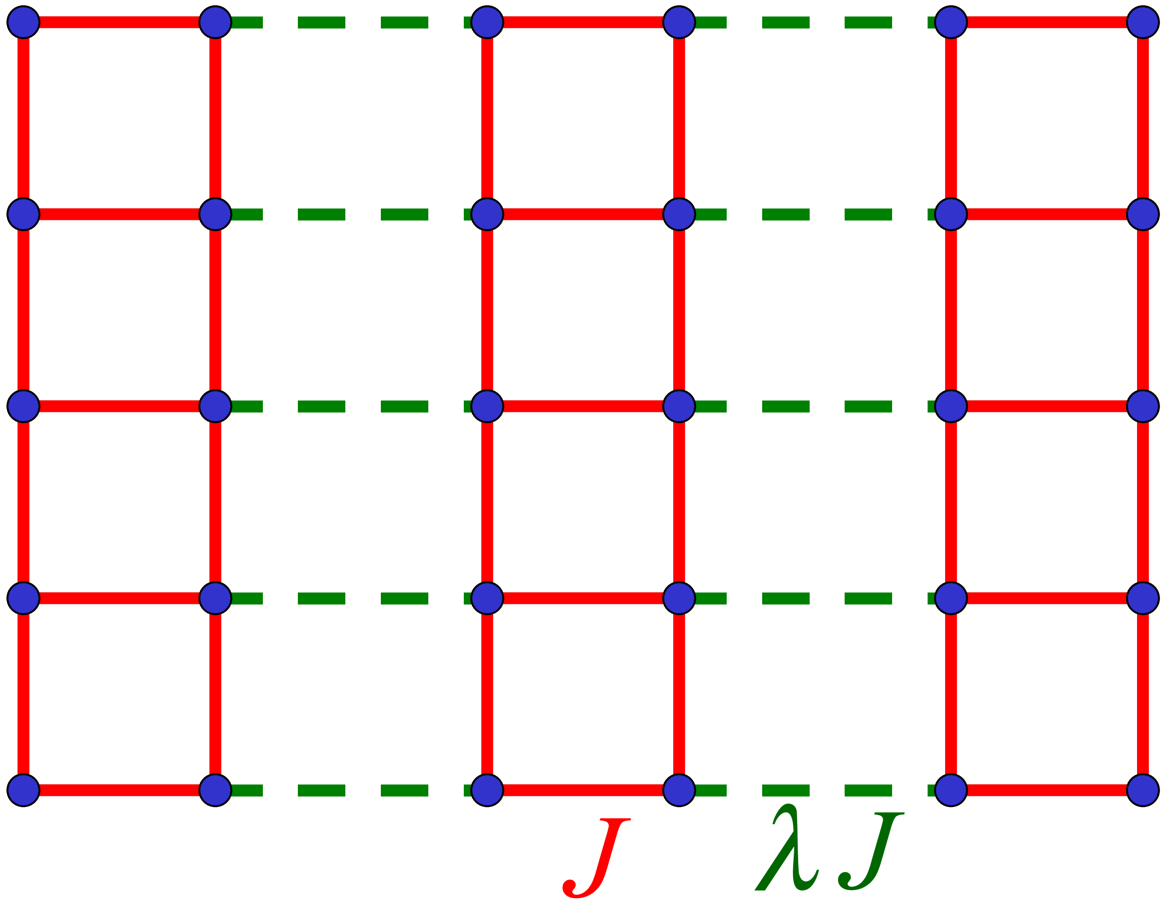
V. Transitions between BCS superconductors with distinct internal Cooper pair wavefunctions.
- A. Quantum field theory
- B. Photoemission and tunnelling experiments on the high temperature superconductors

VI. Conclusions

I.A Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, Phys. Rev. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, cond-mat/0107115.

S=1/2 spins on coupled 2-leg ladders



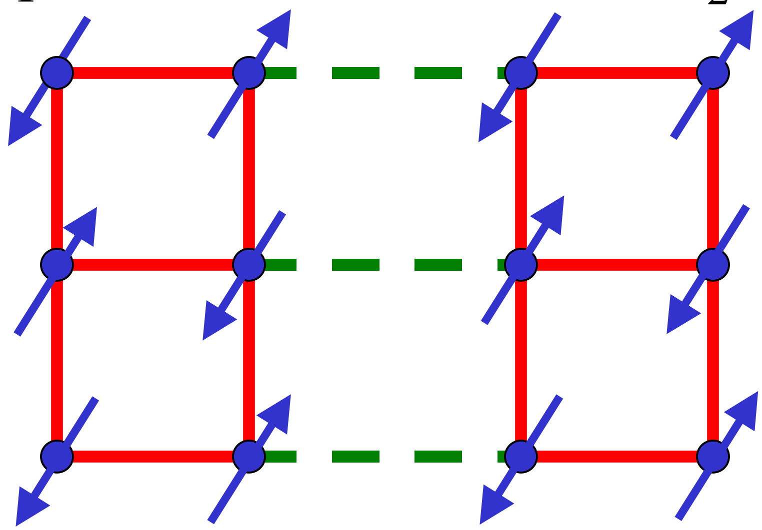
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



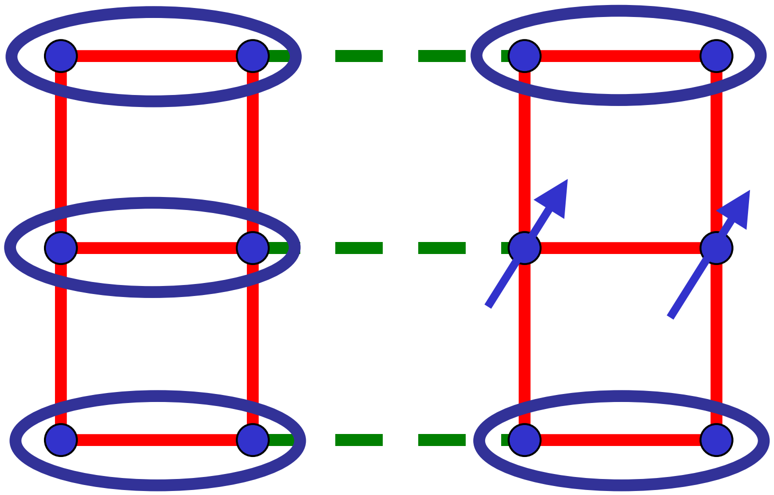
Ground state has long-range magnetic (Neel) order

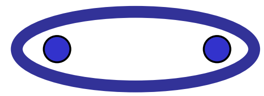
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_k = \sqrt{c_x^{\,2} k_x^{\,2} + c_y^{\,2} k_y^{\,2}}$

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

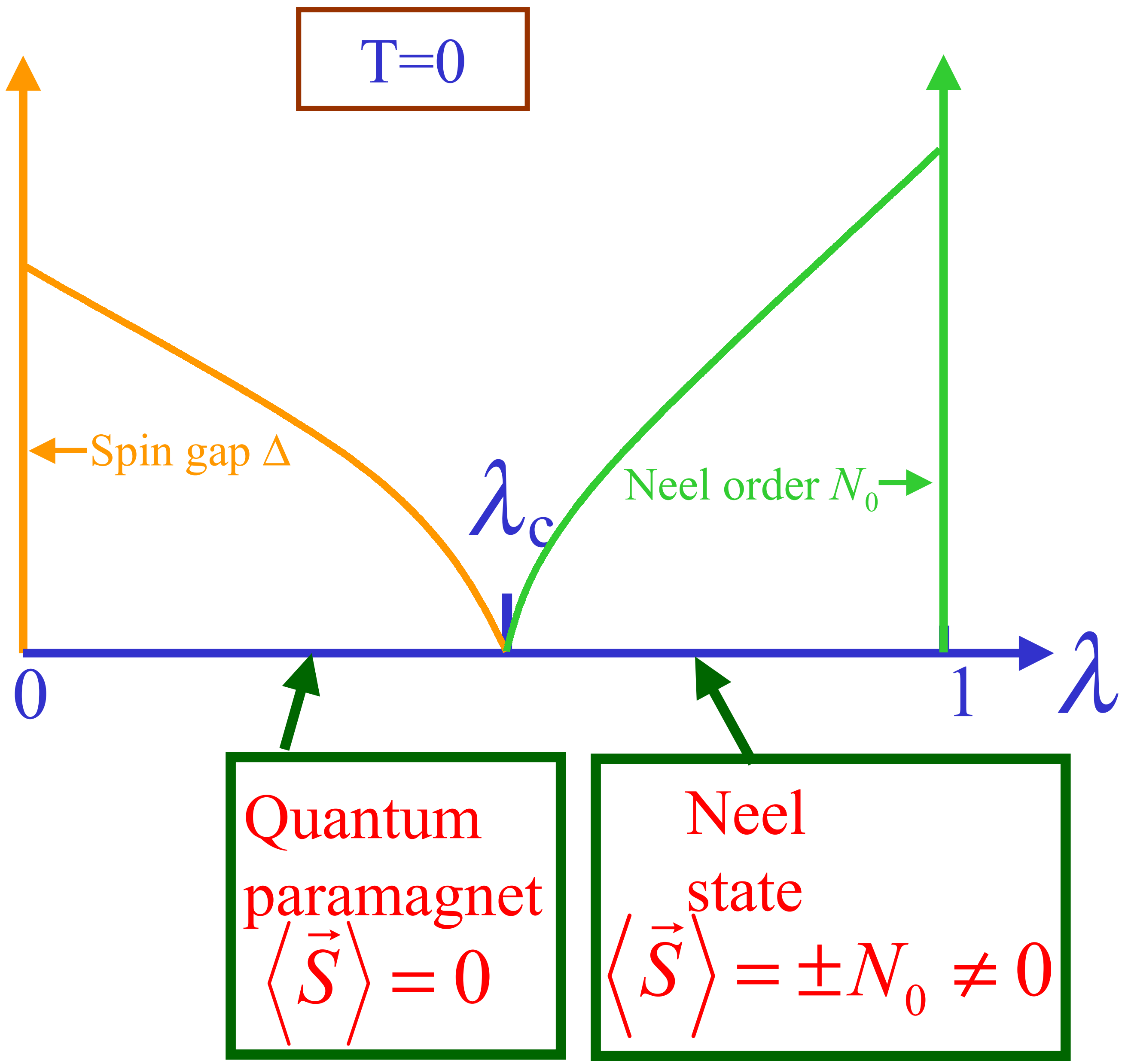
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 ***exciton*** (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$



T=0
Spin gap Δ
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle = \pm N_0 \neq 0$

I.B Coherent state path integral

See Chapter 13 of *Quantum Phase Transitions*, S. Sachdev, Cambridge University Press (1999).

Path integral for a single spin

$$Z = \mathrm{Tr}\left(e^{-H[\boldsymbol{S}]/T}\right)$$

$$= \int \mathcal{D}\boldsymbol{N}(\tau)\,\delta\left(\boldsymbol{N}^2-1\right)\exp\left(-iS\int A_\tau(\tau)d\tau - \int d\tau H\left[S\boldsymbol{N}(\tau)\right]\right)$$

$A_\tau(\tau)d\tau =$ Oriented area of triangle on surface of unit sphere bounded by $\boldsymbol{N}(\tau), \boldsymbol{N}(\tau+d\tau)$, and a fixed reference $\boldsymbol{N}_0$

Action for lattice antiferromagnet

$$Z = \int \mathcal{D}\boldsymbol{n}(x,\tau)\,\delta\left(\boldsymbol{n}^2-1\right)\exp\left(-iS\sum_j\int \eta_j A_\tau(x_j,\tau)d\tau - \frac{1}{2g}\int d^d x\left(\left(\partial_\tau \boldsymbol{n}\right)^2 + c^2\left(\nabla_x \boldsymbol{n}\right)^2\right)\right)$$

$\eta_j = \pm 1$ identifies sublattices

I.C Quantum field theory for critical point

λ close to λ_c

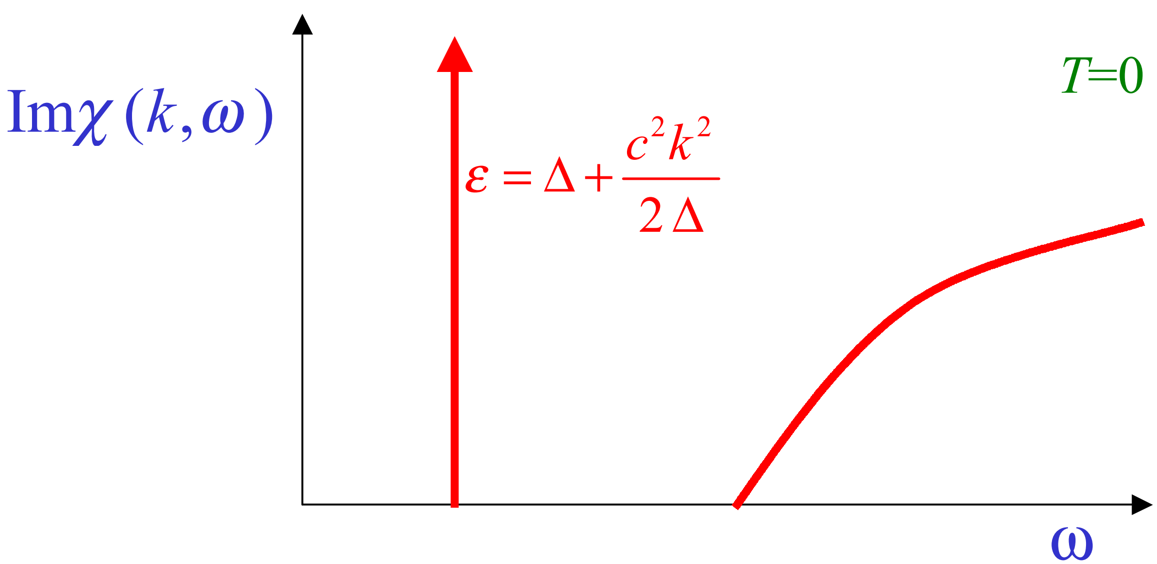
$$S_b = \int d^2 x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$ → $\lambda < \lambda_c$

$r < 0$ → $\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)
→ spin-1 collective mode



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II. Quantum XY model in one dimension

