Tuning antiferromagnetic quantum criticality in the cuprates by a magnetic field

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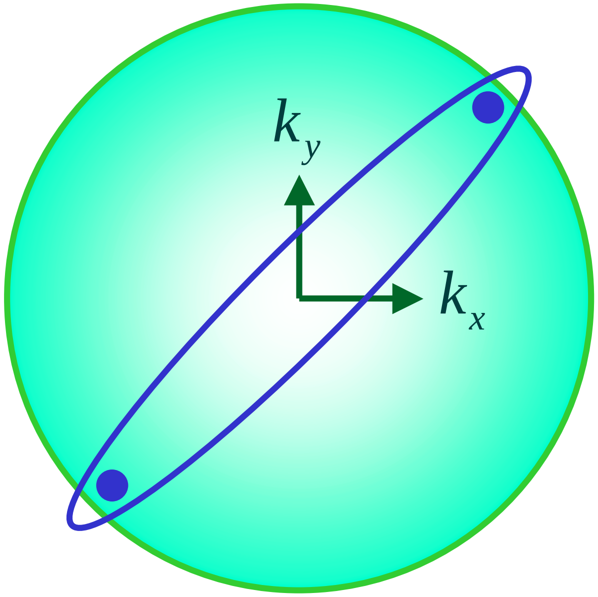
Talk online at
http://pantheon.yale.edu/~subir
(Search for “*Sachdev*” on Google)

Central message on theory of cuprate superconductivity

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid



Pair wavefunction

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Some *low* temperature properties of the cuprate superconductors appear to be qualitatively similar to those predicted by BCS theory.

Central message on theory of cuprate superconductivity (contd)

Superconductivity in a doped Mott insulator

When superconductivity is disrupted at low temperatures, BCS theory predicts that the Fermi surface will reappear.

Instead, we obtain states characterized by competing order parameters.

Theory and experiments indicate that the most likely competing orders are spin and "charge" density waves.

Superconductivity can be suppressed globally by a strong magnetic field or large current flow.

Competing orders are also revealed when superconductivity is suppressed locally, near impurities or around vortices.

S. Sachdev, *Phys. Rev.* B **45**, 389 (1992); N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992);
D.P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang *Phys. Rev. Lett.* **79**, 2871 (1997);
K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).

Outline

I. Introduction to theory of competing orders and quantum criticality
   A. Quantum Ising Chain.
   B. Coupled Ladder Antiferromagnet.

II. Spin density wave (SDW) order in LSCO
   A. Quantum criticality at finite temperature.
   B. Tuning order and transitions by a magnetic field.

III. Connections with “charge” order
   STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

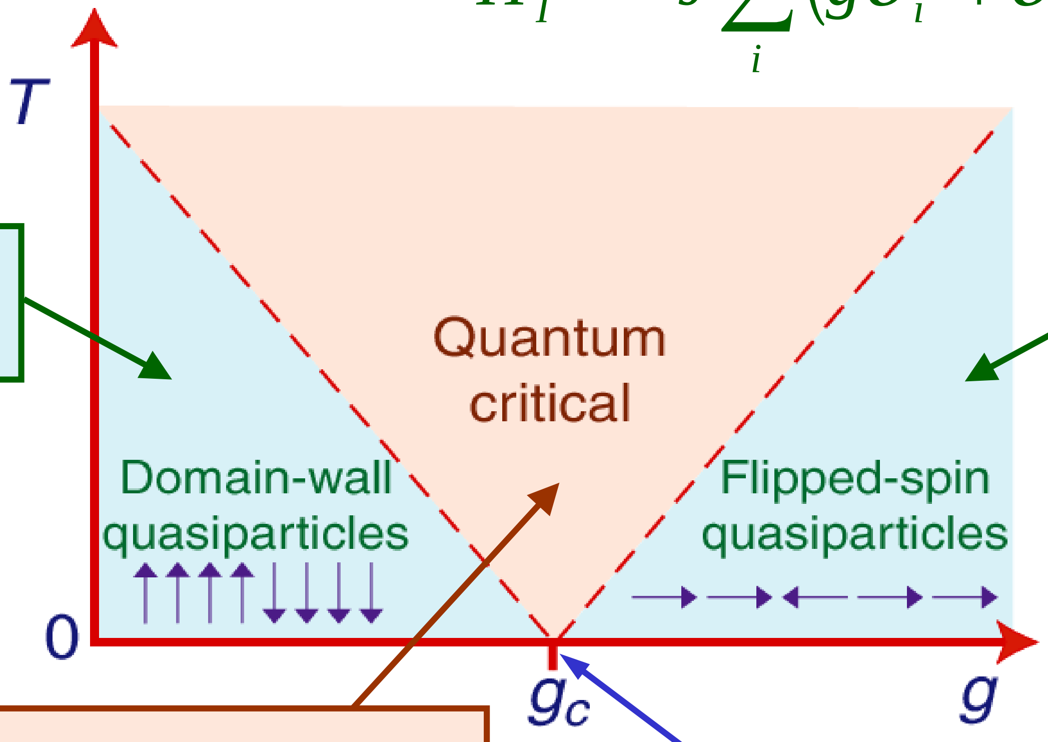
IV. Microstructure of “charge” order:
   Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods.

V. Conclusions

I.A Quantum Ising Chain

$$H_I = -J\sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics



Quasiclassical dynamics

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle \right)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle \right)$$

$$\chi(\omega) = \frac{i}{\hbar}\sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4}\left(1 - i\omega/\Gamma_R + \ldots\right)}$$

$$\Gamma_R = \left(2\tan\frac{\pi}{16} \right)\frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{|j-k|^{1/4}}$$

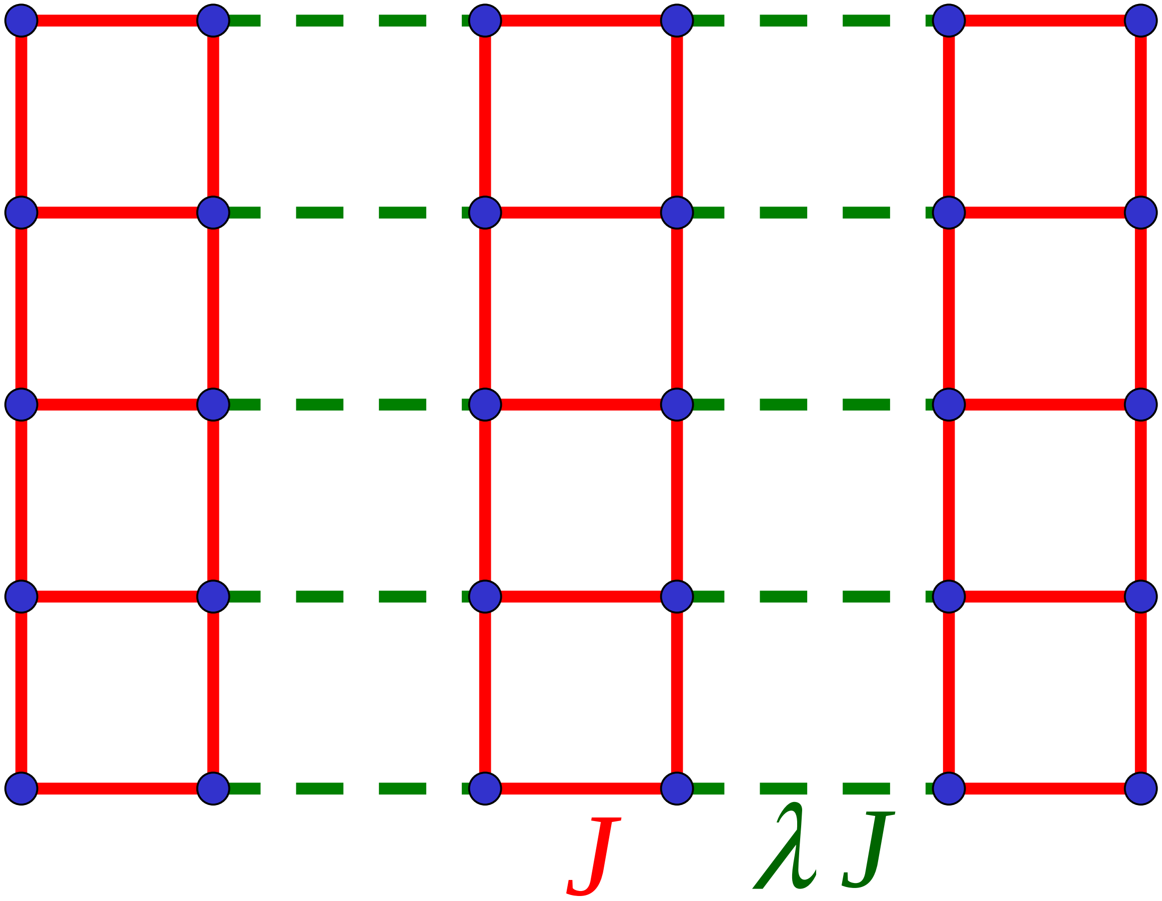
P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

I.B Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen,
Phys. Rev. B **59**, 115 (1999).

S=1/2 spins on coupled 2-leg ladders



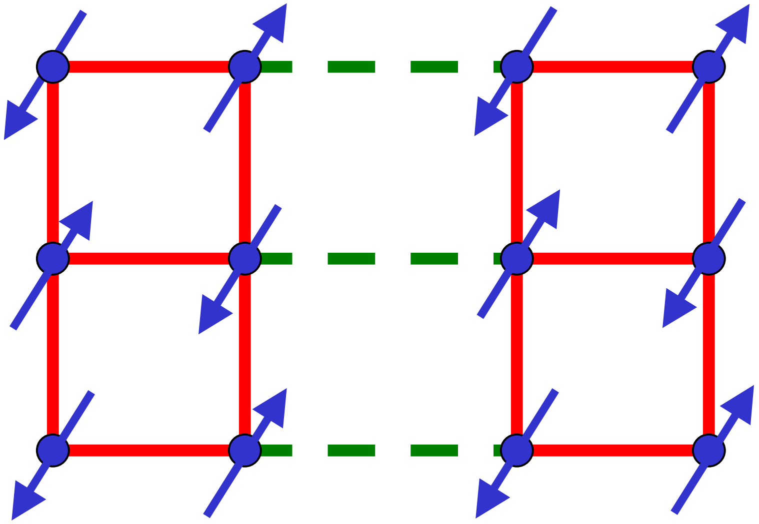
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

$\boxed{\lambda \text{ close to } 1}$

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



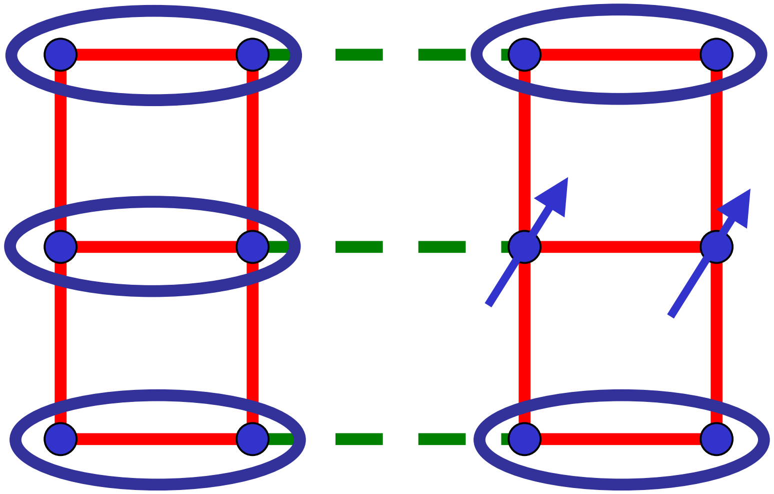
Ground state has long-range magnetic (Neel) order