S. Sachdev and K. Park, cond-mat/0108214

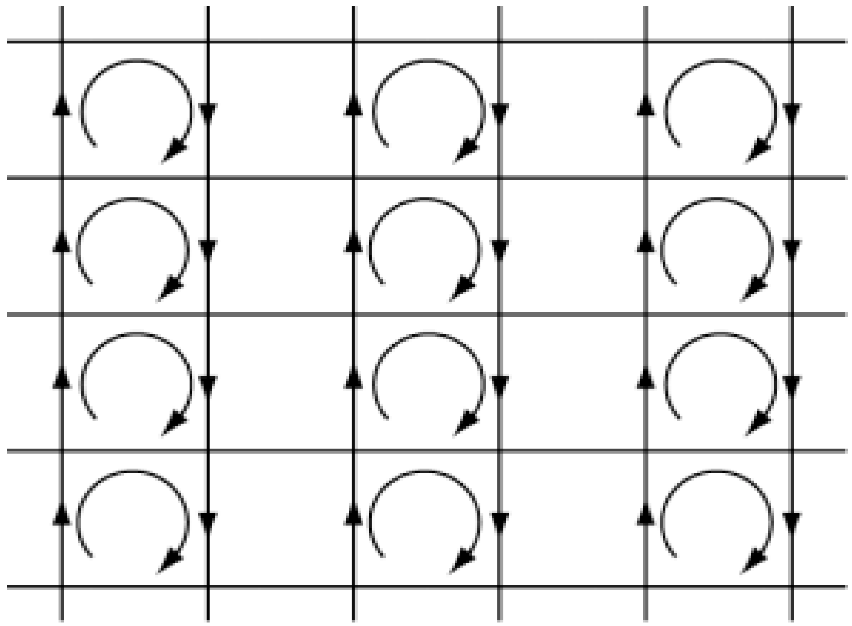
$$H_{XY} = J_1 \sum_j (S_{xj}S_{x,j+1} + S_{yj}S_{y,j+1} + \lambda S_{zj}S_{z,j+1}) + J_2 \sum_j (S_{xj}S_{x,j+2} + S_{yj}S_{y,j+2} + \lambda S_{zj}S_{z,j+2})$$

Write

$$\mathbf{n}_j = (\cos(\theta_j), \sin(\theta_j), 0)$$

where j is now a discrete spacetime index upon a square lattice.

Structure of Berry phase terms.



$$S\sum_j \eta_j A_{j\tau} = S\sum_j \ell_{\bar{\jmath}} \epsilon_{\mu\nu} \Delta_\mu A_{j\nu}$$

where $\ell_{\bar{\jmath}} = 0$ ($\ell_{\bar{\jmath}} = 1$) on even (odd) columns.

For XY model, $S\epsilon_{\mu\nu}\Delta_\mu A_{j\nu} = (2\pi S) \times$ vortex number

Vortices in odd columns carry a factor $(-1)^{2S}$.

$\underline{S \text{ integer}}$.

$$Z'_{XY} = \prod_j \int d\theta_j \exp\left(\frac{1}{g}\sum_j \cos(\Delta_\mu \theta_j)\right)$$

where $\mu = x, \tau$.

This is the action for a "classical" XY model in $D = 2$.

Displays Kosterlitz-Thouless transition.

Dual height model for KT transition:

$$Z'_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp\left(-\frac{1}{2g}\sum_j (\Delta_\mu \theta_j - 2\pi m_{j\mu})^2\right)$$

This is the Villain periodic Gaussian form. Poisson summation leads to

$$Z'_{XY} = \sum_{\{p_{\bar{j}}\}} \exp\left(-\frac{g}{2}\sum_{\bar{j}} (\Delta_\mu p_{\bar{j}})^2\right)$$

Height (or roughening) model on the square lattice

All heights are integers.

For XY model, the A 'flux' measures vortex number.
In the periodic Gaussian formulation

$$S\epsilon_{\mu\nu}\Delta_\mu A_{j\nu} = \pi\epsilon_{\mu\nu}\Delta_\mu m_{j\nu}$$

So with Berry phases partition function of $S = 1/2$ quantum XY model is

$$Z_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp\left(-\frac{1}{2g}\sum_j (\Delta_\mu \theta_j - 2\pi m_{j\mu})^2 + i\pi \sum_j \ell_{\bar{\jmath}}\epsilon_{\mu\nu}\Delta_\mu m_{j\nu}\right)$$

Vortices in odd columns contribute a factor (-1) to the partition function
Weights are not positive.
Dual height model for Z_{XY}

$$Z_{XY} = \sum_{\{p_{\bar{\jmath}}\}} \exp\left(-\frac{g}{2}\sum_{\bar{\jmath}} \left(\Delta_\mu p_{\bar{\jmath}} - \frac{1}{2}\Delta_\mu \ell_{\bar{\jmath}}\right)^2\right)$$

Height model in which the heights are integers (half-integers) on even (odd) columns.

Phases of height model

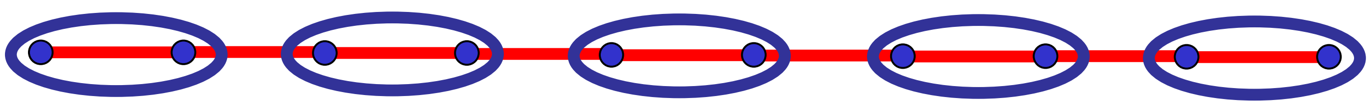
Rough interface

Tomonaga-Luttinger liquid with power-law spin correlations:

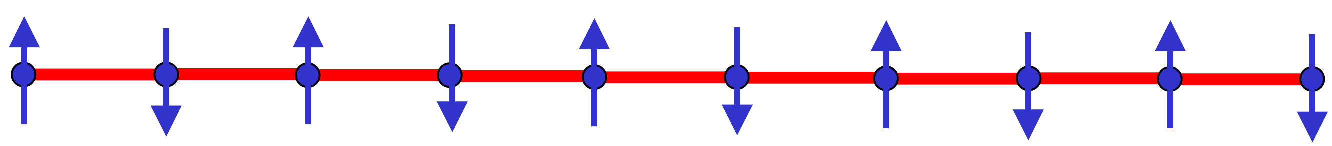
$$\langle n_{ix} n_{jx} \rangle = \langle n_{iy} n_{jy} \rangle \sim \frac{1}{|i-j|^{g/(2\pi)}}$$
$$\langle n_{iz} n_{jz} \rangle \sim \frac{1}{|i-j|^{2\pi/g}}$$

Smooth interface

All correlations decay exponentially → there is a gap to all excitations
$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle$ has a definite value: any such definite value breaks a discrete symmetry of the Hamiltonian.
$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle = 0, 1/2$ (plus integer) are states with **bond-centered charge order** *i.e.* neighboring links have valence bonds with distinct probabilities

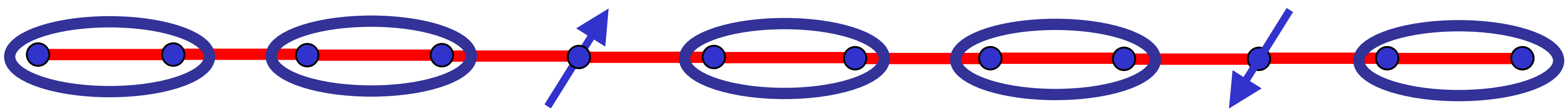


$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle = 1/4, 3/4$ (plus integer) are the two states with Ising antiferromagnetic order.

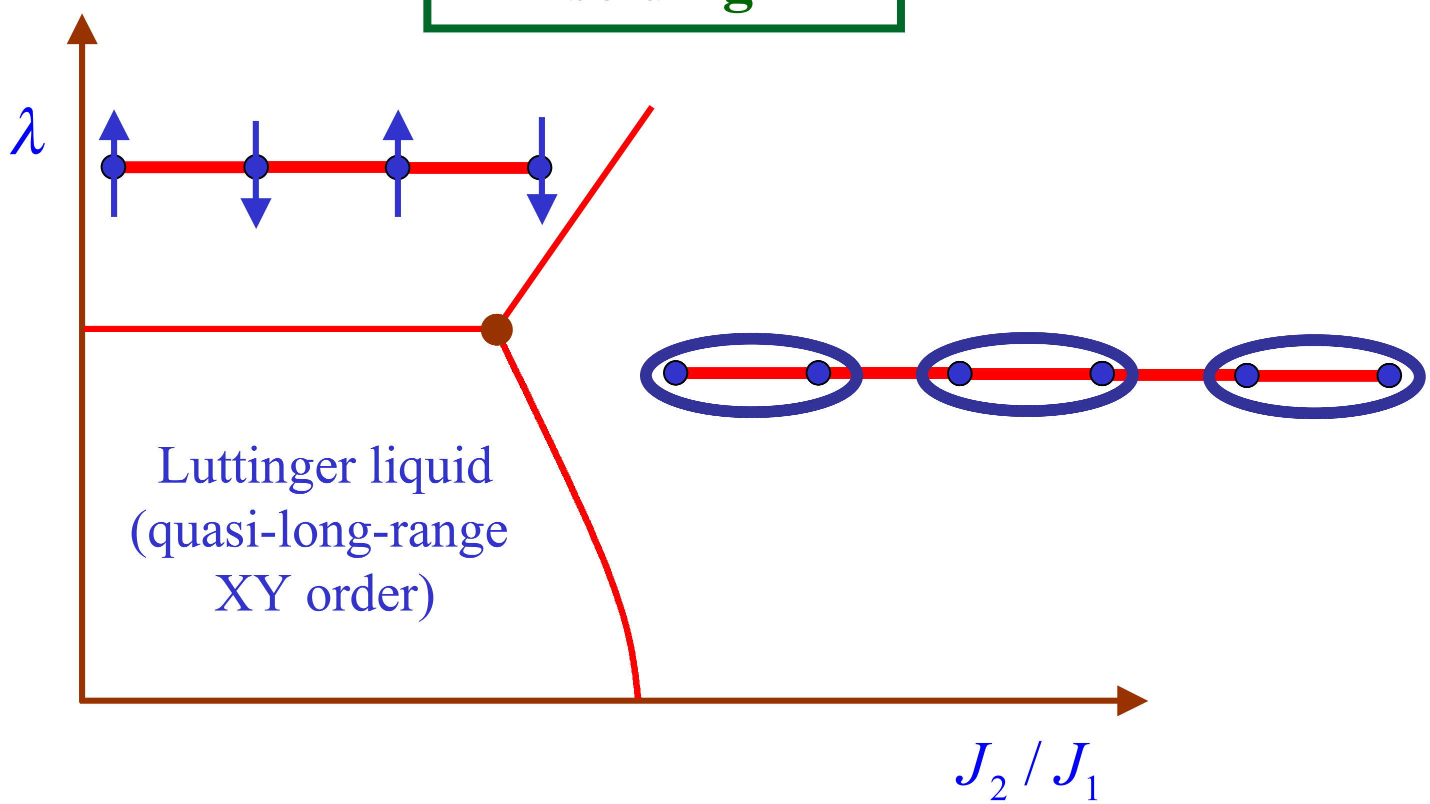


Excitations of paramagnet with bond-charge-order

Deconfined S=1/2 spinons



Phase diagram



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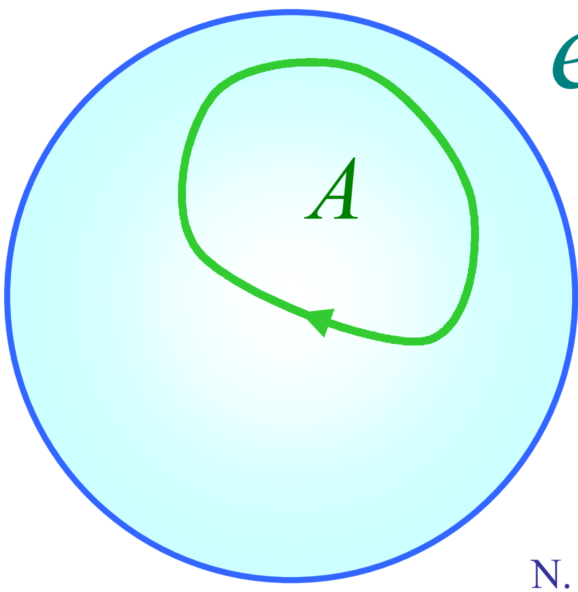
III. Berry phases and the square lattice antiferromagnet

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action: $$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 \right) + V\left(\phi_\alpha^2 \right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



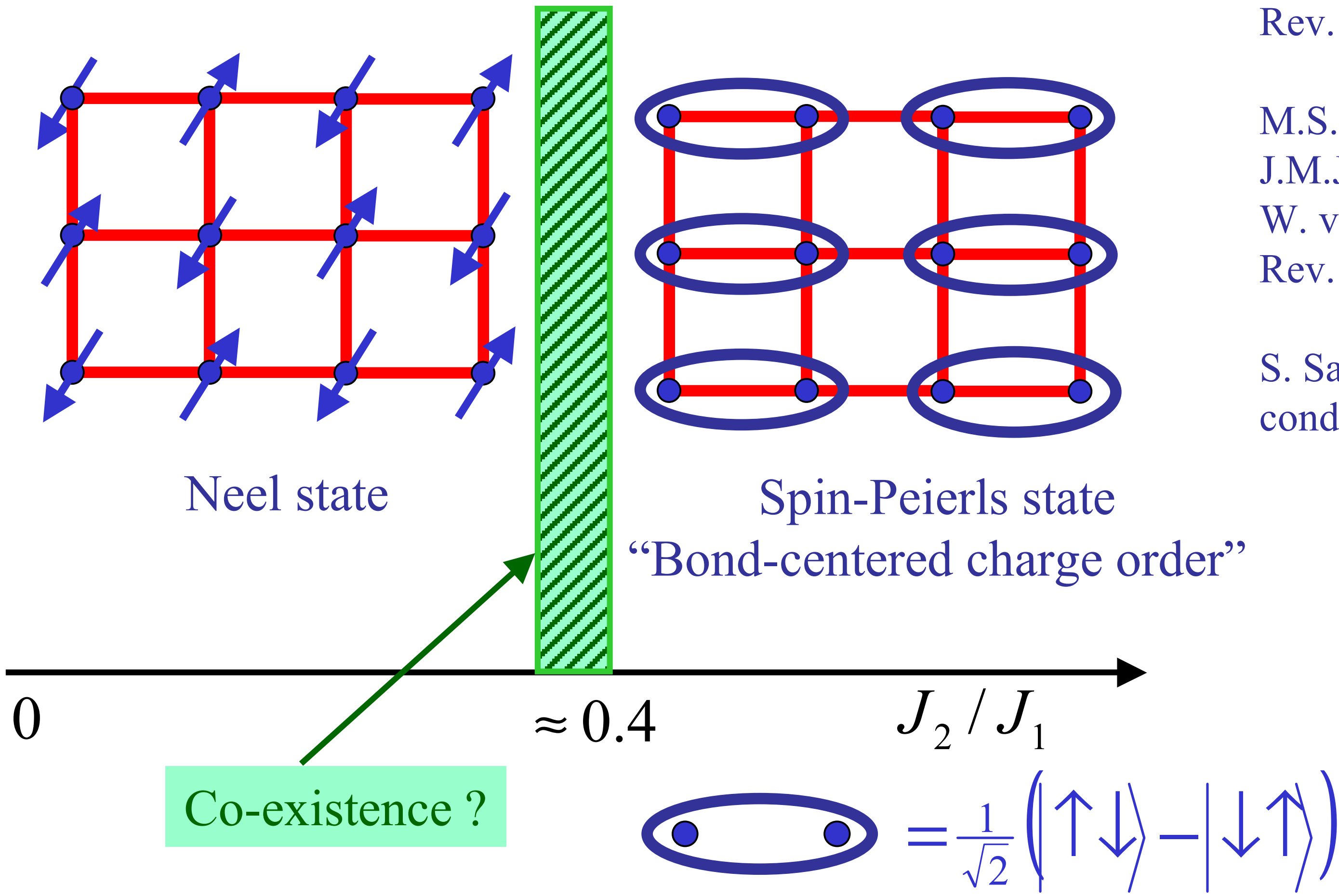
$$e^{iSA}$$

Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
"Dual order parameter"

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Square lattice with first (J_1) and second (J_2) neighbor exchange interactions



N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

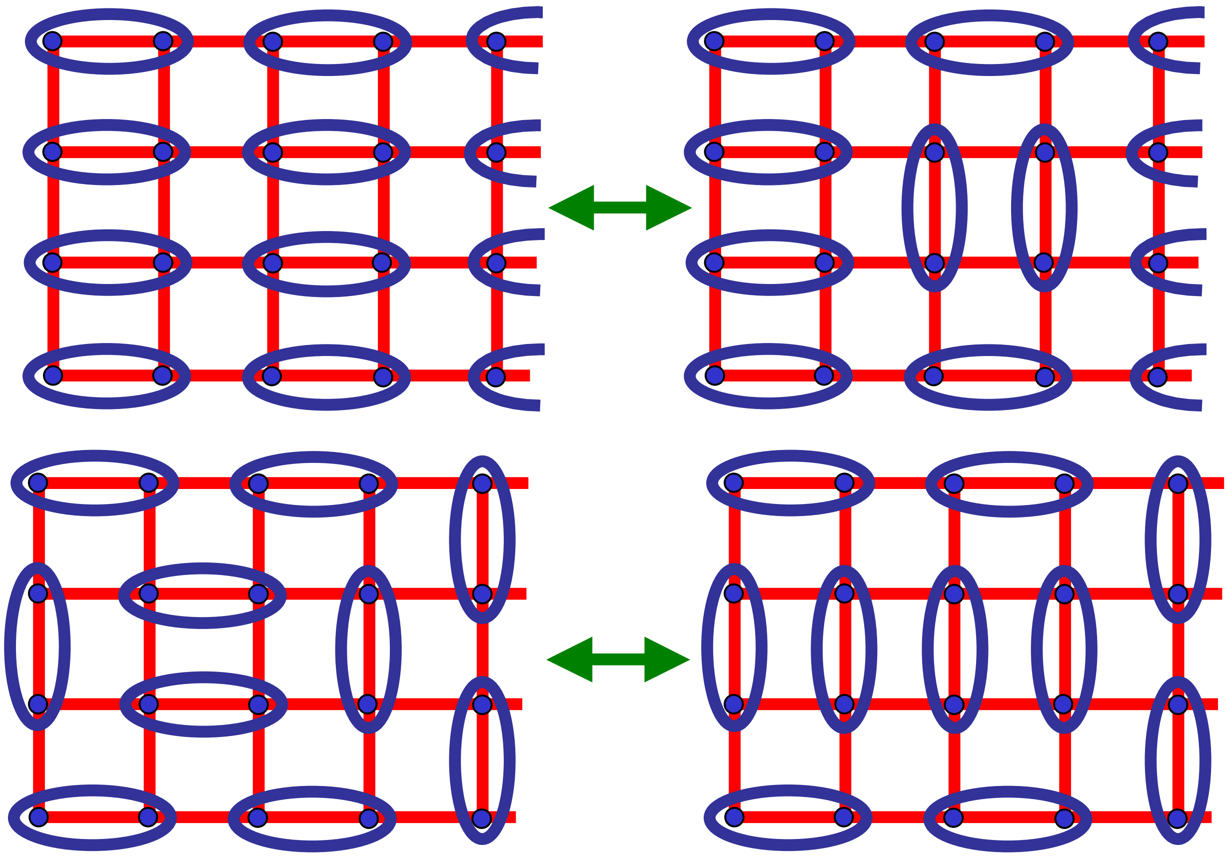
O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).

S. Sachdev and K. Park, cond-mat/0108214.

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

These effects lead to a broken square lattice symmetry near the transition to the Neel state.

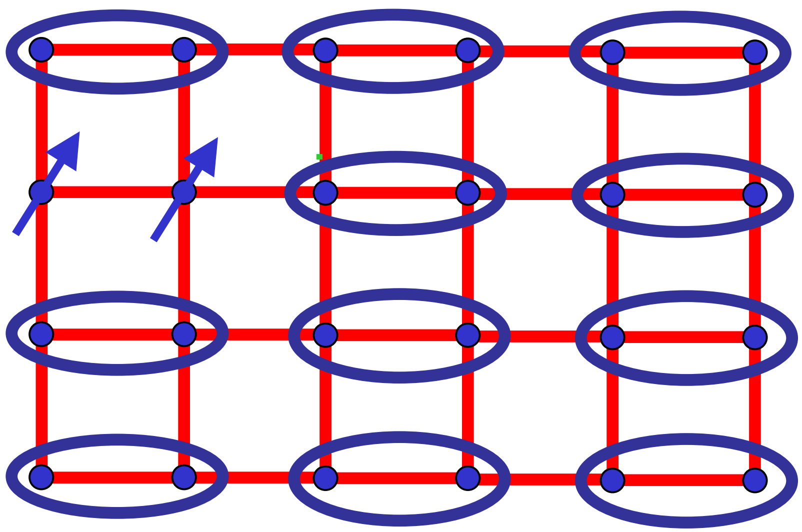
N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

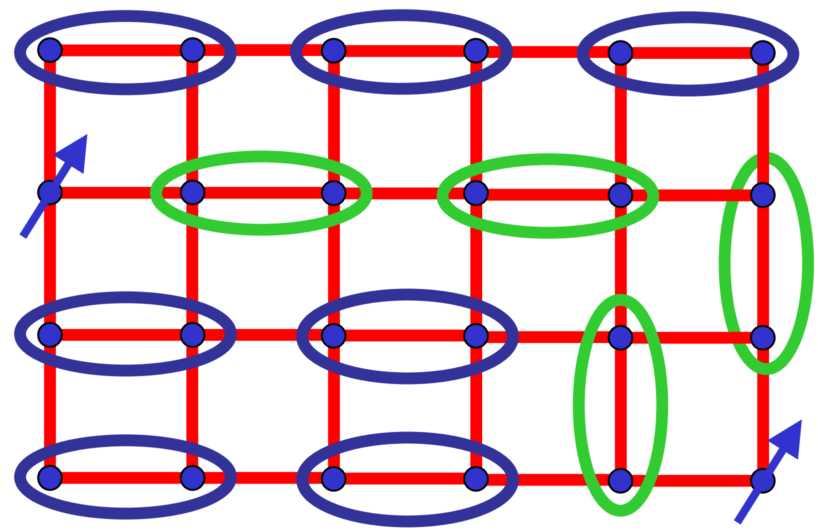
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

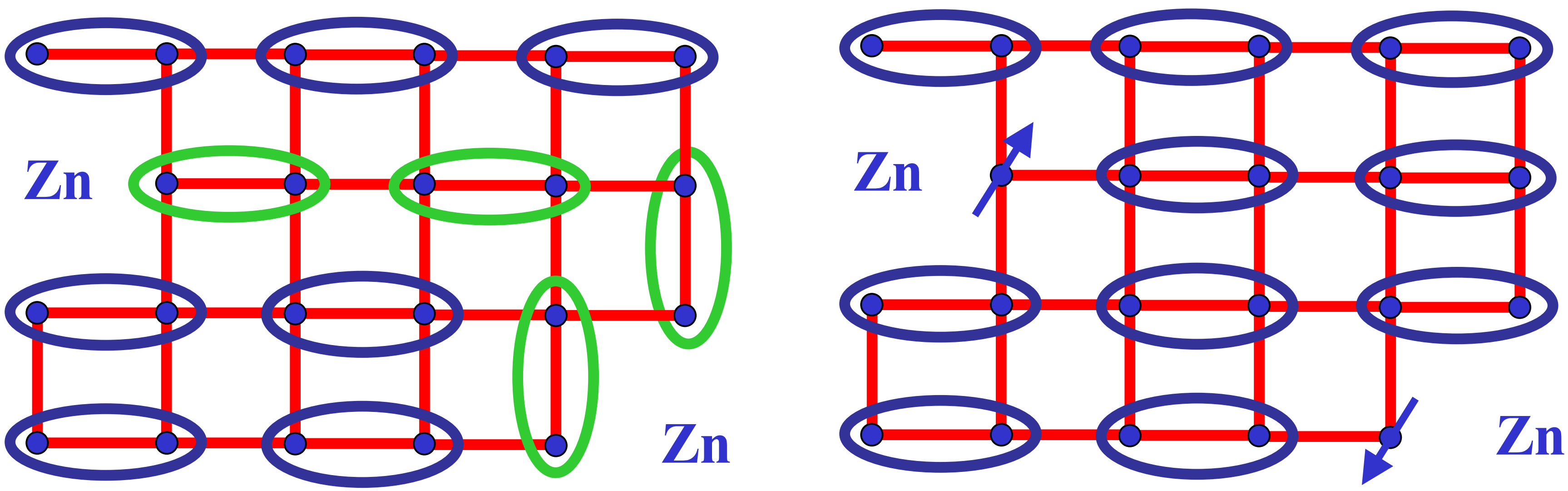
$\Delta \rightarrow$ Spin gap





S=1/2 spinons are ***confined*** by a linear potential.

Effect of static non-magnetic impurities (Zn or Li)



Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).
J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001).

Field theory of bond order

Discretize coherent state path integral on a cubic lattice in spacetime:

$$Z = \prod_j \int d\mathbf{n}_j \delta(\mathbf{n}_j^2 - 1) \exp\left(-\frac{1}{2g} \sum_{j,\mu} \mathbf{n}_j \cdot \mathbf{n}_{j+\hat{\mu}} - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

where $\mu = x, y, \tau$, and we assume henceforth that $S = 1/2$.
For large g, perform a "high temperature" expansion to obtain an effective action for the $A_{j\mu}$. This action must be invariant under the 'gauge' transformation

$$A_{j\mu} \to A_{j\mu} - \Delta_\mu \gamma_j$$

associated with the change in choice of $\mathbf{n}_0$ (γ_j is the oriented area of the spherical triangle formed by $\mathbf{n}_j$ and the two choices for $\mathbf{n}_0$). Also, it should be invariant under

$$A_{j\mu} \to A_{j\mu} + 4\pi$$

because area of triangle is uncertain modulo 4π.
Simplest large g effective model

$$Z = \prod_j \int dA_{j\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos\left(\frac{1}{2} \epsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda} \right) - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

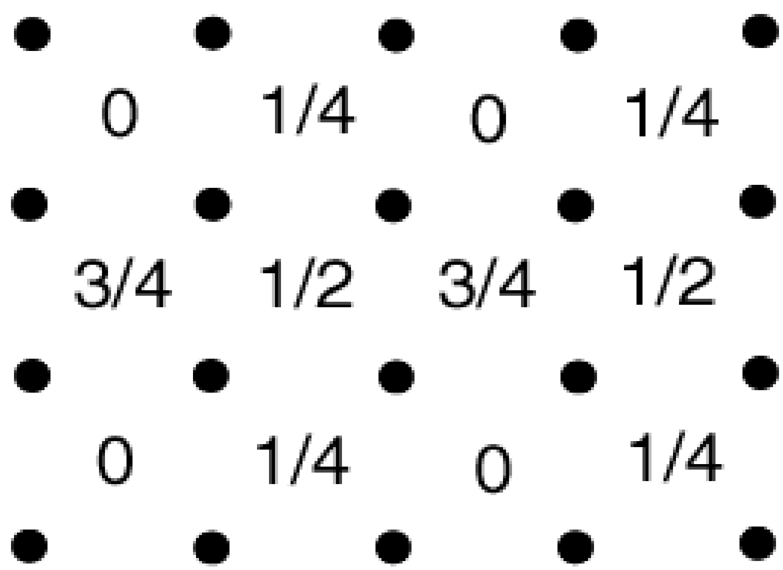
with $e^2 \sim g^2$.
This is compact QED in 2+1 dimensions with Berry phases.