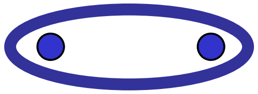
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Paramagnetic ground state

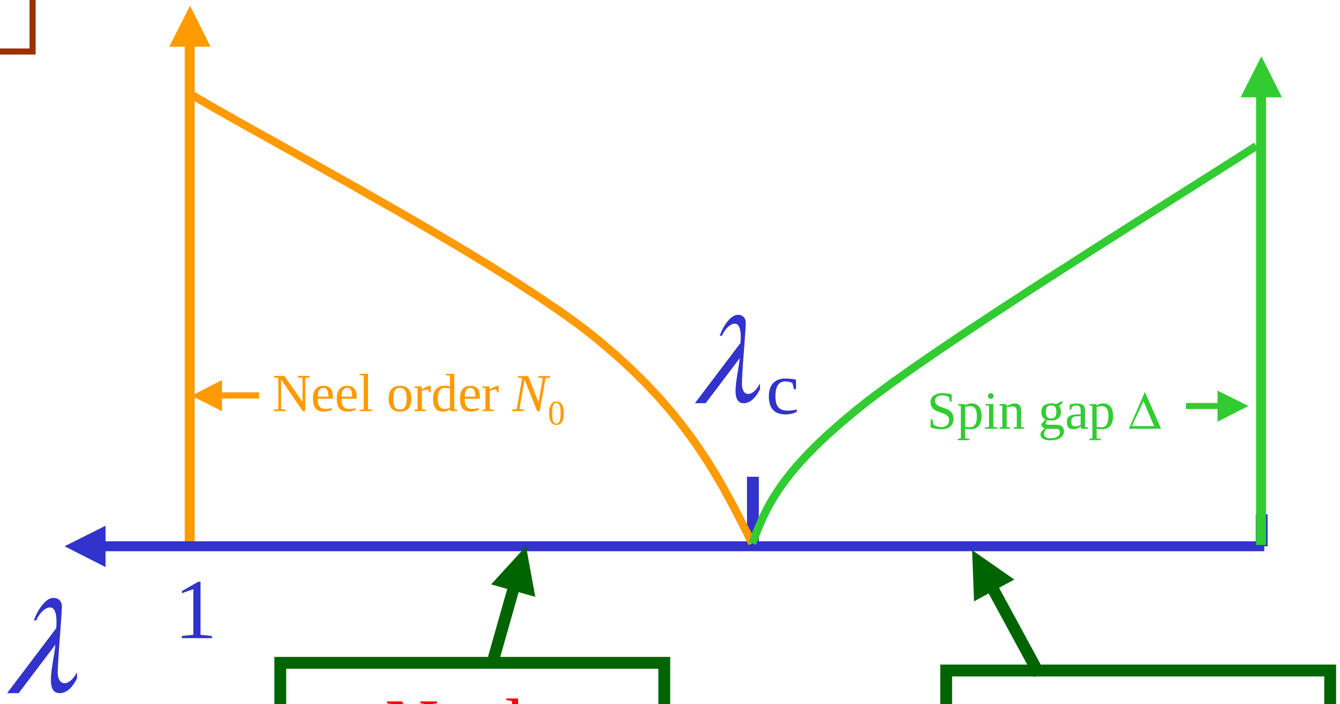
$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: *S*=1 ***exciton*** (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$

T=0

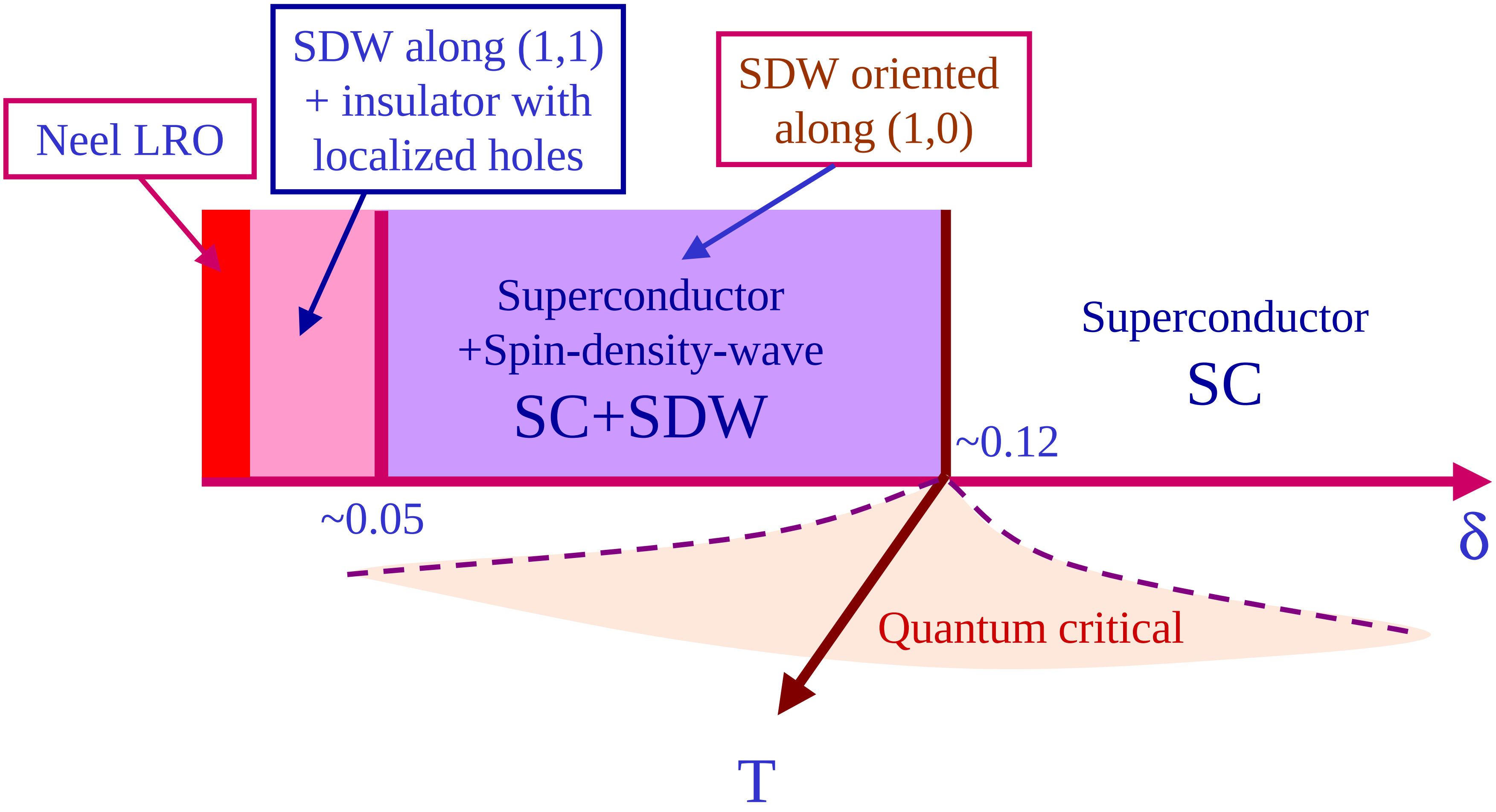


Neel
state
$\langle \vec{S} \rangle = N_0$

Quantum
paramagnet
$\langle \vec{S} \rangle = 0$

Theory for a system with strong interactions: describe both phases by expanding in the deviation from the quantum critical point between them.

II.A Phase diagram of LSCO at zero temperature measured by neutron scattering



Neel LRO

SDW along (1,1)
+ insulator with
localized holes

SDW oriented
along (1,0)

Superconductor
+Spin-density-wave
SC+SDW

Superconductor
SC

~0.12

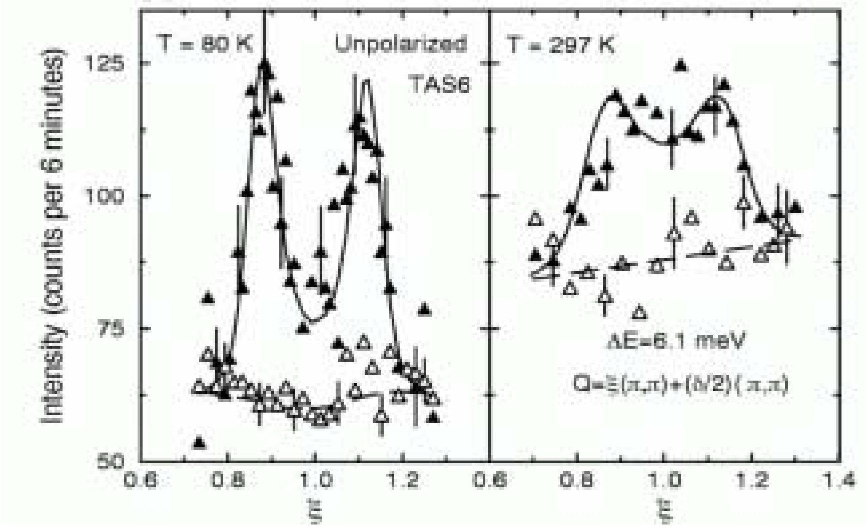
~0.05

δ

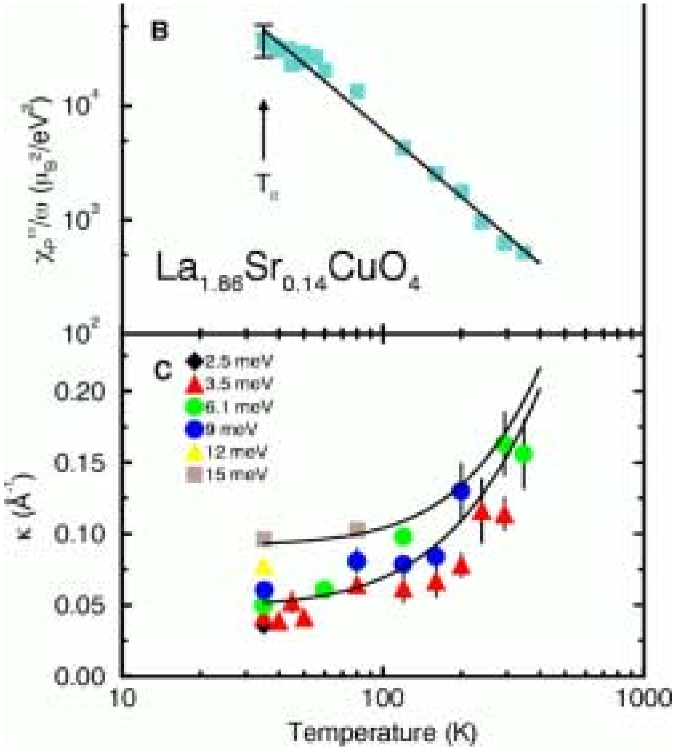
Quantum critical

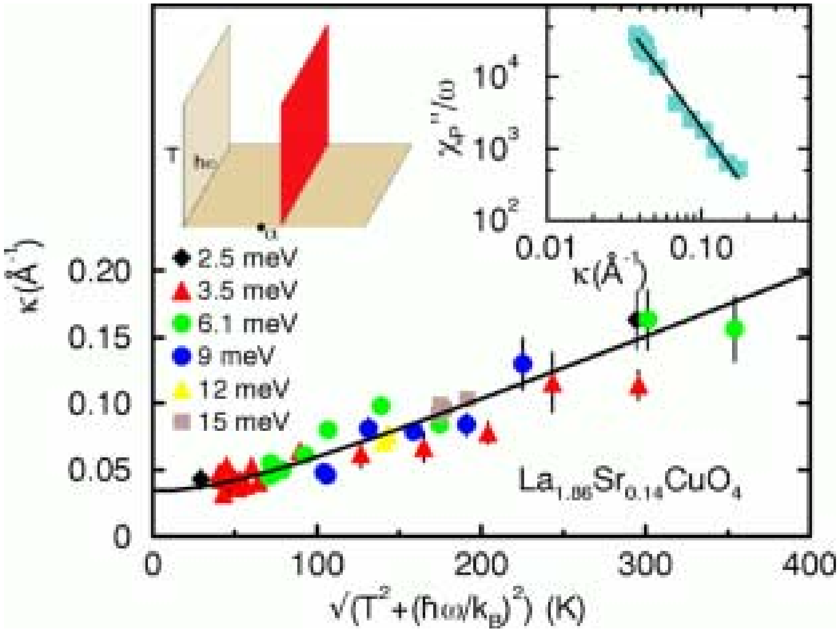
T

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.