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields

$$Z = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2}\sum_{\bar{\jmath}}\left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}}\right)^2\right)$$

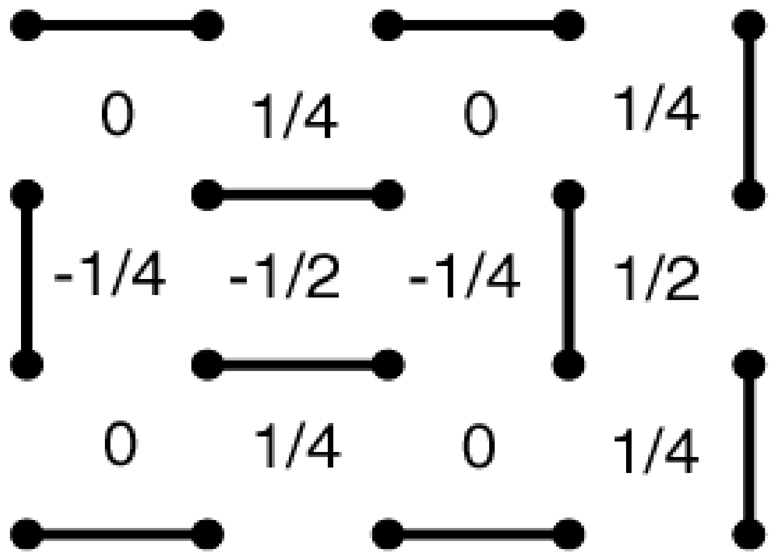
with $h_{\bar{\jmath}}$ integer.

Height model in 2+1 dimensions with 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

⟹ 2+1 dimensional height model is the path integral of the quantum dimer model



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

⟹ There is a definite average height of the interface

⟹ **Ground state has bond-charge order.**

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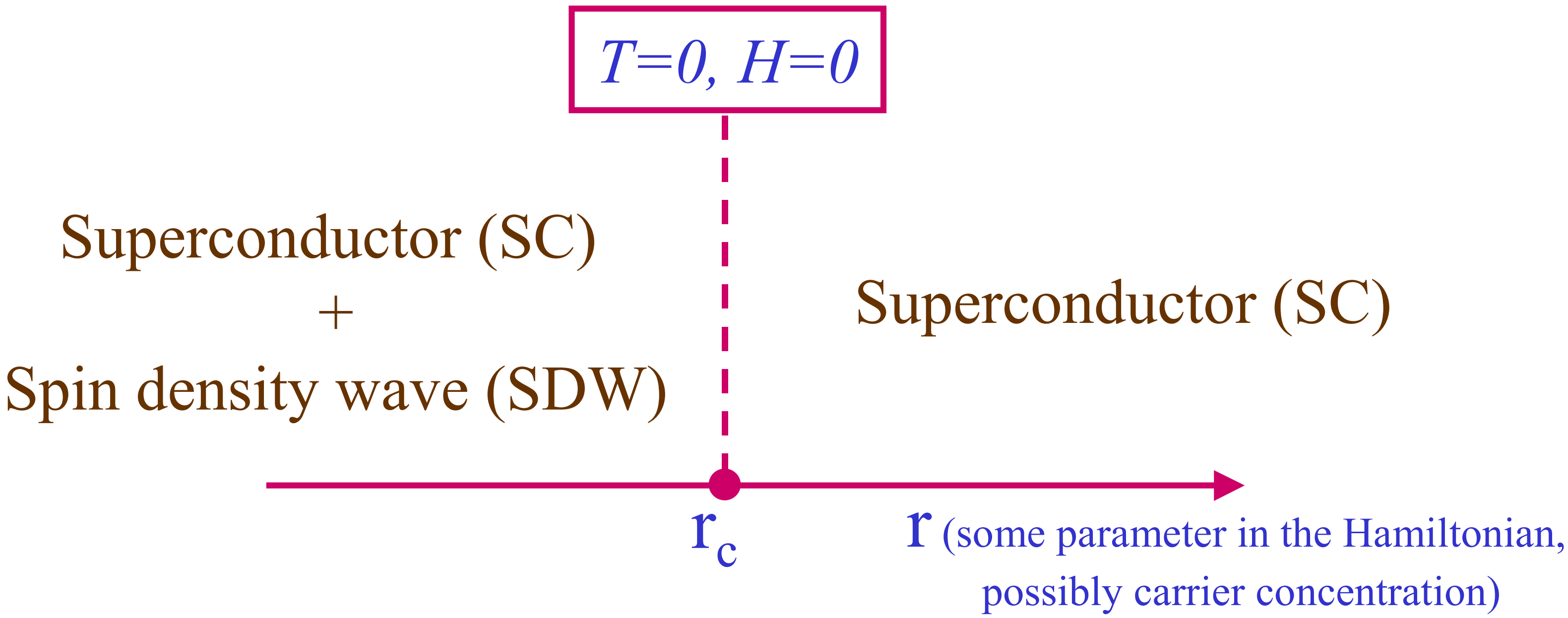
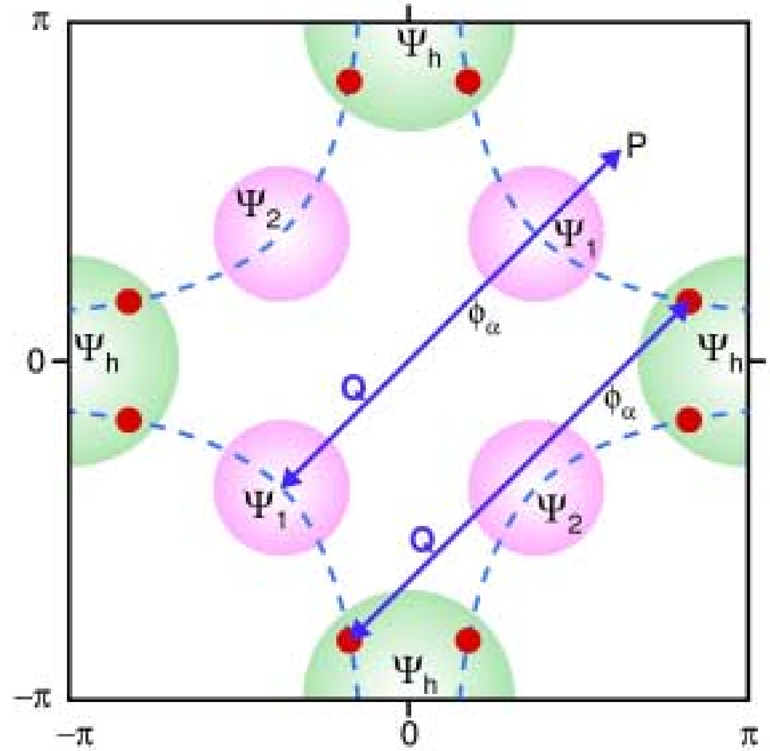


Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001).
Y. Sidis, C. Ulrich, P. Bourges, *et al*., cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al*., preprint.

Structure of quantum theory

- Charge-order is not critical: can neglect Berry phases.



- Generically, momentum conservation prohibits decay of $S=1$ exciton ϕ_α into $S=1/2$ fermionic excitations at low energies. Virtual pairs of fermions only renormalize parameters in the effective action for ϕ_α .
- Zeeman coupling only leads to corrections at order H^2
- Simple Landau theory couplings between ϕ_α and superconducting order ψ are allowed (S.-C. Zhang, Science **275**, 1089 (1997)), *e.g.*:

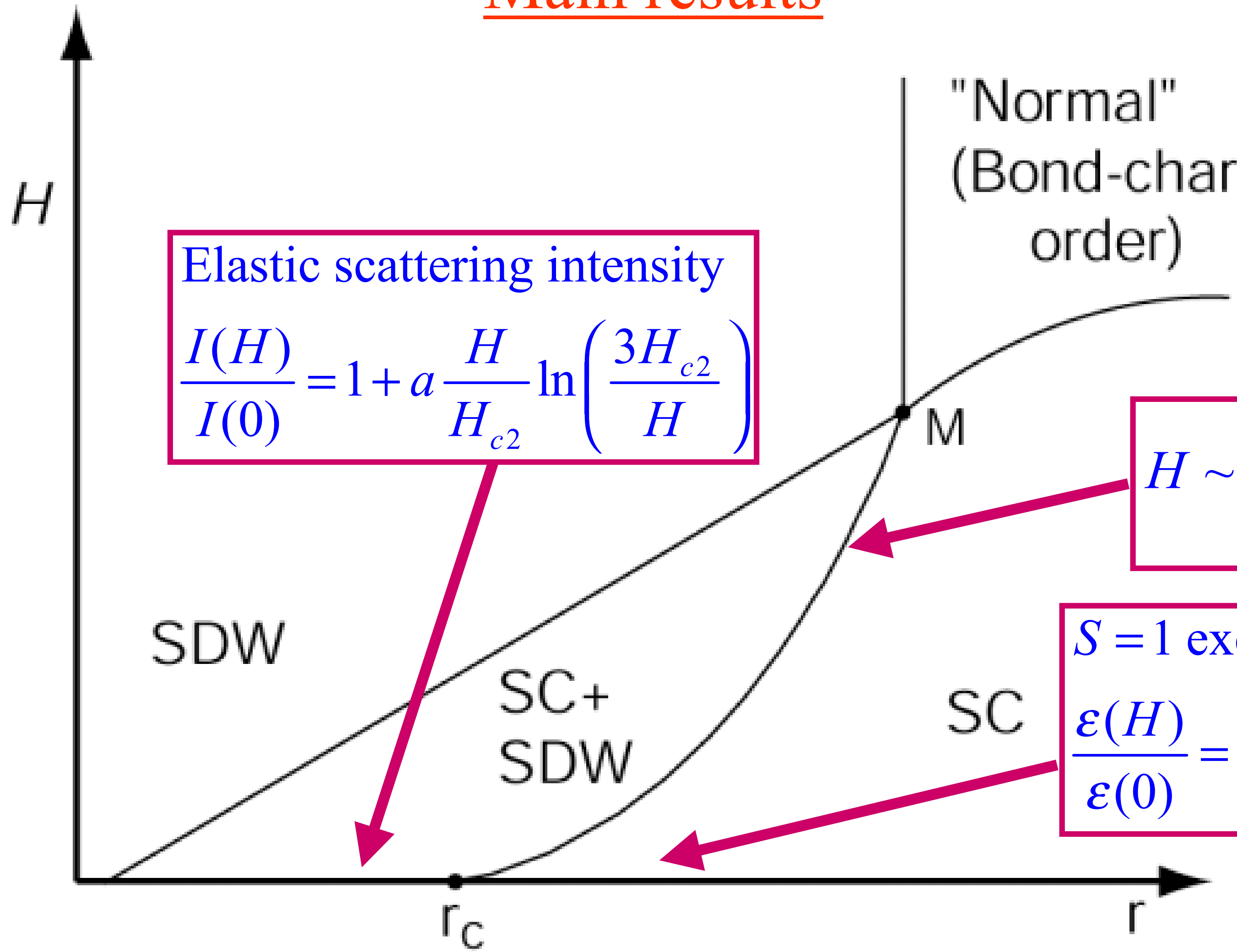
$$V\left(\phi_\alpha^2\right) \rightarrow V\left(\phi_\alpha^2\right) + \lambda \phi_\alpha^2 \left|\psi\right|^2$$

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

Main results

(extreme Type II superconductivity)

$T=0$



Elastic scattering intensity

$$\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(r-r_c)}{\ln\left(1/(r-r_c)\right)}$$

$S=1$ exciton energy

$$\frac{\varepsilon(H)}{\varepsilon(0)} = 1 - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

- All functional forms are **exact**.
- Similar results apply to other competing orders *e.g.* SC + staggered flux

E. Demler, S. Sachdev, and Y. Zhang, Phys. Rev. Lett. **87**, 067202 (2001).

- Theory should account for quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a static Ginzburg-Landau theory

Action $\quad F_{GL}/T + \mathcal{S}_b + \mathcal{S}_c$

$$F_{GL} = \int d^2x \left[-|\psi|^2 + \frac{|\psi|^4}{2} + \left|\left(\nabla_x - iA\right)\psi\right|^2 \right]$$

$$\mathcal{S}_c = \int d^2x d\tau \left[\frac{v}{2} \phi_\alpha^2 |\psi|^2 \right]$$

$$\mathcal{S}_b = \int d^2x \int_0^{1/T} d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

Self-consistent Hartree theory of quantum spin fluctuations (large N limit)

$$\chi(x,x',\omega_n)\delta_{\alpha\beta} = \left\langle \phi_\alpha(x,\omega_n)\phi_\beta(x',-\omega_n)\right\rangle$$

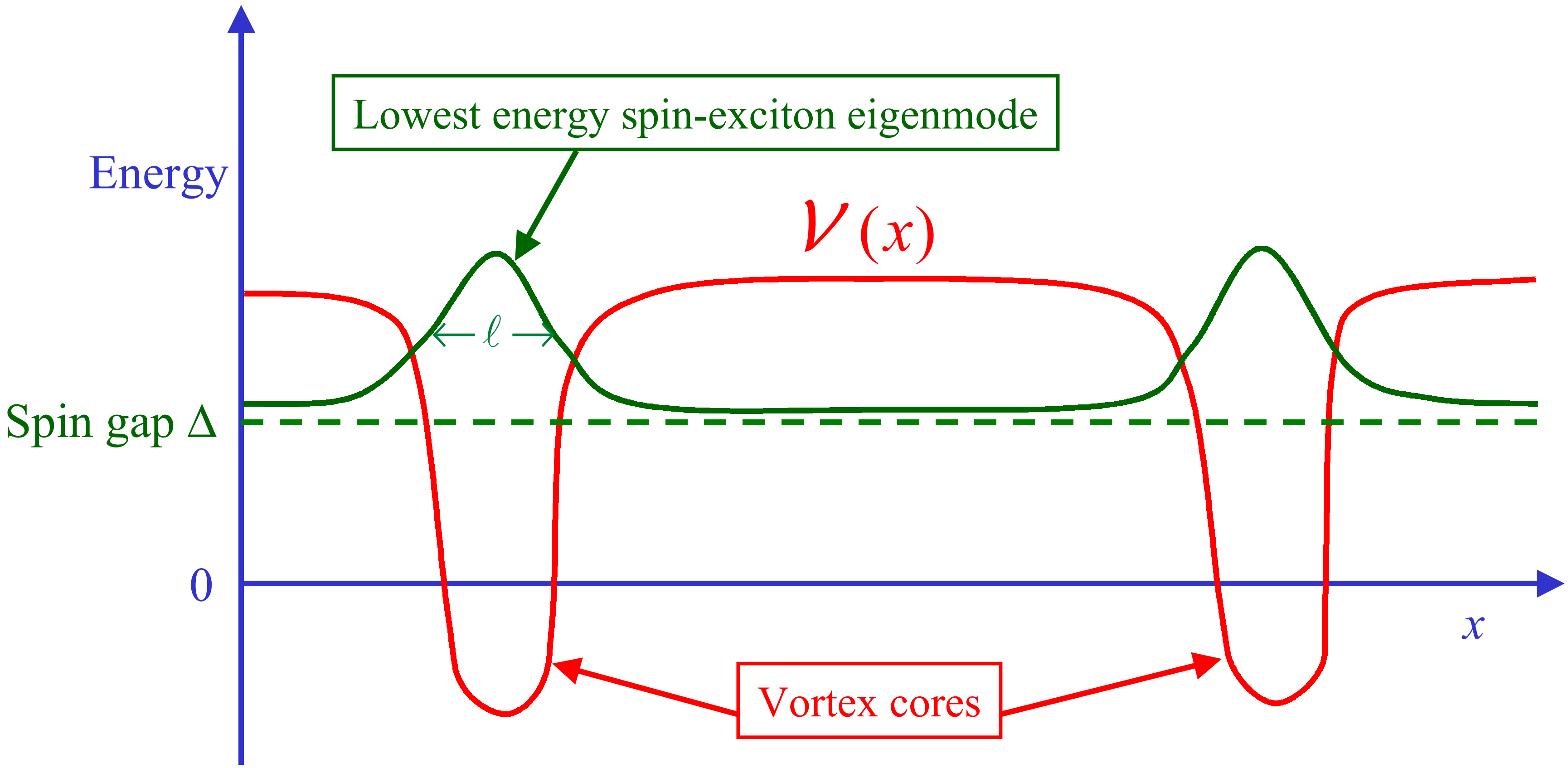
$$\left(\omega_n{}^2 - c^2\nabla_x^2 + \mathcal{V}(x)\right)\chi(x,x',\omega_n) = \delta(x-x')$$

$$\mathcal{V}(x) = r + v\left|\psi(x)\right|^2 + (NgT/6)\sum_{\omega_n}\chi(x,x,\omega_n)$$

$$\left[-1+\left|\psi(x)\right|^2 - \left(\nabla_x - i\vec{A}\right)^2\right]\psi(x) + (NvT/2)\sum_{\omega_n}\chi(x,x,\omega_n)\psi(x) = 0$$

$\mathcal{V}(x) \rightarrow$ local classical energy of spin fluctuations; can become negative in vortex cores for $v > 0$.

However, spin gap remains finite because of quantum fluctuations

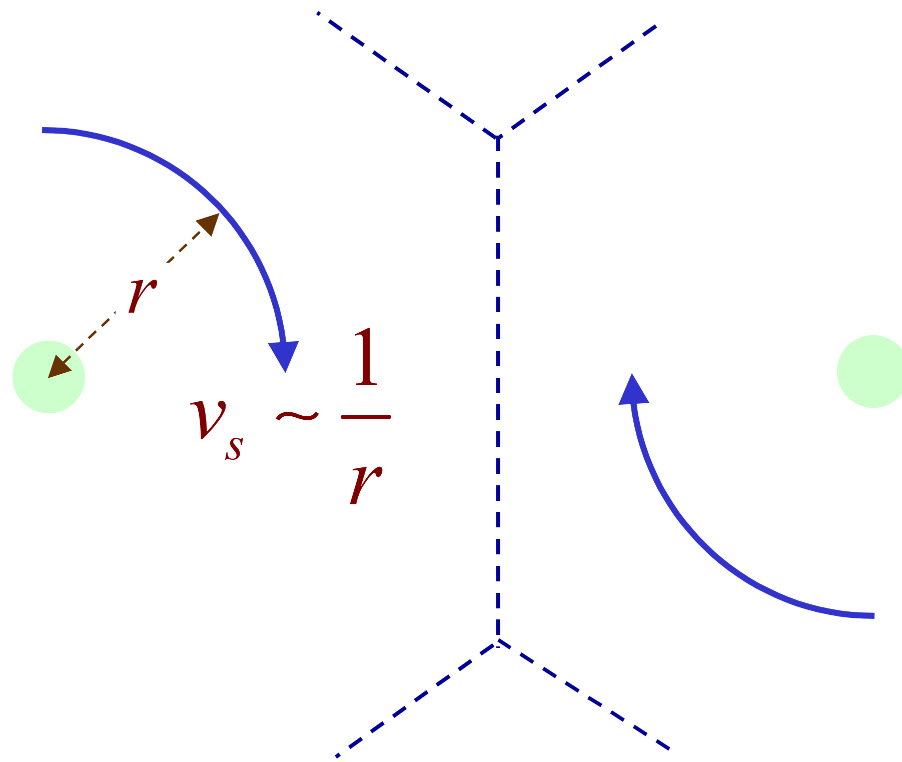


As $\Delta \to 0$, $\ell \to \infty$, because of self interaction, g, of spin excitations.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L765 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

See D. P. Arovas *et al*., Phys. Rev. Lett. **79**, 2871 (1997) for a different viewpoint.

Influence of $\psi(x)$ on extended spin eigenmodes:

$$|\psi(x)| = 1 - \frac{1}{2x^2}$$ outside each vortex core because of superflow kinetic energy

$$\left\langle |\psi(x)|^2 \right\rangle = 1 - \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

In SC phase, spin gap obeys:

$$\Delta(H) = \Delta(0) - \frac{24\pi c^2 v}{Ng\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

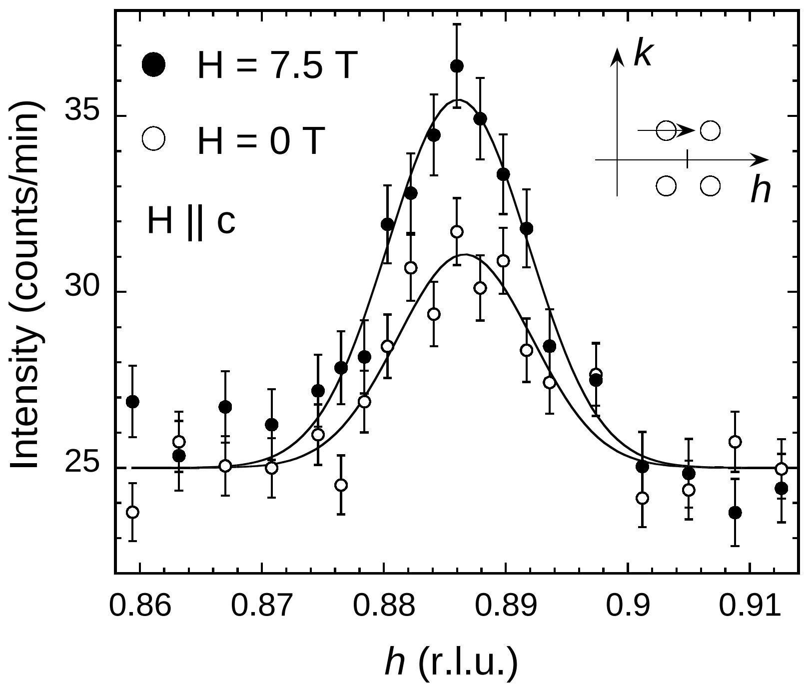
In SC+SDW phase, intensity of elastic scattering obeys:

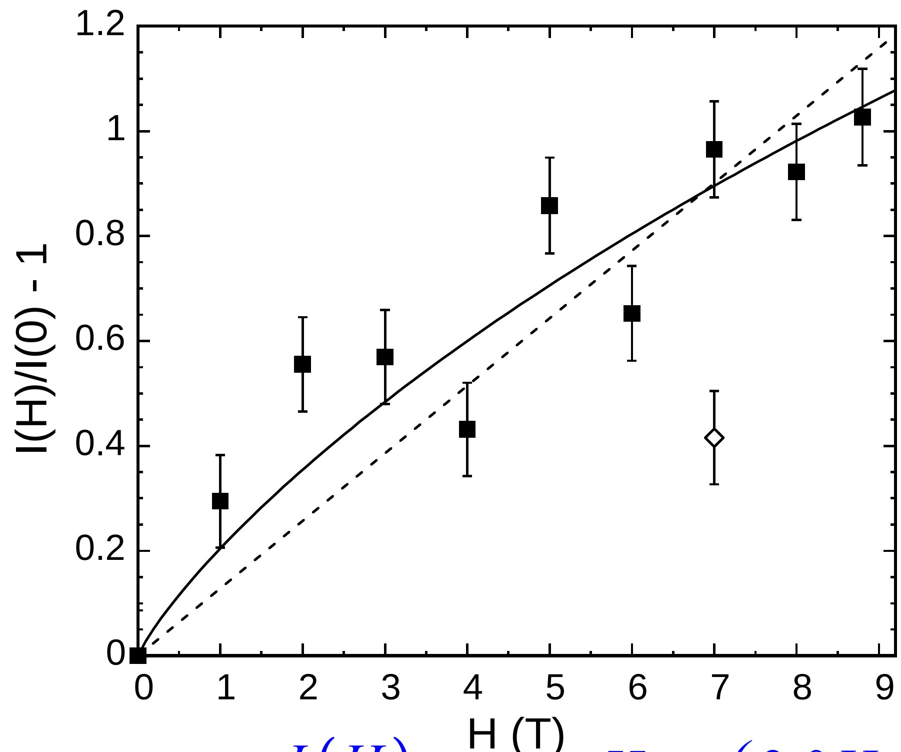
$$I(H) = I(0) + \frac{6v}{g\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas,

M. A. Kastner, and R.J. Birgeneau, preprint.





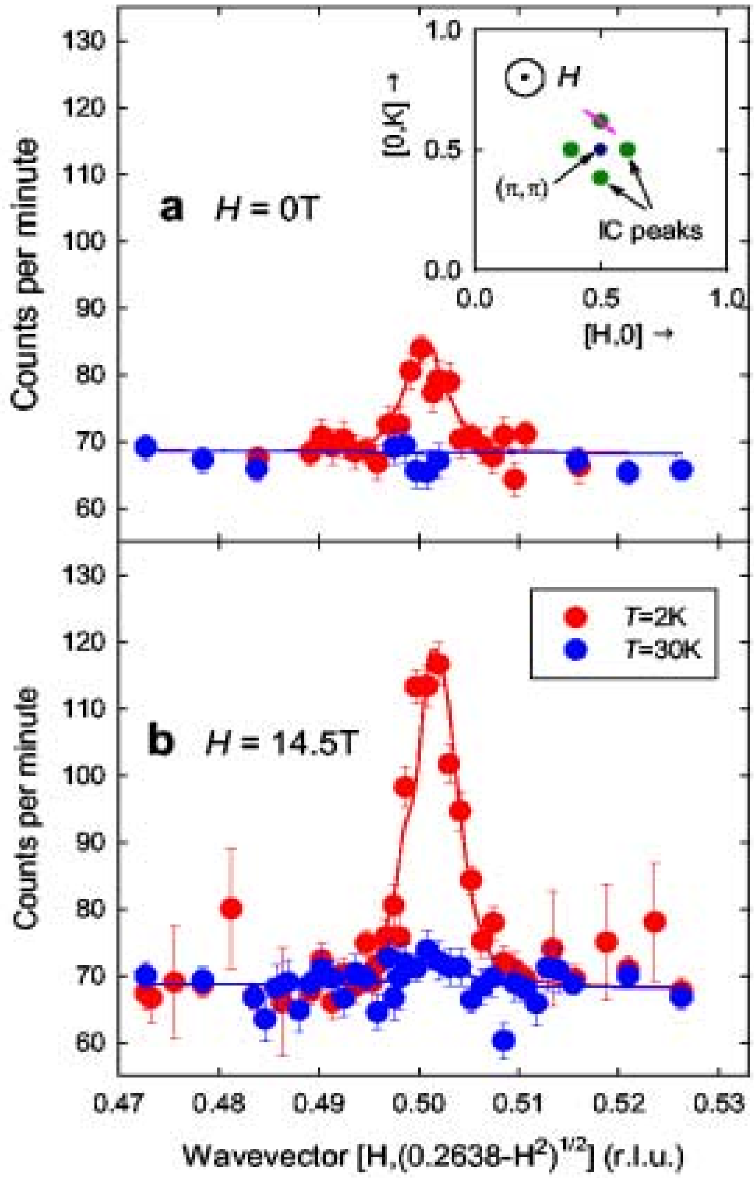
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

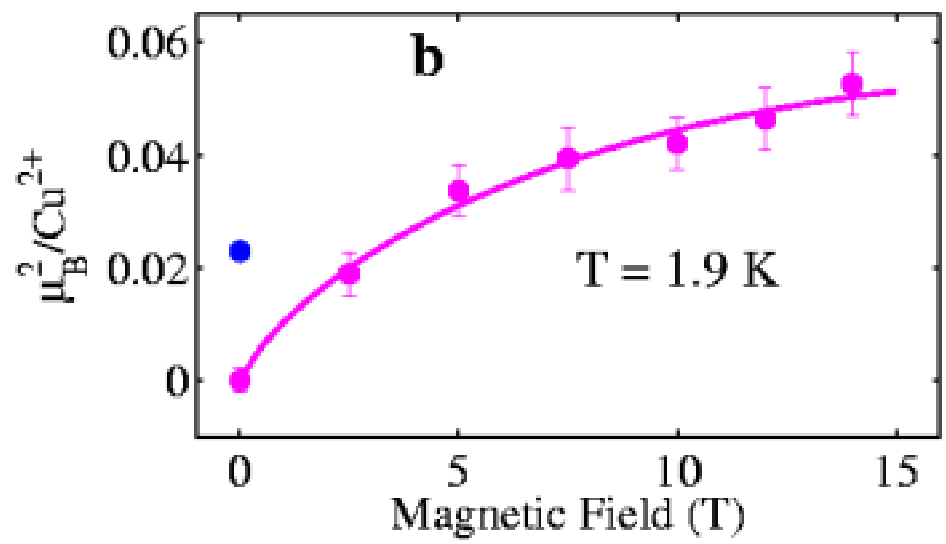
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*