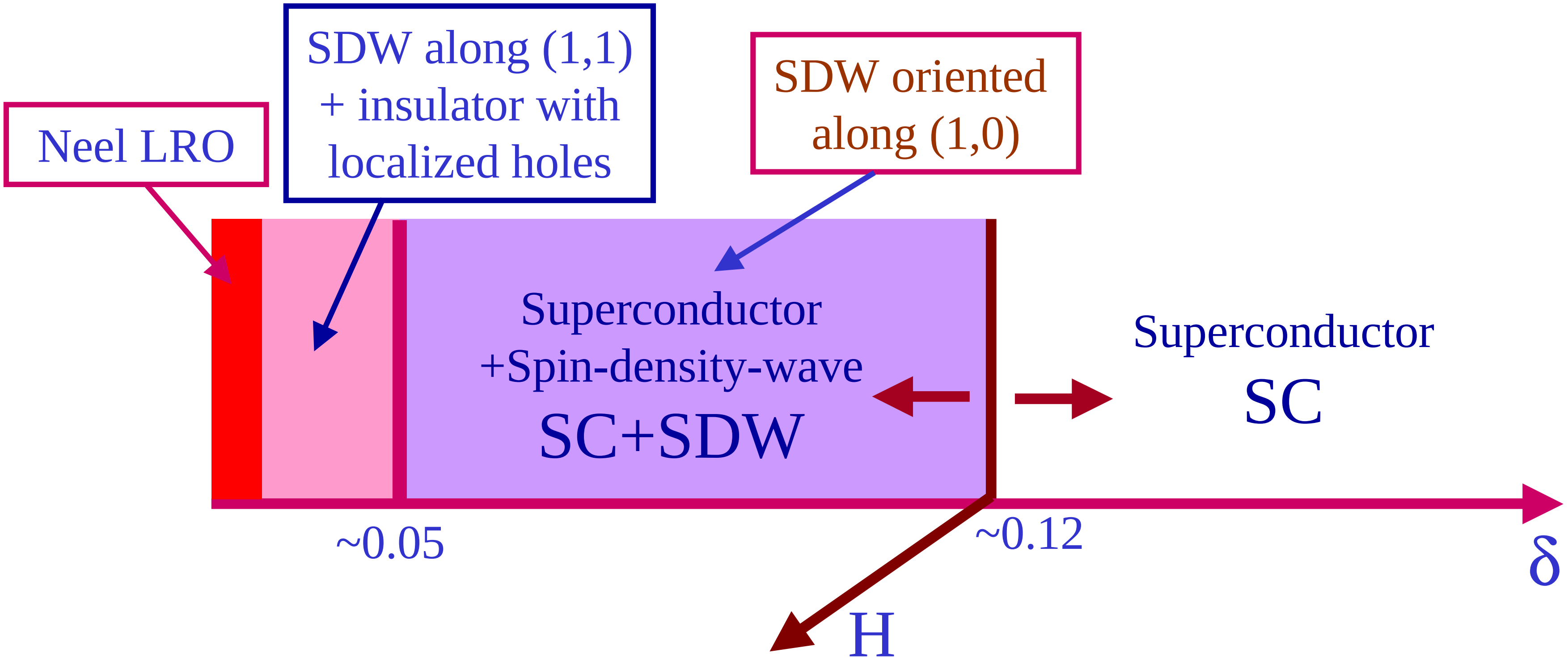
Neutron scattering measurements of dynamic spin correlations in $La_{1.86}Sr_{0.14}CuO_4$ at $\boldsymbol{K} \approx \left(3\pi/4,\pi\right)$: T and ω dependent divergence scaling as a function of $\hbar\omega/k_BT$





G. Aeppli, T.E. Mason, S,M. Hayden, H.A. Mook, and J. Kulda, *Science* **278**, 1432 (1998).

II.B Phase diagram of LSCO at zero temperature measured by neutron scattering



Neel LRO

SDW along (1,1)
+ insulator with
localized holes

SDW oriented
along (1,0)

Superconductor
+Spin-density-wave
SC+SDW

Superconductor
SC

~0.05

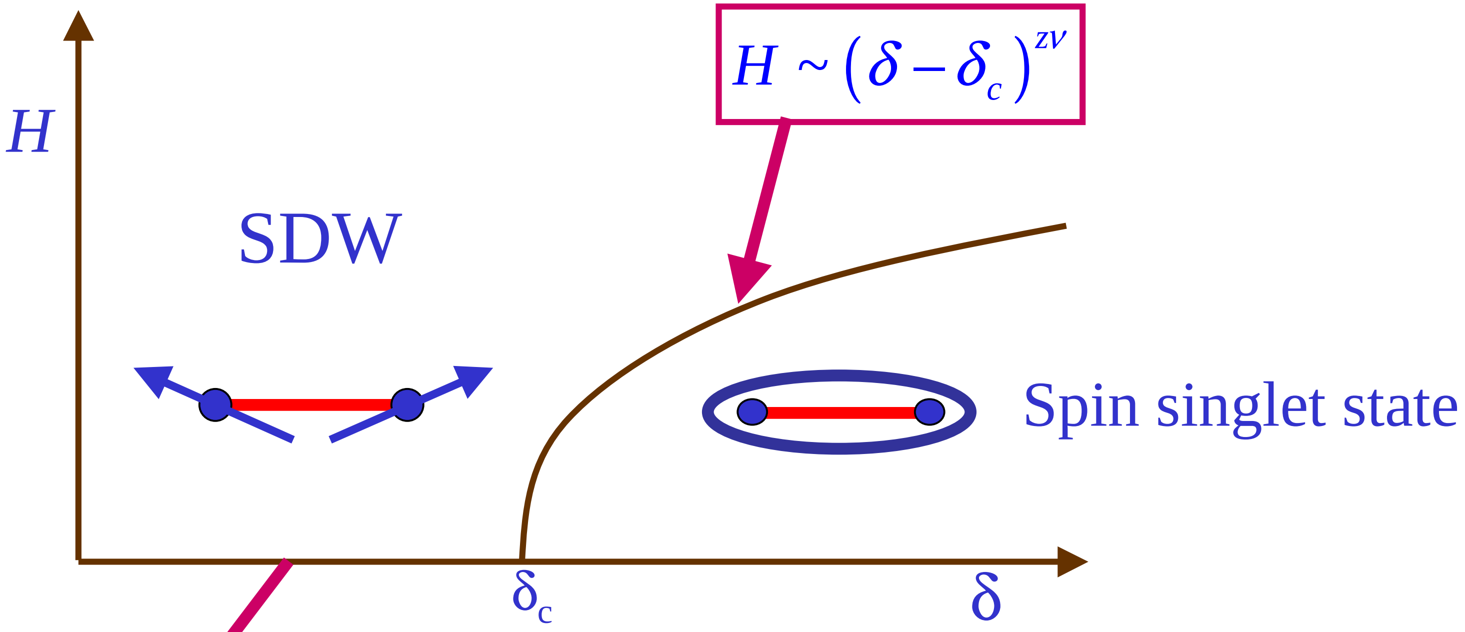
~0.12

δ

H

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Effect of the Zeeman term: precession of SDW order about the magnetic field



$$H \sim (\delta - \delta_c)^{z\nu}$$

Elastic scattering intensity

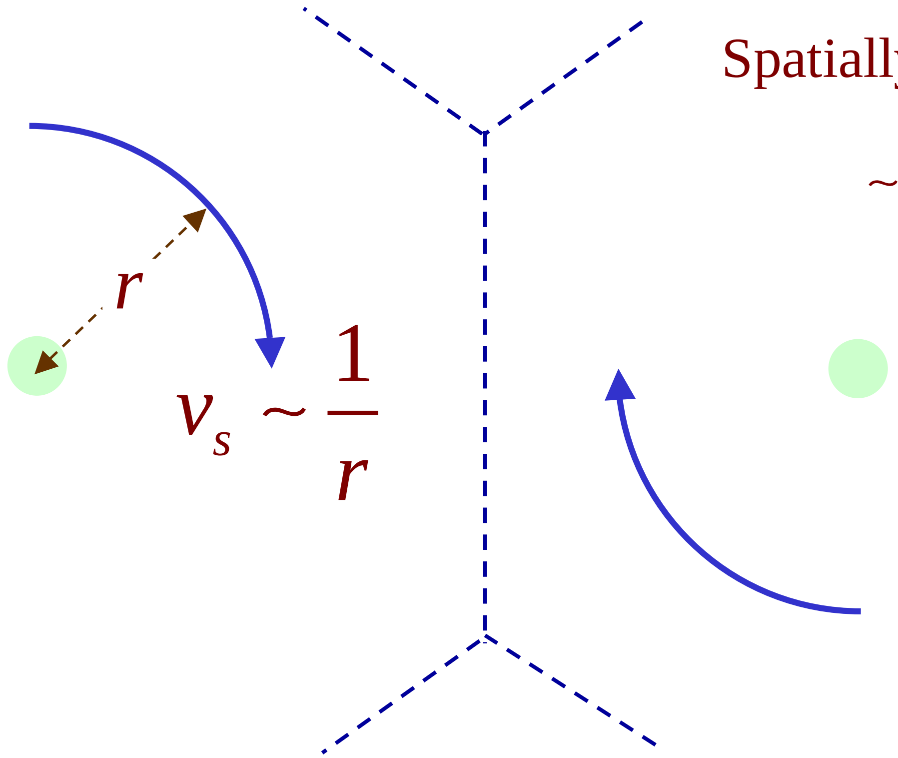
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