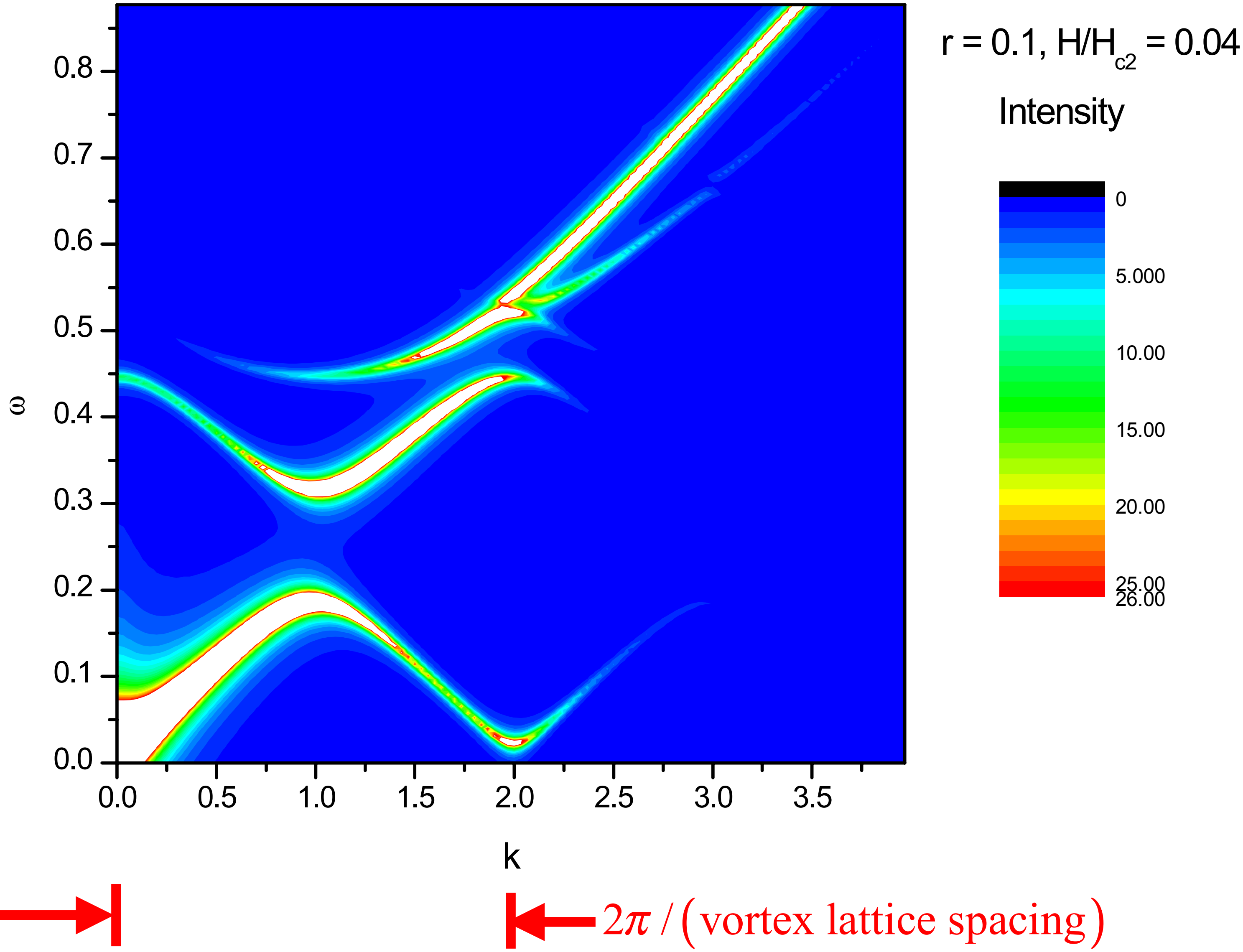




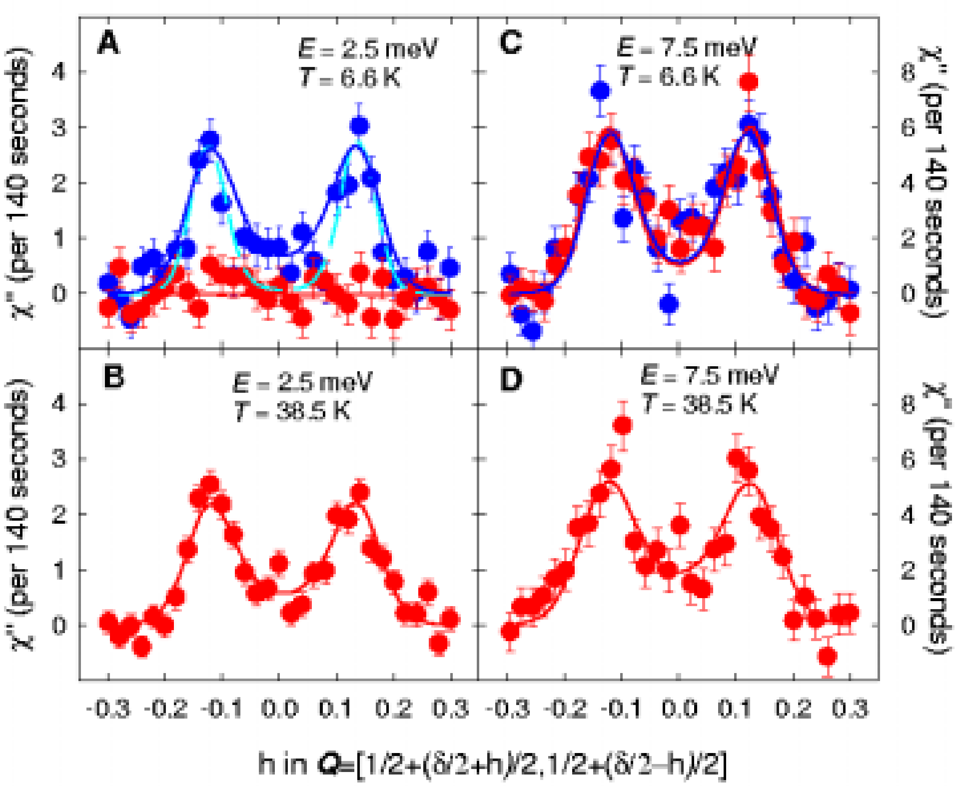
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Presence of vortex lattice leads to supermodulation in the spin exciton spectrum

Computation of spin susceptibility $\chi''(k,\omega)$ in self-consistent large N theory of ϕ_α fluctuations in a vortex lattice



$2\pi/(\text{vortex lattice spacing})$

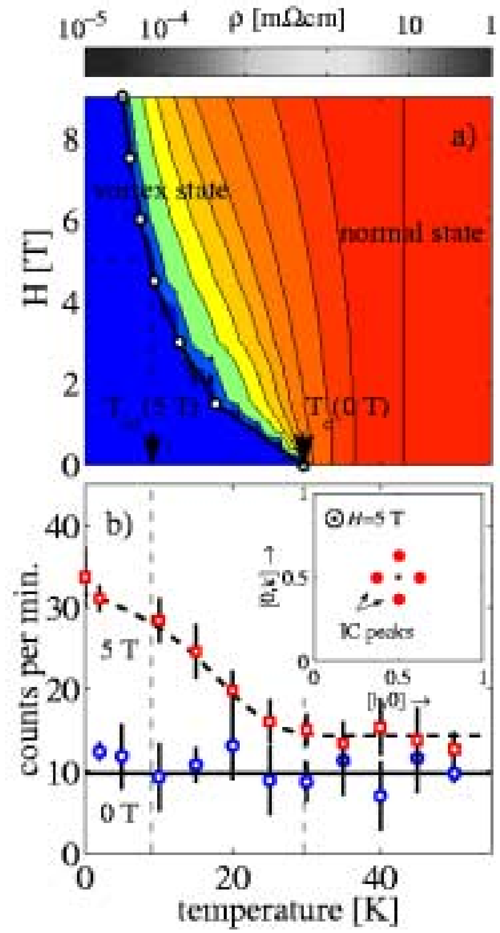


Neutron scattering off $La_{2-x}Sr_xCuO_4$

($x = 0.163$, ***SC phase***)

in H=0 (red dots) and H=7.5T (blue dots).

B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001)



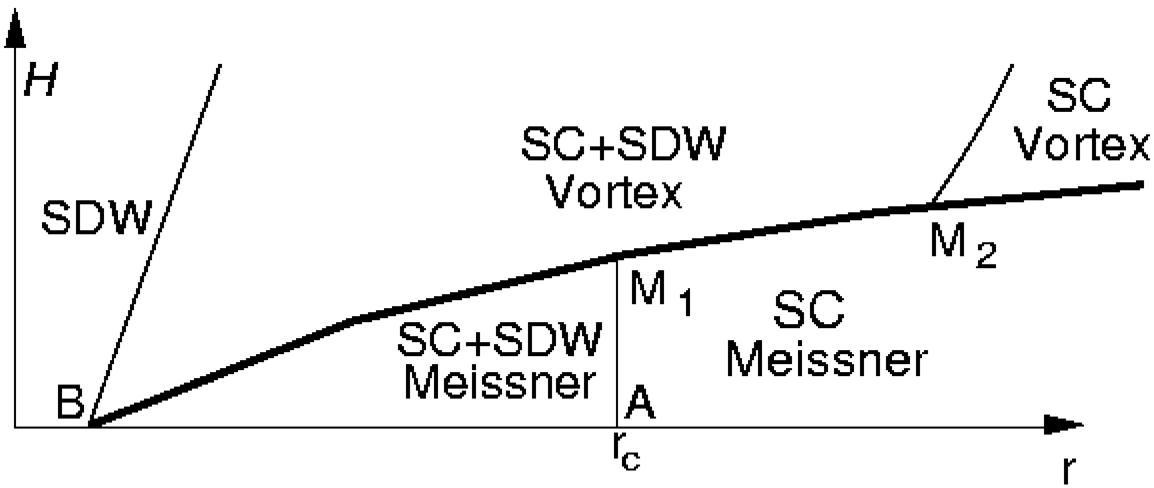
Elastic neutron scattering off $La_{2-x}Sr_xCuO_4$

($x = 0.10$, ***SC + SDW phase***)

in H=0 (blue dots) and H=5T (red dots).

B. Lake, H. Ronnow *et al*., cond-mat/0104026

Consequences of a finite London penetration depth (finite κ)



Outline

I. Coupled Ladder Antiferromagnet
   A. Ground states in limiting regimes
   B. Coherent state path integral
   C. Quantum field theory for critical point

II. Berry phases in one dimension
   S=1/2 quantum XY model.

III. Berry phases in two dimensions
   Bond-centered charge ("spin-Peierls") order.

IV. Magnetic transitions in *d*-wave superconductors
   Effect of an applied magnetic field

V. Transitions between BCS superconductors with distinct internal Cooper pair wavefunctions.
   A. Quantum field theory
   B. Photoemission and tunnelling experiments on the high temperature superconductors

VI. Conclusions

V. Quantum transitions between BCS superconductors

From numerical/analytical/RG studies of the square lattice Hubbard model we know that ground state has

$\Rightarrow$ $d_{x^2-y^2}$ superconductivity near half filling

C.J. Halboth and W. Metzner Phys. Rev. Lett. **85**, 5162 (2000).

$\Rightarrow$ d_{xy} superconductivity for small electron density

M.A. Baranov and M. Yu Kagan, Z. Phys. B **86**, 237 (1992).
M.A. Baranov, A.V. Chubukov, and M. Yu Kagan, Int. J. Mod. Phys. B **6**, 2471 (1992).

We model this phenomenologically:

$$H = \sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J_1 \sum_{j,\mu} \mathbf{S}_j \cdot \mathbf{S}_{j+\hat{\mu}} + J_2 \sum_{j,\nu} \mathbf{S}_j \cdot \mathbf{S}_{j+\hat{\nu}}$$

where the sum on μ is over x, y, that on ν is over $x+y$, $x-y$,

$$\mathbf{S}_j \equiv \frac{1}{2} c_{j\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'}$$

with $\vec{\sigma}$ the Pauli matrices, and the dispersion

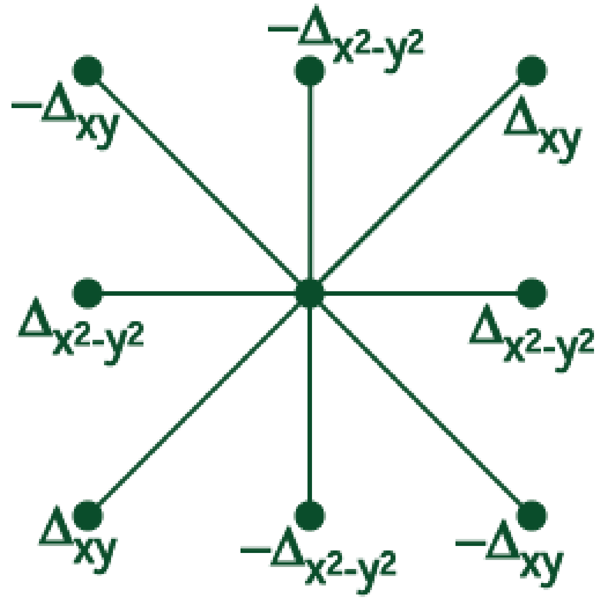
$$\varepsilon_k = -2t_1(\cos(k_x) + \cos(k_y)) - 2t_2(\cos(k_x+k_y) + \cos(k_x-k_y)) - \mu$$

BCS mean-field theory

$$H_{BCS} = \sum_k \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} \quad - \quad \frac{J_1}{2} \sum_{j,\mu} \Delta_\mu (c^\dagger_{j\uparrow} c^\dagger_{j+\hat{\mu},\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{j+\hat{\mu},\uparrow}) + \text{h.c.}$$
$$- \quad \frac{J_2}{2} \sum_{j,\nu} \Delta_\nu (c^\dagger_{j\uparrow} c^\dagger_{j+\hat{\nu},\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{j+\hat{\nu},\uparrow}) + \text{h.c.}$$

We choose

$$\begin{aligned} \Delta_x &= -\Delta_y \equiv \Delta_{x^2-y^2} \\ \Delta_{x+y} &= -\Delta_{x-y} \equiv \Delta_{xy} \end{aligned}$$



The complex numbers $\Delta_{x^2-y^2}$ and Δ_{xy} are to be determined by minimizing the ground state energy per site

$$E_{BCS} = J_1 |\Delta_{x^2-y^2}|^2 + J_2 |\Delta_{xy}|^2 - \int \frac{d^2k}{4\pi^2} \left[E_k - \varepsilon_k \right]$$

where the fermionic quasiparticle dispersion is

$$E_k = \left[\varepsilon_k^2 + \left| J_1 \Delta_{x^2-y^2} (\cos k_x - \cos k_y) + 2 J_2 \Delta_{xy} \sin k_x \sin k_y \right|^2 \right]^{1/2}$$

The energy only depends upon the relative phase between $\Delta_{x^2-y^2}$ and Δ_{xy}.

Evolution of ground state

$\Delta_{x^2-y^2} \neq 0$

$\Delta_{xy} = 0$

$d_{x^2-y^2}$ superconductor

$\Delta_{x^2-y^2} \neq 0$

$\Delta_{xy} \neq 0$

$$\arg\left(\Delta_{xy}\right) = \arg\left(\Delta_{x^2-y^2}\right) \pm \frac{\pi}{2}$$

$d_{x^2-y^2} + i d_{xy}$ superconductor

$\Delta_{x^2-y^2} = 0$

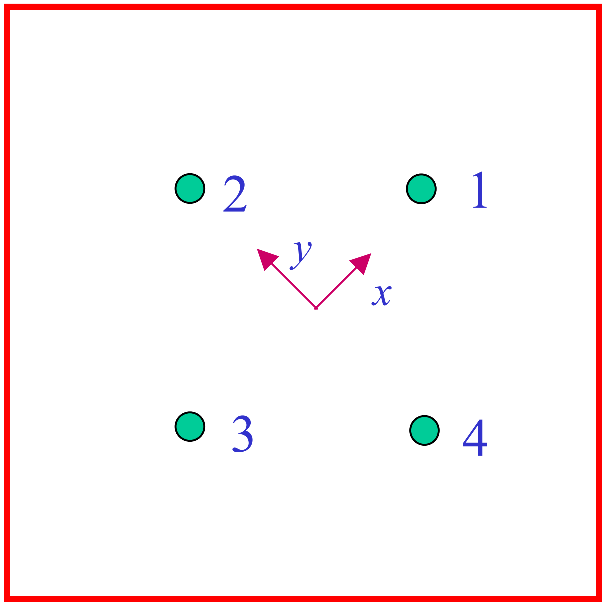
$\Delta_{xy} \neq 0$

d_{xy} superconductor

J_2 / J_1

BCS theory fails near quantum critical points

Field theory for transition from $d_{x^2-y^2}$ to $d_{x^2-y^2}+id_{xy}$ superconductivity



Gapless Fermi Points in a d-wave superconductor at wavevectors $(\pm K, \pm K)$

K=0.391π

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^\dagger \\ f_{1\downarrow} \\ -f_{3\uparrow}^\dagger \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^\dagger \\ f_{2\downarrow} \\ -f_{4\uparrow}^\dagger \end{pmatrix}$$

$$S_\Psi = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_1$$

$$+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_2$$

Ising order parameter for transition $\phi \sim i\Delta_{xy}$

Coupling to low energy fermions

$$i\lambda \int d^2x d\tau \left[\phi \left(f_{1\uparrow}^\dagger f_{3\downarrow}^\dagger - f_{1\downarrow}^\dagger f_{3\uparrow}^\dagger - f_{2\uparrow}^\dagger f_{4\downarrow}^\dagger - f_{2\downarrow}^\dagger f_{4\uparrow}^\dagger \right) + \text{h.c.} \right]$$

$$= \int d^2x d\tau \left[\lambda\phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right]$$

Action for low energy fluctuations near critical point

$$S = \int d^2x d\tau \left[\frac{1}{2} (\partial_\tau \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 - \frac{s}{2}\phi^2 + \frac{u}{24}\phi^4 \right.$$
$$+ \Psi_1^\dagger \left(\partial_\tau + i v_F \partial_x \tau^z + i v_\Delta \partial_y \tau^x \right) \Psi_1$$
$$+ \Psi_2^\dagger \left(\partial_\tau + i v_F \partial_y \tau^z + i v_\Delta \partial_x \tau^x \right) \Psi_2$$
$$\left. + \lambda\phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right]$$

{For $v_F = v_\Delta$ terms with fermions

$$= \overline{\Psi}_1 \gamma_\mu \partial_\mu \Psi_1 + \overline{\Psi}_2 \gamma_\mu \partial_\mu \Psi_2 + \lambda\phi\left(\overline{\Psi}_1 \Psi_1 - \overline{\Psi}_2 \Psi_2\right)$$

Chiral symmetry breaking in the Higgs-Yukawa model}

Momentum shell renormalization group equations

$$\frac{d\lambda}{d\ell} = \frac{(3-d)}{2}\lambda - C_1\lambda^3$$

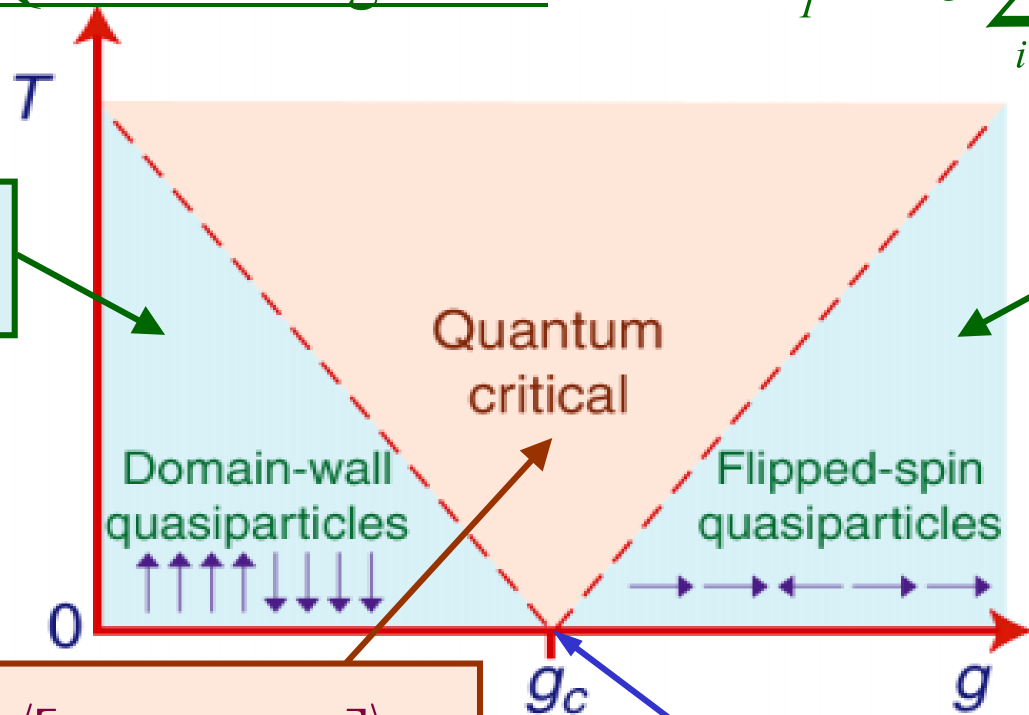
$$\frac{du}{d\ell} = (3-d)u - C_2u^2 + C_3\lambda^4 - C_4\lambda^2 u$$

where C_{1-4} are functions of v_F / v_Δ; similar flow equation for v_F / v_Δ. Critical point is controlled by fixed point $u = u^*,\ \lambda = \lambda^*, v_F^* = v_\Delta^*$, which obeys hyperscaling, and is Lorentz invariant.

Finite temperature crossovers near a quantum critical point: Quantum Ising Chain

$$H_I = -J \sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics



Quasiclassical dynamics

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4}\left(1 - i\omega / \Gamma_R + \ldots\right)}$$

$$\Gamma_R = \left(2\tan\frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{|j-k|^{1/4}}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

In quasiparticle regimes of Ising chain, spin relaxation rate $\sim e^{\Delta/T}$

In a Fermi liquid, quasiparticle relaxation rate $\sim T^2$

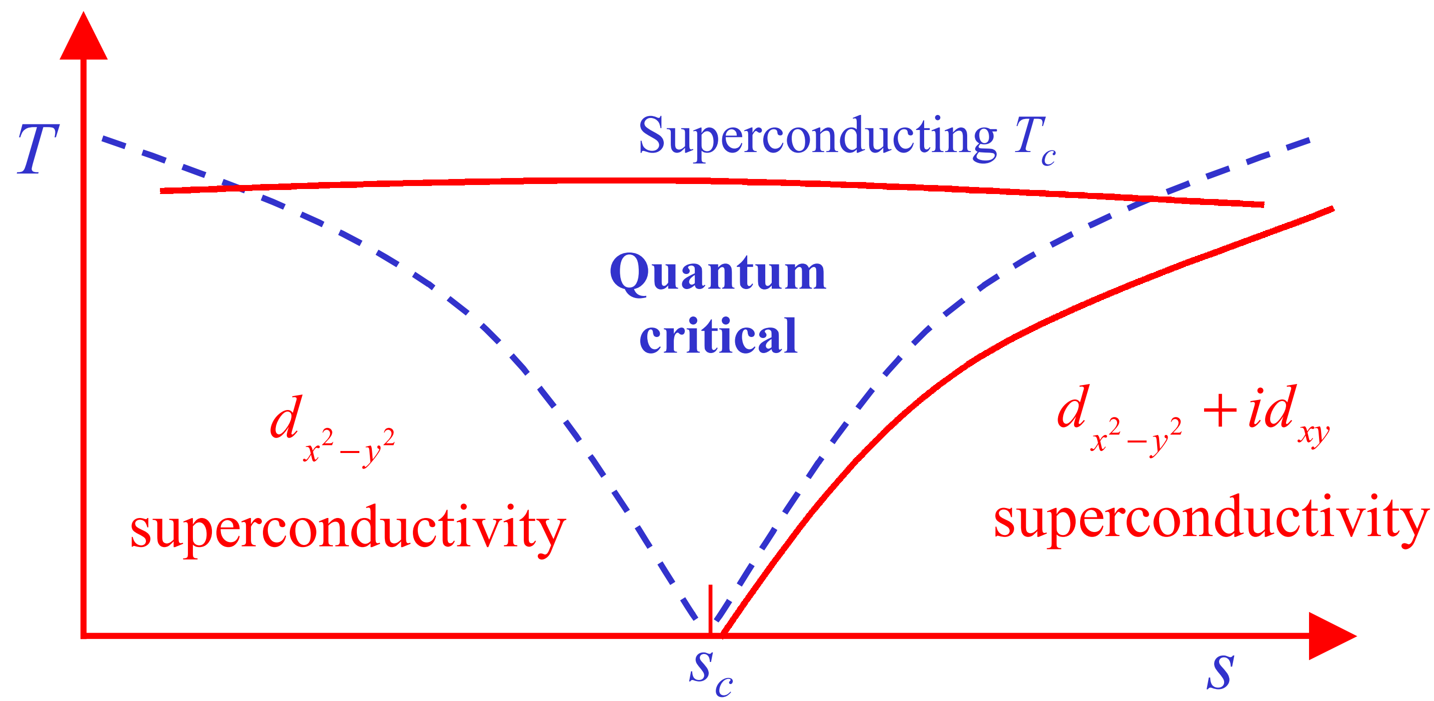
In a BCS d-wave superconductor, quasiparticle relaxation rate $\sim T^3$

In quantum critical region associated with a second-order phase transition with hyperscaling properties $\Rightarrow$

Relaxation dynamics with rate $\sim k_B T / \hbar$

$$\chi(\omega) = \frac{C}{T^{2-\eta}} \Phi\left(\frac{\hbar\omega}{k_B T}\right)$$

Crossovers near transition in *d*-wave superconductor

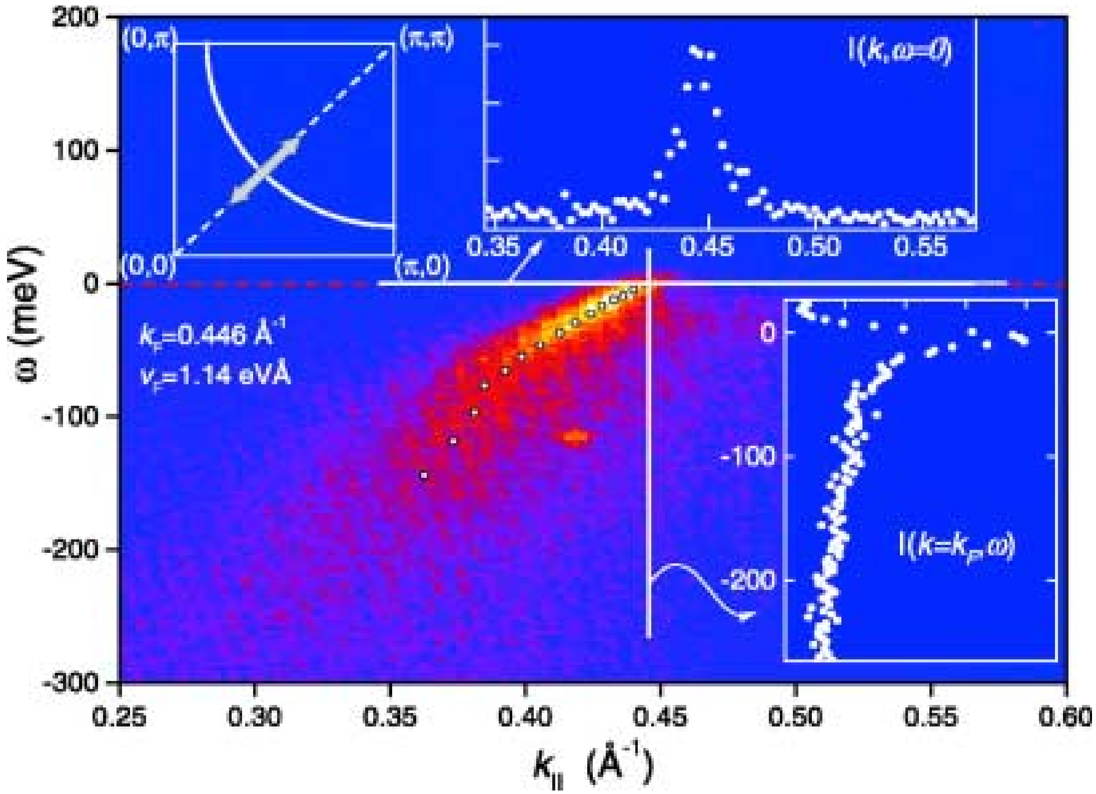


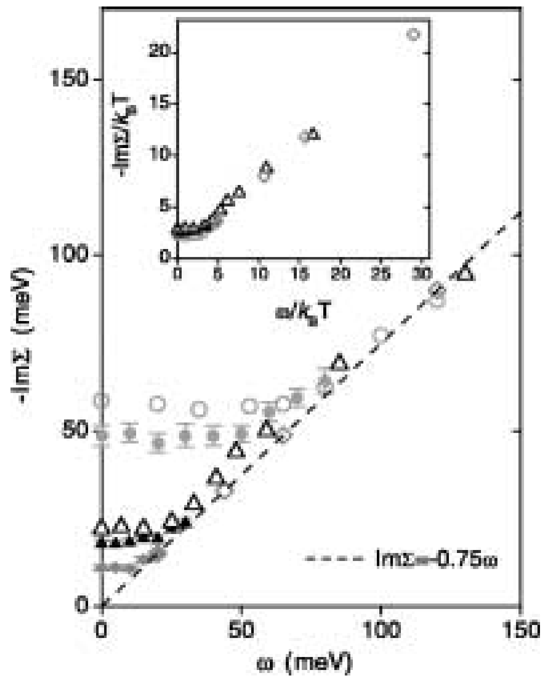
S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000).