$$v_s \sim \frac{1}{r}$$

Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

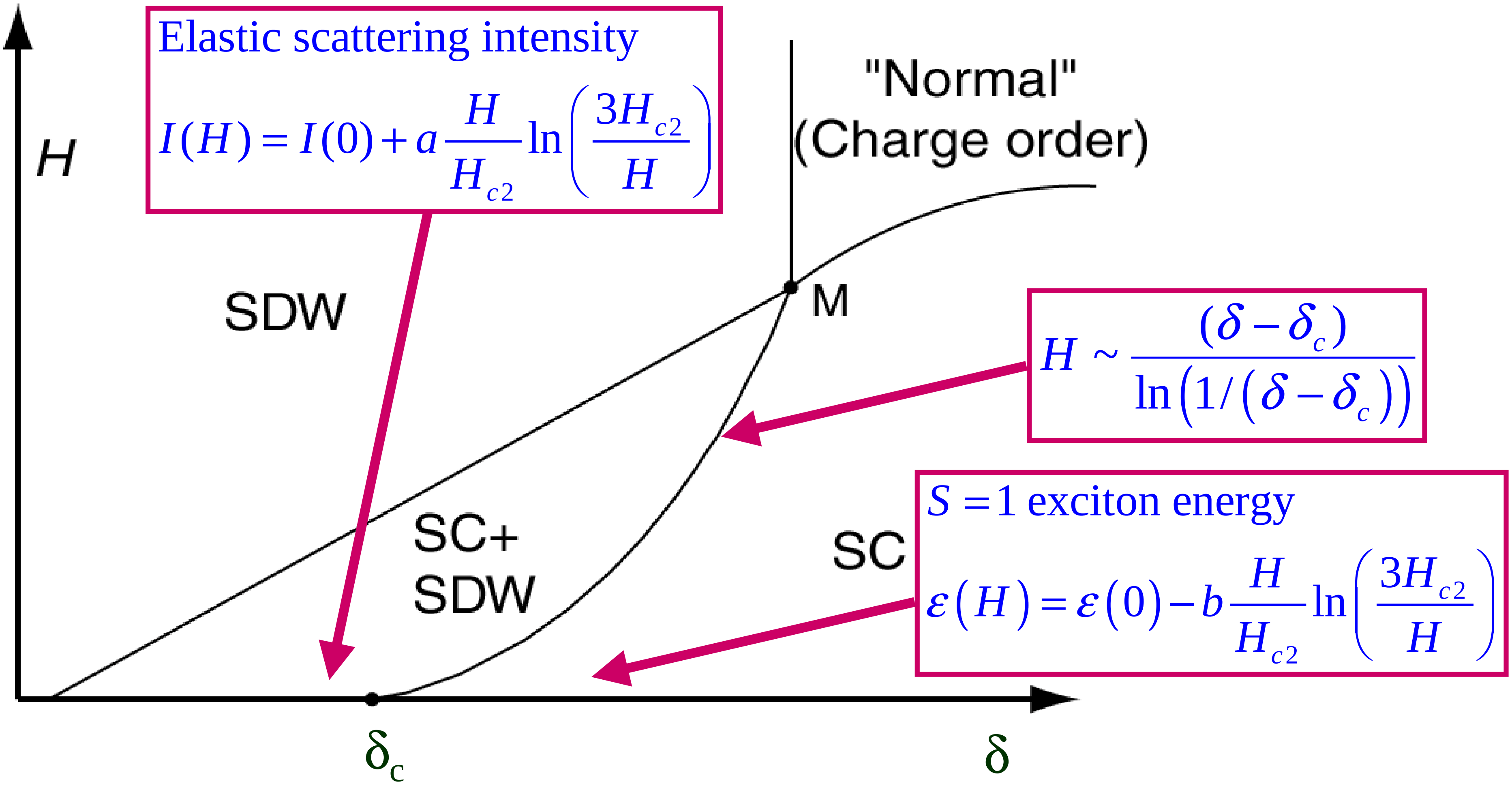
The presence of the field replaces δ by

$$\delta_{\mathit{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Main results

T=0



Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(\delta-\delta_c)}{\ln\left(1/(\delta-\delta_c)\right)}$$

$S=1$ exciton energy

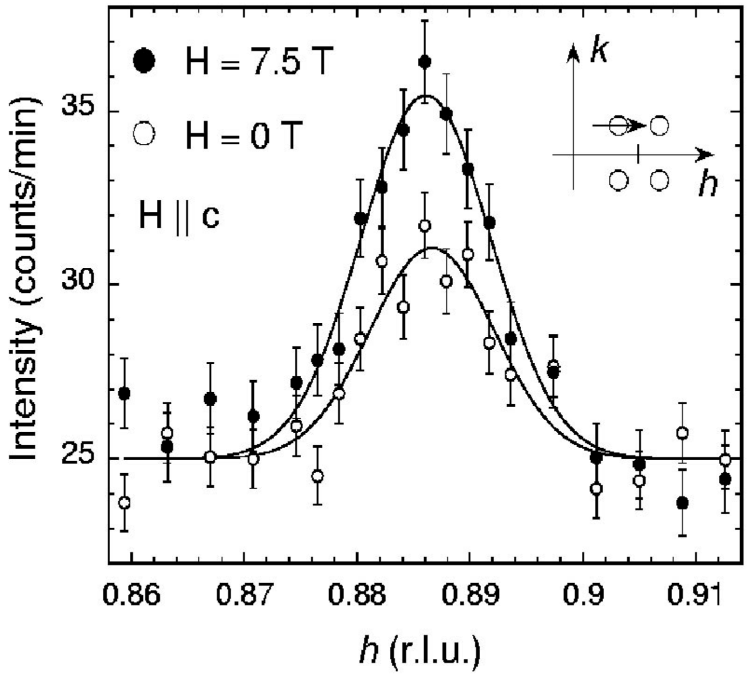
$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

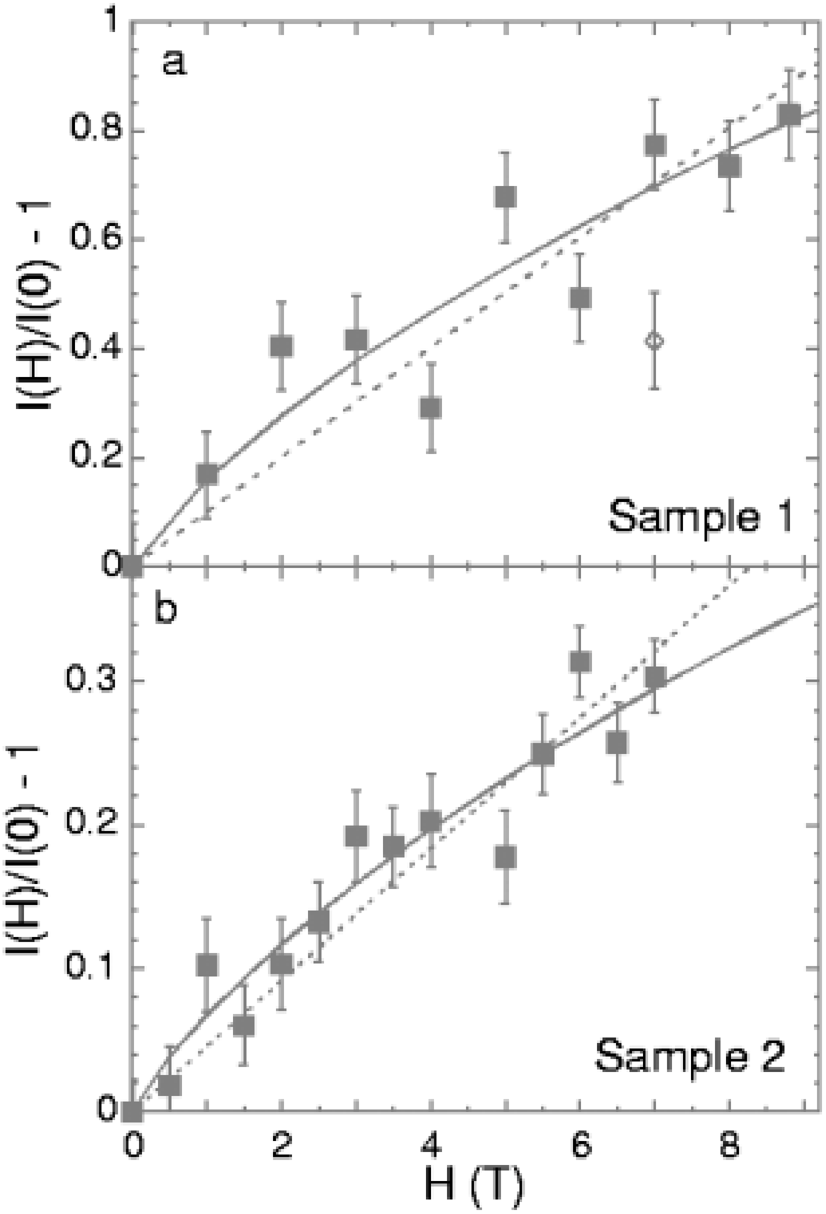
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La$_2$CuO$_{4+y}$

B. Khaykovich, Y. S. Lee, S. Wakimoto,
K. J. Thomas, M. A. Kastner,
and R.J. Birgeneau, cond-mat/0112505.



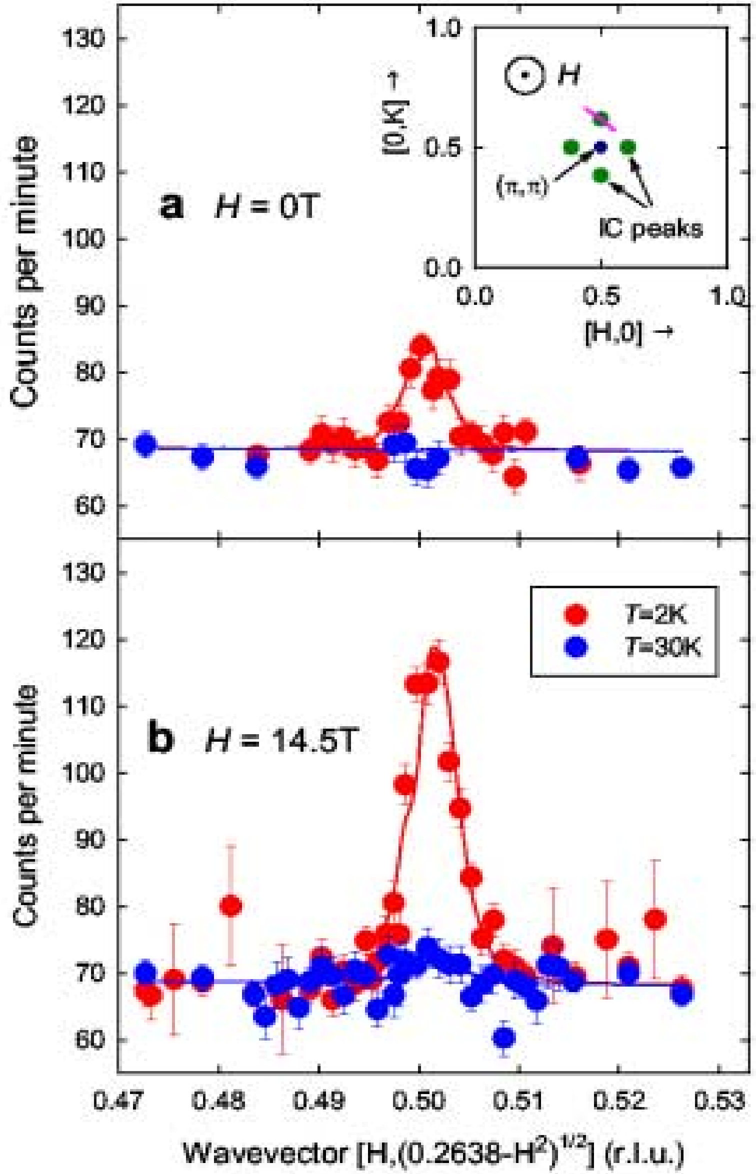


Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

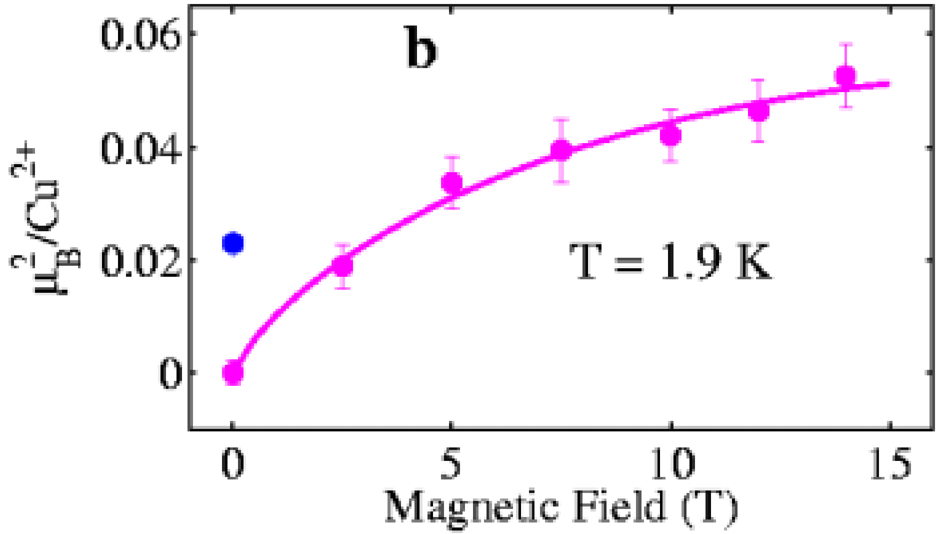
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen,
G. Aeppli, K. Lefmann, D. F. McMorrow,
P. Vorderwisch, P. Smeibidl, N. Mangkorntong,
T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason,
Nature, **415**, 299 (2002).



Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

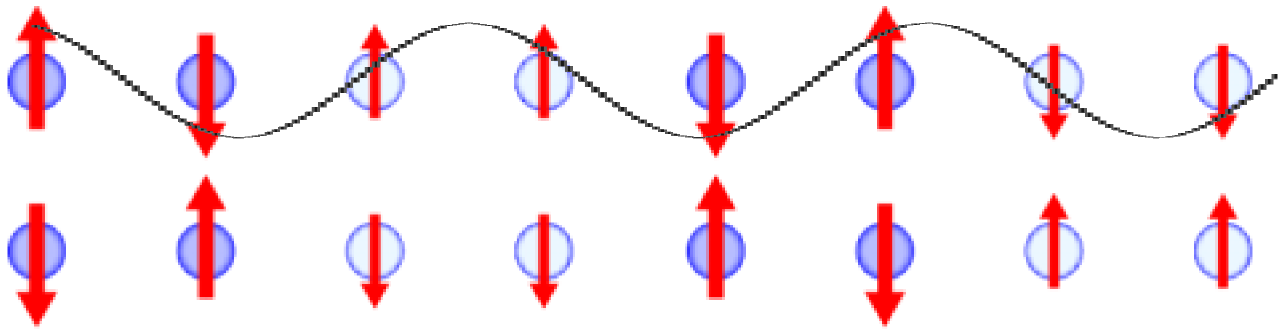
III. Connections with "charge" order

Spin density wave order parameter for general ordering wavevector

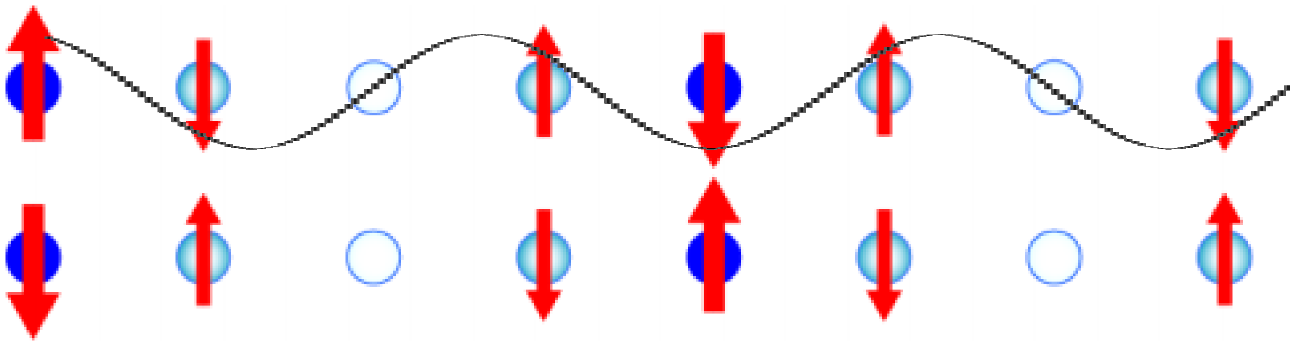
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K} = (3\pi/4, \pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\boxed{\Phi_\alpha = e^{i\theta} n_\alpha}$



Bond-centered



Site-centered

A longitudinal spin density wave necessarily has an accompanying modulation in the site charge densities, exchange and pairing energy per link etc. at half the wavelength of the SDW

“Charge” order: periodic modulation in local observables invariant under spin rotations and time-reversal.

Order parmeter $\sim \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r})$

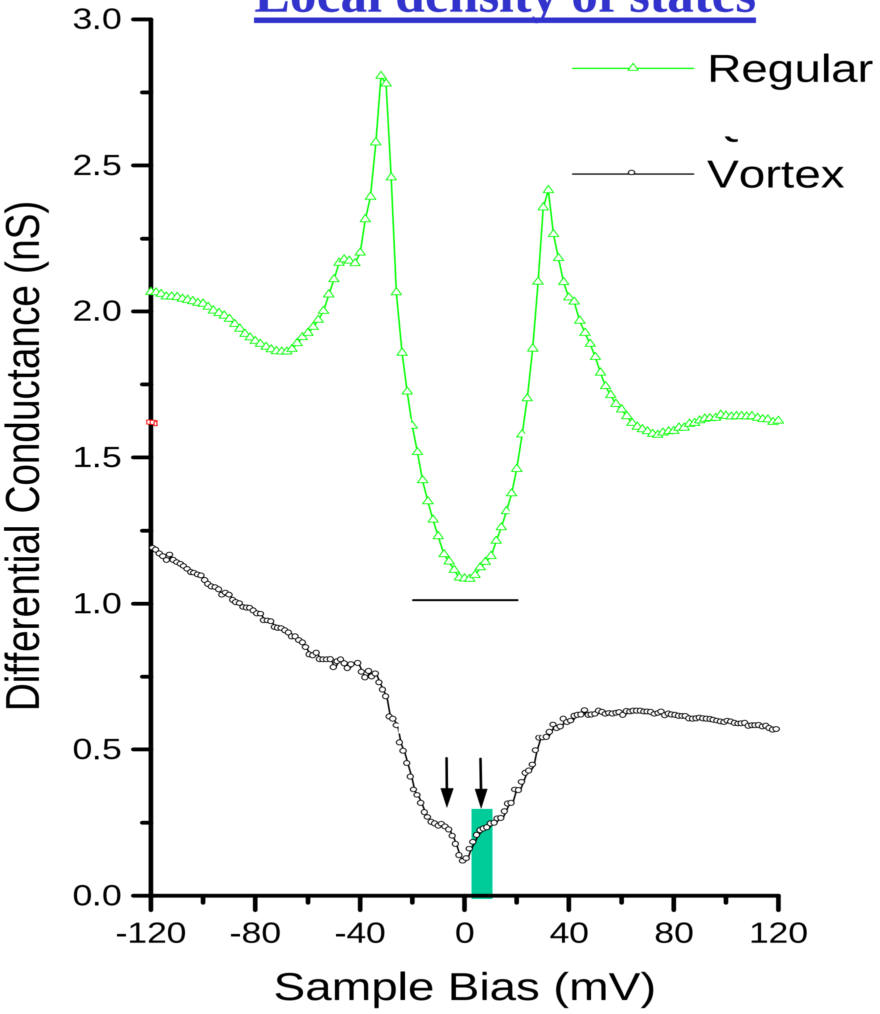
$$\delta\rho(\boldsymbol{r}) \propto S_{\alpha}^{2}(\boldsymbol{r}) = \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

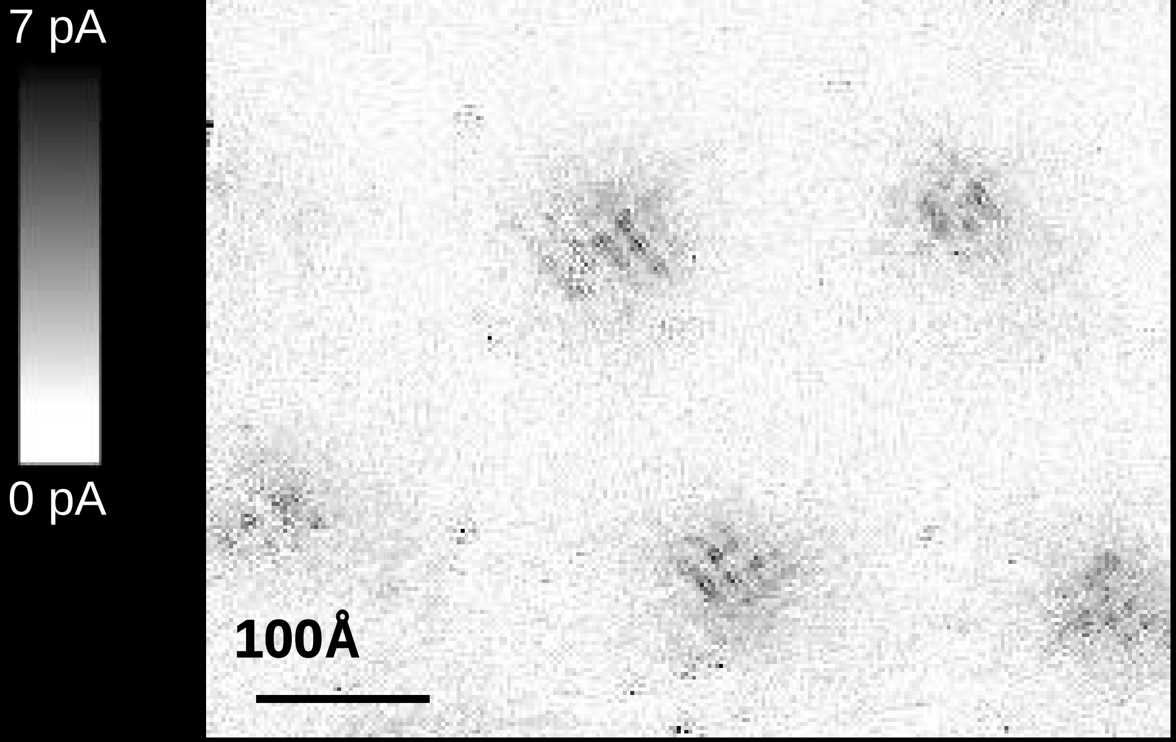


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

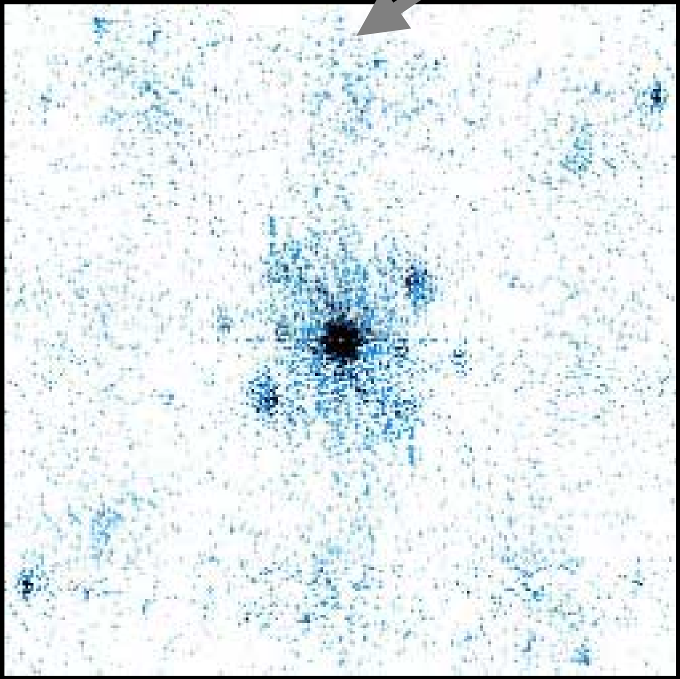
(1meV to 12 meV) at B=5 Tesla.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

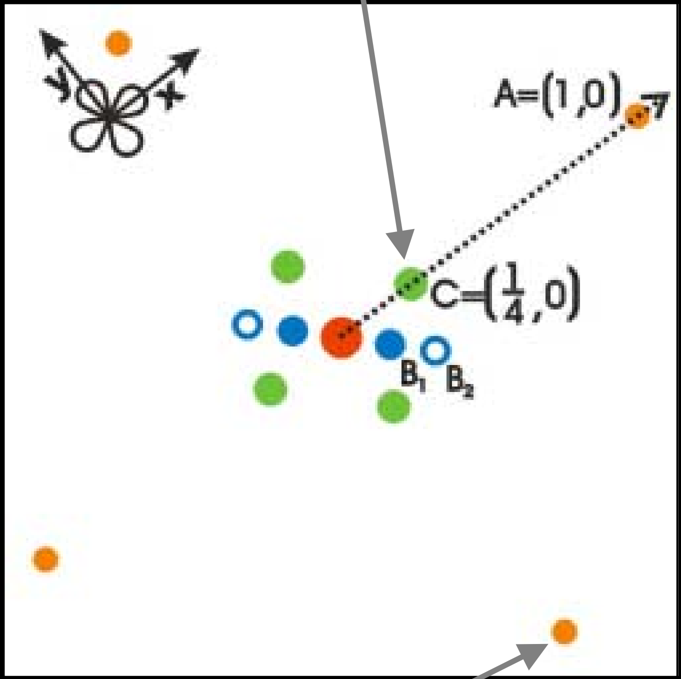


J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan,
S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis,
***Science* 295, 466 (2002).**

Fourier Transform of Vortex-Induced LDOS map



K-space locations of vortex induced LDOS



K-space locations of Bi and Cu atoms

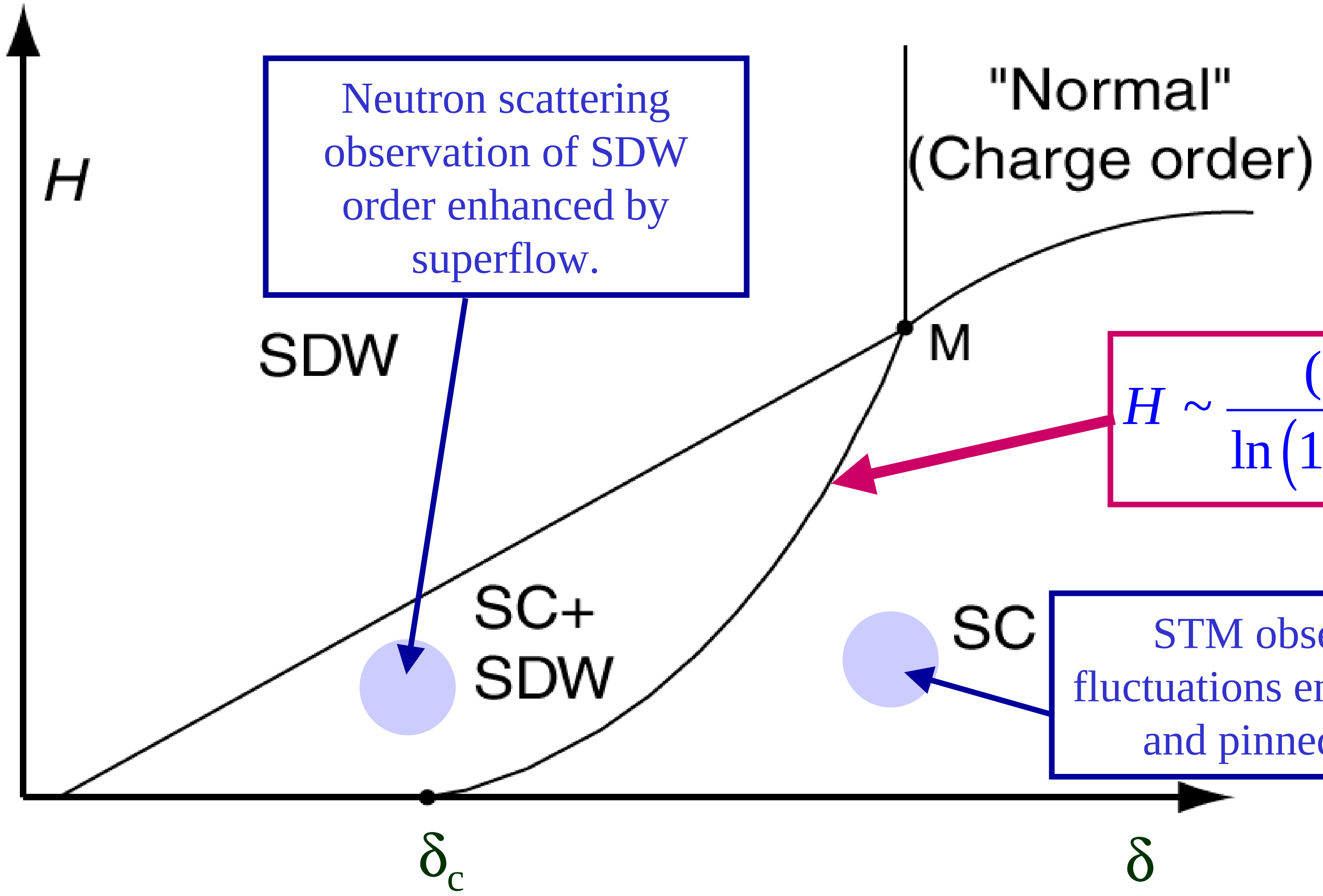
Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

J. Hoffman *et al. Science*, **295**, 466 (2002).

Summary of theory and experiments

(extreme Type II superconductivity)

$T=0$



Neutron scattering observation of SDW order enhanced by superflow.

$$H \sim \frac{(\delta-\delta_c)}{\ln\left(1/(\delta-\delta_c)\right)}$$

STM observation of CDW fluctuations enhanced by superflow and pinned by vortex cores.

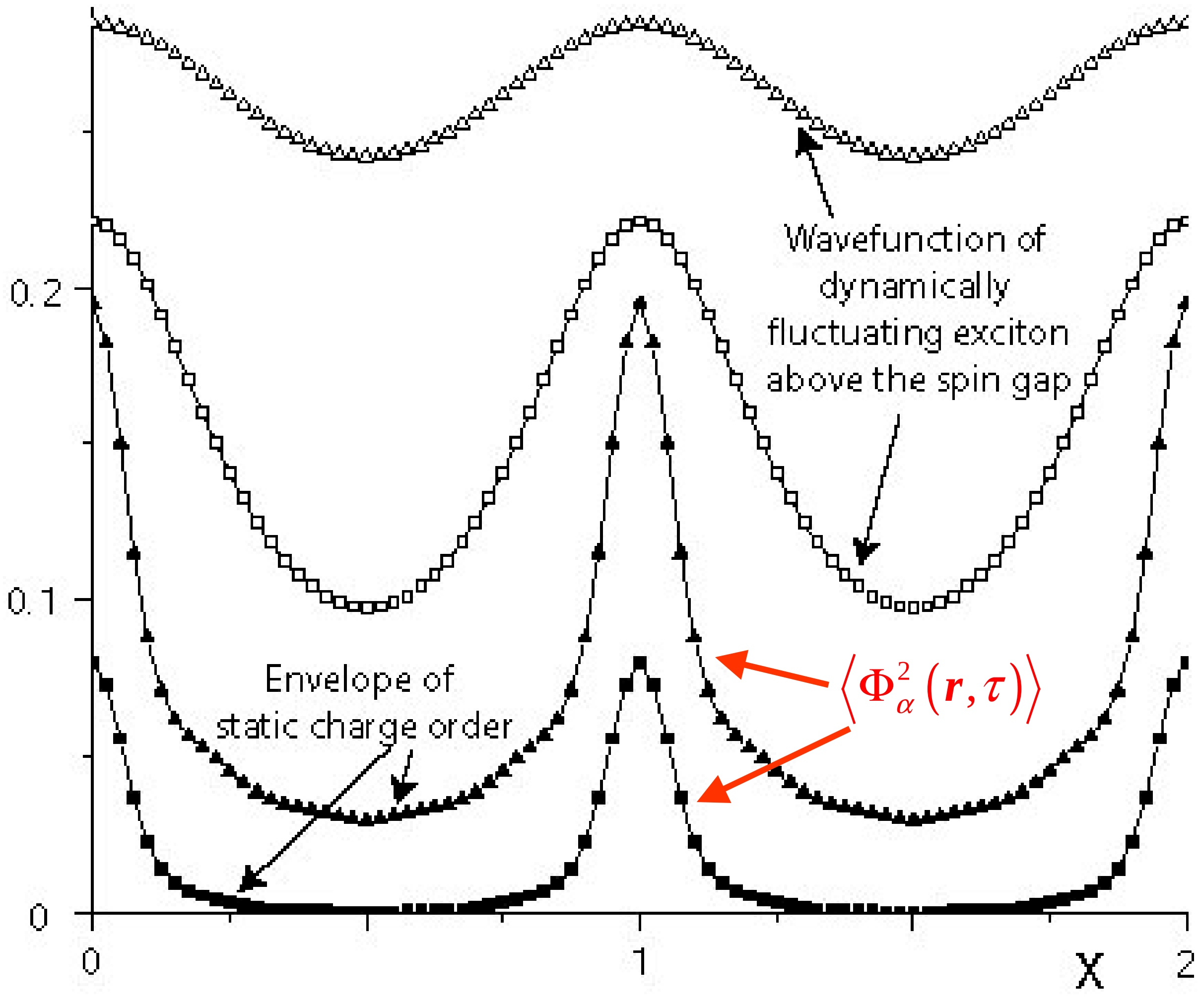
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Quantitative connection between the two experiments ?

Pinning of CDW order by vortex cores in SC phase

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

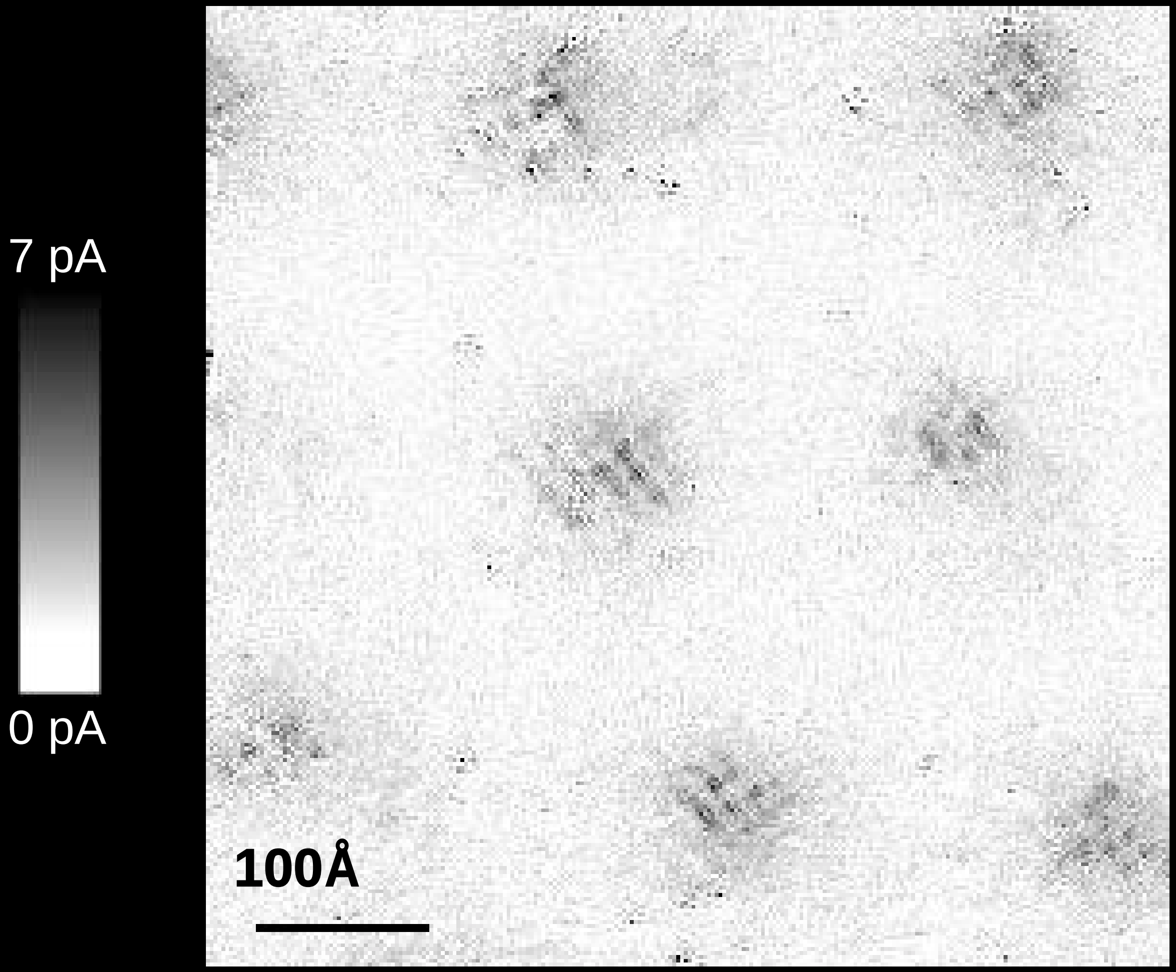
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\,\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2$$



□→ low magnetic field

△→ high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



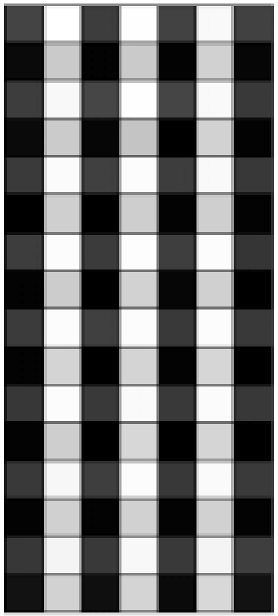
**J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan,
S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis,
Science 295, 466 (2002).**

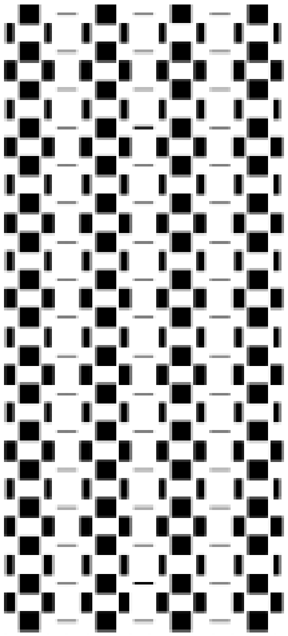
IV. Microstructure of the charge order: magnetic transitions in Mott insulators and superconductors

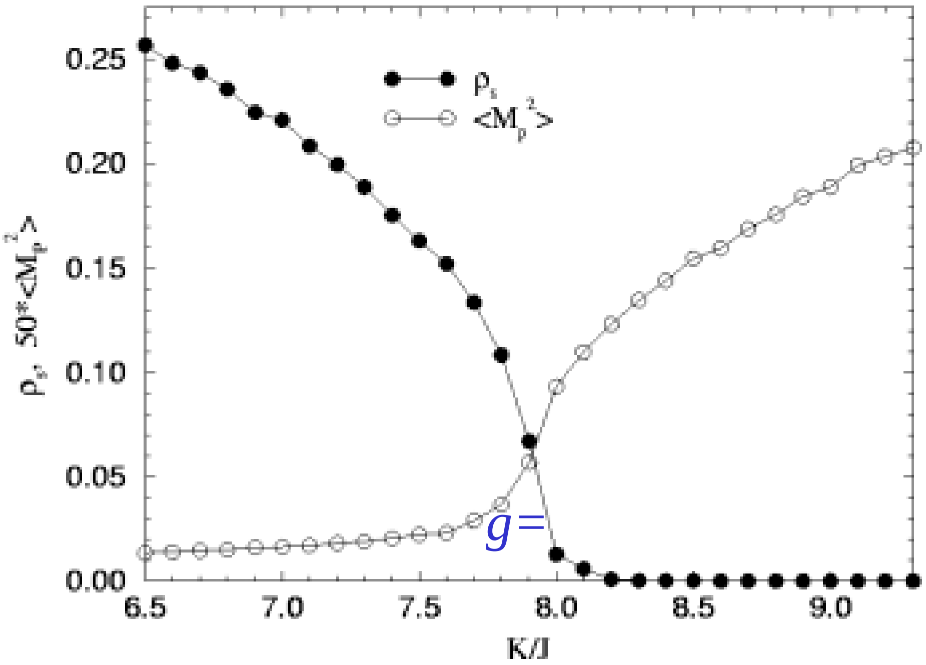
Bond-centered stripe order in a frustrated S=1/2 XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in *S=1/2* antiferromagnet with full square lattice symmetry



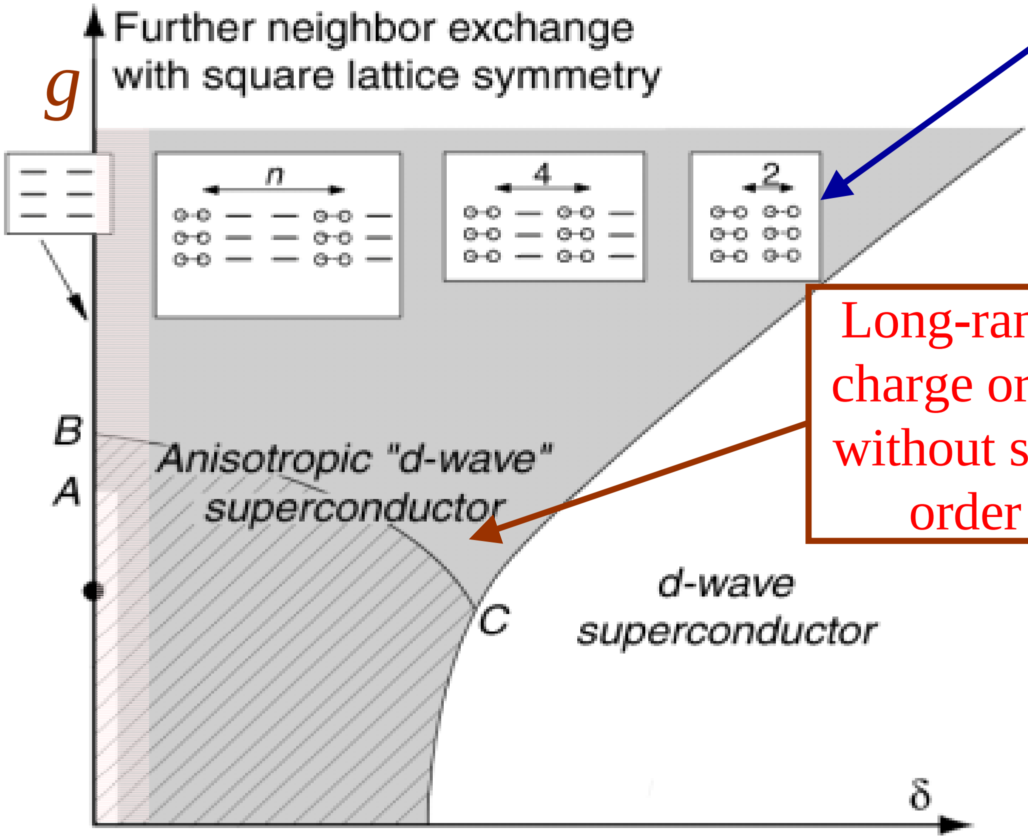




$$H = -2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle \subset \square} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989);
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

III. Charge order in the superconductor.



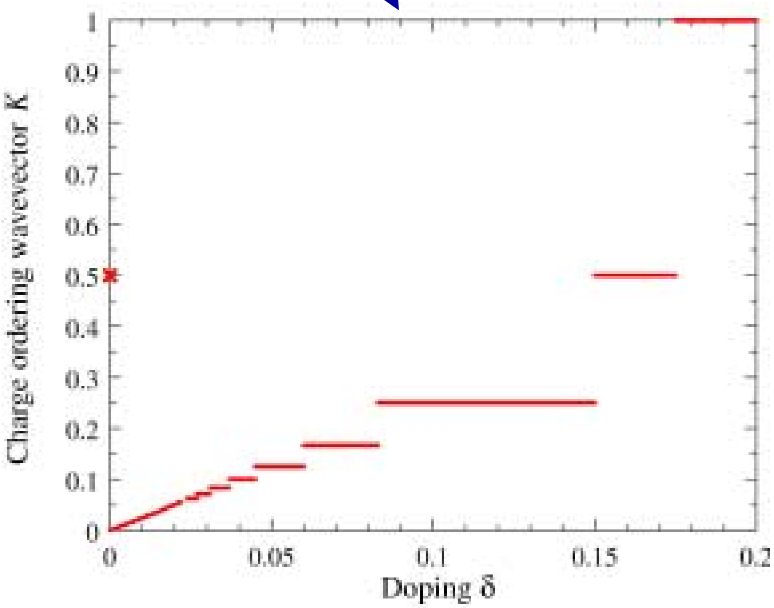
Long-range charge order without spin order

Hatched region --- spin order
Shaded region ---- charge order

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).

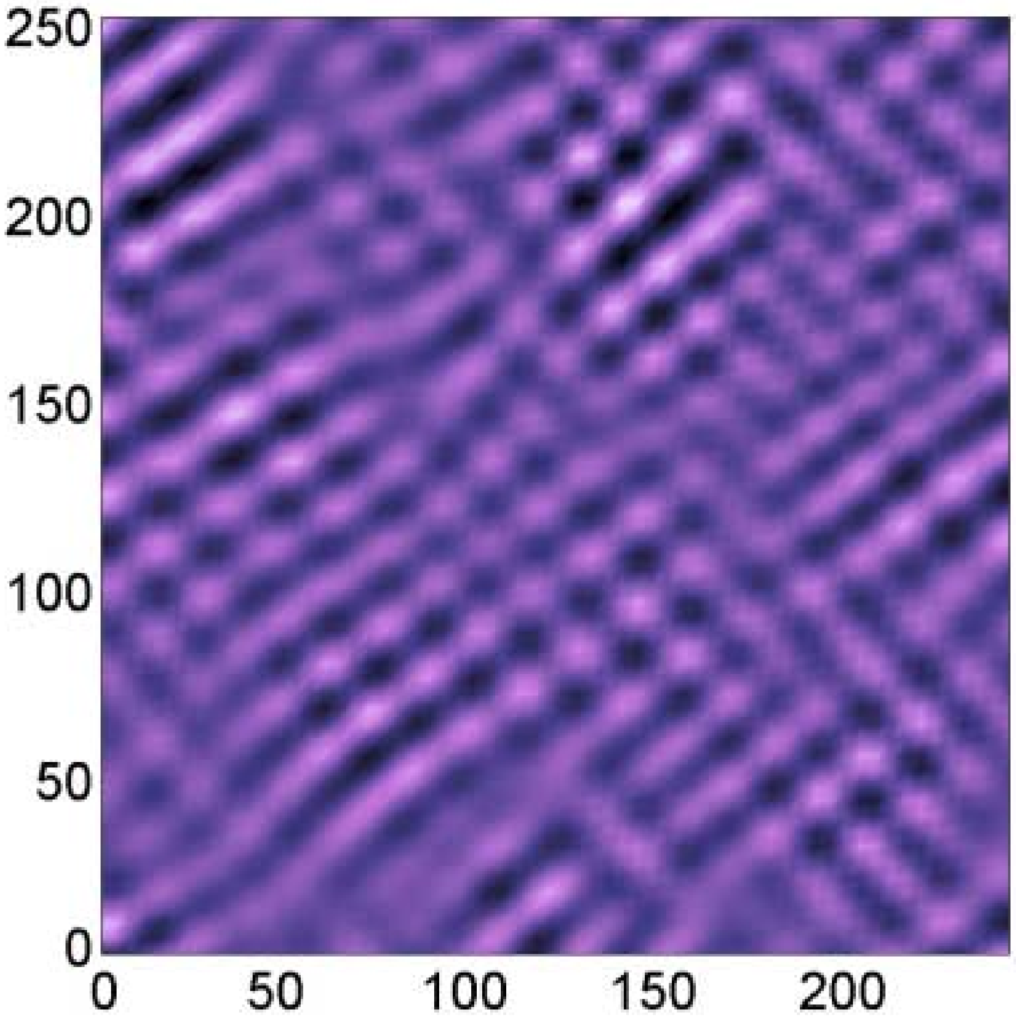
“Large *N*” theory in region with preserved spin rotation symmetry

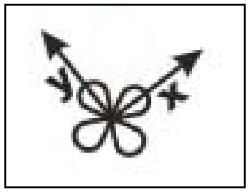
S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).



Charge order is bond-centered and has an even period.

IV. STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

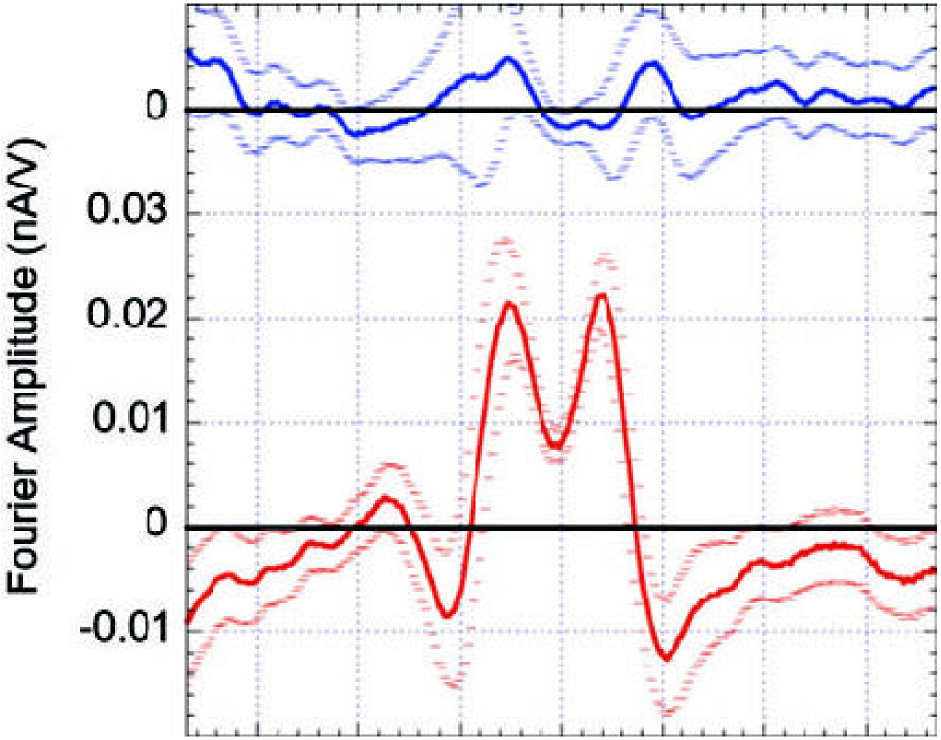




Charge order period = 4 lattice spacings

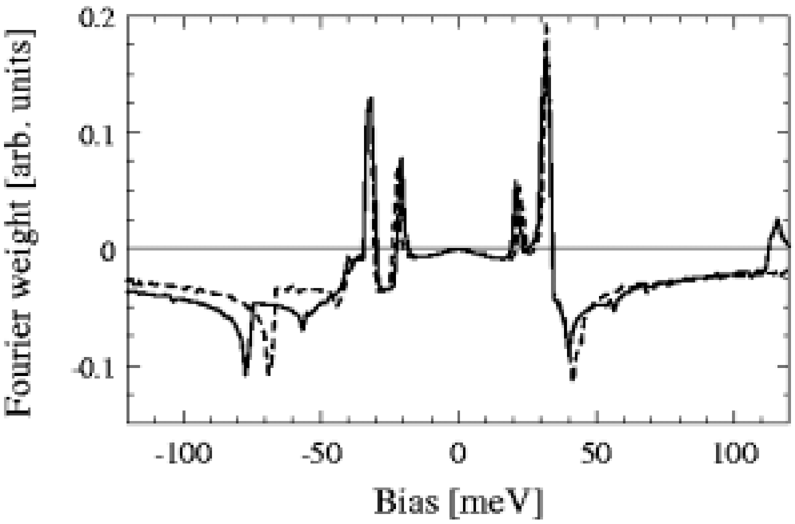
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, cond-mat/0204284.
D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, cond-mat/0204011

IV. Neutron scattering observation of static charge order in $YBa_2Cu_3O_{6.35}$

(spin correlations are dynamic)

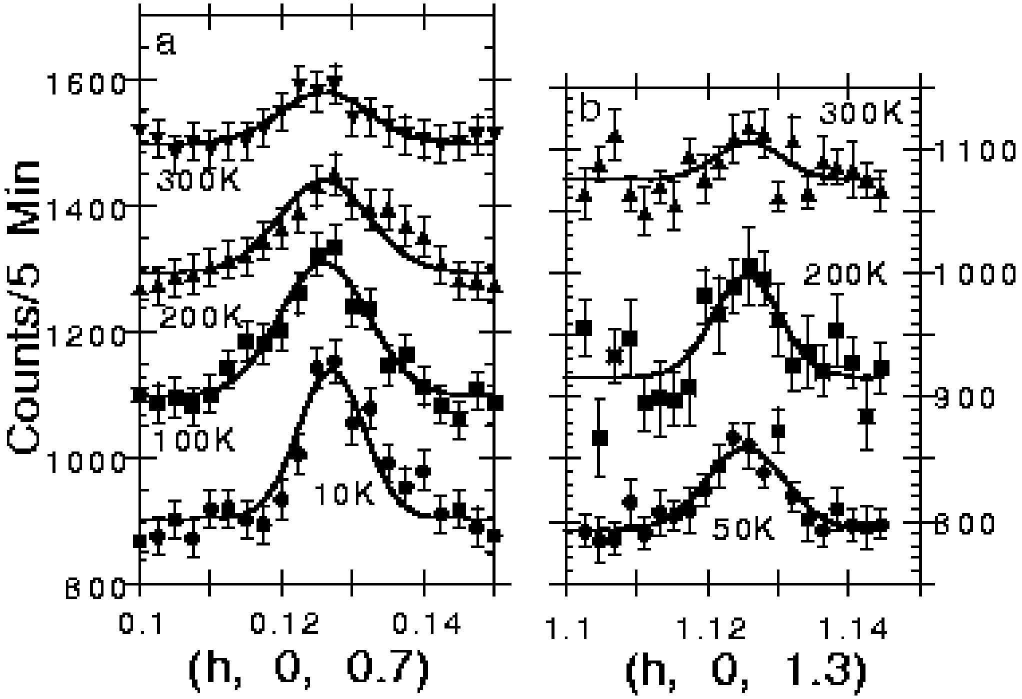
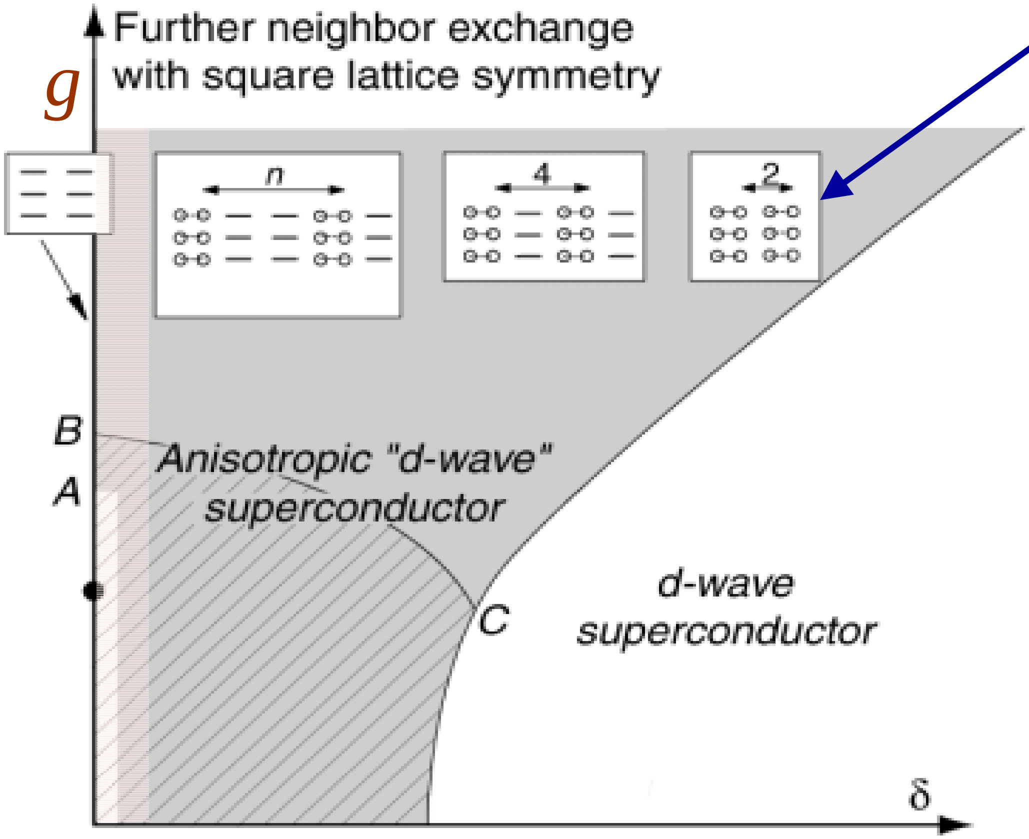


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

Charge order period
= 8 lattice spacings

H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

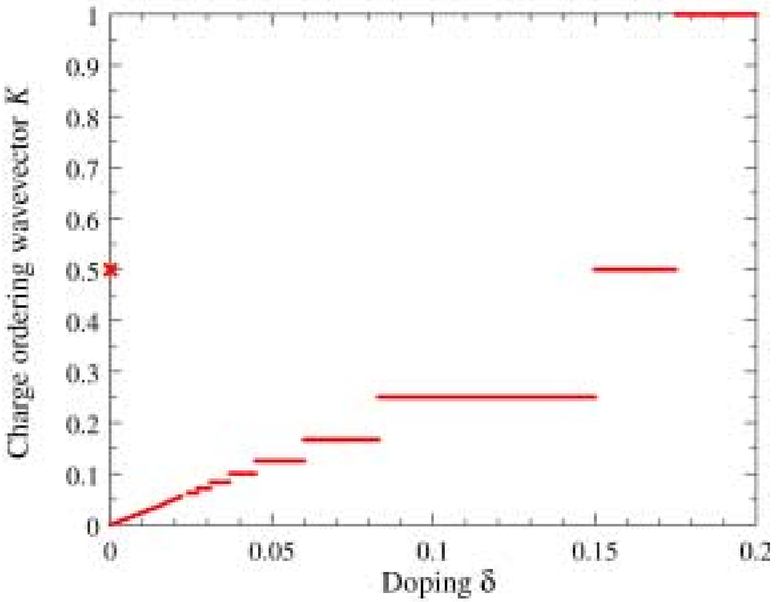
Charge order in the superconductor



Hatched region --- spin order
Shaded region ---- charge order

"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has charge-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?