Damping of Nodal Quasiparticles

Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))





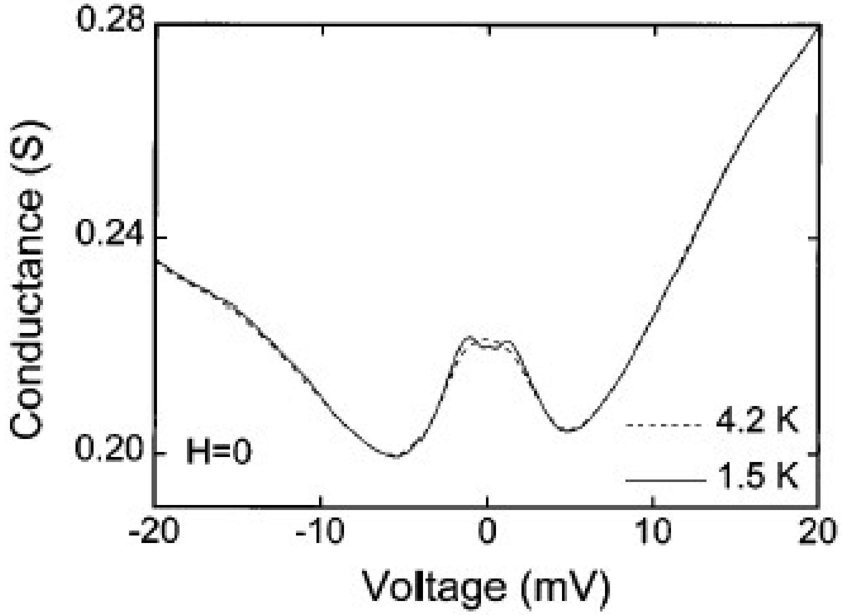
$$\mathrm{Im}\,\Sigma \sim k_B T$$

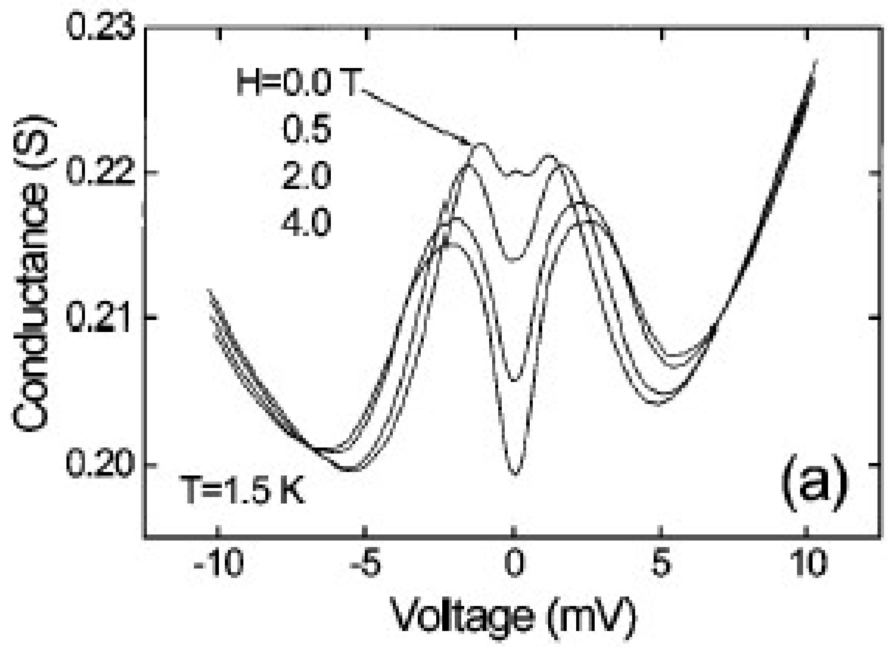
Yoram Dagan and Guy Deutscher, cond-mat/0106128

Observations of splitting of the ZBCP

Spontaneous splitting (zero field)

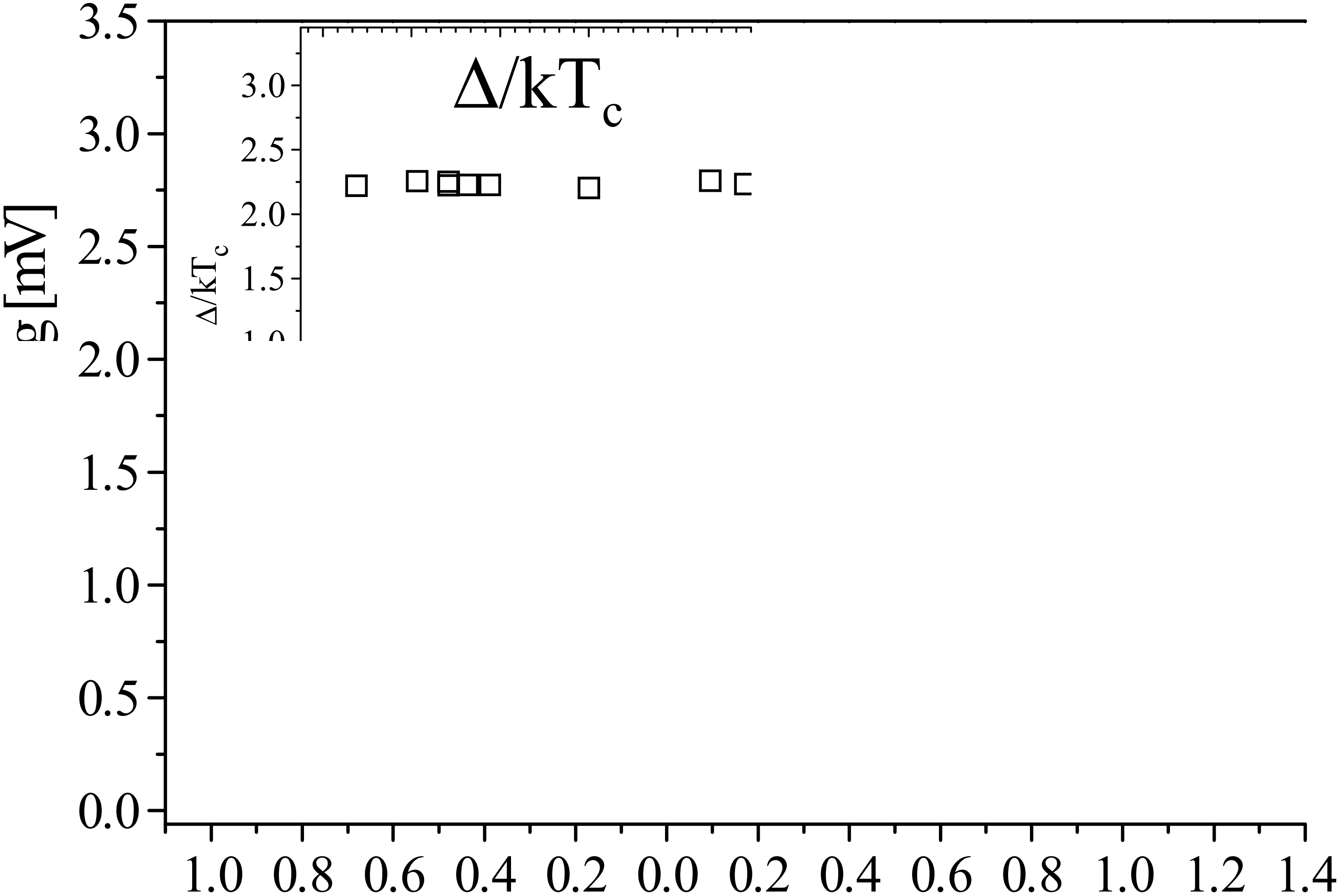
Magnetic field splitting





Covington, M. *et al.* Observation of Surface-Induced Broken Time-Reversal Symmetry in $YBa_2Cu_3O_{7-\delta}$ Tunnel Junctions, Phys. Rev. Lett. **79**, 277-281 (1997)

Zero Field splitting and χ^{-1} versus $[\Delta_{max}-\Delta]^{1/2}$ All YBCO samples



Conclusions I: Phase transitions of BCS superconductors

Examined general theory of all possible candidates for zero momentum, spin-singlet order parameters which can induce a second-order quantum phase transitions in a *d*-wave superconductor

Only cases

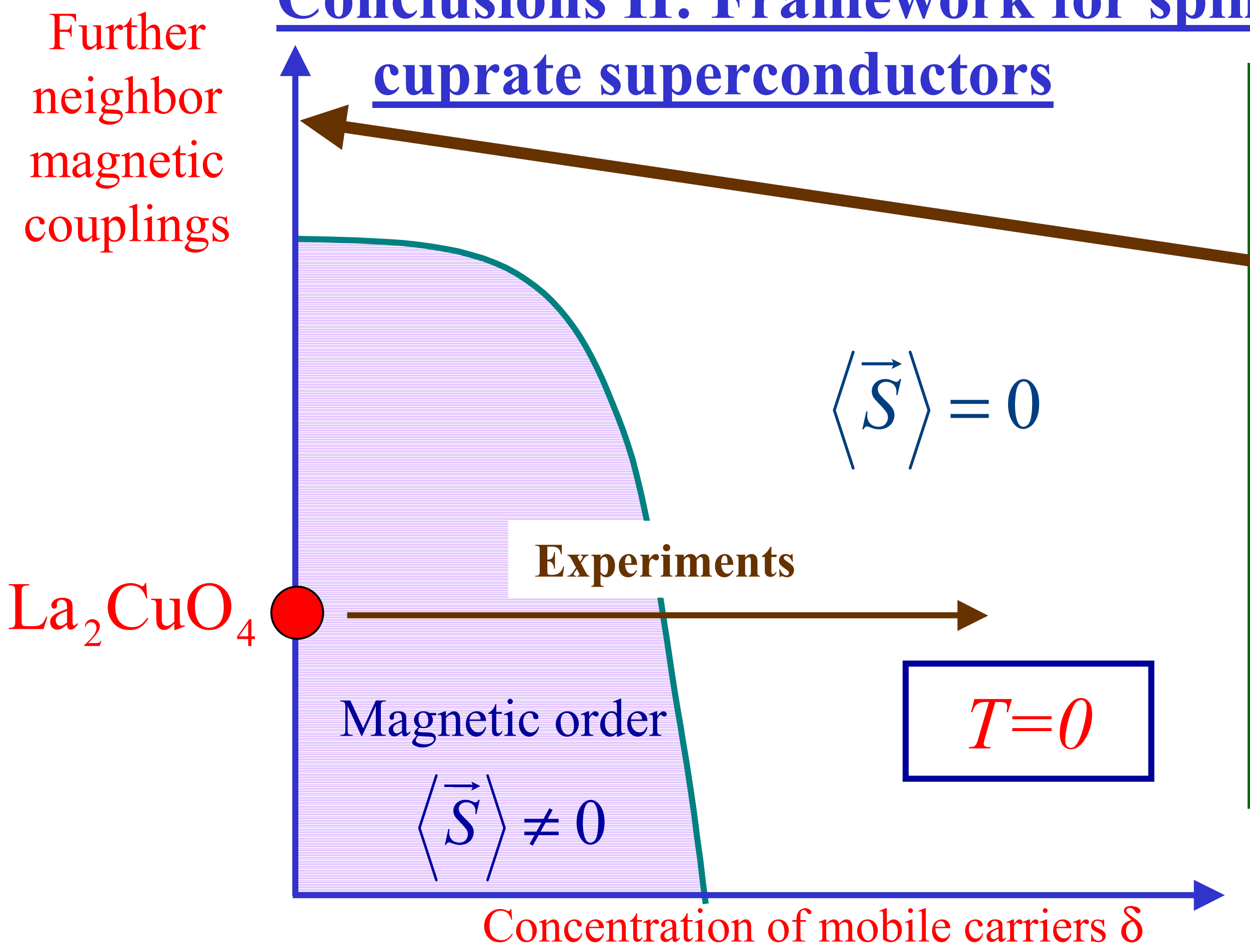
(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions, and so are candidates for producing damping $\sim k_B T$ of nodal fermions.

Independent evidence for (B) from tunneling experiments.

Conclusions II: Framework for spin/charge order in cuprate superconductors



Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. S=1/2 moments near non-magnetic impurities

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations